



BSI Standards Publication

Railway applications — Wheel-rail contact geometry parameters — Definitions and methods for evaluation

National foreword

This British Standard is the UK implementation of EN 15302:2021. It supersedes BS EN 15302:2008+A1:2010, which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee RAE/1/-/8, Railway Applications - Vehicle/Track Interaction.

A list of organizations represented on this committee can be obtained on request to its committee manager.

Contractual and legal considerations

This publication has been prepared in good faith, however no representation, warranty, assurance or undertaking (express or implied) is or will be made, and no responsibility or liability is or will be accepted by BSI in relation to the adequacy, accuracy, completeness or reasonableness of this publication. All and any such responsibility and liability is expressly disclaimed to the full extent permitted by the law.

This publication is provided as is, and is to be used at the recipient's own risk.

The recipient is advised to consider seeking professional guidance with respect to its use of this publication.

This publication is not intended to constitute a contract. Users are responsible for its correct application.

© The British Standards Institution 2021
Published by BSI Standards Limited 2021

ISBN 978 0 580 98206 4

ICS 45.060.01; 17.040.20

Compliance with a British Standard cannot confer immunity from legal obligations.

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 31 October 2021.

Amendments/corrigenda issued since publication

Date	Text affected
------	---------------

English Version

Railway applications - Wheel-rail contact geometry parameters - Definitions and methods for evaluation

Applications ferroviaires - Paramètres géométriques
du contact roue-rail - Définitions et méthodes de
détermination

Bahnanwendungen - Parameter der Rad-Schiene
Kontaktgeometrie - Definitionen und
Berechnungsmethoden

This European Standard was approved by CEN on 2 August 2021.

CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration. Up-to-date lists and bibliographical references concerning such national standards may be obtained on application to the CEN-CENELEC Management Centre or to any CEN member.

This European Standard exists in three official versions (English, French, German). A version in any other language made by translation under the responsibility of a CEN member into its own language and notified to the CEN-CENELEC Management Centre has the same status as the official versions.

CEN members are the national standards bodies of Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Republic of North Macedonia, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and United Kingdom.



EUROPEAN COMMITTEE FOR STANDARDIZATION
COMITÉ EUROPÉEN DE NORMALISATION
EUROPÄISCHES KOMITEE FÜR NORMUNG

CEN-CENELEC Management Centre: Rue de la Science 23, B-1040 Brussels

Contents

Page

European foreword.....	7
Introduction	8
1 Scope	9
2 Normative references.....	9
3 Terms and definitions	10
4 Symbols and abbreviations	11
5 Overview of the process for determining contact parameters.....	12
6 Description of wheel and rail profiles	12
6.1 General.....	12
6.2 Uncertainty of the measuring systems.....	14
7 Plausibility check and processing of measured wheel and rail profiles.....	15
8 Determining the wheel-rail contact positions and contact functions.....	16
8.1 General.....	16
8.2 Determining the rolling radius difference function	16
8.3 Other wheel-rail contact geometry functions.....	17
9 Determining the equivalent conicity and the related nonlinearity parameter	17
9.1 Background to equivalent conicity.....	17
9.1.1 Mathematical description of the kinematic lateral wheelset motion	17
9.1.2 Determining the wavelength of a coned wheelset	18
9.2 Determining the equivalent conicity	19
9.3 Determining the nonlinearity parameter	19
10 Determining the rolling radii coefficient.....	20
10.1 Background and definition	20
10.2 Determining point E for the calculation of the rolling radii coefficient.....	22
11 Other wheel-rail contact parameters	23
12 Testing of calculation software for contact geometry parameters.....	24
12.1 Overview	24
12.2 Validation of the calculation algorithms	24
12.3 Assessment of the smoothing process.....	24
13 Assessment of the complete process for determination of wheel-rail contact parameters	28
13.1 General.....	28
13.2 Reproducibility of contact parameter determination based on rail profile measurement.....	28
13.2.1 Manual rail profile measuring devices.....	28
13.2.2 Vehicle based rail profile measuring systems	29
13.3 Reproducibility of contact parameter determination based on wheel profile measurement.....	30
13.3.1 Manual wheel profile measuring devices.....	30
13.3.2 Ground based wheel profile measuring systems.....	30
Annex A (informative) Example of presentation of contact geometry functions.....	32

Annex B (informative) Derivation of the kinematic equation of wheelset motion.....	33
Annex C (informative) Determination of the lateral peak displacements	36
Annex D (informative) Method for determining the wavelength of the wheelset motion by two-step integration of the nonlinear differential equation	38
D.1 General	38
D.2 Step 1.....	38
D.3 Step 2.....	38
Annex E (informative) Method for determining the wavelength of the wheelset motion by direct integration of the nonlinear differential equation	40
Annex F (informative) Method for determining the equivalent conicity by linear regression of the Δr function	41
F.1 General	41
F.2 Concerns regarding the method.....	41
Annex G (informative) Method for determining linearization parameters by harmonic linearization	43
G.1 General	43
G.2 Concerns regarding the method.....	44
Annex H (informative) Handling of special cases of the Δr function.....	45
Annex I (normative) Reference profiles for testing	48
I.1 General	48
I.2 Wheel A.....	49
I.2.1 Drawing.....	49
I.2.2 Analytic definition.....	49
I.2.3 Cartesian coordinates	50
I.3 Wheel B.....	52
I.3.1 Drawing.....	52
I.3.2 Analytic definition.....	52
I.3.3 Cartesian coordinates	53
I.4 Wheel C.....	55
I.4.1 Drawing.....	55
I.4.2 Analytic definition.....	55
I.4.3 Cartesian coordinates	56
I.5 Wheel H	58
I.5.1 Drawing.....	58
I.5.2 Analytic definition.....	58
I.5.3 Cartesian coordinates	59
I.6 Wheel I.....	61
I.6.1 Drawing.....	61

I.6.2	Analytic definition	61
I.6.3	Cartesian coordinates.....	62
I.7	Rail A.....	64
I.7.1	Drawing	64
I.7.2	Analytic definition	64
I.7.3	Cartesian coordinates.....	65
Annex J (normative)	Calculation results with reference profiles.....	67
J.1	General.....	67
J.2	Wheel A/Rail A	68
J.2.1	Representation of contact points, diagrams of Δr , $\Delta \tan \gamma$, $\tan \gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track.....	68
J.2.2	Numerical values for Δr function	69
J.2.3	Numerical values for $\tan \gamma_e$ function.....	70
J.3	Wheel B/Rail A	72
J.3.1	Representation of contact points, diagrams of Δr , $\Delta \tan \gamma$, $\tan \gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track.....	72
J.3.2	Numerical values for Δr function	73
J.3.3	Numerical values for $\tan \gamma_e$ function.....	74
J.4	Wheel C/Rail A	76
J.4.1	Representation of contact points, diagrams of Δr , $\Delta \tan \gamma$, $\tan \gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track.....	76
J.4.2	Numerical values for Δr function	77
J.4.3	Numerical values for $\tan \gamma_e$ function.....	79
J.5	Wheel H/Rail A.....	81
J.5.1	Representation of contact points, diagrams of Δr , $\Delta \tan \gamma$, $\tan \gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track.....	81
J.5.2	Numerical values for Δr function	82
J.5.3	Numerical values for $\tan \gamma_e$ function.....	83
J.6	Wheel I/Rail A	85
J.6.1	Representation of contact points, diagrams of Δr , $\Delta \tan \gamma$, $\tan \gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track.....	85
J.6.2	Numerical values for Δr function	86
J.6.3	Numerical values for $\tan \gamma_e$ function.....	87
J.7	Modified Wheel A (–2 mm on left wheel diameter)/Rail A.....	89
J.7.1	Representation of contact points, diagrams of Δr , $\Delta \tan \gamma$, $\tan \gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track.....	89
J.7.2	Numerical values for Δr function	90

