

Physical testing of rubber —

Part 5: Guide to the application of rubber testing to finite element analysis

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Foreword

This British Standard has been prepared by Technical Committee PRI/22.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 43 and a back cover.

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1 Scope

This part of BS 903 gives recommendations for test procedures and guidance on appropriate methods for determining model parameters from test data for use in finite element analysis (FEA) of rubber. It covers stress-strain characterization, mechanical failure, friction, thermal properties and heat build-up.

It is applicable to solid vulcanized rubbers of hardness 20 to 80 IRHD for which the deformation is predominantly elastic, and to cellular materials formed using such rubbers. It might prove useful for other materials which can be deformed elastically to large strain.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS 903-A1, *Physical testing of rubber — Part A1: Determination of density*.

BS 903-A2, *Physical testing of rubber — Part A2: Method for determination of tensile stress-strain properties*.

BS 903-A4, *Physical testing of rubber — Part A4: Determination of compression stress-strain properties*.

BS 903-A10 (BS ISO 132), *Rubber, vulcanized or thermoplastic — Part A10: Determination of flex cracking and crack growth (De Mattia)*.

BS 903-A14, *Physical testing of rubber — Part A14: Method for determination of modulus in shear or adhesion to rigid plates — Quadruple shear method*.

BS 903-A21, *Physical testing of rubber — Part A21: Determination of rubber to metal bond strength*.

BS 903-A42 (BS ISO 3384), *Physical testing of rubber — Part A42: Determination of stress relaxation in compression at ambient and at elevated temperatures*.

BS 903-A51, *Physical testing of rubber — Part A51: Determination of resistance to tension fatigue*.

BS 903-A61 (BS ISO 15113), *Physical testing of rubber — Part A61: Determination of the frictional properties*.

BS 7608, *Code of practice for fatigue design and assessment of steel structures*.

BS EN 12667, *Thermal performance of building materials and products — Determination of thermal resistance by means of guarded hot plate and heat flow meter methods — Products of high and medium thermal resistance*.

BS ISO 34-1, *Physical testing of rubber — Determination of tear strength — Part 1: Trouser, angle and crescent test pieces*.

BS ISO 4664, *Rubber — Guide to the determination of dynamic properties*.

BS ISO 23529, *Rubber — Physical test methods — Preparation and conditioning of test pieces and preferred test conditions*.

3 Terms and definitions

For the purposes of this British Standard, the following terms and definitions apply.

3.1

finite element analysis

FEA

numerical method of analysing a product in which the numerical calculations are carried out for discrete, linked elements of the product

NOTE FEA is usually carried out using a commercial software package which allows visual simulation of the product to aid creation of the finite element model and viewing of the results of the analysis.

3.2

deformation mode

deformation arising through a particular relationship of the principal extension ratios

NOTE Examples are uniaxial tension, uniaxial compression, pure shear, simple shear or equibiaxial extension.

3.3

deviatoric deformation

deformation involving change of shape with no associated change of volume

3.4

volumetric deformation

deformation involving change of volume with no associated change of shape

3.5

extension ratio

ratio of deformed to undeformed length of a line element of material in a specified direction

NOTE 1 This is also known as a stretch ratio.

NOTE 2 The term extension is used to cover all types of deformation; thus a compression is defined with an extension ratio less than one.

3.6

hyperelastic

<material> deforming to high strains under the application of stresses, and returning to the original shape when the stresses are reduced, such that there is no loss of energy

3.7

incompressible

<material> deforming without a change in volume

NOTE For infinitesimal strains, incompressibility is equivalent to a Poisson's ratio of 0.5. For large deformations, the condition is that $\lambda_1\lambda_2\lambda_3 = 1$.

