

Glossary of

Terms used in operational research

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Foreword

This British Standard has been prepared under the direction of Technical Committee MS/2. It was decided that the time was opportune to produce a glossary of Operational Research (OR) terms in view of the wide application of OR techniques in industry, commerce and the public services. This standard complements several related BSI glossaries dealing with management and scientific disciplines.

The term Operational Research (OR) was first used to describe the application of scientific methods to military operations and is known as operations research or management science in North America. In the late 1940s, many civilian OR groups were set up in industries such as steel, coal and oil, where the mathematical approach was applied to operations in areas such as purchasing, stock control and production scheduling. Operational Research then spread rapidly into all areas of manufacturing and service industry, the financial sector and government departments. A number of academic OR departments were founded and OR is now taught both as a subject in itself and as part of more broadly based studies such as business, accountancy, computing and engineering. A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 38, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

Introduction

Concepts of Operational Research

The management science of Operational Research (OR) is entirely practical in intention and its objective is to propose solutions for complex problems arising in the direction and management of large systems of people, machines, materials and money in industry, business, government and community services. The distinctive OR approach is to develop a scientific model of the system under review, incorporating measurements of factors such as chance and risk, with which to predict the outcomes of alternative decisions, strategies or controls. OR should therefore enable management to determine its policy and actions scientifically. The purpose of this standard is to promote the use of standardized terminology for Operational Research as an aid to effective communications, particularly in business studies and where OR techniques have come into more general use.

There is a continuing debate within Operational Research circles as to what OR actually is, because the words “operational” and “research” can both be misleading. “Operational” suggests that OR ought to be concerned with ongoing operations, the tactical as opposed to the strategic; whereas “research” paradoxically suggests almost the opposite, namely that OR should be concerned with possible future operations which cannot be put into effect until certain necessary analysis, involving models, experimentation and pilot studies has been satisfactorily completed. In order to avoid these semantic difficulties, some organizations, while making use of the OR approach and techniques, have replaced the term Operational Research with other phrases such as Management Science, Management Services, Systems Analysis, or Information Science. In many occupations and professions, one or two particular techniques may be very important, while the rest remain unused.

As originally conceived, an Operational Research team was composed of experts drawn from various fields such as general management, finance, marketing, logistics, manufacturing and quality assurance. Cross-fertilization of ideas and pooling of expertise was encouraged, so that complex and apparently intractable problems might be solved. Many of the early OR successes resulted from the application of relatively simple techniques such as linear regression and experimental design in situations where these techniques had not previously been applied. In due course OR became a subject in its own right and the OR expert emerged.

The OR approach could be described first in terms of the scientific method where hypotheses are formulated and tested and second by the standard approach to problem solving using the following process:

- define the problem and formulate it in such a way that a quantitative approach is possible;
- develop an appropriate model;
- collect relevant data;
- validate the model by checking that it produces the same answer using the same data found in the real world;
- evaluate the options by running hypothetical, forecast or what-if data through the model and interpreting results;
- select the preferred option;
- prepare appropriate documentation and/or computer programs;
- implement.

A central concept within OR is that of a model, by which the OR practitioner reaches a deeper understanding of the problem area. The purpose of the model is to generate practical solutions with regard to difficulties of implementation. As one of the pioneers of OR expressed it, the third-best solution that can be produced tomorrow may be more useful than the second-best in a month, or the best solution that could take years or may be too difficult to put into operation. In OR terms, a model may be a set of equations, a computer program, occasionally an analogue device, or indeed any operating description, preferably quantitative, of the dynamics of the situation. Where this cannot be represented mathematically, it may be simulated. The relationships modelled may be on a one-to-one basis, where each element in the real world corresponds to an element in the model. More usually the relationships are on a one-to-many basis, for example, where one number may represent all the invoices passed in a month. Model is to OR as metaphor is to language.

The complexity of an OR problem frequently arises from the fact that there is no evident structure in it and no obvious method of solution. Various internal and external pressures, such as industrial relations and legal or contractual constraints, may be involved and may require modifications to an otherwise good solution. There may be no solution (in the mathematical sense of the word) to an ill-defined problem, but enlightenment may be gained by a structured approach. The first task of OR is therefore to explore the problem situation itself to determine its structure. The factors that affect the problem and are affected by it, should then be defined and assessed quantitatively if possible. The aim should be to clarify the precise nature of the problem and make it evident to all the parties involved.