J.7.3	Numerical values for $\tan\gamma_e$ function	91
J.8	Modified Wheel B (–2 mm on left wheel diameter)/Rail A	93
J.8.1	Representation of contact points, diagrams of Δr , $\Delta\tan\gamma$, $\tan\gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track	93
J.8.2	Numerical values for Δr function	94
J.8.3	Numerical values for $\tan\gamma_e$ function	95
J.9	Modified Wheel H (–2 mm on left wheel diameter)/Rail A	97
J.9.1	Representation of contact points, diagrams of Δr , $\Delta\tan\gamma$, $\tan\gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track	97
J.9.2	Numerical values for Δr function	98
J.9.3	Numerical values for $\tan\gamma_e$ function	99
J.10	Modified Wheel I (–2 mm on left wheel diameter)/Rail A	101
J.10.1	Representation of contact points, diagrams of Δr , $\Delta\tan\gamma$, $\tan\gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track	101
J.10.2	Numerical values for Δr function	102
J.10.3	Numerical values for $\tan\gamma_e$ function	103
J.11	(Right Wheel A – Left Wheel B)/Rail A	105
J.11.1	Representation of contact points, diagrams of Δr , $\Delta\tan\gamma$, $\tan\gamma_e$ functions and representation of kinematic rolling movement of the wheelset on track	105
J.11.2	Numerical values for Δr function	106
J.11.3	Numerical values for $\tan\gamma_e$ function	107
Annex K (normative)	Tolerances on equivalent conicity for testing calculations	109
K.1	General	109
K.2	Wheel A/Rail A	110
K.2.1	Diagram	110
K.2.2	Numerical values	111
K.3	Wheel B/Rail A	113
K.3.1	Diagram	113
K.3.2	Numerical values	114
K.4	Wheel C/Rail A	116
K.4.1	Diagram	116
K.4.2	Numerical values	117
K.5	Wheel H/Rail A	119
K.5.1	Diagram	119
K.5.2	Numerical values	120
K.6	Wheel I/Rail A	122
K.6.1	Diagram	122

K.6.2	Numerical values.....	123
K.7	Modified Wheel A (–2 mm on left wheel diameter)/Rail A.....	125
K.7.1	Diagram	125
K.7.2	Numerical values.....	126
K.8	Modified Wheel B (–2 mm on left wheel diameter)/Rail A.....	128
K.8.1	Diagram	128
K.8.2	Numerical values.....	129
K.9	Modified Wheel H (–2 mm on left wheel diameter)/Rail A	131
K.9.1	Diagram	131
K.9.2	Numerical values.....	132
K.10	Modified Wheel I (–2 mm on left wheel diameter)/Rail A.....	134
K.10.1	Diagram	134
K.10.2	Numerical values.....	135
K.11	(Right Wheel A – Left Wheel B)/Rail A	137
K.11.1	Diagram	137
K.11.2	Numerical values.....	138
	Bibliography.....	140

European foreword

This document (EN 15302:2021) has been prepared by Technical Committee CEN/TC 256 “Railway applications”, the secretariat of which is held by DIN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by April 2022, and conflicting national standards shall be withdrawn at the latest by April 2022.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN shall not be held responsible for identifying any or all such patent rights.

This document supersedes EN 15302:2008+A1:2010.

The main changes with respect to the previous edition are listed below:

- Extension of the Scope;
- Introduction of new wheel-rail contact geometry parameters (rolling radii coefficient, nonlinearity parameter);
- Additional methods for evaluation of equivalent conicity;
- Improvement of the description of the reference profiles;
- Additional reference wheel profile C;
- Reference results based on analytical solutions;
- Hints for plausibility checking of measured profiles;
- Revised assessment of the smoothing process;
- New assessment of the complete process.

This document has been prepared under a mandate given to CEN by the European Commission and the European Free Trade Association.

Any feedback and questions on this document should be directed to the users’ national standards body. A complete listing of these bodies can be found on the CEN website.

According to the CEN-CENELEC Internal Regulations, the national standards organisations of the following countries are bound to implement this European Standard: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Republic of North Macedonia, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and the United Kingdom.

Introduction

This document is based on the UIC Code 519 OR submitted to CEN by the International Union of Railways (UIC) and which has been revised by CEN/TC 256/WG 10 “Vehicle/Track Interaction”.

The wheel-rail contact geometry is fundamental for explaining the dynamic running behaviour of a railway vehicle, as well as the quasi-static behaviour in curves. Among the parameters which influence the dynamic behaviour of a rail vehicle, the equivalent conicity plays an essential role since it allows for the satisfactory characterization of the wheel-rail contact geometry on tangent track and on very large-radius curves. A wheelset describes a waveform while running on a track. Klingel's theory, valid for massless wheelsets with conical profiles, states that the waveform is sinusoidal and its wavelength depends on the cone angle of the wheel profile.

Real wheel profiles are not pure cones, but have changing cone angles across the tread, leading to a nonlinear dependency of the rolling radius difference on the lateral movement of the wheelset on the track. The wavelength of the wheelset movement according to the nonlinear kinematic equations of motion may be calculated by solving numerically this formula or by specific methods for linearization of the rolling radius difference function. Equivalent conicity is evaluated by comparison of this wavelength with the equivalent wavelength of a conical wheelset according to Klingel's formula or by calculating the conicity from the linearized rolling radius difference function.

It is important to have a clear specification for the evaluation of wheel-rail contact geometry parameters, which are used in European and national standards and documents (legal and technical).

The objective is to ensure that the results for the determined parameters are consistent. However, it is possible to use different evaluation procedures to those given in this document, provided that the procedure used leads to the determination of wheel-rail contact parameters in accordance with the calculation results using the reference profiles specified in Annex I. A validation process is given in this document to be used in order to determine whether or not an evaluation procedure can achieve the specified reference results.

Technical background will be given in a Technical Report published after the publication of this document.

1 Scope

This document establishes definitions and evaluation methods for wheel-rail contact geometry parameters influencing the vehicle running dynamic behaviour:

- the rolling radius difference between the two wheels of a wheelset (Δr -function) which serves as a basis for all further calculations;
- the equivalent conicity function from which are derived:
 - a single equivalent conicity value for a specified amplitude which is relevant for the assessment of vehicle running stability on straight track and in very large radius curves according to EN 14363;
 - the nonlinearity parameter which characterizes the shape of this function and is related to the vehicle behaviour particularly in the speed range close to the running stability limit;
- the rolling radii coefficient which is used to describe the theoretical radial steering capability of a wheelset in a curved track.

Additional information is given about the relationship between the contact angles of the two wheels of a wheelset ($\Delta \alpha$ -function) and about the roll angle parameter.

NOTE Out of the presented parameters only those related to the contact angle are relevant for independently rotating wheels of wheel pairs.

Descriptions of possible calculation methods are included in this document. Test case calculations are provided to achieve comparable results and to check the proper implementation of the described algorithms.

To validate alternative methods not described in this document acceptance criteria are given for the equivalent conicity function. This includes reference profiles, profile combinations, tolerances and reference results with tolerance limits.

This document also includes minimum requirements for the measurement of wheel and rail profiles as well as of the parameters needed for the transformation into a common coordinate system of right- and left-hand profiles.

This document does not define limits for the wheel-rail contact geometry parameters and gives no tolerances for the rail profile and the wheel profile to achieve acceptable results.

For the application of this document some general recommendations are given.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

EN 13231-2:2020, *Railway applications — Track — Acceptance of works — Part 2: Acceptance of reprofiling rails in plain line, switches, crossings and expansion devices*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <https://www.electropedia.org/>
- ISO Online browsing platform: available at <https://www.iso.org/obp>

NOTE This document uses the standard European notation for numeric values with “comma” (,) as the decimal point and “space” () as the thousands delimiter. Thus, for example 2,5 is to be understood as two and one-half and 1 500 as one thousand five hundred.

3.1 equivalent conicity

tangent of the cone angle of a wheelset with coned wheels whose kinematic movement has the same wavelength as the given wheelset for a certain amplitude of the lateral wheelset movement

3.2 nonlinearity parameter

local slope of the equivalent conicity function between two specified wheelset displacement amplitudes

3.3 radial steering index

ratio between the curve radius negotiable without longitudinal creepage and the actual curve radius of the track section to describe the radial steering capability of a wheelset in a track section

3.4 rolling radii coefficient

relationship describing the capability of a wheel-rail contact geometry to provide the rolling radii difference needed for a wheelset to negotiate an actual curve without longitudinal creepage and flange contact

Note 1 to entry: This parameter is related to the radial steering index.