3.8

model

<finite element> computer simulation of a product or structure, containing all the information necessary to carry out a finite element analysis

3.9

model

<material> description, expressed mathematically, of some aspect of the behaviour of the material, which is implemented in the FEA package

3.10

principal strain axis

one of three orthogonal directions within the material along which the deformation results only in changes in length of a line element of the material without any rotation of the element, other than that associated with any rigid body rotation

3.11

principal stress

stress acting on a surface element of material normal to a principal stress axis

3.12

principal extension ratio

λ_i

ratio of deformed to undeformed length of an element of material in the direction of one of the principal strain axes

3.13

principal stress axis

one of three orthogonal directions in the material with the property that the stress acting on a surface normal to it has the same direction

3.14**stable**

<model> showing an increase in stress for an increase in extension

NOTE This guide applies only to elastic materials for which unstable models are not physically possible.

3.15**stiffness**

ratio of force to deflection of a test piece or product

NOTE Dynamic stiffness is the ratio of the amplitude of the periodic force to that of an applied sinusoidal deformation.

3.16**strain energy function**

mathematical expression for the strain energy per unit unstrained volume (strain energy density) of the hyperelastic material in terms of its deformation

3.17**strain invariant**

I_1, I_2, I_3

function of the principal extension ratios which is independent of the choice of co-ordinate axes

NOTE The most commonly used are defined in Clause 4.

4 Symbols

The following symbols are used with the following definitions throughout this part of BS 903. Other symbols are defined in the clauses in which they appear.

I_1	is the first strain invariant, defined such that $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$;
I_2	is the second strain invariant, defined such that $I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2$;
I_3	is the third strain invariant, defined such that $I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2$;
W	is the strain energy density, as defined in 3.16 ;
$\lambda_1, \lambda_2, \lambda_3$	are the three principal extension ratios.

5 Introduction to FEA

FEA has become a popular tool for design engineers and others. It provides a means of obtaining a computer simulation of the behaviour of a product from which useful quantitative predictions may be made. Some of the uses of FEA for products containing rubber are:

- to simulate the response of the product under a system of applied forces or deflections;
- to optimize the design (shape) of a component;
- to identify regions where failure can occur;
- to estimate the sealing pressure of a seal;
- to optimize the cure time and temperature of a bulky product.

The widespread use of FEA among engineers has been aided by the development of a number of commercial FEA software packages, which enable those without a specialist knowledge of mathematics or numerical analysis methods to carry out FEA. As well as the program which performs the analysis (the solver), many such software packages also contain programs which allow a visual simulation of the product on the computer screen (the pre- and post-processor). These are very useful for generating the correct input for the analysis (the FEA model) and for viewing the results, such as the deformed shape of the product, or the temperature distribution following a thermal analysis.

Relatively simple, so-called “linear”, FEA packages are available and are suitable for carrying out analyses where the strains are small, such as stress analysis of metals. Two difficulties make such packages unsuitable for analysing rubber products: they assume that the strains are very small, and that the material is compressible (Poisson’s ratio significantly less than 0.5). Instead, for mechanical analyses of rubber products, so called “non-linear” FEA packages are required. Non-linear packages are also required for most transient thermal analyses, such as simulating heat flow during vulcanization.

As with any computer simulation, the accuracy of the analysis is heavily dependent on the quality of the information supplied to the computer. The skill of the FEA practitioner lies in selecting the best analysis method, and also in using sensible models for describing the behaviour of the materials, with accurately measured values of the model parameters. Adequate models for some features of rubber behaviour are not yet provided with any existing commercial FEA package, notably for:

- a) dynamic properties of rubber (see 6.3);
- b) the Mullins effect (see 6.3);
- c) heat build-up (see Clause 10).

The purpose of this guide is to give some background information on the material models available in FEA packages for modelling rubber, and to recommend suitable test methods for measuring the model parameters.

6 Stress-strain behaviour

6.1 Solid (incompressible) rubbers

6.1.1 Introduction

Traditionally, rubber is modelled as a perfect hyperelastic material. This means that features such as set, hysteresis and strain softening are ignored. Commercial FEA packages are gradually introducing models which do take account of these features, but such models do not always provide a realistic description of the material behaviour, so should be used with caution. Further details are given in 6.3.

