

Railway applications — Method for determining the equivalent conicity

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National foreword

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Railway applications - Method for determining the equivalent conicity

Applications ferroviaires - Méthode de détermination de la conicité équivalente

Bahnanwendungen - Verfahren zur Bestimmung der äquivalenten Konizität

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Foreword

This document (EN 15302:2008+A1:2010) has been prepared by Technical Committee CEN/TC 256 “Railway applications”, the secretariat of which is held by DIN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by May 2011, and conflicting national standards shall be withdrawn at the latest by May 2011.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

This document includes Amendment 1, approved by CEN on 2010-09-28.

This document supersedes EN 15302:2008.

The start and finish of text introduced or altered by amendment is indicated in the text by tags **A1** **A1**.

A1 This document has been prepared under a mandate given to CEN/CENELEC/ETSI by the European Commission and the European Free Trade Association, and supports essential requirements of EU Directive 2008/57/EC.

For relationship with EU Directive 2008/57/EC, see informative Annex ZA, which is an integral part of this document. **A1**

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Introduction

This European Standard is based on the UIC Code 519 OR submitted to CEN by the International Union of Railways (UIC) and which has been revised by CEN/TC 256/WG 10 "Vehicle/Track Interaction".

The contact geometry is fundamental to explain the dynamic running behaviour of a railway vehicle. Among the parameters by which the dynamic behaviour of a rail vehicle is characterised, the conicity plays an essential role since it allows the satisfactory appreciation of the wheel-rail contact on tangent track and on very large-radius curves (when operated with low cant deficiencies). A wheelset with conical profiles describes a waveform while running on a track. Klingel's theory states that the wavelength depends on the cone angle of the wheel profile and the distance between contact patches.

For practical wheel profiles with changing cone angles along the profile it is possible to evaluate the wavelength of the wheelsets movement by integration of the function of rolling radius difference depending on the lateral movement of the wheelset on the track. Equivalent conicity is evaluated by comparison of this wavelength with the one evaluated according to Klingel's theory.

It is necessary to have a clear procedure for the evaluation of equivalent conicity, which is used in European and national standards and documents (legal and technical).

The results need to be consistent. However it is possible to use different evaluation procedures to those given in this European Standard, provided that the procedure used leads to the determination of an equivalent conicity in accordance with the calculation results using reference profiles specified in Annex E.

To confirm whether an alternative evaluation procedure can achieve the results specified in this European Standard, three aspects of the process need to be evaluated in a benchmark process given in this European Standard and outlined below in Steps 1, 2 and 3:

In Step 1, tables of reference profiles in Annex D are applied to the interpolation and calculation algorithm which allows the location of the contact points in order to calculate the rolling radius difference as a function of the lateral position of the wheelset. Starting from this function the equivalent conicity is calculated as a function of the amplitude of the oscillation. A comparison of the achieved results with the reference results in Annex E and a defined field of allowed tolerances in Annex F determine the acceptance or rejection of the assessed evaluation procedure (see Figure 1).

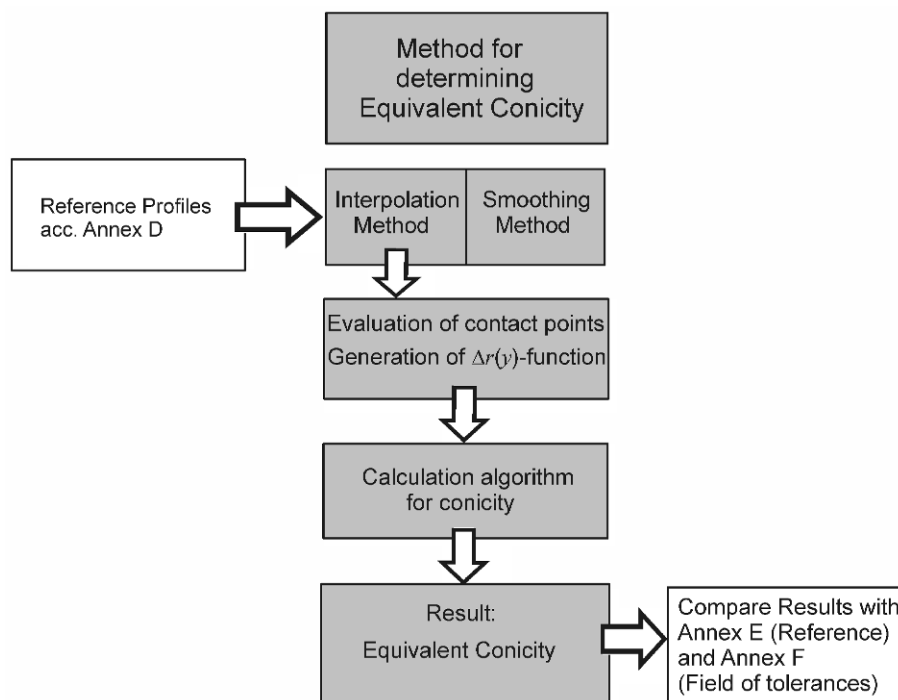


Figure 1 — Benchmark process, Step 1

In Step 2, random errors given in Annex G are added to the reference profiles in Annex D and are applied to the smoothing and interpolation algorithm. A comparison of the achieved results with the reference results including the field of tolerances in Annex F allows the assessment of the evaluation procedure (see Figure 2).

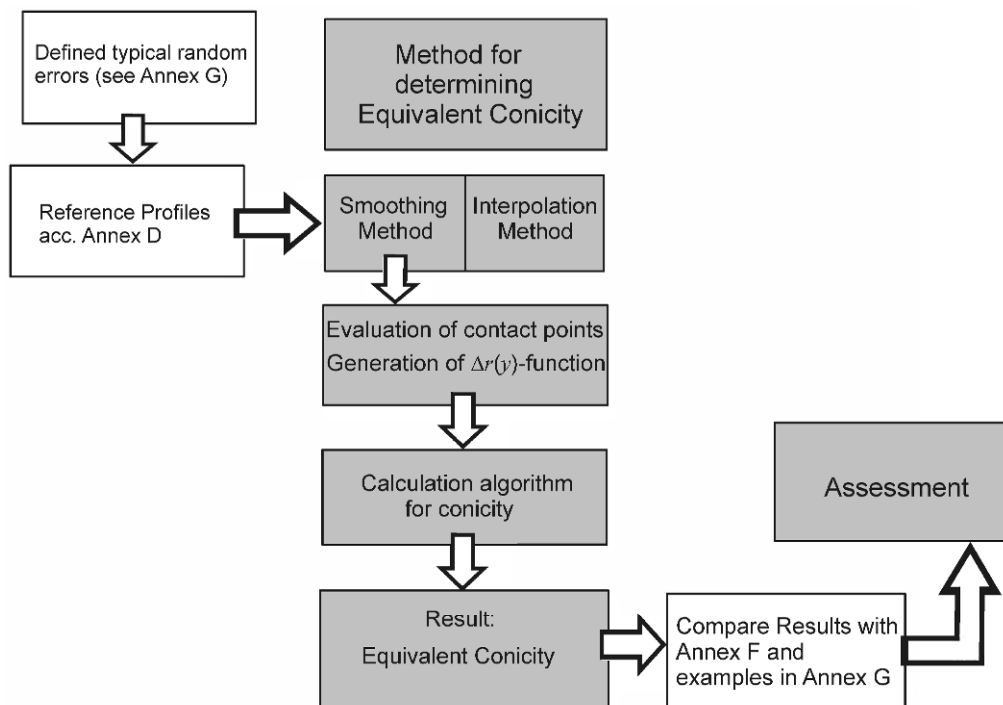


Figure 2 — Benchmark process, Step 2

In Step 3, the tolerances of the measuring system used are compared with the random errors applied in Step 2 in order to assess their influence on the results.

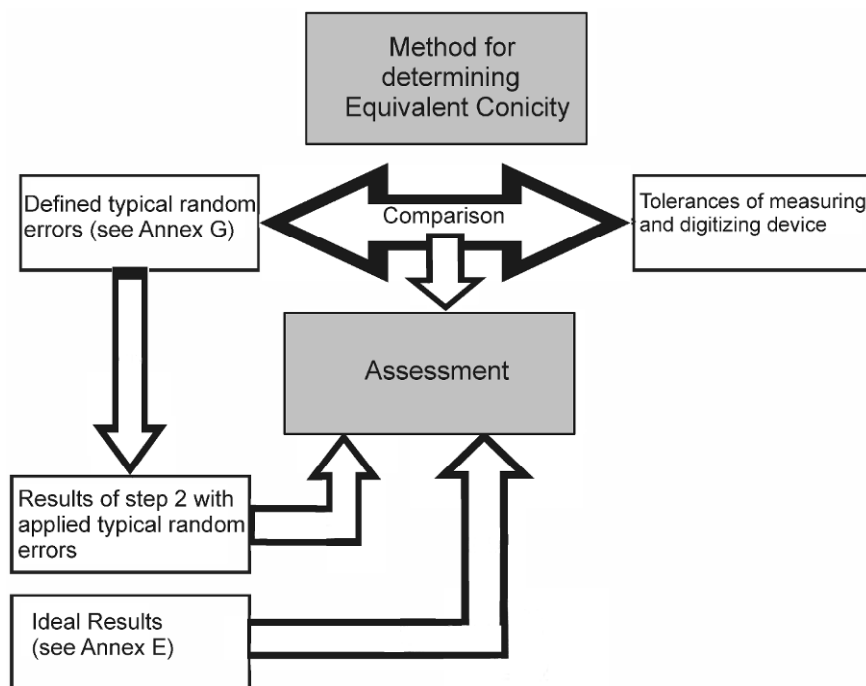


Figure 3 — Benchmark process, Step 3

1 Scope

This European Standard establishes an evaluation procedure for determining equivalent conicity. A benchmark calculation is specified to achieve comparable results on a consistent basis for the equivalent conicity, which may be calculated by different methods not given in this European Standard. This European Standard also proposes possible calculation methods. Informative examples of the use of the Klingel formula (see Annex B) and linear regression of the Δr -function (see Annex C) are included in this European Standard.