3.5 uncertainty

refer to the definition of expanded uncertainty with a coverage factor equal to 2 as defined in ISO/IEC Guide 98-3:2008-09 (JCGM/WG1/100)

Note 1 to entry: The uncertainty as defined corresponds to a confidence level of about 95 % of a normal distribution.

3.6 reproducibility

degree of agreement between the values of successive measurements of the same parameter made under varying conditions using the same measurement and interpretation methods

4 Symbols and abbreviations

For the purposes of this document, the following symbols apply.

$2b_A$	nominal contact point spacing (defined as 1 500 mm for standard gauge)
D_0	tread datum position; location on the wheel tread, 70 mm (for standard gauge) from the internal face of the wheel
x	displacement of the wheelset in the longitudinal direction of the track
y	displacement of the wheelset in the lateral direction of the track (at top of rail level)
ψ	angle of the wheelset movement in the x - y -plane
ds	curve length of the path corresponding to the angle $d\psi$
V	speed of forward movement of the vehicle
R_{WS}	local radius of the wheelset path
r	mean rolling-radius of both wheels
r_0	radius of the wheels when the wheelset is centred on the track
r_1	rolling-radius of the right-hand wheel
r_2	rolling-radius of the left-hand wheel
Δr	difference of the rolling-radius between right-hand and left-hand wheels
y_{em}	lateral displacement where $\Delta r = 0$
y_{emin}	minimum value of lateral displacements
y_{emax}	maximum value of lateral displacements
$\hat{y}$	amplitude of the wave
λ	wavelength of the wheelset movement
γ	contact angle; angle between the tangent at the wheel-rail contact point and the track plane
$\Delta \tan \gamma$	difference of the tangents of the contact angles between right-hand and left-hand wheels
$\tan \gamma_e$	equivalent conicity
N_P	nonlinearity parameter
Δr_E	rolling radius difference available for kinematic rolling (rolling without slip)
R_E	minimum curve radius for kinematic rolling
q_E	radial steering index
ρ_E	rolling radii coefficient
ε_e	contact angle parameter
φ	roll angle of the wheelset around the longitudinal axis
σ_e	roll angle parameter
σ_{err}	standard deviation of random profile errors
NOTE	Some additional symbols not included in the list above are explained in the section where they are used.

5 Overview of the process for determining contact parameters

Figure 1 gives an overview of the process for determining the contact parameters described in this document. The figure also shows the clauses of this standard where more information on particular steps of the process is given, including possible options.

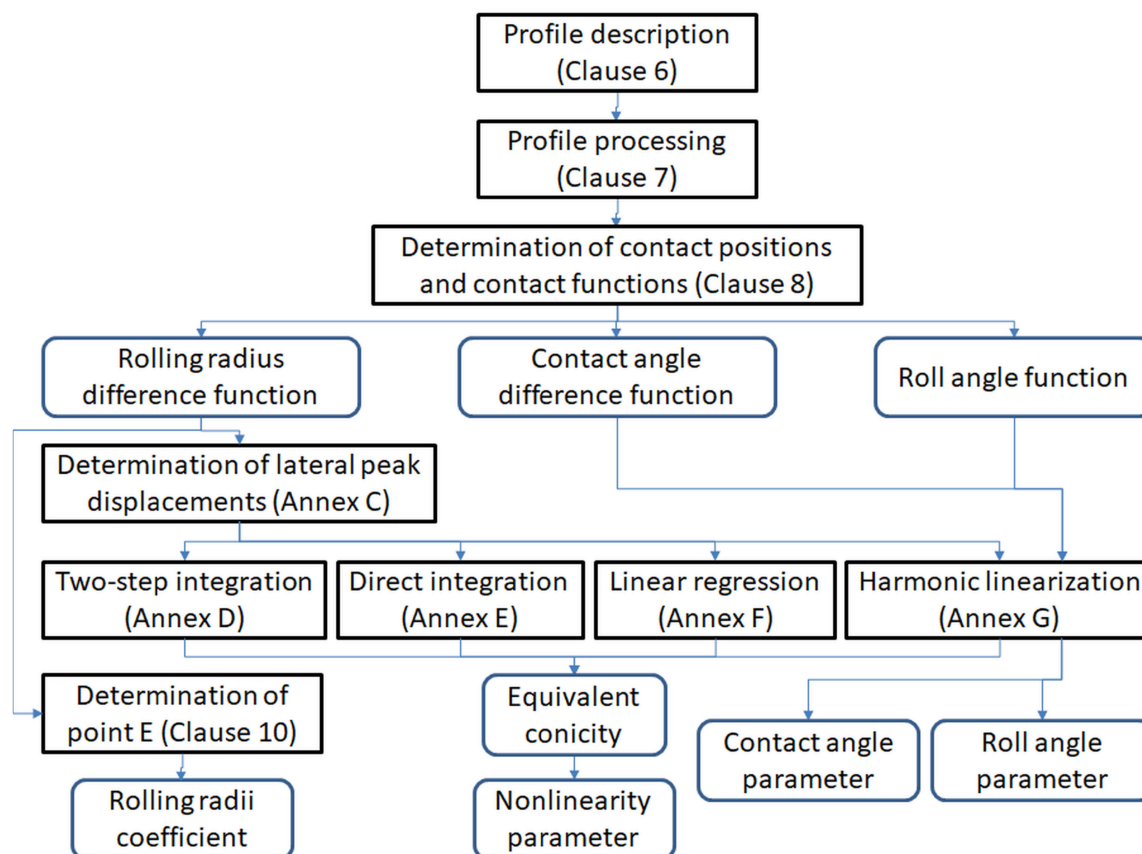


Figure 1 — Process of contact geometry parameter determination

6 Description of wheel and rail profiles

6.1 General

The determination of wheel-rail contact geometry parameters requires knowledge of the shapes of the wheel and rail profiles to be assessed as well as their relative position including:

- wheel back-to-back distance,
- wheel diameters, if relevant,
- track gauge,
- profile orientation in relation to the track plane (rail inclination, wheel inclination due to axle bending).

NOTE 1 If there is significant difference in wheel diameter (more than 2 mm) the resulting roll effect may need to be included if not covered by the measuring system.

NOTE 2 The bending of the axle under the load generally is relevant. Therefore, the measurements are usually made for representative load conditions near to the contact point.

In case of measuring systems which are not able to include the wheel inclination due to axle bending, the orientation may be determined by other methods such as static calculations. In some cases, it may be possible to use already known bending angles representative for the vehicle considered.

When the profiles are determined by measurement, special-purpose devices can be used, such as wheel and rail profile measuring devices or automatic measuring systems carried aboard special railbound vehicles for rail profiles or ground based systems for wheel profiles.

The measurement devices shall be able to provide the profile coordinates with a maximum 0,5 mm spacing along the arc of the profile.

NOTE 3 In areas with high profile curvature, a lower spacing may be required in order to get an accurate profile shape.

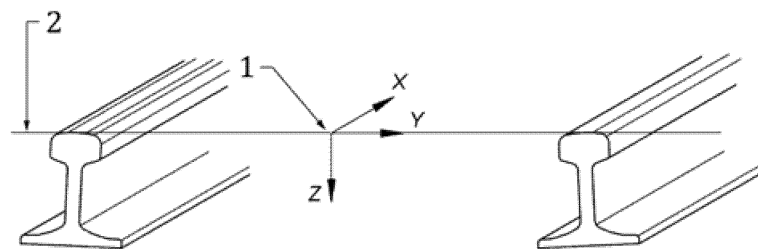
It shall be reported whether the profiles were measured in the loaded or unloaded condition.

NOTE 4 For freight vehicles contact geometry is potentially more significantly influenced by the load. Therefore, if the equivalent conicity is to be used to investigate running behaviour, the contact geometry is sometimes considered in empty and laden condition.

When theoretical profiles are used the inclination shall be considered.

Independent of the source of the profiles (theoretical or measured), the two rails of the railway track shall be referred to a track-related coordinate system oriented such that the x -axis is longitudinal to the track, the y -axis tangential to the upper surface of the rail heads and the z -axis perpendicular to both axes, see Figure 2. The two wheels of the wheelset shall be referred to a single coordinate system with axes oriented in analogous directions.