When all parties involved in the problem situation agree that it is adequately formulated, then the operational researcher can employ the methods and techniques of OR. These are merely the tools that the operational researcher uses on the way to solving problems and implementing their solution; some of these tools are surprisingly powerful. Although the bulk of this standard is devoted to terms used in these techniques, it is essential to understand that OR is far more than a collection of techniques.

The role of OR is not finished when proposals for resolving the original problem are accepted; an important aspect is implementation, which almost invariably raises difficulties of its own, for example the need to adjust administrative procedures, to change attitudes, skills and established practices.

Arrangement

Sections 1 to 9 of this British Standard cover particular topics of Operational Research and within these sections a wide range of terms is defined as follows:

- section 1 defines the fundamental and generic terms used in Operational Research;
- section 2 defines terms used in mathematical programming;
- section 3 defines terms used in queuing theory;
- section 4 defines terms used in simulation;
- section 5 defines terms used in gaming;
- section 6 defines terms used in forecasting;
- section 7 defines terms used in graph theory;
- section 8 defines terms used in decision theory; and
- section 9 defines terms used in soft operational research.

In some cases, where two or more terms have the same meaning and one of them is preferred, the preferred term is printed in bold type (as are all single terms) and the others are printed in ordinary type. Italic type denotes a term that is defined elsewhere in this glossary.

Other British Standard glossaries such as BS 3138, BS ISO 3534, BS 3811, BS 4335, BS EN ISO 8402 and BS 5191 include some OR terms that are relevant to their respective disciplines. It is envisaged that in due course a new standard BS 6079 *Guide to project management* will be published which will also contain OR terms that are relevant.

Section 1. Fundamental terms

No.	Term	Definition
1.1 Terms and definitions		
11001	bottom-up	An approach that starts at the level of greatest detail or lowest hierarchical level and proceeds up towards the level of least detail or highest hierarchical level.
11002	decision variable	A quantity that varies, or a vector of such quantities, that is under the control of the decision maker.
11003	heuristics	A systematic procedure involving informal trial and error solutions. NOTE The solution is narrowed down by using experience, rather than only theoretical knowledge, to reject obviously impractical routes.
11004	model	A representation of reality that reproduces the essential features or elements of the <i>system</i> or <i>entity</i> being studied.
11005	Operational Research (OR)	The application of the methods of science to complex problems arising in the management of large systems of people, machines, materials and money in industry, business, government and community services. NOTE 1 The distinctive approach is to develop a <i>model</i> of the <i>system</i> , incorporating measures of factors such as chance and <i>risk</i> , with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically. NOTE 2 The practical application of OR makes use of certain specific disciplinary methods such as probability, statistics, <i>mathematical programming</i> and queuing theory.
11006	optimizing	1) Obtaining the best possible mathematical solution to a given set of equations. 2) Obtaining the best possible solution within the constraints of a problem where the situation cannot be completely described by mathematical equations.
11007	parameter	A value circumscribing a problem, that is constant within a process, but may be changed for subsequent processes.
11008	satisficing	Achieving an acceptable solution that is not necessarily the best achievable solution.
11009	sensitivity analysis (general)	Investigation of the behaviour of the solution as a result of variations of the <i>parameters</i> .
11010	steady state (general)	The state of the <i>system</i> when its variables have converged to values independent of time. NOTE See also 45005.
11011	system	A coherent and logical arrangement of principles, people, data, material, components, equipment and/or procedures.
11012	top-down	An approach that starts at the level of least detail or highest hierarchical level and proceeds downwards to the level of greatest detail or lowest hierarchical level.
11013	validation	The process of checking by comparing a <i>model</i> with the real world.
11014	verification	The process of checking how consistent a <i>model</i> is in itself.