This European Standard includes reference profiles, profile combinations, tolerances and reference results with tolerance limits, which allow the user to assess the acceptability of a measuring and calculation system including random- and grid- errors of the measuring system. It sets down the principles of calculation that need to be followed but does not impose any particular numerical calculation method.

This European Standard does not define limits for the equivalent conicity and gives no tolerances for the rail profile and the wheel profile to achieve acceptable results for the conicity.

For purposes outside the scope of this European Standard (e.g. simulation of vehicle behaviour) it can be useful or necessary to use more sophisticated theories. These methods are not within the scope of this European Standard.

For the application of this European Standard some general recommendations are given in Annex I.

2 Normative references

The following referenced document is indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

N/A

3 Symbols

For the purposes of this document, the following symbols apply.

x	displacement of the wheelset in the longitudinal direction of the track
y	displacement of the wheelset in the lateral direction of the track
Ψ	angle of the movement in the x - y -plane
e	nominal contact patches distance (approximately 1 500 mm for standard gauge)
λ	wave length
r_0	radius of the wheels when the wheelset is centred on the track
r_1	rolling-radius of the right-hand wheel
r_2	rolling-radius of the left-hand wheel
r	mean rolling-radius of both wheels
Δr	difference of the rolling-radius between right-hand and left-hand wheels
R	local radius of the wheel path
ds	curve length of the path corresponding to the angle $d\Psi$
$\tan\gamma_e$	equivalent conicity
$\tan\gamma_a$	inclination of the wheel and rail profiles in the contact point
$\hat{y}$	amplitude of the wave
y_{em}	y -displacement where $\Delta r = 0$
y_{emin}, y_{emax}	minimum and maximum values of y -displacements
V	speed of forward movement of the vehicle
γ	angle of the tangent to the point of contact between wheel and rail

4 Principle of determining the equivalent conicity

4.1 Integration of the equation of the wheelset movement of a conical profile

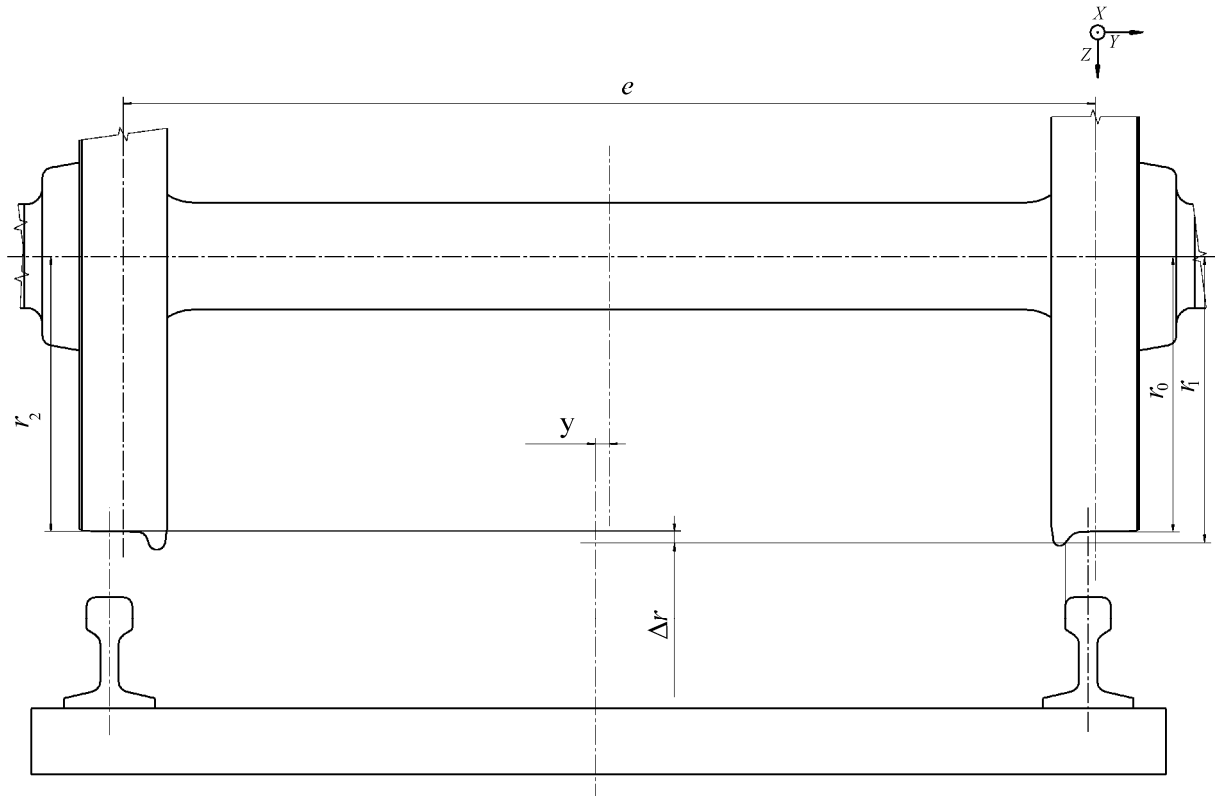


Figure 4 — Dimensions on the wheelset

The kinematic lateral movement of a free wheelset, with no inertia, running on a track, is defined by the following differential equation:

$$\ddot{y} + \frac{V^2}{er_0} \Delta r = 0 \quad (1)$$

Without limiting the conclusions concerning the calculation of the equivalent conicity, the speed of forward movement V of the vehicle can be assumed to be constant for the purposes of this European Standard, such that:

$$V = \frac{dx}{dt} \quad (2)$$

$$\text{hence } \frac{dy}{dt} = V \frac{dy}{dx} \text{ and } \frac{d^2y}{dt^2} = V^2 \frac{d^2y}{dx^2} \quad (3)$$

The differential equation becomes:

$$\frac{d^2y}{dx^2} + \frac{\Delta r}{er_0} = 0 \quad (4)$$

The previously determined $\Delta r = f(y)$ characteristic is used together with the following initial conditions:

$$y = y_0 \quad \text{for } x = 0 \quad (5)$$

$$\frac{dy}{dx} = 0 \quad \text{for } x = 0 \quad (6)$$

This integration, based on the initial amplitude y_0 , leads to a periodic movement of the wheelset with a peak-to-peak amplitude of $2\hat{y}$ and a wavelength λ as presented in Figure 5.

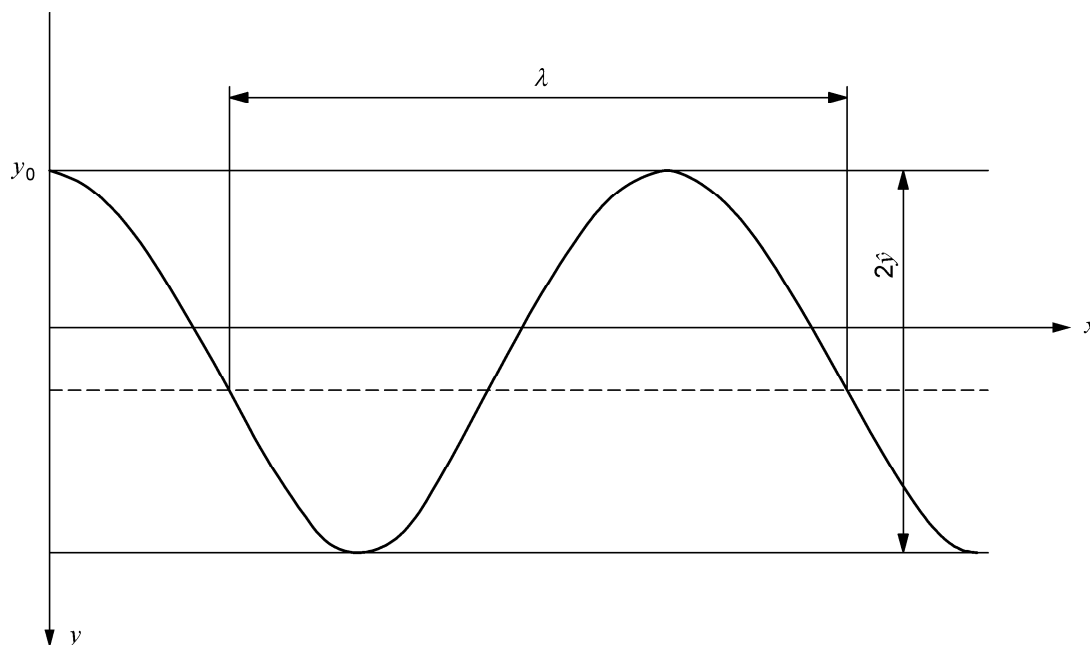


Figure 5 — $y = f(x)$ function

4.2 Determining the wavelength of a conical profile

In the case of a wheelset whose wheels have a conical profile with a constant angle γ , the rolling radius difference is:

$$\Delta r = 2y \tan \gamma \quad (7)$$

The differential Equation (4) then becomes:

$$\frac{d^2 y}{dx^2} + \frac{2 \tan \gamma}{er_0} y = 0 \quad (8)$$

a second order differential equation with constant coefficients whose solution is a sinewave with a wavelength of λ

$$\lambda = 2\pi \sqrt{\frac{er_0}{2 \tan \gamma}} \quad \text{Klingel's formula} \quad (9)$$

4.3 Definition of equivalent conicity for nonlinear profiles

When the wheels do not have a conical profile, linearization methods shall be used so that the linear differential equation can still be applied by replacing $\tan \gamma$ by "equivalent conicity" $\tan \gamma_e$.

