

Guide for the statistical analysis of ageing test data —

Part 1: Methods based on mean values of normally distributed test results

Committees responsible for this British Standard

The preparation of this British Standard was entrusted to Technical Committee GEL/15, Insulating material, upon which the following bodies were represented:

- British Ceramic Research Ltd.
- British Industrial Ceramic Manufacturers' Association
- Electrical and Electronic Insulation Association (BEAMA Ltd.)
- Electricity Association
- Federation of the Electronics Industry
- Rotating Electrical Machines Association (BEAMA Ltd.)
- Transmission and Distribution Association (BEAMA Ltd.)

This British Standard, having been prepared under the direction of the Electrotechnical Sector Board, was published under the authority of the Standards Board and comes into effect on 15 November 1995

© BSI 09-1999

The following BSI references relate to the work on this standard:
Committee reference GEL/15
Draft announced in *BSI News* April 1995

ISBN 0 580 24570 5

Amendments issued since publication

Amd. No.	Date	Comments

Contents

	Page
Committees responsible	Inside front cover
National foreword	ii
1 Scope	1
2 Introduction	1
3 Statistical methods	1
4 Treatment of the results of ageing tests	7
Appendix A References	9
Appendix B List of symbols	11
Figure 1 — Proof tests	12
Figure 2 — Destructive measurement of property	13
Figure 3 — Destructive measurement of property	14
List of references	Inside back cover

National foreword

This British Standard has been prepared by Technical Committee GEL/15. It is identical with IEC 493-1:1974 *Guide for the statistical analysis of ageing test data — Part 1: Methods based on mean values of normally distributed test results*, published by the International Electrotechnical Commission (IEC).

Cross-reference

International Standard

IEC 216-3-1:1990

Corresponding British Standard

BS 5691 *Guide for the determination of thermal endurance properties of electrical insulating materials Part 3 Instructions for calculating thermal endurance characteristics*

Section 3.1:1995 *Calculations using mean values of normally distributed complete data*

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 14, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

1 Scope

This guide gives statistical methods which may be applied to the analysis and evaluation of the results of ageing tests.

It covers numerical methods based on mean values of normally distributed test results.

These methods are only valid under specific assumptions regarding the mathematical and physical laws obeyed by the test data. Statistical tests for the validity of some of these assumptions are also given.

Other parts will cover other statistical methods such as those based on:

- graphical evaluation of test results;
- median values of test results;
- truncated data;
- extreme value statistics.

2 Introduction

Procedures for estimating ageing properties are described in specific test procedures, or are covered by the general documents on test procedures for ageing tests with a specific environmental stress (e.g. temperature, radiation, partial discharges).

In many cases, a certain property is determined as a function of time at different ageing stresses, and a time to failure based on a chosen end-point criterion is found at each ageing stress. A plot of time to failure versus ageing stress may be used to obtain an estimate of the time to failure for similar specimens exposed to a specified stress, or to obtain an estimate of the value of stress which will cause failure in a specified time.

The physical and chemical laws governing the ageing phenomena may often lead to the assumption that a linear relationship exists between the property examined and the ageing time at fixed ageing stresses, or between certain mathematical functions of property and ageing time, e.g. square root or logarithm. Also there may be a linear relationship between time to failure and ageing stress, or mathematical functions of these variables.

The methods described in Sub-clause 3.7 of this guide apply to such cases of linear relationship. The methods are illustrated by the example of thermal ageing, where in the case of a simple chemical process it may be assumed that the degradation obeys the Arrhenius law, i.e. the logarithm of time to failure is a linear function of the reciprocal thermodynamic temperature.

Numerical examples demonstrating the use of the methods in this case are given in IEC Publication 216-3 (under consideration).

It should be noted that computer programmes exist for many of the procedures involved (determination of mean value, variance, regression coefficients) and that a complete computer programme may be easily established.

The methods described in this guide are treated in most textbooks on statistics. References on the individual methods are given in Appendix A, together with references to tables of the statistical distributions to be used in connection with the statistical tests described in this guide.

In some cases, the calculations involved in the evaluation of the test results may be replaced by graphical procedures, which may be easier to perform, although less accurate. References to such methods are given in Appendix A.

NOTE In some cases, the term “end of life” is used instead of what is here termed “failure”, and “life-time” instead of “time to failure”. It is advisable not to use “life” in this context in order to avoid confusion with “service life” of actual equipment.

