

Glass in building — Procedures for goodness of fit and confidence intervals for Weibull distributed glass strength data

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National foreword

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The UK participation in its preparation was entrusted by Technical Committee B/520, Glass and glazing in building, to Subcommittee B/520/4, Properties and glazing methods, which has the responsibility to:

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Glass in building - Procedures for goodness of fit and confidence intervals for Weibull distributed glass strength data

Verre dans la construction - Procédures de validité de l'ajustement et intervalles de confiance des données de résistance du verre au moyen de la loi de Weibull

Glas im Bauwesen - Bestimmung der Biegefestigkeit von Glas - Schätzverfahren und Bestimmung der Vertrauensbereiche für Daten mit Weibull-Verteilung

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This European Standard exists in three official versions (English, French, German). A version in any other language made by translation under the responsibility of a CEN member into its own language and notified to the Management Centre has the same status as the official versions.

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Foreword

This document (EN 12603:2002) has been prepared by Technical Committee CEN/TC 129 "Glass in building", the secretariat of which is held by IBN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by May 2003, and conflicting national standards shall be withdrawn at the latest by May 2003.

In this standard the annexes A, B and C are informative.

According to the CEN/CENELEC Internal Regulations, the national standards organizations of the following countries are bound to implement this European Standard: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom.

Introduction

This European Standard is based on the assumption that the statistical distribution of the attribute taken into consideration can be represented by one single Weibull distribution function, even where in certain cases (e.g. lifetime measurements) mixed distributions have frequently been observed. For this reason, the user of the standard has to check by a goodness of fit test whether the measured data of a sample can be represented by means of one single Weibull function. Only in this case can the hypothesis be accepted and the procedures described in this standard be applied.

The user decides on this question also considering all previous relevant data and the general state of knowledge in the special field. Every extrapolation into ranges of fractiles not confirmed by measured values requires utmost care, the more so the farther the extrapolation exceeds the range of measurements.

NOTE The three-parameter Weibull function is:

$$G(x) = 1 - \exp \left[- \left(\frac{x - x_0}{\theta} \right)^\beta \right] \quad (1)$$

If $x_0 = 0$ is assumed, the two-parameter Weibull function results:

$$G(x) = 1 - \exp \left[- \left(\frac{x}{\theta} \right)^\beta \right] \quad (2)$$

which can be written as:

$$x = \theta \left[\ln \left(\frac{1}{1 - G(x)} \right) \right]^{\frac{1}{\beta}} \quad (3)$$

The calculation can be based either on an uncensored or a censored sample. There are several methods of censoring. In this standard only the following method of censoring is considered:

- given a number $r < n$ of specimens of which attribute values x_i were measured.

1 Scope

This European Standard specifies procedures for the evaluation of sample data by means of a two-parameter Weibull distribution function.

2 Normative references

This European Standard incorporates by dated or undated reference, provisions from other publications. These normative references are cited at the appropriate places in the text, and the publications are listed hereafter. For dated references, subsequent amendments to or revisions of any of these publications apply to this European Standard only when incorporated in its amendment or revision. For undated reference, the latest edition of the publications referred to applies (including amendments).

ISO 2854:1976, *Statistical interpretation of data - Techniques of estimation and tests relating to means and variances*.

ISO 3534, *Statistics - Vocabulary and symbols*.

3 Terms and definitions

For the purposes of this European Standard, the terms and definitions given in ISO 3534 apply.

4 Symbols and abbreviated terms

X attribute taken into consideration;

x, x_i, x_r values of X ;

$G(x)$ distribution function of X = percentage of failure;

x_0, β, θ parameters of the three-parameter Weibull function;

$\wedge$ identification label for point estimators (e.g. $\hat{\beta}, \hat{\theta}, \hat{G}$);

$1-\alpha$ confidence level;

ℓ_i value used in the goodness of fit test;

L value used in the goodness of fit test;

n sample size;

r number of specimens of which attribute values x_i were measured;

NOTE The sample is ordered, i.e. $x_1 \leq x_2 \leq x_3 \dots \leq x_r \leq n$;

f, f_1, f_2 degrees of freedom;

K_n, K_{rn} factors used in estimating $\hat{\beta}$;

- $C_{r,n}$ factor used in estimating $\hat{\theta}$;
- s $\text{int}(0,84n) = \text{largest integer} < 0,84n$;
- η, ξ ordinate and abscissa of the Weibull diagram;
- χ^2 chi-square distribution function;
- y, v, γ auxiliary factors used in estimating the confidence limits of $G(x)$;
- A, B, C constants used in evaluating v ;
- $H(f_2)$ variable used in evaluating γ ;
- $T_{n,\alpha/2}, T_{n,1-\alpha/2}$ coefficients used in estimating the confidence limits of θ ;

Subscripts:

- un lower confidence limit;
- ob upper confidence limit;
- z confidence interval limited on two sides.

5 Goodness of fit

Sort the r values of x into rank ascending order.