By definition, the equivalent conicity is equal to the tangent of the cone angle of a wheelset with coned wheels whose lateral movement has the same kinematic wavelength as the given wheelset (but only on tangent track and on very large-radius curves):

$$\tan \gamma_e = \left(\frac{\pi}{\lambda} \right)^2 2er_0. \quad (10)$$

Annex B and Annex C give two possible methods for determining the equivalent conicity.

5 Description of the reference procedure

5.1 General principles

For the described method of determination the following assumptions are used:

- in the calculation both the wheel and the rail are considered rigid;
- wheels are symmetrical in revolution and are represented by a single profile for each wheel;
- rails are straight, parallel to each other and represented by a single profile for each rail;
- wheel does not penetrate into the rail: only point contacts are considered;
- no account is taken of an axle's rolling (rotation about an axis longitudinal to the track) as the wheelset moves laterally on the track;

NOTE 1 It is recommended to take into account this effect but it should be noted that in most applications up to now and in all calculations of the benchmark in this standard this effect was neglected at the point of contact, the tangent planes to the rail and to the wheel are parallel.

NOTE 2 When using measured profiles the principles for averaging are given in Clause 6, point e) and point f).

For any given wheel and rail profile (theoretical or real), it is possible to determine the equivalent conicity $\tan \gamma_e$ associated with the lateral movement $\hat{y}$ of the wheelset on the track.

This is done by the following procedure:

- a) determine the wheel and rail profiles, either by measurement of real profiles or by a theoretical calculation for theoretical profiles;
- b) determine the $\Delta r = f(y)$ characteristic giving, for each lateral movement y of the wheelset on the track, the difference between the right-hand and the left-hand rolling radius $\Delta r = r_1 - r_2$;
- c) determine the equivalent conicity for a lateral movement $\hat{y}$ of the wheelset on the track.

These three defined steps of the procedure are explained in 5.2 to 5.4.

5.2 Determining the wheel and rail profiles

5.2.1 Principles of measurement

The two rails of the railway track and the two wheels of the wheelset shall be referred to a track-centred coordinate system oriented such that the x -axis is longitudinal, the y -axis transversal and the z -axis vertical.

The wheel and rail profiles shall be characterised in a Cartesian coordinate system (y_i, z_i) such that:

- for the rail, the profile is defined not only on top but also on the inner side;
- for the wheel, the profile is defined not only on the classic wheel tread but also on the outer part and in the area of the wheel flange root.

When the profiles are determined by measurement, special-purpose devices can be used, such as wheel and rail profile measuring apparatus or automatic measuring systems carried aboard special railbound vehicles for rail profiles or track based systems for wheel profiles.

It shall be stated in the report whether the profiles were measured in the loaded or unloaded condition.

When theoretical profiles are used the inclination is to be considered.

5.2.2 Accuracy of the measuring system

To ensure accuracy of the conicity calculation measurement accuracies shall lie within the following tolerances:

- difference of wheel radii (measured at wheel tread plane): 0,2 mm;
- wheel back-to-back distance: 0,5 mm;
- track gauge: 0,5 mm.

Measured data (wheel and rail profiles) shall be consistent with the evaluation (smoothing and calculation).

NOTE 1 Tolerances for the diameter difference between wheels on the same axle are specified in prEN 15313.

NOTE 2 For the assessment of equivalent conicity the sum of the tolerances of wheel back-to-back distance and track gauge is the relevant parameter.

5.3 Determining the rolling radius difference function Δr

The determination of the Cartesian coordinates of the wheels, shall take into account the actual radius of each wheel.

The Cartesian coordinates of the two wheels and the two rails, referred to a track-centred coordinate system, can be computer-processed (smoothing or interpolation in particular) for easier utilisation in the next step of the procedure.

Then, the $\Delta r = f(y)$ characteristic which yields the rolling radius difference between the right and left side for a lateral movement y of the wheelset on the track, shall be worked out relative to a centred position and for lateral movements up to a maximum corresponding to $\Delta r = r_1 - r_2 = 5$ mm, in maximum steps of $\Delta y = 0,2$ mm.

NOTE An example of a $\Delta r = f(y)$ characteristic is presented in Annex A. The maximum lateral displacement for the determination of the geometric functions depends upon the clearance between wheel and rail.

5.4 Determining the equivalent conicity

The computation of the equivalent conicity shall be carried out for the amplitude $\hat{y}$ from the actual movement of the wheelset.

NOTE For practicality, a typical range for the evaluation of the conicity for changes in $\hat{y}$ is 1 mm to 8 mm. An example of graphic representation of this exercise is given in Annex A. The maximum value 8 mm might never be reached if the $\Delta r = f(y)$ characteristic is either highly nonlinear or exhibits substantial jumps.

6 Benchmark calculation

6.1 Overview

It is necessary to validate the method to be used by means of a benchmark calculation with defined test profiles for the whole evaluation system for the equivalent conicity because the following are not defined when using this European Standard in practice:

- sample rate of the profile measurement;
- smoothing procedure;
- integration method for the evaluation of the wavelength.

Both the method of determination of the equivalent conicity and the smoothing procedure for profile data shall be validated by a benchmark calculation with defined reference-profiles combined with random- and grid-errors. This is achieved by a three step process:

- step 1 checks the interpolation and calculation algorithm;
- when Step 1 has been successfully passed, Step 2 applies defined errors to the data of Step 1;
- when Step 2 has been successfully passed Step 3 then checks the whole candidate system.

6.2 Validation of evaluation method

For all steps the calculations shall be done for all combinations of the reference profiles given in Annex E including the influence of asymmetrical Δr -functions produced by different wheel diameters or different wheel profiles on both wheels as given from E.5 to E.9.

If the equivalent conicity is considered at certain amplitude when applying EN 14363 and at this amplitude a rapid change in conicity occurs, further measurements and/or calculations in order to determine the sensitivities and trends of the rapid change shall be carried out. The further measurements will then be included in the assessment of the candidate technique consisting of measuring and calculation algorithm.

NOTE 1 The reference profiles given in Annex D are only defined for benchmark calculations to test the algorithm in a wide range of conditions; they have no practical meaning for a design of a real profile.

Step 1

In Step 1, the interpolation and calculation algorithm to be tested shall be applied to the reference profiles in Annex D. The achieved results shall be compared with the reference results in Annex E and the defined field of allowed tolerances in Annex F. The achieved results shall be within the tolerance field given in Annex F in order for the calculation algorithm to successfully pass Step 1 (see Figure 1).

Step 2

The purpose of Step 2 is to assess the smoothing procedure. It repeats the calculations of Step 1. In Step 2, the smoothing, interpolation and calculation algorithm shall be applied to the reference profiles in Annex D with all errors given in Annex G superimposed.

The effect of a random error and a grid shall be proven by separate calculations for the cases by which an error occurs only at the wheel, only at the rail and at both wheel and rail.

The effects shall be quantified in z -, y - and $(z + y)$ -directions depending on the measuring system. For non Cartesian measuring systems appropriate coordinates should be used.

For each case a sufficient number of calculations (≥ 20) shall be done for different randomized errors related to the grid origin position and random limit (see Annex H).

The assessment of the evaluation procedure (see Figure 2) shall be performed by a comparison of the achieved results with the reference results including the field of tolerances in Annex F.

NOTE 2 The illustrations given in Annex G represent results of typical calculations. These results are considered acceptable.

Step 3

Step 3 checks that the whole system and process of the candidate technique will generate results that are sufficiently accurate. It repeats the calculations of Step 2 using the tolerances of the measuring and digitizing system to be assessed instead of the prescribed errors in Step 2.

NOTE 3 The illustrations given in Annex G represent results of typical calculations. These results are considered acceptable.

Annex A (informative)