3 Statistical methods

3.1 Statistical distributions and parameters

The distribution of a stochastic variable X is described by the distribution function:

$$F(x) = P(X \leq x) \quad (3.1.1)$$

where $P(X \leq x)$ is the probability that the value of X is $\leq x$. Here $F(x)$ goes from 0 to 1 and is a never-decreasing function of x . If $F(x)$ is a continuous function of x , then the probability density is determined as:

$$f(x) = \frac{dF(x)}{dx} \quad (3.1.2)$$

The distribution may be characterized by parameters, of which the most important are:

- the mean value:

$$\xi = \int_{-\infty}^{\infty} x f(x) dx \quad (3.1.3)$$

- the median value $\tilde{\xi}$, determined by:

$$F(\tilde{\xi}) = \int_{-\infty}^{\tilde{\xi}} f(x) dx = 0.5, \text{ and} \quad (3.1.4)$$

- the variance:

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \xi)^2 f(x) dx \quad (3.1.5)$$

The median bisects the distribution in equal parts, so that half the population has $X \leq \tilde{\xi}$. If the distribution is symmetric about $\tilde{\xi}$, then $\tilde{\xi} = \xi$.

The square root of the variance is termed the standard deviation σ .

3.2 Estimates of parameters

From a sample of n stochastic independent specimens from a population, estimates of the parameters of the population (see Sub-clause 3.1) may be derived.

An estimate of the mean value of the population (3.1.3) is calculated as the average of the individual sample values:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad (3.2.1)$$

where:

x_i = the individual sample values ($i = 1, 2, \dots, n$)

If the sample values are arranged in ascending order of magnitude:

$(x_{(1)}, x_{(2)}, \dots, x_{(n)})$,

the central value of the sample (i.e. the middle value for n odd or the average of the two values nearest the middle for n even) is an estimate of the population median ξ according to equation (3.1.4)

$$\tilde{x} = x_{\frac{n+1}{2}} \text{ (} n \text{ odd) or} \quad (3.2.2a)$$

$$\tilde{x} = \frac{1}{2} x_{\frac{n}{2}} + \frac{1}{2} x_{\frac{n}{2}+1} \text{ (} n \text{ even)} \quad (3.2.2b)$$

An estimate of the variance of the population (3.1.5) is the sample variance:

$$\begin{aligned} s^2 &= \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \\ &= \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} \\ &= \frac{n \sum x_i^2 - (\sum x_i)^2}{n(n-1)} \end{aligned} \quad (3.2.3)$$

$n-1 = f$ is called number of degrees of freedom of s^2 .

3.3 Significance tests

When a hypothesis has been established about the correct value of a population parameter ε , this hypothesis may be tested by a statistical test, where a sample estimate of the parameter, e , is compared with the hypothetical value. The range of possible values of the estimate e is divided into two parts, one containing the values which are least probable under the hypothesis, corresponding to a total probability α (e.g. 0.05) of getting a value of e in this range if the hypothesis is true, and one containing the values most possible under the hypothesis, corresponding to a probability of $1 - \alpha$.

The range of the least probable values is called the rejection range; if the sample yields an estimate within this range, the hypothesis is rejected. The other range is called the acceptance range, and if the sample estimate is within this range, the hypothesis is accepted. This implies that if the hypothesis is true, the probability of making a correct decision by accepting the hypothesis is $1 - \alpha$ (e.g. 95 %) and the probability of making a false decision by rejecting the hypothesis although true is α (e.g. 5 %).

The decision whether to accept or reject the hypothesis is said to be made on significance level α .

3.4 Confidence limits

The estimate of a population parameter (also called point estimate) gives only an optimal evaluation of the parameter value, but no information on the probable uncertainty in this value. For this purpose, confidence limits may be calculated.

From the sample estimate e , limits e_1 and e_2 are calculated in each case, which, with a stated probability (confidence) $1 - \alpha$, include the true value of the parameter ε , where possibly either $e_1 = -\infty$, or $e_2 = +\infty$. This implies that in 100 $(1 - \alpha)$ % (e.g. 95 %) of the cases, the interval from e_1 to e_2 actually includes the true value ε , whereas in 100 α % (e.g. 5 %) of the cases, it does not.

Confidence limits may be bilateral or unilateral.

Bilateral $(1 - \alpha)$ confidence limits e_1 and e_2 are two values below and above e which, with confidence $1 - \alpha$ include the true value ε .

A lower unilateral $(1 - \alpha)$ confidence limit e_1 is a value (below e) which, with confidence $1 - \alpha$, is smaller than the true value ε (in this case $e_2 = +\infty$).

An upper unilateral $(1 - \alpha)$ confidence limit e_2 is a value (above e) which, with confidence $1 - \alpha$, is greater than the true value ε (in this case $e_1 = -\infty$).

3.5 Distribution types

3.5.1 The normal distribution

The normal (Gaussian) distribution is defined by:

$$f(x) = \frac{\exp \{ -(x - \xi)^2 / 2\sigma^2 \}}{\sqrt{2\pi}\sigma^2} \quad (3.5.1)$$

and is completely characterized by its mean value ξ and variance σ^2 .

The standardized normal distribution:

$$f(u) = \frac{\exp \frac{-u^2}{2}}{\sqrt{2\pi}} \quad (3.5.2)$$

where:

$$u = \frac{x - \xi}{\sigma} \quad (3.5.3)$$

and the corresponding distribution function $F(u)$ have been tabulated.