Compute for each value from $i = 1$ to $i = r - 1$:

$$\ell_i = \frac{\ln(x_{i+1}) - \ln(x_i)}{\ln \left[\frac{\ln \left(\frac{4(n-i-1)+3}{4n+1} \right)}{\ln \left(\frac{4(n-i)+3}{4n+1} \right)} \right]} \quad (4)$$

Compute the quantity:

$$L = \frac{\sum_{i=\lfloor r/2 \rfloor + 1}^{r-1} \frac{\ell_i}{\lfloor (r-1)/2 \rfloor}}{\sum_{i=1}^{\lfloor r/2 \rfloor} \frac{\ell_i}{\lfloor r/2 \rfloor}} \quad (5)$$

where the symbol $\lfloor r/2 \rfloor$ is used to denote the largest integer less than or equal to $r/2$.

Reject the hypothesis that the data is from a Weibull distribution at the α significance level if:

$$L \geq F_{\alpha} \left(2 \lfloor (r-1)/2 \rfloor, 2 \lfloor r/2 \rfloor \right) \quad (6)$$

The values of the fractiles of the F distribution can be found for example in Table IV of ISO 2854:1976.

6 Point estimators for the parameters β and θ of the distribution

6.1 Censored sample

$$\hat{\beta} = \frac{n \kappa_{r;n}}{r \ln x_r - \sum_{i=1}^r \ln x_i} \quad (7)$$

$$\hat{\theta} = \exp \left[\ln x_r - C_{r;n} \frac{1}{\hat{\beta}} \right] \quad (8)$$

The factors $\kappa_{r;n}$ and $C_{r;n}$ are listed in Table 1 and Table 2.

Table 1 — Coefficient $\kappa_{r,n}$

n	r/n								
	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
5				0,2231		0,4813		0,8018	
10		0,1054	0,2172	0,3369	0,4667	0,6098	0,7715	0,9616	1,202
20	0,0513	0,1583	0,2721	0,3944	0,5277	0,6756	0,8448	1,048	1,316
30	0,0684	0,1759	0,2904	0,4137	0,5482	0,6979	0,8697	1,077	1,357
40	0,0770	0,1848	0,2996	0,4233	0,5584	0,7090	0,8822	1,092	1,378
50	0,0821	0,1901	0,3051	0,4291	0,5646	0,7158	0,8898	1,101	1,391
60	0,0855	0,1936	0,3088	0,4330	0,5687	0,7202	0,8949	1,108	1,400
70	0,0879	0,1961	0,3114	0,4357	0,5717	0,7235	0,8985	1,112	1,406
80	0,0898	0,1980	0,3134	0,4378	0,5739	0,7259	0,9012	1,115	1,410
90	0,0912	0,1995	0,3149	0,4394	0,5756	0,7277	0,9033	1,118	1,414
100	0,0924	0,2007	0,3162	0,4407	0,5770	0,7292	0,9050	1,120	1,417
κ_p	0,10265	0,21129	0,32723	0,45234	0,58937	0,74274	0,92026	1,1382	1,4436
d_1	-1,0271	-1,0622	-1,1060	-1,1634	-1,2415	-1,3540	-1,5313	-1,8567	-2,6929
d_2	0,000	0,030	0,054	0,089	0,145	0,242	0,433	0,906	2,796
Asymptotic estimate for large n : $\kappa_{r,n} = \kappa_p + d_1/n + d_2/n^2$									

Table 2 — Coefficient $C_{r,n}$

n	r/n								
	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
10	-2,880	-1,826	-1,267	-0,8681	-0,5436	-0,2574	0,0120	0,2837	0,5846
20	-2,547	-1,658	-1,147	-0,7691	-0,4548	-0,1727	0,0979	0,3776	0,7022
30	-2,444	-1,605	-1,108	-0,7364	-0,4253	-0,1443	0,1269	0,4098	0,7446
40	-2,394	-1,578	-1,089	-0,7202	-0,4106	-0,1301	0,1415	0,4262	0,7664
50	-2,365	-1,562	-1,077	-0,7105	-0,4018	-0,1216	0,1503	0,4360	0,7796
60	-2,345	-1,522	-1,069	-0,7040	-0,3959	-0,1159	0,1562	0,4426	0,7885
70	-2,331	-1,544	-1,064	-0,6994	-0,3917	-0,1118	0,1604	0,4473	0,7949
80	-2,321	-1,539	-1,060	-0,6959	-0,3886	-0,1088	0,1635	0,4509	0,7998
90	-2,313	-1,534	-1,056	-0,6932	-0,3861	-0,1064	0,1660	0,4537	0,8035
100	-2,307	-1,531	-1,054	-0,6911	-0,3841	-0,1045	0,1679	0,4559	0,8065
c_p	-2,2504	-1,4999	-1,0309	-0,67173	-0,36651	-0,08742	0,18563	0,47589	0,83403
a_1	-5,5743	-3,0740	-2,2859	-1,9301	-1,7619	-1,7114	-1,7727	-2,0110	-2,7773
a_2	-7,201	-1,886	-0,767	-0,335	-0,091	0,111	0,369	0,891	2,825
Asymptotic estimate for large n : $C_{r,n} = c_p + a_1/n + a_2/n^2$									

6.2 Uncensored (complete) sample

$$\hat{\beta} = \frac{n \kappa_n}{\frac{s}{n-s} \sum_{i=s+1}^n \ln x_i - \sum_{i=1}^s \ln x_i} \quad (10)$$

$$\hat{\theta} = \exp \left[\frac{1}{n} \sum_{i=1}^n \ln x_i + 0,5772 \frac{1}{\beta} \right] \quad (11)$$

The factors κ_n are listed in Table 3.