NOTE 5 The relative position and orientation of the two profiles is relevant. Therefore, the use of a measuring system with a single profile measuring head without reference between the two sides can lead to large uncertainties of the calculated contact parameters.



Key

- 1 Track-related coordinate system
- 2 Top of rail level

Figure 2 — Track-related coordinate system (consistent with EN 13848-1)

It is recommended to provide the input data (theoretical or measured) related to the following coordinate systems.

For the rails the origin of the coordinate system is defined so that $y = 0$ at the middle of the measured track gauge and $z = 0$ at the top of the rails.

For the wheels the origin of the coordinate system is defined so that $y = 0$ at the middle of the wheel back to back distance and $z = 0$ at D_0 for both wheels. Any significant wheel diameter difference shall be included as an offset in the rolling radius difference function, see 8.2.

The wheel and rail profiles shall be characterized such that:

- for the rail, the profile is defined not only on the top but also on the inner side (gauge face) at least down to 14 mm below the top of the rail,
- for the wheel, the profile is defined not only on the wheel tread but also on the outer part and in the area of the wheel flange root down to at least 10 mm below D_0 ,
- any significant radius difference between the two wheels of a wheelset (measured at the tread datum positions of the profiles) is taken into account.

The numerical resolution of the profile data shall be consistent with the evaluation process (smoothing and calculation). If profile data are given with low numerical resolution, the shape may become step-like with repeated samples of identical amplitude and jumps which are much larger than in reality. This will lead to unrealistic results for the contact parameters. It is therefore recommended to provide profile data with a high numerical resolution (number of digits), e.g. with $1 \cdot 10^{-3}$ mm. This is clearly beyond the precision of the measurement system but prevents problems in the calculation of contact geometry parameters.

For the following steps in the procedure the profile coordinates shall be provided sorted along the arc, in order to give a continuous profile.

6.2 Uncertainty of the measuring systems

The uncertainty of the measuring system shall be quantified as a combination of the uncertainties of the following parameters:

- a) coordinates of a single profile;
- b) relative position of left and right profile (vertical and lateral distance, rotation around longitudinal axis).

These uncertainties shall be determined in laboratory tests and be provided by the supplier of the measuring system. One possible test for rail profile measurements is described in EN 13231-2:2020, Annex C. If it is not practical to check the measuring system in the laboratory, an in-field cross-check with a reference measuring instrument is required.

If the profiles and the relative position parameters are not measured with the same system then the interface between the systems shall be included in the uncertainty analysis (for example the influence of different spacing of data points, adjustment of the different measuring systems, etc.).

In order to limit the uncertainty of the calculated contact parameters, measurement uncertainties of the relative position parameters shall not exceed the following values:

- difference of wheel radii (measured at the tread datum positions of the profiles): 0,2 mm;
- wheel back-to-back distance: 0,5 mm;
- track gauge: 0,5 mm.

NOTE 1 Tolerances for the diameter difference between wheels on the same axle are specified in EN 15313 for new and re-profiled wheelsets.

NOTE 2 For the assessment of equivalent conicity the sum of the tolerances of wheel back-to-back distance and track gauge is the relevant parameter.

7 Plausibility check and processing of measured wheel and rail profiles

For the measurement of wheel and rail profiles a lot of measuring instruments with different measuring principles and differing sensitivity to measuring conditions are available. Before using measured profiles for contact geometry parameter calculations, the following pre-processing steps, which may be additional to the already implemented checks in the measuring systems, shall be carried out:

- plausibility check;
- elimination of outlier points;
- profile smoothing and/or interpolation.

The plausibility check tests if the relevant wheel-rail contact zone is measured completely. Typically, the following ranges are relevant, see Figure 3:

- for wheel profiles the range from the top of the flange to the point on the tread 90 mm from the flange thickness measuring point;
- for rail profiles the range from the gauge measuring point to the point 2 mm below the top of the rail on the field side.

Other plausibility checks such as for maximum spacing between adjacent points are recommended as well.

Profiles failing a plausibility check shall be excluded from the analysis.

Dimensions in millimetres

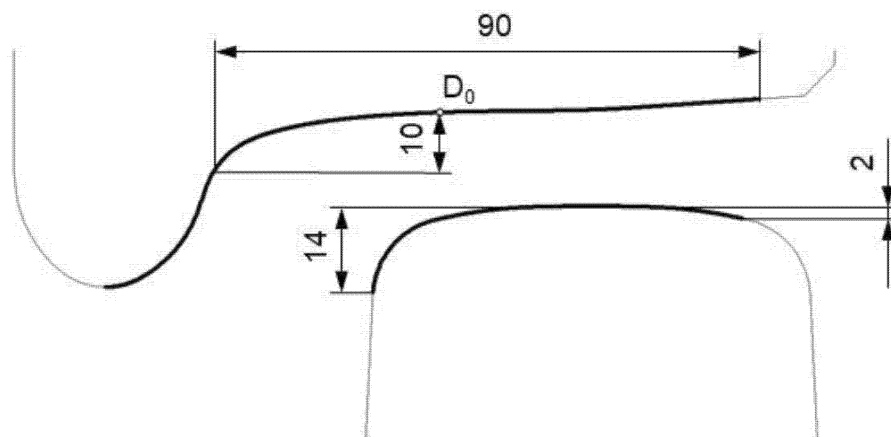


Figure 3 — Typical relevant wheel-rail contact zones

In the next step it may be necessary to detect and eliminate profile points which do not fit the continuous shape of the profile. These outliers can be identified for example by calculating the angle formed between lines joining each data point and the two adjacent points or by comparing the distance of each profile point to a smoothed profile.

The Cartesian coordinates of the two wheels and the two rails, referred to the track-related coordinate system, can be processed (smoothing or interpolation in particular) for easier utilization in the following

steps of the procedure. The validation test described in Clause 12 shall be used in order to check the quality of the smoothing or interpolation method.

8 Determining the wheel-rail contact positions and contact functions

8.1 General

For the described method of determination, the following assumptions are used:

- wheels are symmetrical in revolution and are represented by a single profile for each wheel;
- the wheel profiles are considered for the wheelset position perpendicular to the tangent of the track;
- rails are straight, parallel to each other and represented by a single profile for each rail;
- the lateral displacement of the wheelset as used in this document is considered at the top of rail level;
- in the calculation the complete wheelset and the track are considered rigid and only point contacts are considered;
- no account is taken of an axle's roll angle (rotation about an axis longitudinal to the track) as the wheelset moves laterally on the track.

Elastic contact is more realistic than rigid contact however, it is more difficult to evaluate and it is dependent on the vertical wheel force. Therefore, it is permitted to use elastic contact. In this case the discontinuities in the Δr function caused by contact point jumps are smoothed.

Including the roll angle will influence the Δr function significantly only at large wheelset amplitudes with contact at the flange. Hence, it can be neglected, however it is permitted to include this effect.

8.2 Determining the rolling radius difference function

The first stage for calculating the contact geometry functions is to determine the contact point positions at each wheel and rail as a function of the lateral wheelset displacement.

The rolling radius difference function $\Delta r = f(y)$ characterizes the difference between the right-hand and the left-hand rolling radius $\Delta r = r_1 - r_2$ depending on the lateral movement y of the wheelset on the track.

The determination of the rolling radius difference function shall take into account the actual radius of each wheel in case of a significant difference of the wheel diameters.

Then, the $\Delta r = f(y)$ characteristic, which yields the rolling radius difference between the right and left side for a lateral movement y of the wheelset on the track, shall be worked out relative to a centred position and for lateral movements up to a maximum. For the calculation of equivalent conicity the maximum shall be chosen so that the rolling radius difference function includes $|\Delta r| = |r_1 - r_2| \geq 5$ mm, in maximum steps of $\Delta y = 0,2$ mm. For the calculation of the rolling radii coefficient the function shall include $|\Delta r| = |r_1 - r_2| \geq 10$ mm, in maximum steps of $\Delta y = 0,1$ mm. The resulting function shall have only one Δr value for each y value.

NOTE An example of a $\Delta r = f(y)$ characteristic is presented in Figure A.1. The maximum lateral displacement for the determination of the geometric functions depends upon the lateral clearance between wheel and rail.