6.1.2 Hyperelastic models

Commercial FEA packages normally provide a choice of hyperelastic models expressed as strain energy functions, and the user is required to input one or more parameters of the function. The strain energy function is a mathematical expression for the amount of energy stored in the material, arising from work done in deforming it. In its most general form, it may be written as:

$$W = W(I_1, I_2, I_3) \quad (1a)$$

or

$$W = W(\lambda_1, \lambda_2, \lambda_3) \quad (1b)$$

where the symbols have the meanings given in Clause 4. Stress-strain equations may be obtained directly from equation (1) by differentiation and, conversely, experimental stress-strain curves may be used to fit the parameters of the strain energy function. The necessary equations are given in Annex A.

Many FEA packages also provide a curve-fitting procedure in which the user may enter tables of experimental stress-strain data and the program provides the best-fit values of the parameters for the chosen function (see 6.7). Alternatively, the function need not have an explicit form, but values required to fit a particular experimental data set may be obtained numerically (see 6.1.4.7).

6.1.3 Choosing a hyperelastic model

6.1.3.1 If a model has a large number of parameters which are fitted from a limited amount of test data it is possible that the model will be unstable and unrepresentative of any real material for strains or deformation modes outside those covered by the test data. Therefore the model with the fewest number of fitted parameters which is able to give a satisfactory description of the rubber behaviour should be used.

6.1.3.2 Hyperelastic models do not account for the effects of set, hysteresis, strain softening and frequency. However, these features usually have a significant effect on the stress-strain behaviour of filled rubbers and thus experimental stress-strain curves used to fit the parameters of the hyperelastic model are dependent on these features. It is unnecessary and unwarranted to attempt to obtain a fit to the experimental data which is much better than the quality of the experimental data, bearing in mind its dependence on features which cannot be modelled. Furthermore, other imperfections and approximations in the FEA can also outweigh imperfections in the material model.

6.1.3.3 For solid rubbers, the bulk modulus is a few thousand times larger than the shear modulus so it is normally reasonable to model the material as fully incompressible. Mathematically, incompressibility is defined by setting $I_3 = 1$. Hence, the strain energy function for an incompressible hyperelastic material is a function of only two variables and is normally expressed as:

$$W = W(I_1, I_2) \quad (2a)$$

or

$$W = W(\lambda_1, \lambda_2) \quad (2b)$$

In **6.1.4** all the models have been expressed in a form which neglects compressibility. However, most commercial FEA programs allow the user to enter a value for the bulk modulus, and take account of compressibility with an additional volumetric term in the strain energy function. Thus they assume that the volumetric deformation does not influence the deviatoric deformation. There is little experimental evidence to indicate the form which the volumetric term should take; in any case, for most analyses, the volumetric deformation will be very small compared to the deviatoric deformation so its exact form is unlikely to be critical. Exceptions are given in **6.5.1**. Most FEA programs consider cellular rubbers or rubbers with a cellular filler, such as cork, separately, and modelling these is discussed in **6.2**.

6.1.3.4 It is often useful to assume that the strain energy density is a function of I_1 only. Hence:

$$W = W(I_1) \quad (2c)$$

This assumption has been validated for filled natural rubber [1]. If this assumption is made at the outset, a considerable saving in the number of experimental tests required to characterize the rubber is possible since, in principle, a single test geometry is all that is required to fit the function $W(I_1)$, it not even being necessary to have a preconceived functional form with a fixed number of parameters. For further elaboration on this point, see **6.4.1**. However, further tests serve to validate the assumption for a particular rubber and provide a check of the accuracy of the test.

6.1.3.5 Some models are designed to model particular features of the stress–strain curve such as the high modulus at low strains, observed in filled rubbers, or the high modulus at high strains due to finite extensibility of the long chain molecules. Not all models will give a good representation of all the features of the stress–strain behaviour of a particular rubber. The model and fitting procedure should be chosen to give a good representation of the behaviour in the strain range of interest.