Section 2. Mathematical programming

No.	Term	Definition
2.1 Programming methodology		
21001	convex programming	Optimization of a <i>convex function</i> over the <i>region</i> where it is convex. NOTE 1 <i>Linear programming</i> and <i>quadratic programming</i> are special cases of <i>convex programming</i>. NOTE 2 The problems P1, P2 below are <i>convex programming</i> problems: P1 min $f(x)$, subject to $x \in R$, $R = \{x \mid g_i(x) \leq 0, i = 1, 2, \dots, m\}$ P2 max $h(x)$, subject to $x \in R$ where $f(x)$ is a <i>convex function</i>; $h(x)$ is a <i>concave function</i>; $g_i(x)$, $i = 1, 2, \dots, m$ are <i>convex functions</i> whereby R is a convex set.
21002	integer programming	Linear programming with the added restriction that all the variables are constrained to be integers. NOTE The general mathematical formulation of the <i>integer programming</i> problem is the set of matrix relationships as follows: min (or) max $f(x) = c^T x$ subject to $Ax \leq b$ and $x \geq 0$ and x integer for all x_j; $j = 1, 2, \dots, m$
21003	linear programming	The general problem of <i>optimizing</i> a linear function of several variables subject to a number of <i>constraints</i> that are linear in these variables and a subset of which restrict the variables to be non-negative. NOTE The general mathematical formulation of the <i>linear programming</i> problem is the set of matrix relationships as follows: min (or) max $f(x) = c^T x$ subject to $Ax \leq b$ and $x \geq 0$.
21004	mathematical programming	The general problem of <i>optimizing</i> a function of several variables subject to a number of <i>constraints</i> .
21005	mixed (integer) programming	<i>Linear programming</i> with the added restriction that some variables are constrained to be integers. NOTE The general mathematical formulation of the <i>mixed programming</i> problem is the set of matrix relationships as follows: min (or) max $f(x) = c^T x$ subject to $Ax \leq b$ and $x_j \geq 0$; $j = 1, 2, \dots, n$ and x_j integer for $j = 1, 2, \dots, k$ and $k < n$

No.	Term	Definition
21006	non-linear programming	An inclusive term covering all types of constrained optimization problems except those problems where the <i>objective function</i> and the <i>constraints</i> are all linear. NOTE 1 Special types of <i>non-linear programming</i> for which theories have been developed are <i>convex programming</i>, <i>concave programming</i> and <i>quadratic programming</i>. NOTE 2 The general mathematical formulation of the <i>non-linear programming</i> problem is the set of matrix relationships as follows: $\begin{aligned} &\max \text{ (or) } \min f(x) \\ &\text{subject to} \\ &\quad g_i(x) \leq b \\ &\text{and} \\ &\quad x \geq 0 \\ &\text{and} \\ &\quad \text{where } f(x) \text{ and } g(x) \text{ are non-linear functions of } x. \end{aligned}$
21007	quadratic programming	Maximization/minimization, subject to linear <i>constraints</i>, of an <i>objective function</i> that is a <i>concave/convex</i> quadratic function. NOTE The general mathematical formulation of the quadratic programming problem for, say, the minimization case is the set of matrix relationships as follows: $\begin{aligned} &\min f(x) = p^T x - \frac{1}{2} x^T Q x \\ &\text{subject to} \\ &\quad Ax \leq b \\ &\text{and} \\ &\quad x \geq 0 \\ &\text{where} \\ &\quad p \text{ is an } n\text{-vector of coefficients;} \\ &\quad Q \text{ is an } n \times n \text{ matrix of quadratic form.} \end{aligned}$
21008	separable programming	A method of formulating certain suitable <i>non-linear programming</i> problems such that each non-linear function appearing may be expressed as the sum of separate linear or non-linear functions of single variables. $g(x_1, x_2, \dots, x_n) = g_1(x_1) + g_2(x_2) + \dots g_n(x_n).$ The simplex algorithm is appropriately modified for the solution of such problems.
21009	stochastic programming	A generalization of <i>linear programming</i> where any of the coefficients may be probabilistic.

No.	Term	Definition
2.2 Specific programming terminology		
22001	constraint	An equation or inequality relating the variables in an optimization problem.
22002	feasible solution	A solution satisfying the <i>constraints</i> of a <i>mathematical programming</i> problem.
22003	infeasible solution	A solution to a <i>mathematical programming</i> problem where <i>constraints</i> , specifically the non-negativity <i>constraints</i> of the problem, have not all been satisfied.
22004	iteration	A single <i>cycle</i> of operations in the algorithm used to solve the problem.
22005	objective function	The combination of the variables of a <i>mathematical programming</i> problem whose value is to be maximized or minimized subject to the <i>constraints</i> of the problem.
22006	optimal solution optimum solution	A <i>set</i> of values of the variables in a <i>mathematical programming</i> problem that optimizes the <i>objective function</i> and satisfies the problem <i>constraints</i> .
22007	redundant constraint	A <i>constraint</i> in a <i>mathematical programming</i> problem that for any <i>feasible solution</i> will never be binding because one or more <i>constraints</i> in the problem are equally or more restrictive.
22008	sensitivity analysis (in LP) post-optimal analysis ranging analysis	Investigation of the behaviour of the <i>optimal solution</i> as a result of variations in coefficient in the <i>objective function</i> or <i>constraint</i> values of a problem.