8.3 Other wheel-rail contact geometry functions

The contact angle difference function $\Delta \tan \gamma = f(y)$ characterizes the difference between the tangents of the right-hand and the left-hand contact angle $\Delta \tan \gamma = \tan \gamma_1 - \tan \gamma_2$ depending on the lateral displacement y of the wheelset on the track. For the calculation of the contact angle difference function the contact angles on left and right wheels are defined in such a way that the contact angle on the flange is positive on both wheels (around 70°).

The roll angle function $\varphi = f(y)$ characterizes the rotation of the wheelset around the longitudinal axis depending on the lateral displacement y of the wheelset on the track. If the roll angle function is needed for a particular application this requires the rolling of the wheelset around the longitudinal axis to be taken into account.

NOTE There are other parameters describing wheel-rail contact characteristics which may be useful but are not covered by this document, e.g. contact point movement, contact bandwidth change rate and contact concentration.

9 Determining the equivalent conicity and the related nonlinearity parameter

9.1 Background to equivalent conicity

9.1.1 Mathematical description of the kinematic lateral wheelset motion

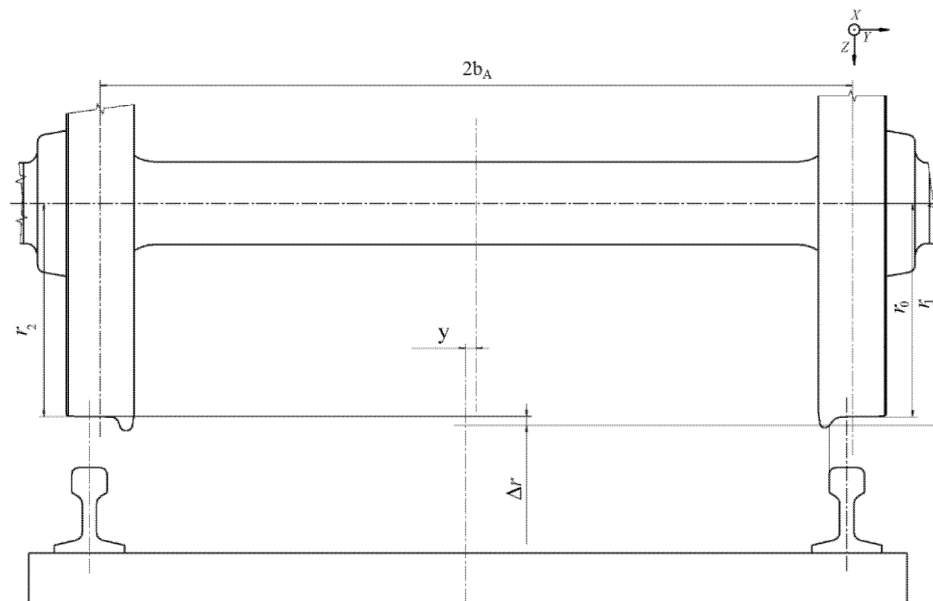


Figure 4 — Dimensions on the wheelset

The kinematic lateral movement of a free wheelset (Figure 4), with no inertia, running on a track, is given by the following differential Formula (1):

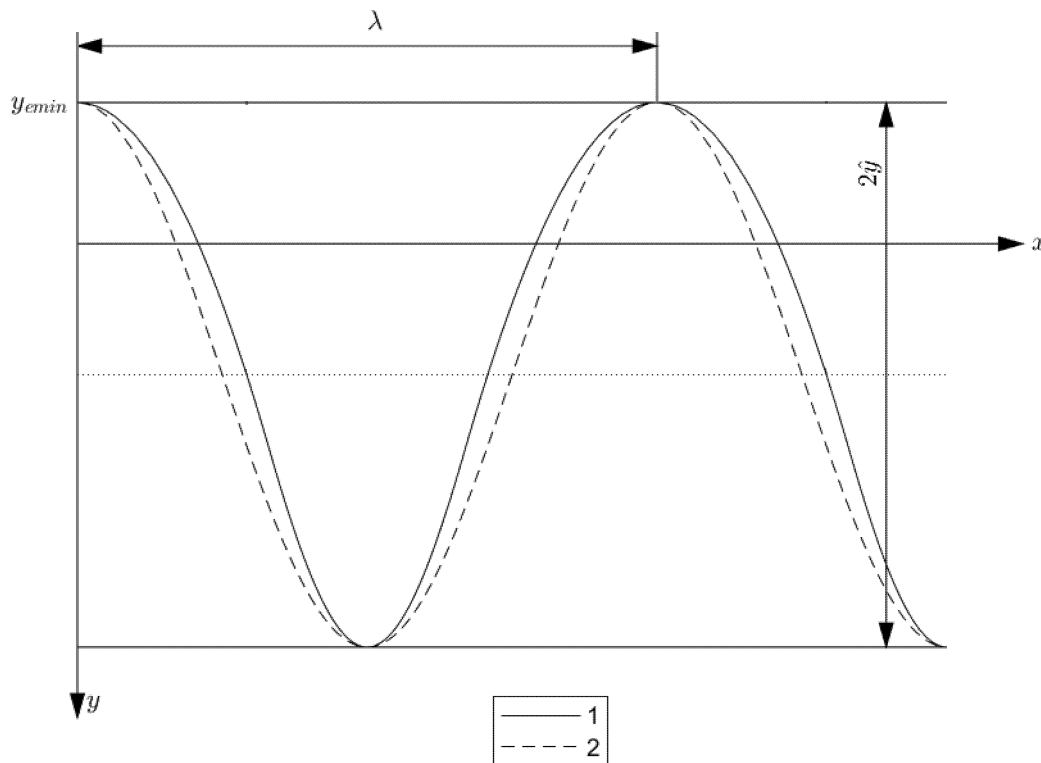
$$\frac{d^2 y}{dx^2} + \frac{\Delta r(y)}{2b_A r_0} = 0 \quad (1)$$

The previously determined $\Delta r = f(y)$ characteristic is used together with the following initial conditions:

$$y = y_{emin} \quad \text{for } x = 0 \quad (2)$$

$$\frac{dy}{dx} = 0 \quad \text{for } x = 0 \quad (3)$$

The integration of the kinematic equation of motion (1) using the initial conditions (2) and (3) leads to a periodic movement of the wheelset with a peak-to-peak amplitude of $2\hat{y}$ and a wavelength λ as presented in Figure 5.



Key

- 1 kinematic motion of a wheelset
- 2 sinusoidal motion of the equivalent coned wheelset

Figure 5 — $y = f(x)$ function

9.1.2 Determining the wavelength of a coned wheelset

In the case of a wheelset whose wheels have a conical profile with a constant angle γ , the rolling radius difference is linear with:

$$\Delta r(y) = 2y \tan \gamma \quad (4)$$

Formula (1) then becomes:

$$\frac{d^2 y}{dx^2} + \frac{\tan \gamma}{b_A r_0} y = 0 \quad (5)$$

a second order differential equation with constant coefficients whose solution is a sinewave with a wavelength of λ

$$\lambda = 2\pi \sqrt{\frac{b_A r_0}{\tan \gamma}} \quad (6)$$

NOTE Formula (6) is also known as Klingel's formula.

9.2 Determining the equivalent conicity

When the wheels do not have a conical profile, various methods are used so that the linear differential equation can still be applied by replacing $\tan \gamma$ by “equivalent conicity” $\tan \gamma_e$.

By definition, the equivalent conicity is equal to the tangent of the cone angle of a wheelset with coned wheels whose kinematic movement has the same wavelength as the given wheelset for a particular amplitude of the lateral wheelset movement:

$$\tan \gamma_e(\hat{y}) = \left(\frac{2\pi}{\lambda} \right)^2 b_A r_0 \quad (7)$$

There are different methods for determining the wavelength of the kinematic wheelset motion:

- two-step integration of the nonlinear differential equation of motion (see Annex D);
- direct integration of the nonlinear differential equation of motion (see Annex E);

or for determining the equivalent conicity directly:

- linear regression of the Δr function (see Annex F);
- harmonic linearization (see Annex G).

The latter two methods are not directly based on the definition of equivalent conicity (same wavelength as the lateral movement of the equivalent conical wheelset). However, these methods are generally a good approximation and can be useful in many circumstances.

NOTE 1 Annex H gives information about how to handle special cases of the Δr function.