6.1.4 Appraisal of incompressible hyperelastic models

6.1.4.1 Introduction

All the models listed in **6.1.4.2** to **6.1.4.6** are available in at least one commercial FEA package. The advantages and disadvantages of each model are discussed, so that the FEA practitioner can make a sensible choice for any specific application. Examples of fits of some of these models to an experimental data set, to demonstrate some of the issues discussed in **6.1.3** and **6.1.4** are given in Annex C.

6.1.4.2 Mooney-Rivlin models

6.1.4.2.1 These models are truncations of the infinite Taylor series expansion for a strain energy function given by [2]:

$$W = \sum_{i,j=0}^{\infty} C_{ij}(I_1 - 3)^i(I_2 - 3)^j \quad (3)$$

where

W , I_1 and I_2 are defined in Clause 4;

C_{ij} are constants.

6.1.4.2.2 The neo-Hookean model [3] is obtained from equation (3) by setting all coefficients other than C_{10} to zero. Hence, the strain energy function is given by:

$$W = C_{10}(I_1 - 3) \quad (4)$$

This model gives linear behaviour in simple shear with the one constant being equal to half the shear modulus. This model is recommended for general use in FEA, when precise details of the non-linear behaviour are unknown or can be neglected. The model may be derived from the statistical theory of rubber elasticity, giving $C_{10} \approx NkT$ where N is the number of active chains in the network, k is Boltzmann's constant and T is absolute temperature, and thus has a molecular, rather than purely phenomenological, basis.

6.1.4.2.3 The Mooney model [4] is given by:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (5)$$

This model was developed to provide a more accurate representation of the behaviour of unfilled rubber in uniaxial tension than the neo-Hookean model, which tends to predict too high a stress at moderate strains if fitted at small strain. However, the Mooney model gives a poorer representation of the behaviour in compression. Also, fits to a limited amount of experimental data sometimes yield unstable functions (see 6.1.3.1). Hence, the Mooney model is not normally recommended.

6.1.4.2.4 The Yeoh model [5] is given by:

$$W = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3 \quad (6)$$

This model is capable of modelling some of the non-linear features of rubber. Normally C_{10} and C_{30} are positive and C_{20} is negative. Where this criterion is satisfied, the model will normally be stable but this is not assured. It is recommended when a more accurate representation of non-linear stress-strain behaviour than can be provided by the neo-Hookean model is required.

6.1.4.2.5 Commercial FEA programs often allow use of functions with more terms of the infinite series. Usually, these functions contain terms in I_1 and I_2 . Due to the large number of fitting constants, it is generally possible to obtain a good fit to a particular set of experimental data. However, these functions are liable to give unstable representations of the behaviour for deformations outside the scope of the fit, unless an extensive range of data is used for the fit, and careful checks of stability are made. Furthermore, inelastic effects, such as hysteresis and stress softening, are often significant, so precise agreement with particular sets of experimental data is usually unwarranted. Hence, these models are not normally recommended.

6.1.4.3 The Ogden model

The Ogden strain energy function [6] takes the form:

$$W = \sum_{n=1}^{\infty} \frac{\mu_n}{\alpha_n} \left\{ \lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + (\lambda_1 \lambda_2)^{-\alpha_n} - 3 \right\} \quad (7)$$

where

μ_n and α_n are constants.

The coefficients, μ_n and α_n , need not be integers or positive. Stability can be assured if $\mu_n \alpha_n > 0$ for all n . The Ogden function reduces to the neo-Hookean form if $n = 1$ and $\alpha_1 = 2$. A better representation of the behaviour might be achieved with more terms, but care is needed to ensure that the behaviour is sensible for strains and deformation modes other than those used for the fit. Three terms are generally sufficient.

6.1.4.4 Models that give a high stiffness at large strains

6.1.4.4.1 The models in **6.1.4.4.2** and **6.1.4.4.3** are specifically formulated to give a very high stiffness at large strains, corresponding to the finite extensibility of the rubber molecules.