2.3 Programming terms with wider application

23001	branch and bound method	A method of solving an optimization problem in which the space of <i>feasible solutions</i> is repeatedly partitioned into smaller and smaller subsets (branching) and the lower bound (for minimum problems) is calculated (bounding). After each partitioning those subsets with bounds exceeding a known <i>feasible solution</i> are excluded from further partitioning.
23002	cutting plane method	A method of problem solving in which the initial problem is relaxed by removing a restriction. If this does not fathom the original problem the relaxation is then tightened by adding a new <i>constraint</i> .
23003	fathomed problem	A problem that either has no <i>feasible solution</i> , or no solution with an <i>objective function</i> better than some predetermined value, or its optimum solution is revealed by analysis.
23004	relaxation, principle of	The principle that when a problem is relaxed to another problem, every <i>feasible solution</i> to the original problem should be a <i>feasible solution</i> to the relaxed problem. NOTE This leads to the following results. a) If there is no <i>feasible solution</i> of the relaxed problem, then there is no <i>feasible solution</i> to the original problem. b) The optimum value of the relaxed problem is at least as good as that of the original problem. c) If an <i>optimal solution</i> of the relaxed problem is a <i>feasible solution</i> of the original problem, it is an <i>optimal solution</i> of the original problem.
23005	separation, principle of	The principle that a problem is separated into sub-problems (descendants) if every <i>feasible solution</i> to the problem is a <i>feasible solution</i> of one and only one of the descendants and if every <i>feasible solution</i> of every descendant is a <i>feasible solution</i> of the problem.
23006	stationary policy	A policy that always prescribes the same action whenever the <i>system</i> is in a given <i>state</i> .

No.	Term	Definition
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2.4 Linear, integer and mixed programming terminology

COMMENT. Applications in several apparently unconnected problem areas often have very similar mathematical structures and therefore almost identical methods of solution. It is customary to name all such problems after the first published one. For example, the *travelling salesman problem* is encountered in many managerial and scientific guises all of which share the same approach to the solution.

Many *linear programming* (LP) problems having special structures can be solved with algorithms that are more efficient than those needed for general LP problems. These problems also tend to be known by their first published application that might be quite irrelevant to another application. Examples of specially named problems are terms 24027 and 24028 and also network flow problem, *assignment problem*, routing problem, machine shop problem, cutting problem, knapsack problem.

24001	artificial variable	An auxiliary variable introduced into an equality <i>constraint</i> at the start of the <i>simplex method</i> of <i>linear programming</i> , to provide an identity basis required to initiate the procedure.
24002	assignment problem	A special case of the <i>transportation problem</i> where in the solution the variables x_{ij} are allowed to take only the values 0 and 1. NOTE 1 The problem is usually interpreted as one of assigning n persons to n jobs in an optimal manner. NOTE 2 The problem can be stated as an LP problem or as a <i>transportation problem</i> but can be solved via a special algorithm.
24003	basic feasible solution	A basic solution that satisfies the <i>constraints</i> of a <i>linear programming</i> problem, including the non-negativity <i>constraints</i> .
24004	basic solution	A solution to a <i>linear programming</i> problem where the number of variables with non-zero values is, at most, equal to the number of <i>constraints</i> in the problem.
24005	basic variable	A variable not initially set to zero in a basic solution. NOTE A basic variable corresponds to one of the column vectors in the <i>basis</i> .
24006	basis	In linear programming an $m \times m$ non-singular matrix consisting of m columns of the problem matrix. NOTE The requirement that the <i>basis</i> is non-singular means that the chosen column vectors have to be linearly independent.
24007	binary variable zero-one variable go-no go alternative logical variable	A variable that takes one of two possible values, zero or one.
24008	complementary slackness	The duality property that states that when, in the <i>optimal solution</i> of the primal, a <i>slack variable</i> has a positive value (i.e. its associated <i>constraint</i> is not binding) then the dual variable corresponding to this primal <i>constraint</i> is zero in the <i>optimal solution</i> of the dual and vice versa.
24009	cycling	In the <i>linear programming</i> solution procedure, the repetitive occurrence of a sequence of <i>basic feasible solutions</i> during which the value of the <i>objective function</i> does not improve.
24010	decomposition (in LP)	A method for subdividing a large <i>linear programming</i> problem into smaller, more manageable linear programs, solving the smaller programs in the standard way and then combining these solutions to solve the original problem. NOTE Linear programs that qualify in particular for <i>decomposition</i> are those whose variables can be separated into classes $X, Y, Z, \dots$ and the <i>constraints</i> into classes $C_0, C_1, C_2, \dots$ such that the <i>constraints</i> in class C_0 can involve all the variables, <i>constraints</i> in class C_1 involve only the variables in X , the <i>constraints</i> in class C_2 only involve the variables in Y , the <i>constraints</i> in class C_3 involve only the variables in Z and so on. The subsets of <i>constraints</i> each involving only part of the variables are called sub-programs while the subset of <i>constraints</i> in C_0 is called the master program.