The equivalent conicity shall be determined for a specified range of the amplitude $\hat{y}$. Annex C provides a method to derive the minimum and maximum lateral wheelset displacements as a function of the amplitude.

NOTE 2 A typical range for the evaluation of the equivalent conicity for changes in $\hat{y}$ is 1 mm to 5 mm using a step size of $\Delta \hat{y} = 0,5$ mm. For cases with very large wheelset clearance it may be necessary to include larger amplitudes $\hat{y}$. An example of graphical representation of this exercise is given in Figure A.1, Figure A.2 and Figure A.3 of Annex A.

9.3 Determining the nonlinearity parameter

The dynamic behaviour of a vehicle depends not only on the equivalent conicity value for a particular wheelset displacement amplitude, but on the overall properties of the contact geometry wheelset-track. The equivalent conicity value for a specified wheelset displacement amplitude is well suited to characterize the vehicle performance regarding instability. A second parameter called the nonlinearity parameter is used in addition to the equivalent conicity with the aim of improving the characterization of the contact geometry wheelset-track. It is related to vehicle behaviour at speeds around the critical speed and to the vehicle's sensitivity to the lateral excitation.

The nonlinearity parameter N_p is defined as the slope of the equivalent conicity function between the equivalent conicity value $\tan\gamma_e(\hat{y} - \Delta\hat{y})$ and the value $\tan\gamma_e(\hat{y} + \Delta\hat{y})$:

$$N_p(\hat{y}, \Delta\hat{y}) = \frac{\tan\gamma_e(\hat{y} + \Delta\hat{y}) - \tan\gamma_e(\hat{y} - \Delta\hat{y})}{2\Delta\hat{y}} \quad [1/\text{mm}]$$

Typically the nonlinearity parameter is calculated for the wheelset displacement amplitude $\hat{y} = 3 \text{ mm}$ and the width of the interval $\Delta\hat{y} = 1 \text{ mm}$. In this case N_p characterizes the slope of the conicity function between wheelset displacement amplitudes of 2 mm and 4 mm.

NOTE Simulation studies show that vehicle behaviour at speeds around the instability limit is usually related to the shape of the equivalent conicity function, as represented by the nonlinearity parameter. If the equivalent conicity function has a positive nonlinearity parameter, the wheelset limit cycle appears suddenly and grows quickly to large amplitudes, often up to flange contact. If the equivalent conicity function has a negative nonlinearity parameter, the wheelset limit cycle starts at lower speed with a small wheelset amplitude that grows slowly with increasing speed. In both cases the instability limit according to EN 14363 is reached at a similar speed. The described phenomenon also influences the vehicle behaviour at speeds before the instability limit is reached. In this case, an equivalent conicity function with negative N_p leads to higher lateral forces between wheel and rail and larger lateral oscillations than an equivalent conicity function with positive N_p .

10 Determining the rolling radii coefficient

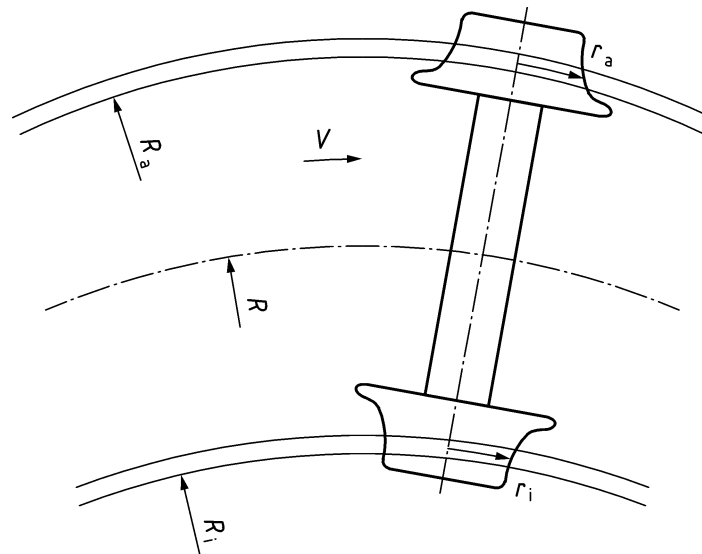
10.1 Background and definition

The behaviour of a vehicle in a curve is determined by a number of factors including the vehicle suspension characteristics and the wheel rail contact geometry. The rolling radii coefficient is one way to characterize the wheel rail contact geometry in a curve. Reporting the information may be useful in order to understand the curving behaviour of a vehicle.

NOTE 1 The rolling radii coefficient is another presentation of the radial steering index that was introduced previously in UIC 518 and EN 14363.

The two wheels of a wheelset are rigidly connected and they are therefore forced to rotate with the same angular velocity when running through a curve.

The running surfaces of the two wheels are usually profiled so that, if the wheelset is moved towards the outer side of the curve, the rolling radius of the outer wheel becomes larger than the rolling radius of the inner wheel (Figure 6). The larger the lateral displacement becomes, the larger becomes the rolling radius difference Δr .

**Key**

R	mean curve radius	V	vehicle speed
R_a	curve radius of outer rail	r_a	radius at outer wheel contact point
R_i	curve radius of inner rail	r_i	radius at inner wheel contact point

Figure 6 — Wheelset rolling through a curve

Due to the slightly larger curve radius of the outer rail compared to the inner rail, the outer wheel needs to roll a greater distance than the inner wheel. To at least compensate for these distance differences and to achieve a radial position of a free and unsuspended wheelset, an appropriate rolling radius difference between outer and inner wheel is necessary:

$$\frac{r_a}{r_i} \geq \frac{R_a}{R_i} \Rightarrow \frac{r_0 + \Delta r_a}{r_0 - \Delta r_i} \geq \frac{R + b_A}{R - b_A}$$

Regarding only first order effects it can be shown that:

$$r_a - r_i = \Delta r \geq r_0 \frac{2b_A}{R}$$

or

$$R \geq r_0 \frac{2b_A}{\Delta r}$$

As the rolling radii coefficient is used to indicate whether this condition can be achieved without flange contact for a given curve radius and a given wheel-rail profile combination, it is necessary to determine a rolling radius difference Δr_E at a point E of the rolling radius difference function before reaching flange contact, see 10.2. The smallest corresponding curve radius R_E is given by:

$$R_E = r_0 \frac{2b_A}{\Delta r_E}$$

The rolling radii coefficient is defined as:

$$\rho_E = \frac{R}{R_E} - 1 = \frac{R}{r_0 \cdot 2b_A} \cdot \Delta r_E - 1$$

Meaning:

- if $\rho_E \geq 0$ radial steering is possible;
- if $\rho_E < 0$ radial steering is not possible but flange contact will occur before a rolling radius difference Δr , big enough for the curve in question, is achieved.

NOTE 2 The radial steering index specified in UIC 518 and EN 14363 is defined as:

$$q_E = \frac{R_E}{R}$$

meaning:

- if $q_E \leq 1$ radial steering is possible;
- if $q_E > 1$ radial steering is not possible but flange contact will occur before a rolling radius difference Δr , big enough for the curve in question, is achieved.

Therefore, the rolling radii coefficient can also be obtained from a radial steering index by:

$$\rho_E = \frac{1}{q_E} - 1$$

The radial steering index q_E assumes a positive slope of the $\Delta r(y)$ function. If $\Delta r_E = 0$ then R_E and q_E are mathematically not defined. It is an advantage of the rolling radii coefficient ρ_E , that it allows the assessment of all real contact conditions to indicate a good or a bad curving behaviour (positive or negative values of Δr_E and $\Delta r_E = 0$).

10.2 Determining point E for the calculation of the rolling radii coefficient

By definition, the rolling radii coefficient ρ_E depends on the curve radius R_E calculated from the rolling radius difference Δr_E that is available for kinematic rolling (rolling without slip).

The definition of the corresponding point E on the Δr function assumes that radial steering of the wheelset is possible until the point of discontinuity (point A°) is reached, see Figure 7. The defined y-coordinate of point E is $y_E = y_{A^\circ} - 1$ mm for left hand curves and $y_E = y_{A^\circ} + 1$ mm for right hand curves.

Depending on the curve direction, either the right or the left part of the Δr function shall be examined, because the wheelset will (in general) follow the outer rail rather than the inner rail of the curve. For a left hand curve, the right part of the Δr function is investigated and vice versa.