6.1.4.4.2 The Gent strain energy function [7] takes the form:

$$W = -\frac{1}{2}G(I_m - 3)\ln\{1 - (I_1 - 3)/(I_m - 3)\} \quad (8)$$

where

G is a constant (the shear modulus at small strain);

I_m is a constant (the maximum possible value of I_1). Typically I_m is about 100.

This model is devised to reduce to the neo-Hookean model at small strains. It provides a straightforward way, with only one extra constant, of modelling rubber at high strains, but it is important to ensure that I_m is large enough never to be exceeded in the analysis; otherwise the analysis will fail.

6.1.4.4.3 The Arruda-Boyce model [8] is given by:

$$W = G \sum_{i=1}^n \frac{C_i}{\lambda_m^{2i-2}} (I_1^i - 3^i) \quad (9)$$

where

G is a constant (the shear modulus at small strain);

λ_m is a constant (the maximum possible extension ratio of a single molecule);

C_i are the constants arising from a series expansion of an inverse Langevin function.

Thus, $C_1 = 1/2$; $C_2 = 1/20$; $C_3 = 11/1050$; $C_4 = 19/7000$; $C_5 = 519/673750$.

This model also reduces to the neo-Hookean model at small strain.

6.1.4.5 Model derived from molecular considerations

The Kilian or Van der Waals model [9] is given by:

$$W = G \left\{ -(\lambda_m^2 - 3)[\ln(1 - \eta) + \eta] - \frac{2}{3}a \left(\frac{\tilde{I} - 3}{2} \right)^{\frac{3}{2}} \right\} \quad (10)$$

where

$$\tilde{I} = (1 - \beta)I_1 + \beta I_2 \quad (11)$$

$$\eta = \sqrt{(\tilde{I} - 3)/(\lambda_m^2 - 3)} \quad (12)$$

G and λ_m have the definitions given in **6.1.4.4.3**

a is the global interaction parameter which may be estimated from:

$$\alpha = \frac{2C_{01}}{3G} + \frac{\lambda_m^2}{\lambda_m^3 - 1} \quad (13)$$

where

C_{01} is the second Mooney-Rivlin parameter;

β is a linear mixture parameter, combining I_1 and I_2 into one parameter. Hence $0 < \beta < 1$.
If $\beta = 0$, the model is a function of I_1 only.

This model is an attempt to derive a strain energy function from molecular considerations. However, its rather complicated form and the relatively large number of parameters make it less convenient than some other models, so it is not recommended for general use.

6.1.4.6 Marlow model [10]

This model does not assume an explicit form for the strain energy function, except that it is a function of I_1 only. Instead, the strain energy density is calculated numerically from the data from one experimental test, interpolating between data points to give a complete description of the material. The strain energy density is assumed to be a function of I_1 only in order to enable predictions in a general strain state to be made.

6.1.4.7 User-defined models

Commercial FEA packages often allow the user to define a strain energy function by writing a user-subroutine. The user is required to provide expressions, in terms of I_1 , I_2 and user-defined parameters, for W and its derivatives with respect to I_1 and I_2 .

6.1.4.8 Summary

The best choice of model depends on the purpose of the analysis, the range of strains present in the component and the availability of experimental characterization data. For many purposes, the neo-Hookean model is sufficient. Only one parameter, the shear modulus, need be measured or estimated. An improved fit may generally be achieved with the Yeoh model. If the application involves large strains (greater than about 200 %) the Gent or Arruda-Boyce model are recommended. For these latter three models, a characterization test in a single mode of deformation is sufficient. The three-term Ogden model, and higher order Mooney-Rivlin type models require more characterization tests. They can give a precision comparable to, or perhaps better than, the Yeoh or Gent models, but only if the fitting is carried out with care to avoid instabilities; for this reason they are not recommended.