Example of presentation of Δr function and conicity

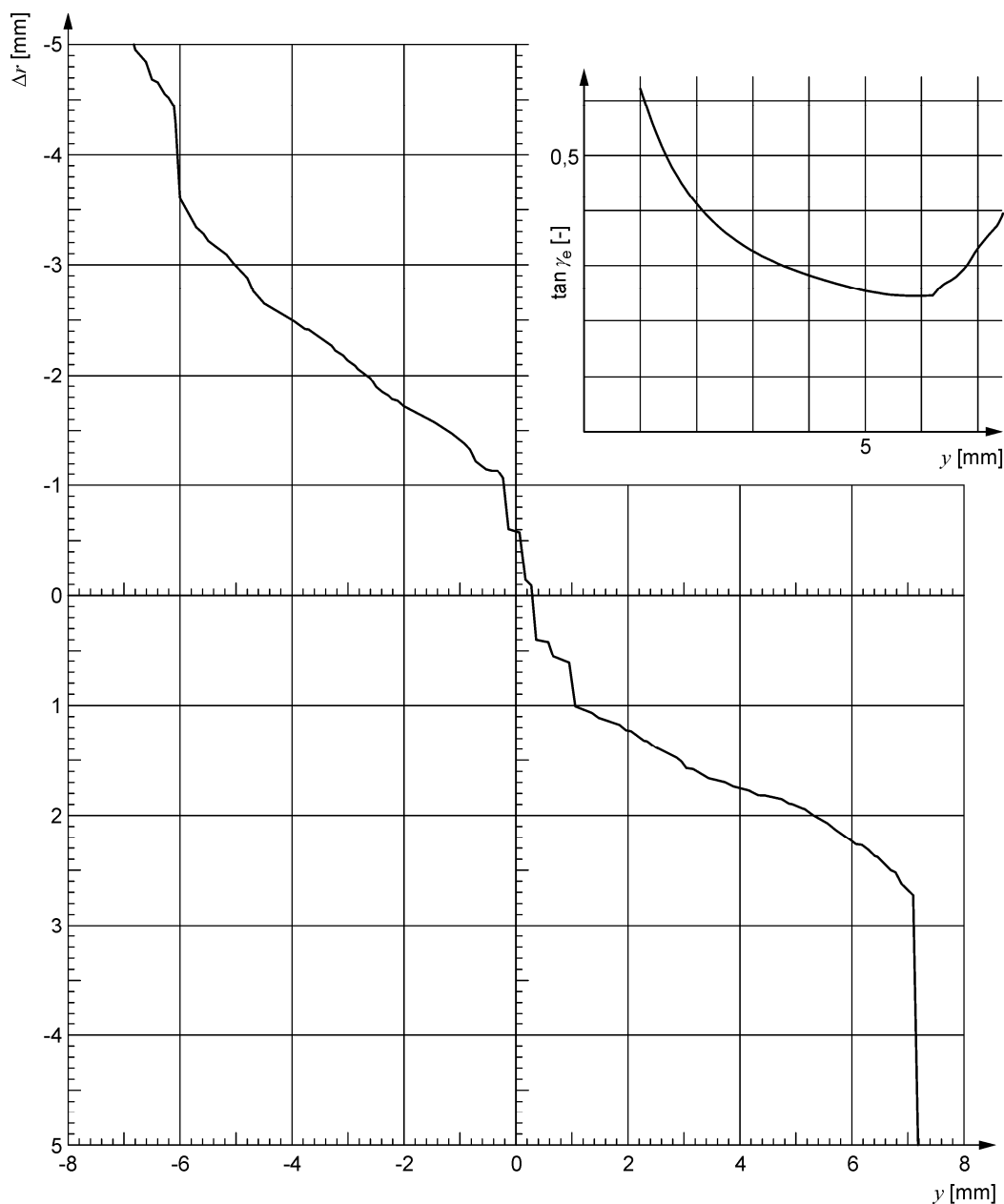


Figure A.1 — $\Delta r = f(y)$ function and $\tan \gamma_e = f(y)$

Annex B (informative)

Example of method for determining the equivalent conicity by integration of the nonlinear differential equation

B.1 Principle

The movement of the wheelset's centre of gravity on the track can be formalised on the basis of the angle:

$$\Psi = \frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} \quad (\text{B.1})$$

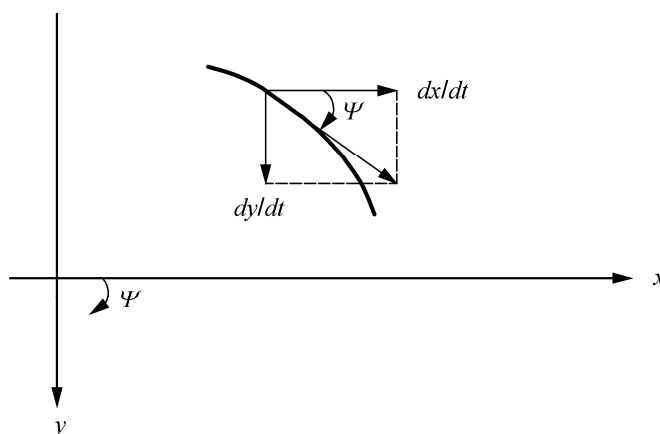


Figure B.1 — Representation of dx , dy

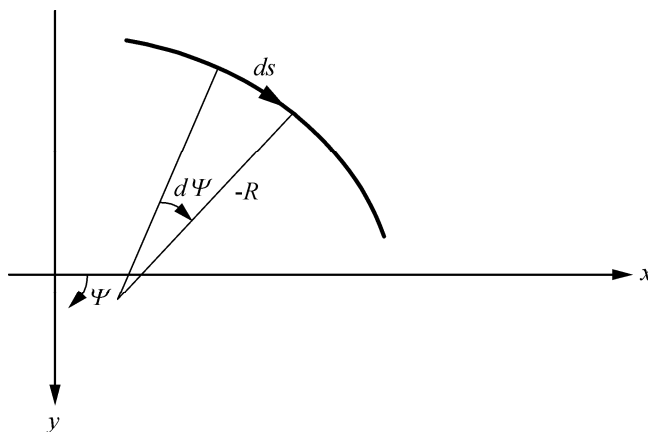


Figure B.2 — Representation of ds , $d\Psi$

$$ds = -R d\Psi$$

for a small angle Ψ , $ds \cong dx$

$$dx = -R d\Psi$$

(B.2)

and, taking into account Equation (B.1), this yields:

$$\Psi = \frac{dy}{dx} = - \frac{dy}{R d\Psi}$$

hence

$$\Psi d\Psi = - \frac{dy}{R} \quad (\text{B.3})$$

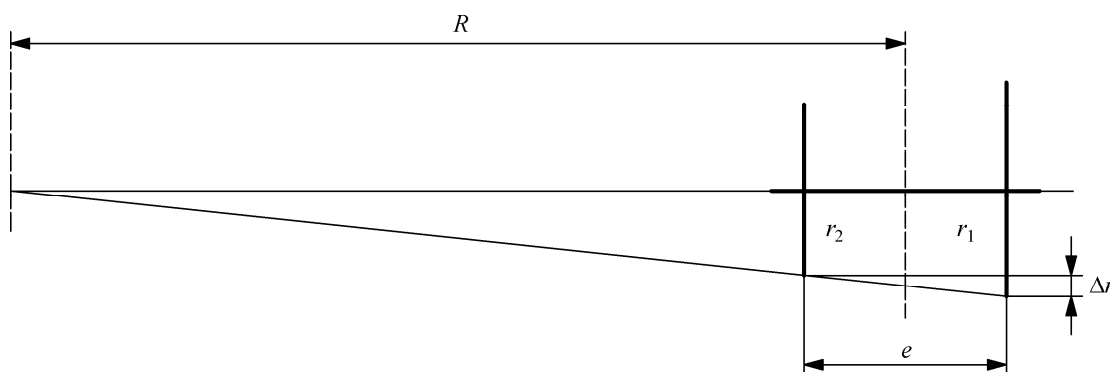


Figure B.3 — Representation of r_1 , r_2 , e

$$\frac{R}{e} = \frac{\frac{r_1 + r_2}{2}}{r_1 - r_2}$$

where

$$\frac{r_1 + r_2}{2} = r_0 \quad \text{is the nominal radius of each wheel}$$

$$r_1 - r_2 = \Delta r$$

e is the distance between the left and right contact points (approximately 1 500 mm for standard gauge).

Thus:

$$R = \frac{er_0}{\Delta r} \quad (\text{B.4})$$

and, by replacing R in (B.3), we get:

$$\Psi d\Psi = \frac{-\Delta r dy}{er_0} \quad (\text{B.5})$$

giving, by integration:

$$\frac{\Psi^2}{2} = -\frac{1}{er_0} \left[\int \Delta r dy + C \right] \quad (\text{B.6})$$

This integration serves to obtain the movement of the wheelset's centre of gravity on the track and in particular the path corresponding to half of the wavelength starting from y_{emin} (for which $\Psi_{\text{emin}} = 0$) and going up to y_{emax} (for which $\Psi_{\text{emax}} = 0$).

The integral $\int \Delta r dy$ needs only to be calculated once for sufficiently large amplitudes to be able to cover the domain that will be needed for the remainder of the determination (Figure B.6).

The wheelset's movement on the track is then obtained with the help of the following integration:

$$dx = \frac{1}{\Psi} dy \text{ gives } x = \int \frac{dy}{\Psi}$$

NOTE Equation (B.5) is equivalent to the differential equation $\frac{d^2 y}{dx^2} + \frac{\Delta r}{er_0} = 0$

As

$$\Psi = \frac{dy}{dx}$$

$$\frac{d\Psi}{dx} = \frac{d^2 y}{dx^2}$$

The differential equation becomes:

$$\frac{d\Psi}{dx} + \frac{\Delta r}{er_0} = 0$$

$$\text{where } dx = \frac{dy}{\Psi}$$

hence

$$\frac{\Psi}{dy} d\Psi + \frac{\Delta r}{er_0} = 0$$

such that

$$\Psi d\Psi = -\frac{\Delta r dy}{er_0}$$

B.2 Steps of the procedure

In practice this method is applied in the following way:

- Based on the function $\Delta r = r_1 - r_2 = f(y)$, find the value y_{em} that corresponds to $\Delta r = 0$ (Figure B.5).
- Calculate the $S(y) = - \int \Delta r \cdot dy$ function, starting from y_{em} in steps of $dy = + 0,1$ mm to $+ y$ and from y_{em} in steps of $dy = - 0,1$ mm to $- y$ (lower diagram in Figure B.6).
- Determine the corresponding amplitudes y_{emin} and y_{emax} and calculate the mean lateral movement $\hat{y}$ (Figure B.6).
- Find the functions $y_{emin} = f(\hat{y})$ and $y_{emax} = f(\hat{y})$ which allow to determine, for a given lateral movement of the wheelset $2 \hat{y}$, the corresponding minimum and maximum amplitudes y_{emin} and y_{emax} (Figure B.7).
- Compute the equivalent conicity $\tan \gamma_e$ for a given movement $\hat{y}$:
 - find the constant C of Equation (6) (Figure B.8) such that $\psi_{emin} = 0$ for the corresponding y_{emin}
 - calculate the angle ψ by integrating Equation (5) to give:

$$\psi = \sqrt{\frac{-2}{er_0} \left[\int \Delta r dy + C \right]}$$

in steps of $dy = 0,1$ mm

- calculate the abscissa of the wheelset movement:

$$x = f(y) = \int \frac{dy}{\Psi} \text{ between } y_{emin} \text{ and } y_{emax}$$

which allows to find the wavelength λ of the wheelset's kinematic motion.