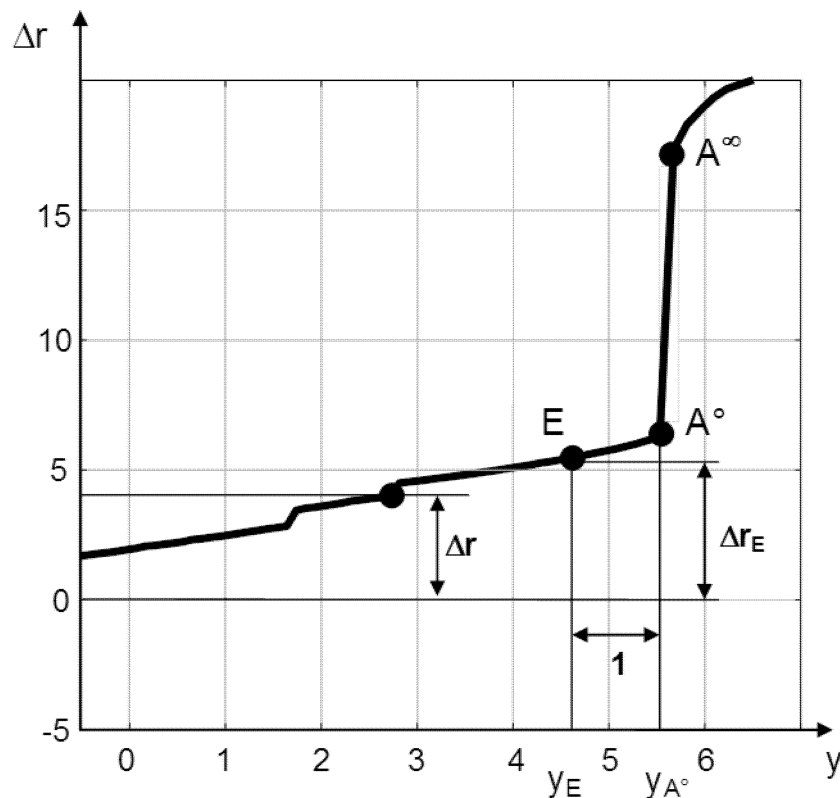


Figure 7 — Example of $\Delta r(y)$ function showing points related to the calculation of ρ_E for a left hand curve

NOTE During evaluation of different real wheel-rail profile combinations it was observed, that Δr functions are possible, which show negative Δr_E values for right and left hand curve.

The basis for calculating the rolling radii coefficient is the rolling radius difference function $\Delta r(y)$. The calculation of $\Delta r(y)$ is defined in 8.2.

The Δr function shall be checked for discontinuities. The discontinuity is found by the following procedure:

- starting from $\Delta r = 0$ search for the first pair of points $A^\circ (y_{A^\circ}, \Delta r_{A^\circ})$ and $A^\infty (y_{A^\infty}, \Delta r_{A^\infty})$ in the range $0 \leq \Delta r \leq 10$ mm for left hand curves and $-10 \text{ mm} \leq \Delta r \leq 0$ for right hand curves respectively with $|y_{A^\circ} - y_{A^\infty}| = 0,1$ mm and $|\Delta r_{A^\circ} - \Delta r_{A^\infty}| \geq 3$ mm subject to the constraints $y_{A^\circ} < y_{A^\infty}$ for left hand curves $y_{A^\infty} < y_{A^\circ}$ for right hand curves;
- if there is no discontinuity then the point A° is defined as the last point before $|\Delta r| = 10$ mm when starting at $y = 0$.

11 Other wheel-rail contact parameters

For simulation purposes it may be useful to use linearized parameters for the contact angle difference function and the roll angle function. These parameters are:

- contact angle parameter ε_θ ;

- roll angle parameter σ_e .

The harmonic linearization method for determining these parameters is described in Annex G.

12 Testing of calculation software for contact geometry parameters

12.1 Overview

To confirm whether calculation software can achieve the results specified in this document, the software shall be evaluated as described below.

Both the method of determination of the equivalent conicity and the rolling radii coefficient as well as the smoothing procedure for profile data shall be checked by benchmark calculations with defined reference-profiles. This is achieved by a two-step process:

- Step 1 validates the calculation algorithms for the contact geometry parameters;
- Step 2 evaluates the smoothing algorithm for defined errors in order to provide information about the possible range of its application with respect to the measuring uncertainty of a measuring system.

This document does not provide benchmark calculations for the following contact geometry parameters: nonlinearity parameter, contact angle parameter and roll angle parameter.

12.2 Validation of the calculation algorithms

For this step the calculations shall be done for all combinations of the reference profiles given in Annex I including the influence of asymmetrical Δr functions produced by different wheel diameters or different wheel profiles on the two wheels.

NOTE 1 The reference profiles given in Annex I are only defined for benchmark calculations to test the algorithms in a wide range of conditions; they have no practical meaning for a design of a real profile.

The calculation (and interpolation) algorithms are applied to the reference profiles in Annex I. This allows the determination of the contact points in order to calculate the rolling radius difference as a function of the lateral position of the wheelset. Starting from this function the equivalent conicity function is calculated as well as the value of Δr_E which leads to the rolling radii coefficient.

For the equivalent conicity function a comparison of the achieved results for $\hat{y} = 1$ mm to 6 mm with the reference results in Annex J and a defined field of allowed tolerances given in Annex K validates (or not) the assessed algorithms.

For the validation of the algorithm for the determination of the rolling radii coefficient the Δr_E values for the different profile combinations shall be compared with the reference results given in Annex J and the tolerance of $\pm 0,15$ mm.

NOTE 2 The reference results have been calculated analytically (without step size) but the difference from calculations with step sizes according to 8.2 is not significant.

NOTE 3 For methods with elastic contact the applied wheel load in the above assessment is usually reduced.

12.3 Assessment of the smoothing process

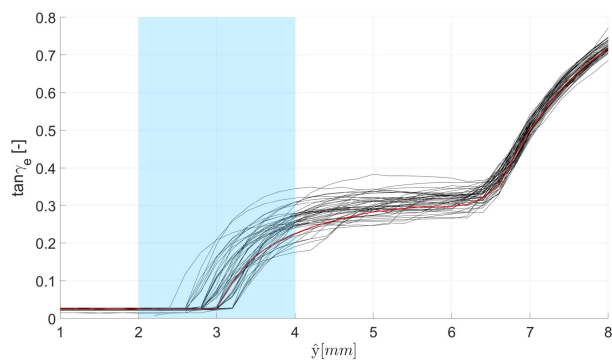
In this step, random errors are added to reference rail profiles and reference wheel profiles in order to represent the effect of measuring uncertainties on the profile coordinates. Then the smoothing and calculation algorithms of the software are applied and the effect of the profile disturbances on the resulting wheel-rail contact parameters, equivalent conicity and rolling radii coefficient, is quantified by comparison with the results for the nominal reference profiles as obtained by the same software

(reference functions). The results provide information about the sensitivity of the output parameters to input errors and can be used for indicating the range of application for which the tested software is suitable.

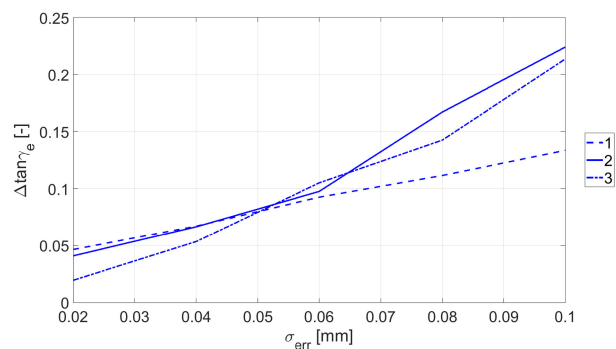
As an initial check if the smoothing process is suitable, it shall be demonstrated that the criteria described in 12.2 are still fulfilled for the nominal reference profiles with smoothing applied.