6.2 Cellular rubbers

Rubbers with a small volume fraction of voids can be modelled with the hyperelastic models given in 6.1.4, with a suitable value for the bulk modulus. In addition, special foam models are provided in some FEA programs for modelling isotropic, hyperelastic highly compressible materials. An inherent difficulty in the use of an isotropic model for significantly compressible cellular rubbers is that their behaviour in tension differs markedly from their behaviour in compression. In tension the stiffness rises with increasing strain. In compression, the initial stiffness, at less than about 5 % strain, is the same as in tension, thereafter the cells buckle, giving a plateau where there is little increase in stress with increasing strains. At high compressive strains, corresponding to collapsed cells, the stiffness rises steeply. This means that it is essential that experimental characterization tests are carried out in similar deformations to those applicable in the analysis; for example, if compression dominates the material should be characterized in compression. Both deviatoric tests (see 6.4) and dilatational tests (see 6.5) are required to characterize the material.

6.3 Limitations of hyperelastic models and alternative modelling techniques

6.3.1 General

The traditional assumption that rubber behaves as a perfectly elastic material which may be described by a hyperelastic material model ignores many features of the real behaviour of typical rubbers. The most significant departures from hyperelastic behaviour are:

- a) hysteresis. There is a loss of energy on loading and unloading so that the loading and unloading curves do not follow the same path; this also gives rise to damping or loss factor. For some unfilled rubbers damping is related to creep, a gradual increase in deformation under a load, stress relaxation, a gradual decrease in force under a constant deformation, and recoverable set through viscoelastic theory. For filled rubbers the interrelationships are not simple and the additional departures from elastic behaviour described in b), c) and d) are more evident.
- b) the Mullins effect [11, 12]. The rubber is softer on second and subsequent loadings at strains less than the previous maximum. If strained above the previous maximum there is a marked stiffening near the previous maximum strain as the stress approaches its level for the unstrained material. Most softening occurs on the first cycle but there is some further softening on subsequent cycles. There is usually some slow recovery from this effect but, for the time scales of many applications, it may be considered to be largely irreversible.
- c) permanent set. After deforming and relaxing the rubber does not return to its original dimensions but retains some permanent deformation. For some rubbers, cyclic loading increases the creep above the level expected from tests under constant load; this phenomenon is also akin to set.
- d) the Payne or Fletcher-Gent effect [13, 14]. The dynamic stiffness of rubber tends to fall as the strain amplitude is increased, especially if the rubber incorporates reinforcing filler or is a block copolymer. The high stiffness at small amplitude recovers quite rapidly after subjection of the material to a higher strain amplitude. An associated effect is the relatively high stiffness of a small “dynamic” strain superimposed on a larger “static” strain.

Models which aim to take account of one or more of these features are under development. Some have been implemented in some commercial packages but not all are adequate for the purpose for which they are designed. Furthermore, some FEA packages allow the user to define and implement a model using a user-subroutine, though such an exercise is not trivial. At the moment, it is often better to use a hyperelastic material model, with the rubber characterized under the most appropriate conditions (see 6.4.1, 6.4.2, and 6.4.3). Exceptions are applications involving slow relaxation of forces under a static deformation, such as relaxation in seals, and applications involving damping using viscoelastic materials. For these applications the viscoelastic model described in 6.3.2 is recommended.