In most cases the integration cannot be done in only one step in the range from y_{emin} to y_{emax} . Therefore the wavelength x shall be calculated by adding up $dx = \frac{dy}{\Psi}$, whereby the step of dy should be $\leq 0,1$ mm.

- calculate the equivalent conicity, applying the Klingel formula

$$\tan \gamma_e = \left(\frac{\pi}{\lambda} \right)^2 2er_0$$

- Determine the function $\tan \gamma_e = f(\hat{y})$ by applying Step B.2 e) for $\hat{y}$ amplitudes starting at 1 mm up to the maximum permitted by the $\Delta r = f(y)$ characteristic, with a maximum step $\Delta \hat{y} = 0,5$ mm.

B.3 Special cases

— $\Delta r = f(y)$ characteristic having a negative slope

When the $\Delta r = f(y)$ characteristic has a negative slope, it is necessary to select for y_{em} the most suitable point.

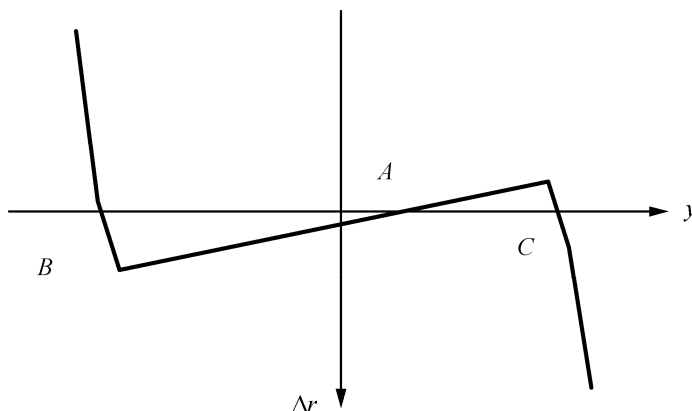


Figure B.4 — $\Delta r = f(y)$ characteristic with negative slope

In the example illustrated in the Figure, instead of choosing Point A for y_{em} choose Point B or Point C according to the initial position of the wheelset.

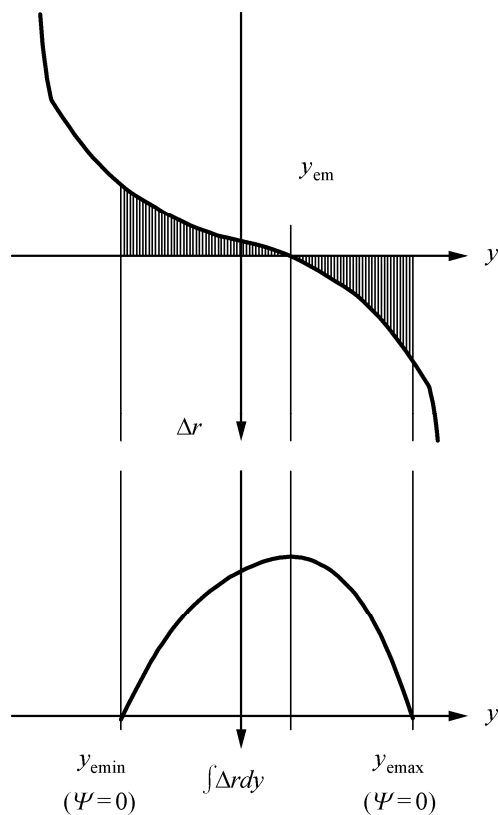


Figure B.5 — Calculation of $\int \Delta r dy$ integral

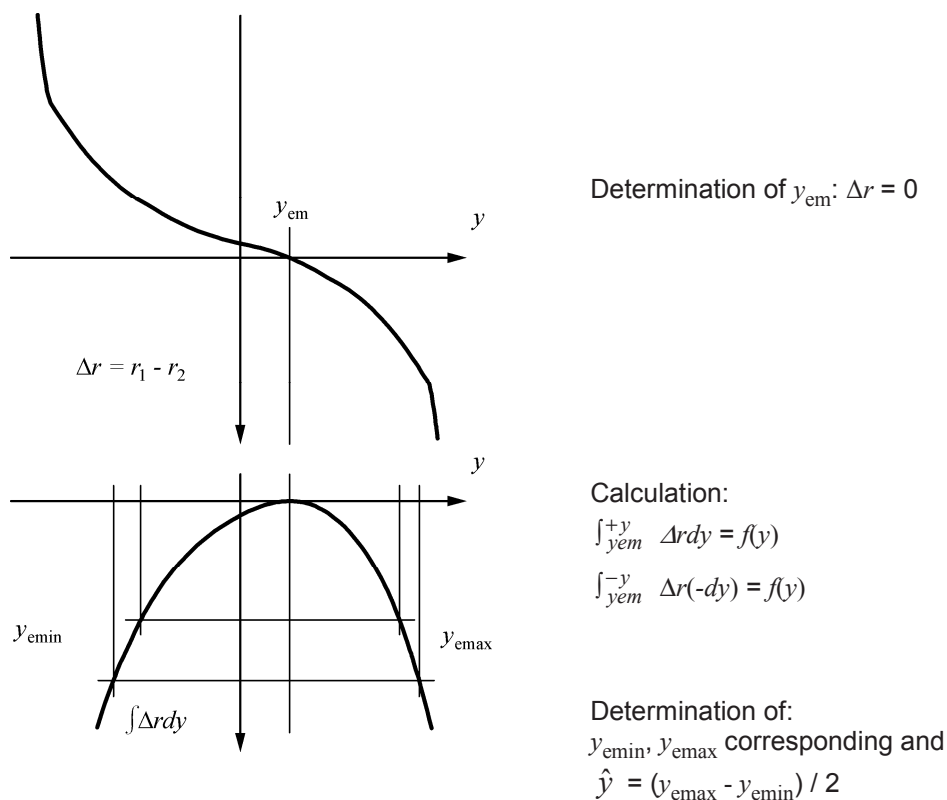


Figure B.6 — Determination of y_{em} , calculation of $\int \Delta r dy$ and determination of $\hat{y}$

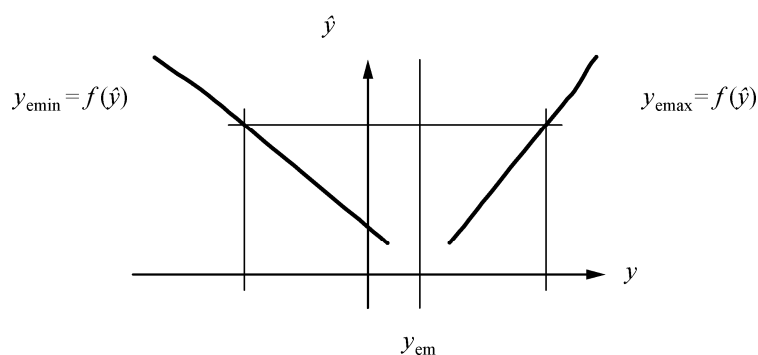


Figure B.7 — Determination of $y_{emmin} = f(\hat{y})$ and $y_{emmax} = f(\hat{y})$ functions

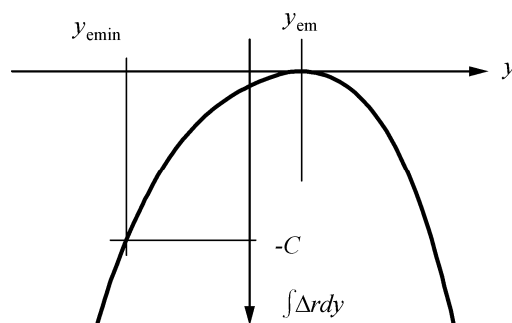


Figure B.8 — Determination of C constant

Annex C (informative)

Example of method for determining the equivalent conicity by linear regression of the Δr function

C.1 Principles

For a linear Δr function the slope of the Δr function is equal to $2\tan\gamma_e$.

For a non linear Δr function the slope of a linear regression gives an approximation of $2\tan\gamma_e$.

For an interval of $2\hat{y}$ the regression of the Δr function shall be done between y_{emin} and y_{emax} , where y_{emin} and y_{emax} are the minimum and maximum amplitude of the real shape of the wave.

For symmetrical Δr functions y_{emin} and y_{emax} are equal to the mean amplitude $\hat{y}$, but for non-symmetrical Δr functions y_{emin} and y_{emax} normally are unequal to $\hat{y}$.

For a mean amplitude $\hat{y}$ the min and max amplitudes y_{emin} and y_{emax} can be calculated by the Δr function as described in Annex B.