No.	Term	Definition
24011	degenerate basic solution	A <i>basic solution</i> where one or more of the basic variables assume a zero value.
24012	dual feasible solution	A solution of the dual of a <i>linear programming</i> problem that satisfies all the <i>constraints</i> in the <i>dual problem</i> . NOTE In general, unless the optimum has been reached, the primal solution corresponding to a <i>dual feasible solution</i> is infeasible and vice versa.
24013	duality	The set of mathematical properties that links the solution of a <i>primal problem</i> with that of its corresponding <i>dual problem</i> , for example the fact that the optimum values of both <i>objective functions</i> are equal.
24014	dual problem	An alternative formulation of a <i>linear programming</i> problem that is obtained by transposing the matrix (such that column vectors in the primal become row vectors in the dual and vice versa) and where the cost coefficients of the primal <i>objective function</i> become <i>constraint</i> values and vice versa, and <i>shadow costs</i> and <i>reduced costs</i> in the primal become the values of the variables in the dual and vice versa. Maximization of the primal corresponds with minimization of the dual and vice versa. NOTE 1 Conventionally a <i>constraint</i> is expressed as a polynomial of the variables concerned and related to a numerical value on the right-hand side (of the equality or inequality sign). NOTE 2 The concept is to convert a maximization problem into a dual minimization problem when the latter is easier to solve. NOTE 3 <i>Primal/dual problems</i> are also encountered in <i>graph theory</i> .
24015	ellipsoid method	An alternative procedure to the <i>simplex method</i> .
24016	free variable	A variable not constrained to integer values only.
24017	integer variable	A variable that assumes whole number values only.
24018	lattice point	A point (set of values) satisfying the integer <i>constraint</i> (in integer and mixed programs).
24019	parametric programming	A method for investigating the effect on an optimal <i>linear programming</i> solution of a sequence of proportionate changes in the elements of a single row or column of the matrix. Most commonly the method is applied to either the <i>objective function</i> row or the <i>constraint</i> column.
24020	primal problem	A programming problem stated in terms of the original variables and <i>constraints</i> .
24021	projective method	An alternative procedure to the <i>simplex method</i> .
24022	reduced cost (of a non-basic variable)	The amount by which, for a non-basic variable, the <i>objective function</i> would change if this variable were to be given unit value. NOTE 1 <i>Reduced costs</i> can be regarded as the <i>shadow prices</i> on the non-negativity <i>constraints</i> of the respective variables. NOTE 2 The <i>reduced costs</i> of basic variables are zero.
24023	revised simplex method	A variant of the <i>simplex method</i> , where the number of calculations necessary to move from one <i>iteration</i> to the next in the simplex procedure is kept as small as possible. Only those calculations are carried out that are needed for the next decision to be made in the problem procedure. NOTE The method is especially suitable for problems where the number of variables is much larger than the number of equations.
24024	shadow price dual price marginal cost marginal price marginal value shadow cost	The amount by which the <i>objective function</i> would change on a particular <i>constraint</i> in a <i>linear programming</i> problem if the right-hand side value of that <i>constraint</i> was increased by one unit.