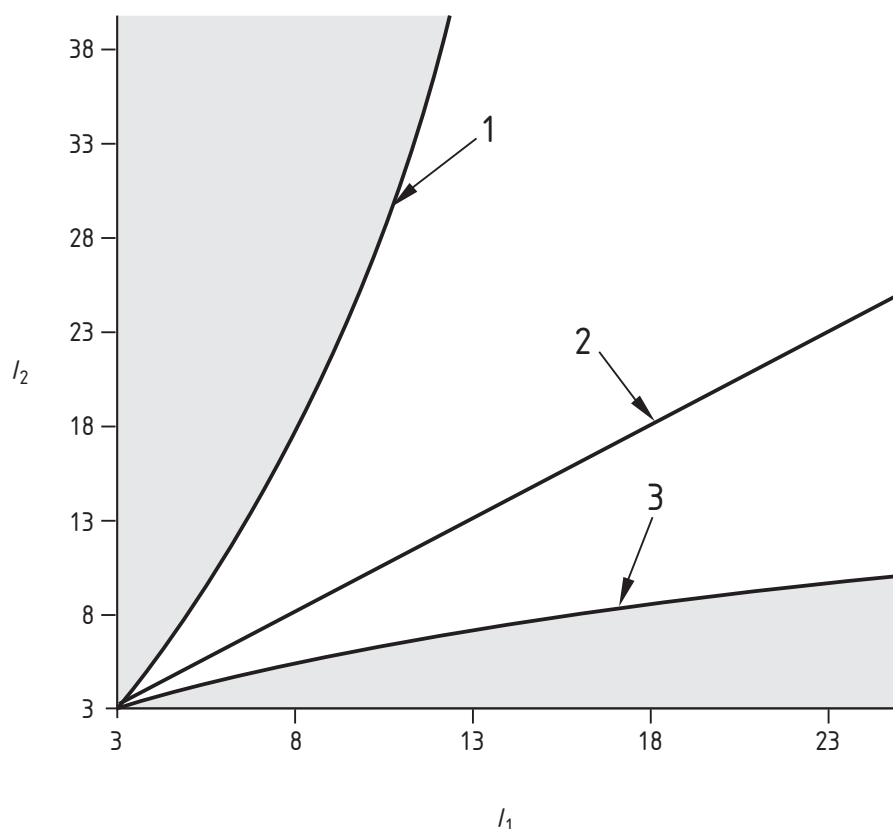
6.3.2 Large strain viscoelasticity

A model which is based on linear viscoelasticity theory, modified to large strains, is implemented in some FEA programs. It is able to model stress relaxation and the frequency dependence of the dynamic modulus. It successfully models only those departures from elasticity given in 6.3.1 a). Experimental data, giving the dependence of modulus with time, from a stress relaxation experiment are usually required. Appropriate test methods are given in 6.6.2. Alternatively, some FEA programs can formulate the model from dynamic properties, such as from a table giving values of storage modulus and loss factor as a function of frequency. Suitable test methods are described in 6.6.3.

6.4 Recommended deviatoric tests for obtaining parameters for hyperelastic models

6.4.1 Introduction

Useful tests are uniaxial tension, uniaxial compression, equibiaxial tension, pure (planar) shear and simple shear. These tests are described in 6.4.4.



Key

- 1 Equibiaxial extension of uniaxial compression
- 2 Pure or simple shear
- 3 Uniaxial tension

NOTE Shaded areas are not physically possible

Figure 1 — Values of I_1 and I_2 for different deformation modes.

Figure 1 shows the values of I_1 and I_2 which are covered by the tests. A comprehensive test schedule would require a fully biaxial test rig, allowing different stretches in two perpendicular directions on a sheet of rubber, in order to map the whole of the I_1 I_2 space, (the area shown unshaded on Figure 1). Ensuring compatibility of strain history and rate effects would be very difficult. Alternatively, a simplifying assumption could be made, such as that W is a function of I_1 only, or that W is a preconceived function of I_1 and I_2 , requiring only a specified number of parameters to be determined. If either assumption is made, a fit may be achieved from a test in a single mode of deformation. However, choosing W as a preconceived function of I_1 and I_2 , and fitting from a test in a single mode of deformation is very dangerous because the fit may be apparently improved by independently adjusting the parameters of both the I_1 and I_2 terms. This effectively increases the number of adjustable parameters and can result in very unrealistic predictions for other modes of deformation, although the use of a stability check should avoid such extreme behaviour.