C.2 Steps of the procedure

In practice this method is applied in the following way:

- Based on the function $\Delta r = r_1 - r_2 = f(y)$, find the value y_{em} that corresponds to $\Delta r = 0$ (Figure B.5).
- Calculate the $S(y) = -\int \Delta r dy$ function, starting from y_{em} in steps of $dy = +0,1$ mm to $+y$ and from y_{em} in steps of $dy = -0,1$ mm to $-y$ (lower diagram in Figure B.6).
- Determine the corresponding amplitudes y_{emin} and y_{emax} and calculate the mean lateral movement $\hat{y}$ (Figure B.6).
- Find the functions $y_{emin} = f(\hat{y})$ and $y_{emax} = f(\hat{y})$ which allow to determine, for a given lateral movement of the wheelset $2\hat{y}$, the corresponding minimum and maximum amplitudes y_{emin} and y_{emax} (Figure B.7).
- Calculate the linear regression of the Δr function in the range between y_{emin} and y_{emax} and calculate the equivalent conicity by $\tan\gamma_e = \frac{B}{2}$ where B = slope of the regression.
- Determine the function $\tan\gamma_e = f(\hat{y})$ by applying Step C.2.e). For $\hat{y}$ amplitudes starting at 1 mm up to the maximum permitted by the $\Delta r = f(y)$ characteristic, with a maximum step $\Delta\hat{y} = 0,5$ mm.

C.3 Particularities

If the $\Delta r = f(y)$ characteristic presents a substantial asymmetry about the origin over the $2\hat{y}$ interval, which can be likened to two straight line segments, it is recommended to apply a regression to each of the parts (positive and negative) to obtain two conicities $\tan\gamma_{e,p}$ and $\tan\gamma_{e,n}$. The equivalent conicity can then be obtained by:

$$\tan\gamma_e = \left[\frac{2}{\frac{1}{\sqrt{\tan\gamma_{e,p}}} + \frac{1}{\sqrt{\tan\gamma_{e,n}}}} \right]^2$$

Annex D (normative)

Reference profiles

D.1 Wheel A

D.1.1 Drawing

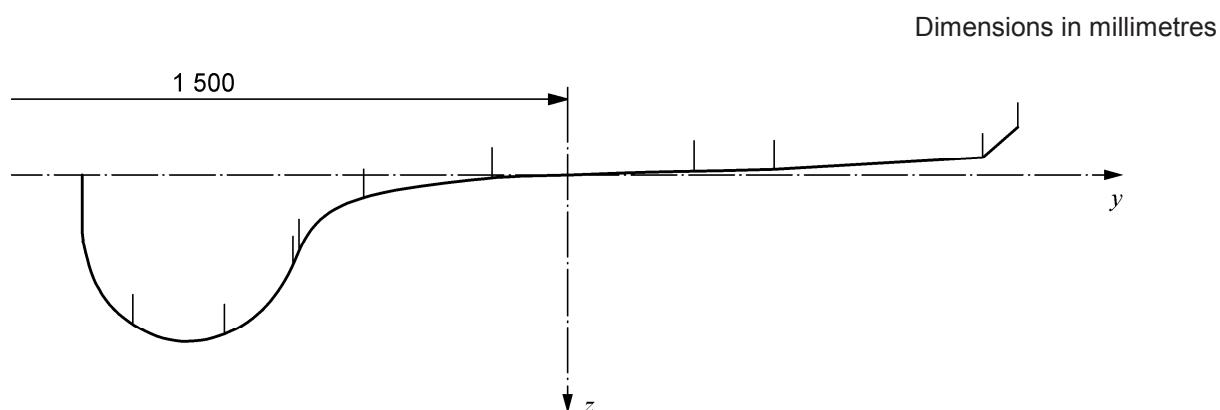


Figure D.1 — Wheel A

D.1.2 Analytic definition

z coordinates of the profile are defined in the following ranges:

From $y = -70$	to $y = -62,764\ 7$	: $z = +9,519\ 3 + (20,5^2 - (y + 49,5)^2)^{0,5}$
From $y = -62,764\ 7$	to $y = -49,662\ 5$	: $z = +16 + (12^2 - (y + 55)^2)^{0,5}$
From $y = -49,662\ 5$	to $y = -39,764\ 5$	: $z = +8,834\ 9 + (20^2 - (y + 58,558\ 3)^2)^{0,5}$
From $y = -39,764\ 5$	to $y = -38,737\ 2$	: $z = -93,576\ 7 - 2,747\ 5 \times y$
From $y = -38,737\ 2$	to $y = -29,459\ 1$	: $z = +17,642\ 1 - (14^2 - (y + 25,581\ 6)^2)^{0,5}$
From $y = -29,459\ 1$	to $y = -10,874\ 9$	: $z = +97,16492 - (96,760\ 5^2 - (y + 2,659\ 4)^2)^{0,5}$
From $y = -10,874\ 9$	to $y = +18,417\ 3$	: $z = +344,508\ 1 - (345^2 - (y - 18,417\ 3)^2)^{0,5}$
From $y = +18,417\ 3$	to $y = +30$	: $z = -0,4919 + 0 \times y$
From $y = +30$	to $y = +60$	: $z = +1,509\ 1 - 0,066\ 7 \times y$
From $y = +60$	to $y = +65$	: $z = +57,507\ 1 - 1 \times y$

D.1.3 Cartesian coordinates

Table D.1 — Wheel profile: R-UIC 519-A — Right wheel

Dimensions in millimetres

y	z	y	z	y	z
-70,000	9,519	-20,000	1,971	30,000	-0,492
-69,000	15,844	-19,000	1,794	31,000	-0,559
-68,000	18,351	-18,000	1,628	32,000	-0,625
-67,000	20,196	-17,000	1,473	33,000	-0,692
-66,000	21,685	-16,000	1,328	34,000	-0,759
-65,000	22,936	-15,000	1,195	35,000	-0,825
-64,000	24,011	-14,000	1,071	36,000	-0,892
-63,000	24,947	-13,000	0,959	37,000	-0,959
-62,000	25,747	-12,000	0,856	38,000	-1,026
-61,000	26,392	-11,000	0,765	39,000	-1,092
-60,000	26,909	-10,000	0,680	40,000	-1,159
-59,000	27,314	-9,000	0,599	41,000	-1,226
-58,000	27,619	-8,000	0,521	42,000	-1,292
-57,000	27,832	-7,000	0,446	43,000	-1,359
-56,000	27,958	-6,000	0,373	44,000	-1,426
-55,000	28,000	-5,000	0,304	45,000	-1,492
-54,000	27,958	-4,000	0,237	46,000	-1,559
-53,000	27,832	-3,000	0,174	47,000	-1,626
-52,000	27,619	-2,000	0,113	48,000	-1,693
-51,000	27,314	-1,000	0,055	49,000	-1,759
-50,000	26,909	0,000	0,000	50,000	-1,826
-49,000	26,403	1,000	-0,052	51,000	-1,893
-48,000	25,821	2,000	-0,101	52,000	-1,959
-47,000	25,157	3,000	-0,147	53,000	-2,026
-46,000	24,401	4,000	-0,191	54,000	-2,093
-45,000	23,538	5,000	-0,231	55,000	-2,159
-44,000	22,548	6,000	-0,268	56,000	-2,226
-43,000	21,402	7,000	-0,303	57,000	-2,293
-42,000	20,052	8,000	-0,335	58,000	-2,360
-41,000	18,411	9,000	-0,363	59,000	-2,426
-40,000	16,291	10,000	-0,389	60,000	-2,493
-39,000	13,576	11,000	-0,412	61,000	-3,493
-38,000	11,178	12,000	-0,432	62,000	-4,493
-37,000	9,541	13,000	-0,449	63,000	-5,493
-36,000	8,290	14,000	-0,464	64,000	-6,493
-35,000	7,284	15,000	-0,475	65,000	-7,493
-34,000	6,456	16,000	-0,483		
-33,000	5,769	17,000	-0,489		
-32,000	5,200	18,000	-0,492		
-31,000	4,733	19,000	-0,492		
-30,000	4,358	20,000	-0,492		
-29,000	4,059	21,000	-0,492		
-28,000	3,782	22,000	-0,492		
-27,000	3,516	23,000	-0,492		
-26,000	3,262	24,000	-0,492		
-25,000	3,019	25,000	-0,492		
-24,000	2,787	26,000	-0,492		
-23,000	2,567	27,000	-0,492		
-22,000	2,357	28,000	-0,492		
-21,000	2,159	29,000	-0,492		

D.2 Wheel B

D.2.1 Drawing

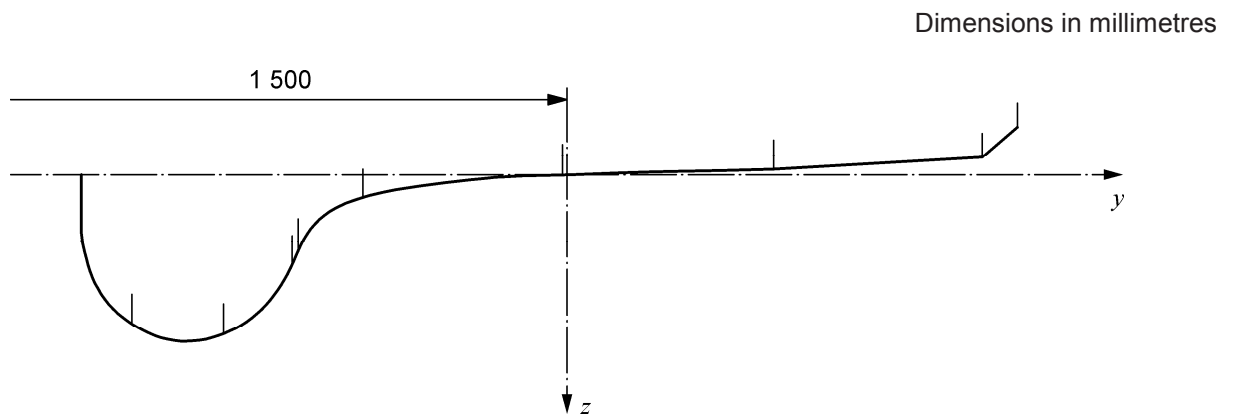


Figure D.2 —Wheel B

D.2.2 Analytic definition

z coordinates of the profile are defined in the following ranges:

From $y = -70$	to $y = -62,764\ 7$	: $z = +9,519\ 3 + (20,5^2 - (y + 49,5)^2)^{0,5}$
From $y = -62,764\ 7$	to $y = -49,662\ 5$	: $z = +16 + (12^2 - (y + 55)^2)^{0,5}$
From $y = -49,662\ 5$	to $y = -39,764\ 5$	: $z = +8,834\ 9 + (20^2 - (y + 58,558\ 3)^2)^{0,5}$
From $y = -39,764\ 5$	to $y = -38,787\ 9$	: $z = -93,576\ 7 - 2,747\ 5 \times y$
From $y = -38,787\ 9$	to $y = -29,350\ 5$	: $z = +17,781\ 4 - (14^2 - (y + 25,632\ 2)^2)^{0,5}$
From $y = -29,350\ 5$	to $y = -0,478\ 7$	: $z = +119,974\ 5 - (120^2 - (y - 2,520\ 4)^2)^{0,5}$
From $y = -0,478\ 7$	to $y = +30$	: $z = +0 - 0,025 \times y$
From $y = +30$	to $y = +60$	: $z = +0,75 - 0,05 \times y$
From $y = +60$	to $y = +65$	: $z = +57,75 - 1 \times y$

D.2.3 Cartesian coordinates

Table D.2 — Wheel profile: R-UIC 519-B — Right wheel

Dimensions in millimetres

<i>y</i>	<i>z</i>	<i>y</i>	<i>z</i>	<i>y</i>	<i>z</i>
-70,000	9,519	-20,000	2,107	30,000	-0,750
-69,000	15,844	-19,000	1,920	31,000	-0,800
-68,000	18,351	-18,000	1,742	32,000	-0,850
-67,000	20,196	-17,000	1,573	33,000	-0,900
-66,000	21,685	-16,000	1,412	34,000	-0,950
-65,000	22,936	-15,000	1,260	35,000	-1,000
-64,000	24,011	-14,000	1,117	36,000	-1,050
-63,000	24,947	-13,000	0,982	37,000	-1,100
-62,000	25,747	-12,000	0,856	38,000	-1,150
-61,000	26,392	-11,000	0,739	39,000	-1,200
-60,000	26,909	-10,000	0,629	40,000	-1,250
-59,000	27,314	-9,000	0,529	41,000	-1,300
-58,000	27,619	-8,000	0,437	42,000	-1,350
-57,000	27,832	-7,000	0,353	43,000	-1,400
-56,000	27,958	-6,000	0,277	44,000	-1,450
-55,000	28,000	-5,000	0,210	45,000	-1,500
-54,000	27,958	-4,000	0,152	46,000	-1,550
-53,000	27,832	-3,000	0,102	47,000	-1,600
-52,000	27,619	-2,000	0,060	48,000	-1,650
-51,000	27,314	-1,000	0,026	49,000	-1,700
-50,000	26,909	0,000	0,000	50,000	-1,750
-49,000	26,403	1,000	-0,025	51,000	-1,800
-48,000	25,821	2,000	-0,050	52,000	-1,850
-47,000	25,157	3,000	-0,075	53,000	-1,900
-46,000	24,401	4,000	-0,100	54,000	-1,950
-45,000	23,538	5,000	-0,125	55,000	-2,000
-44,000	22,548	6,000	-0,150	56,000	-2,050
-43,000	21,402	7,000	-0,175	57,000	-2,100
-42,000	20,052	8,000	-0,200	58,000	-2,150
-41,000	18,411	9,000	-0,225	59,000	-2,200
-40,000	16,291	10,000	-0,250	60,000	-2,250
-39,000	13,576	11,000	-0,275	61,000	-3,250
-38,000	11,221	12,000	-0,300	62,000	-4,250
-37,000	9,610	13,000	-0,325	63,000	-5,250
-36,000	8,373	14,000	-0,350	64,000	-6,250
-35,000	7,377	15,000	-0,375	65,000	-7,250
-34,000	6,557	16,000	-0,400		
-33,000	5,877	17,000	-0,425		
-32,000	5,313	18,000	-0,450		
-31,000	4,851	19,000	-0,475		
-30,000	4,480	20,000	-0,500		
-29,000	4,188	21,000	-0,525		
-28,000	3,921	22,000	-0,550		
-27,000	3,662	23,000	-0,575		
-26,000	3,413	24,000	-0,600		
-25,000	3,173	25,000	-0,625		
-24,000	2,942	26,000	-0,650		
-23,000	2,720	27,000	-0,675		
-22,000	2,506	28,000	-0,700		
-21,000	2,302	29,000	-0,725		

D.3 Wheel H

D.3.1 Drawing

Dimensions in millimetres

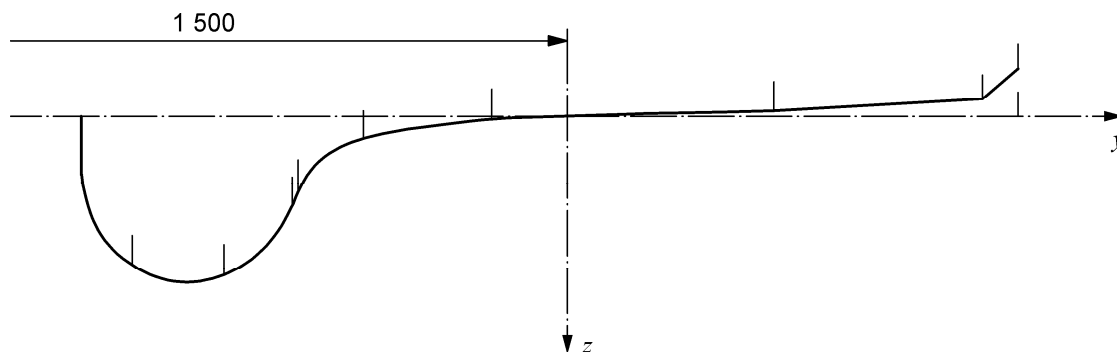


Figure D.3 — Wheel H

D.3.2 Analytic definition

z coordinates of the profile are defined in the following ranges:

From $y = -70$	to $y = -62,764\ 7$	: $z = +9,519\ 3 + (20,5^2 - (y + 49,5)^2)^{0,5}$
From $y = -62,764\ 7$	to $y = -49,662\ 5$	: $z = +16 + (12^2 - (y + 55)^2)^{0,5}$
From $y = -49,662\ 5$	to $y = -39,764\ 5$	: $z = +8,834\ 9 + (20^2 - (y + 58,558\ 3)^2)^{0,5}$
From $y = -39,764\ 5$	to $y = -38,722$	: $z = -93,576\ 7 - 2,747\ 5 \times y$
From $y = -38,722$	to $y = -28,311\ 9$	: $z = +17,600\ 3 - (14^2 - (y + 25,566\ 3)^2)^{0,5}$
From $y = -28,311\ 9$	to $y = -10,229\ 7$	: $z = -1,790\ 2 - 0,2 \times y$
From $y = -10,229\ 7$	to $y = +30$	: $z = +0 - 0,025 \times y$
From $y = +30$	to $y = +60$	: $z = +1,251 - 0,066\ 7 \times y$
From $y = +60$	to $y = +65$	: $z = +57,249 - 1 \times y$

D.3.3 Cartesian coordinates

Table D.3 — Wheel profile: R-UIC 519-H — Right wheel

Dimensions in millimetres

<i>y</i>	<i>z</i>	<i>y</i>	<i>z</i>	<i>y</i>	<i>z</i>
-70,000	9,519	-20,000	2,210	30,000	-0,750
-69,000	15,844	-19,000	2,010	31,000	-0,817
-68,000	18,351	-18,000	1,810	32,000	-0,883
-67,000	20,196	-17,000	1,610	33,000	-0,950
-66,000	21,685	-16,000	1,410	34,000	-1,017
-65,000	22,936	-15,000	1,210	35,000	-1,084
-64,000	24,011	-14,000	1,010	36,000	-1,150
-63,000	24,947	-13,000	0,810	37,000	-1,217
-62,000	25,747	-12,000	0,610	38,000	-1,284
-61,000	26,392	-11,000	0,410	39,000	-1,350
-60,000	26,909	-10,000	0,250	40,000	-1,417
-59,000	27,314	-9,000	0,225	41,000	-1,484
-58,000	27,619	-8,000	0,200	42,000	-1,550
-57,000	27,832	-7,000	0,175	43,000	-1,617
-56,000	27,958	-6,000	0,150	44,000	-1,684
-55,000	28,000	-5,000	0,125	45,000	-1,751
-54,000	27,958	-4,000	0,100	46,000	-1,817
-53,000	27,832	-3,000	0,075	47,000	-1,884
-52,000	27,619	-2,000	0,050	48,000	-1,951
-51,000	27,314	-1,000	0,025	49,000	-2,017
-50,000	26,909	0,000	0,000	50,000	-2,084
-49,000	26,403	1,000	-0,025	51,000	-2,151
-48,000	25,821	2,000	-0,050	52,000	-2,217
-47,000	25,157	3,000	-0,075	53,000	-2,284
-46,000	24,401	4,000	-0,100	54,000	-2,351
-45,000	23,538	5,000	-0,125	55,000	-2,418
-44,000	22,548	6,000	-0,150	56,000	-2,484
-43,000	21,402	7,000	-0,175	57,000	-2,551
-42,000	20,052	8,000	-0,200	58,000	-2,618
-41,000	18,411	9,000	-0,225	59,000	-2,684
-40,000	16,291	10,000	-0,250	60,000	-2,751
-39,000	13,576	11,000	-0,275	61,000	-3,751
-38,000	11,166	12,000	-0,300	62,000	-4,751
-37,000	9,521	13,000	-0,325	63,000	-5,751
-36,000	8,266	14,000	-0,350	64,000	-6,751
-35,000	7,256	15,000	-0,375	65,000	-7,751
-34,000	6,426	16,000	-0,400		
-33,000	5,737	17,000	-0,425		
-32,000	5,166	18,000	-0,450		
-31,000	4,698	19,000	-0,475		
-30,000	4,321	20,000	-0,500		
-29,000	4,028	21,000	-0,525		
-28,000	3,810	22,000	-0,550		
-27,000	3,610	23,000	-0,575		
-26,000	3,410	24,000	-0,600		
-25,000	3,210	25,000	-0,625		
-24,000	3,010	26,000	-0,650		
-23,000	2,810	27,000	-0,675		
-22,000	2,610	28,000	-0,700		
-21,000	2,410	29,000	-0,725		