This problem can only be avoided by carrying out additional tests in other modes of deformation to ensure that the fit gives a realistic dependence for both I_1 and I_2 . Fortunately, it appears that the first option, W as a function of I_1 only, is often satisfactory, and then testing in a single mode of deformation is adequate. In any case, assuming W is a function of I_1 only is robust and will at worst impair the fit rather than lead to very unrealistic predictions in other modes of deformation. Nevertheless, further tests would serve to

validate the assumption that W is a function of I_1 only, provide a check of the experimental accuracy of the tests and enable a greater range of strains to be covered than is possible from a single test. If tests in more than one deformation mode are to be carried out, it is sensible to choose tests which cover as much of the I_1, I_2 space of Figure 1 as possible. To this end, the following points should be taken into account. For an incompressible material:

- a) equibiaxial tension and uniaxial compression are equivalent deformations. No additional information, except for checks of the accuracy of the test methods, and the applicability of an incompressible hyperelastic model, may be obtained from carrying out both tests;
- b) pure (planar) and simple shear are equivalent.

Any two or three tests, which correspond to different lines on Figure 1, provide a reasonably comprehensive evaluation of the I_1 and I_2 dependence of W .

Another consideration when choosing test methods is their experimental simplicity and the range of strain which may be covered. Uniaxial tension, pure shear and simple shear are all straightforward to carry out, and require no sophisticated equipment except a test machine, nor exceptionally precise measurement, although simple shear test pieces require moulding and bonding to end-pieces. These test methods are recommended for providing accurate data with the minimum effort or expense. Uniaxial compression is slightly more difficult to carry out accurately since it requires smooth, parallel test platens, effective lubrication of the test piece and careful measurement of deflection. Moreover, it is not possible to reach high strains. The alternative, an equibiaxial tension test, is also difficult. One technique, equibiaxial straining of a flat sheet (see 6.4.4.4.2), requires a relatively elaborate test jig. The other, inflation of a sheet (see 6.4.4.4.3), also requires some special equipment and demands precise measurements on curved surfaces to be made, thus is liable to be the most inaccurate of the tests at modest strains. However, it is possible to reach very high extensions by this method and thus it is a useful technique for modelling applications involving very large strains.

Test conditions should be chosen to reflect as far as possible the conditions of the product to be modelled. Thus the test speed, test temperature and strain range should, if possible, be comparable to those experienced by the product. The effect of strain rate on stress is normally quite small; a tenfold increase in strain rate typically gives less than 10 % increase in modulus. In respect of strain range it is important to bear in mind that some parts of the product might experience local strains that are much larger than the overall strain, thus the maximum strains in the characterization tests should be large enough to reflect this.

6.4.2 Methods for standardizing strain history effects

Because rubber, especially filled rubber, is not a perfect hyperelastic material, the stress-strain data will be dependent on strain history. Further information on strain history effects such as the Mullins effect are given in 6.3.1. Methods of carrying out characterization tests are recommended as follows.

- a) Use only data from the first loading of a previously unstrained test piece. This technique does not address the phenomenon of softening due to previous straining, and thus will lead to models which overestimate the stiffness of products that have been pre-strained. However, it avoids confusion or error due to an inappropriate prestraining procedure, and is very simple to define and carry out.
- b) Stretch the test piece one or more times to a predetermined strain, then use the data from the next loading. This technique allows for stress-softening. However, the magnitude of the prestrain affects the stress-strain behaviour in subsequent loadings. The time between prestraining and testing has some, though usually a small, effect on the stress-strain behaviour. A single prestrain is representative only of a product in a largely uniform state of strain. Normally, some parts of the product are subjected to higher strains than other parts and thus it is preferable for the prestrain to depend on the strain.
- c) Stretch the test piece a fixed number of times to successively increasing strains, and use only the values at the peak of the final stretch at each strain. A typical test might be to stretch the test piece three times to 5 %, 10 %, 20 %, 50 %, 100 %, 150 %, 200 %, 250 % and 300 % strain. This technique allows for strain softening in a way which takes account of non-uniform strains in a product. Its disadvantages are that a relatively elaborate test procedure is needed and only a few data points are generated, so that the fit to a material model might be rather crude. Also, the stress-strain curve is a composition of many curves, rather than one obtainable directly from the rubber.

Method a) is recommended for applications in which the product experiences occasional deformations of varying magnitude. Method c) is recommended for applications in which the product is subjected to regular cyclic deformations.