D.4 Wheel I

D.4.1 Drawing

Dimensions in millimetres

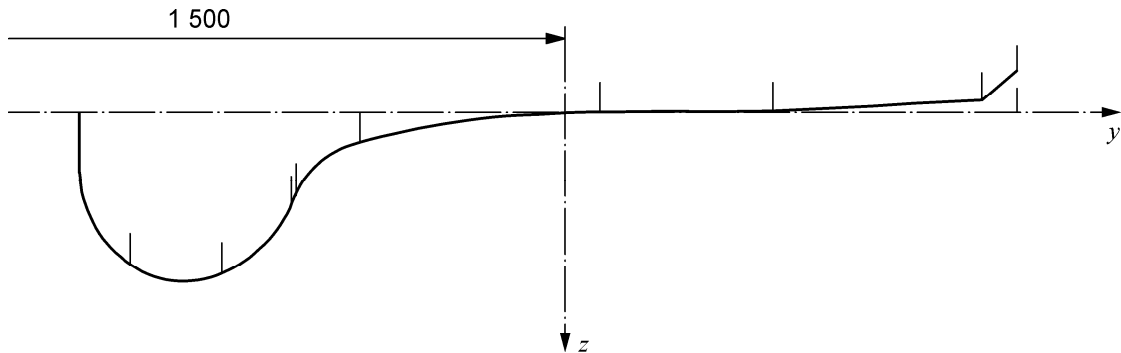


Figure D.4 — Wheel I

D.4.2 Analytic definition

z coordinates of the profile are defined in the following ranges:

From $y = -70$	to $y = -62,764\ 7$	: $z = +9,519\ 3 + (20,5^2 - (y + 49,5)^2)^{0,5}$
From $y = -62,764\ 7$	to $y = -49,662\ 5$	: $z = +16 + (12^2 - (y + 55)^2)^{0,5}$
From $y = -49,662\ 5$	to $y = -39,764\ 5$	: $z = +8,834\ 9 + (20^2 - (y + 58,558\ 3)^2)^{0,5}$
From $y = -39,764\ 5$	to $y = -39,048\ 6$	: $z = -93,576\ 7 - 2,747\ 5 \cdot y$
From $y = -39,048\ 6$	to $y = -29,923\ 1$	: $z = +18,497\ 4 - (14^2 - (y + 25,892\ 9)^2)^{0,5}$
From $y = -29,923\ 1$	to $y = +5$	: $z = +119,895\ 8 - (120^2 - (y - 5)^2)^{0,5}$
From $y = +5$	to $y = +30$	: $z = -0,104\ 2 + 0 \times y$
From $y = +30$	to $y = +60$	: $z = +1,896\ 8 - 0,066\ 7 \times y$
From $y = +60$	to $y = +65$	: $z = +57,894\ 8 - 1 \times y$

D.4.3 Cartesian coordinates

Table D.4 — Wheel profile: R-UIC 519-I — Right wheel

Dimensions in millimetres

<i>y</i>	<i>z</i>	<i>y</i>	<i>z</i>	<i>y</i>	<i>z</i>
-70,000	9,519	-20,000	2,529	30,000	-0,104
-69,000	15,844	-19,000	2,320	31,000	-0,171
-68,000	18,351	-18,000	2,121	32,000	-0,238
-67,000	20,196	-17,000	1,930	33,000	-0,304
-66,000	21,685	-16,000	1,748	34,000	-0,371
-65,000	22,936	-15,000	1,574	35,000	-0,438
-64,000	24,011	-14,000	1,410	36,000	-0,504
-63,000	24,947	-13,000	1,253	37,000	-0,571
-62,000	25,747	-12,000	1,106	38,000	-0,638
-61,000	26,392	-11,000	0,967	39,000	-0,705
-60,000	26,909	-10,000	0,837	40,000	-0,771
-59,000	27,314	-9,000	0,715	41,000	-0,838
-58,000	27,619	-8,000	0,602	42,000	-0,905
-57,000	27,832	-7,000	0,497	43,000	-0,971
-56,000	27,958	-6,000	0,401	44,000	-1,038
-55,000	28,000	-5,000	0,313	45,000	-1,105
-54,000	27,958	-4,000	0,234	46,000	-1,171
-53,000	27,832	-3,000	0,163	47,000	-1,238
-52,000	27,619	-2,000	0,100	48,000	-1,305
-51,000	27,314	-1,000	0,046	49,000	-1,372
-50,000	26,909	0,000	0,000	50,000	-1,438
-49,000	26,403	1,000	-0,038	51,000	-1,505
-48,000	25,821	2,000	-0,067	52,000	-1,572
-47,000	25,157	3,000	-0,088	53,000	-1,638
-46,000	24,401	4,000	-0,100	54,000	-1,705
-45,000	23,538	5,000	-0,104	55,000	-1,772
-44,000	22,548	6,000	-0,104	56,000	-1,838
-43,000	21,402	7,000	-0,104	57,000	-1,905
-42,000	20,052	8,000	-0,104	58,000	-1,972
-41,000	18,411	9,000	-0,104	59,000	-2,039
-40,000	16,291	10,000	-0,104	60,000	-2,105
-39,000	13,578	11,000	-0,104	61,000	-3,105
-38,000	11,468	12,000	-0,104	62,000	-4,105
-37,000	9,975	13,000	-0,104	63,000	-5,105
-36,000	8,810	14,000	-0,104	64,000	-6,105
-35,000	7,864	15,000	-0,104	65,000	-7,105
-34,000	7,084	16,000	-0,104		
-33,000	6,436	17,000	-0,104		
-32,000	5,900	18,000	-0,104		
-31,000	5,462	19,000	-0,104		
-30,000	5,113	20,000	-0,104		
-29,000	4,813	21,000	-0,104		
-28,000	4,522	22,000	-0,104		
-27,000	4,241	23,000	-0,104		
-26,000	3,969	24,000	-0,104		
-25,000	3,706	25,000	-0,104		
-24,000	3,453	26,000	-0,104		
-23,000	3,208	27,000	-0,104		
-22,000	2,973	28,000	-0,104		
-21,000	2,746	29,000	-0,104		

D.5 Rail A

D.5.1 Drawing

Dimensions in millimetres

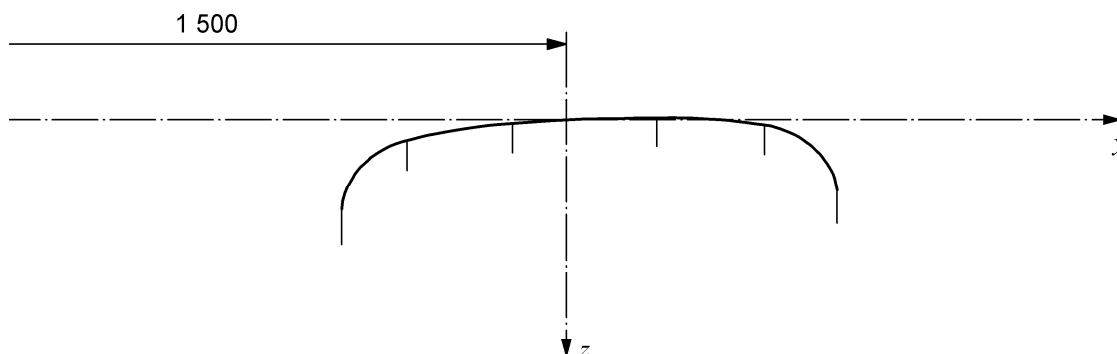


Figure D.5 — Rail A

D.5.2 Analytic definition

z coordinates of the profile are defined in the following ranges:

From $y = -32,639\ 5$	to $y = -23,240\ 1$	: $z = 15,899\ 0 - (13^2 - (y + 19,639\ 5)^2)^{0,5}$
From $y = -23,240\ 1$	to $y = -7,874\ 9$	: $z = 80,277\ 9 - (80^2 - (y + 1,082\ 5)^2)^{0,5}$
From $y = -7,874\ 9$	to $y = +13,098\ 9$	: $z = 299,483\ 5 - (300^2 - (y - 17,596\ 6)^2)^{0,5}$
From $y = +13,098\ 9$	to $y = +28,670\ 9$	: $z = 79,508\ 2 - (80^2 - (y - 14,298\ 2)^2)^{0,5}$
From $y = +28,670\ 9$	to $y = +39,270\ 5$	: $z = 13,598\ 4 - (13^2 - (y - 26,335\ 3)^2)^{0,5}$