For a general assessment of the smoothing process the error profiles are achieved by adding random errors simultaneously to the lateral (y) and to the vertical (z) profile coordinates of the reference rail profile A and the reference wheel profiles A, B, and C given in Annex I. The random error amplitudes are given by centred, double sided normal distributions. The corresponding standard deviations shall be varied in steps of 0,02 mm between 0,02 mm and 0,10 mm. For each error level (standard deviation) a sufficient number of randomly disturbed profiles (≥ 50) shall be generated. The assessment of the smoothing process shall be carried out by separate calculations for the cases with random profile errors applied only to the wheel, only to the rail and to both wheel and rail. For assessment of the smoothing process for limited application, for example assessment of measured rail profiles with one reference wheel profile or assessment of measured wheel profiles with one reference rail profile, alternative reference profiles and profile errors may be used. The chosen cases and their justifications shall be documented.

The effect of the smoothing process in regard to equivalent conicity is quantified by the absolute differences between the conicity functions of the disturbed profiles and the corresponding reference function, see Figure 8 (left). The differences are calculated for lateral wheelset amplitudes in the range between 2 mm and 4 mm (shaded in Figure 8) in steps of 0,2 mm. Finally, the quality number for each error level is given by the 95 %-value of the total distribution of these differences. The quality numbers are plotted against the corresponding profile error level, see Figure 8 (right).



**a) Example for spread of conicity functions
(high error level for wheel B – rail A)**



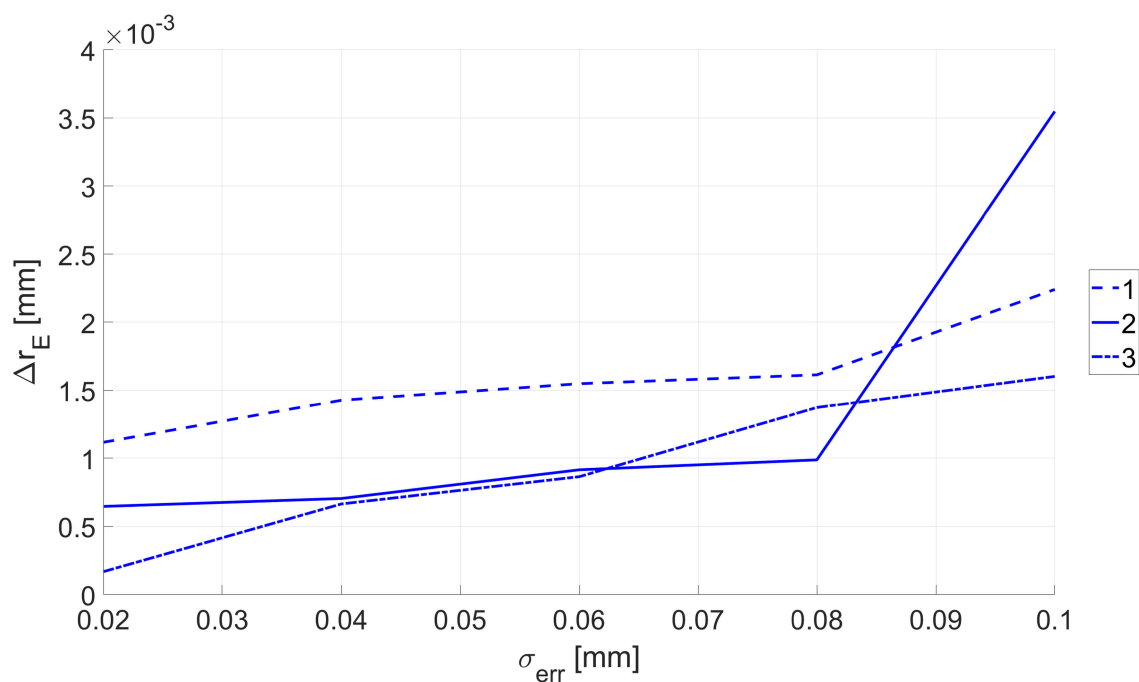
**b) Example of quality numbers for equivalent
conicity as a function of the error level**

Key

- 1 Results for profile combination wheel A – rail A
- 2 Results for profile combination wheel B – rail A
- 3 Results for profile combination wheel C – rail A

Figure 8 — Example of assessment of the smoothing process

The quality number for the rolling radii coefficient is derived from the rolling radius difference functions directly using the rolling radius differences at point E (Δr_E). It is given by the 95 %-value of the absolute differences between the solutions for the disturbed and the reference profiles and is also presented in a diagram as a function of the error level, see Figure 9.

**Key**

- 1 Results for profile combination wheel A – rail A
- 2 Results for profile combination wheel B – rail A
- 3 Results for profile combination wheel C – rail A

Figure 9 — Example of quality numbers for rolling radius differences at point E (Δr_E) as a function of the error level

The results achieved shall be provided in the documentation of the software.

13 Assessment of the complete process for determination of wheel-rail contact parameters

13.1 General

The complete process for determination of the wheel-rail contact parameters from measured data is assessed in several stages:

- the calculation algorithms for equivalent conicity and for rolling radii coefficient are validated, using the benchmark profiles according to 12.2;
- the influence of measurement errors is checked according to 12.3;
- the complete process is evaluated for reproducibility in a field test.

The result of the reproducibility evaluation is an estimate of the uncertainty of the complete process.

The number of measurements to be used for assessing the reproducibility is dependent on the type of measuring system (manual or automatic) but the principles are the same.

Depending on the intended application, the evaluation for the complete process will include assessment of equivalent conicity and/or rolling radii coefficient derived from:

- measured rail profiles and reference wheel profiles;
- reference rail profiles and measured wheel profiles; and/or
- measured rail profiles and measured wheel profiles.

Other wheel-rail contact parameters may also be considered if required.

For the evaluation of equivalent conicity, see 9.2, the value for an amplitude $\hat{y} = 3$ mm is proposed but a different value can be selected if relevant for particular applications.

For the evaluation of the rolling radii coefficient, see Clause 10, the Δr_E value is proposed.

The selection of a reference wheel profile for assessing the track system or a reference rail profile for assessing the wheel system shall be carefully considered so that the influence of the measured profiles can be clearly seen in the resulting contact parameters. For example, when assessing the track system, a conical or cylindrical wheel profile will not provide useful information.

The final evaluation of the contact parameters, e.g. calculation of a sliding mean over 100 m, shall be carried out according to the intended application.

13.2 Reproducibility of contact parameter determination based on rail profile measurement

13.2.1 Manual rail profile measuring devices

In order to determine the reproducibility of the wheel-rail contact parameter evaluations based on field measurements of rail profiles, the results (e.g. equivalent conicity, rolling radii coefficient) of several rail profile measurements on the same track location shall be compared with each other. The selected track locations shall include a wide range of rail wear conditions.

The following conditions shall be covered for each track location where applicable:

- measurement by different trained users (at least three users are recommended);
- both orientations of the measuring system (use of both measuring heads for twinhead systems).

Any conditions not covered by the investigation shall be justified in the report.

After each measurement the measuring system shall be completely removed, turned by 180 degrees and placed again on the track.

In order to get reliable results of the statistical analysis a total of at least 40 measurements is required including a minimum number of 3 track locations and 12 measurements per track location.

The number and characteristics of track locations and the number of measurements considered in the study shall be reported.

To estimate the reproducibility of the determination of contact parameters, the differences to the mean value of each considered parameter shall be calculated at every measuring location. After that the 95th percentile of the absolute values of all differences shall be calculated as an estimate of the uncertainty of the complete process.

13.2.2 Vehicle based rail profile measuring systems

In order to determine the reproducibility of the wheel-rail contact parameter evaluations based on field measurements of rail profiles, the results (e.g. equivalent conicity, rolling radii coefficient) of several rail profile measurements on the same track sections shall be compared with each other.

For vehicle based measuring systems the track sections shall cover a wide range of track design features (curves of various radii and directions, significant cant) and rail wear conditions. For each track condition the total length shall be at least 200 m and contain results from at least 50 measured cross sections. The total length of all track sections is recommended to be at least 5 km.

The following conditions shall be varied for each track section when applicable for the measuring system:

- measuring speed including at least the minimum and maximum possible measuring speeds;
- both measuring directions (forwards/backwards);
- both orientations of the measuring system (turned by 180°).

Any conditions not covered by the investigation shall be justified in the report.

In order to get reliable results of the statistical analysis a minimum number of 8 measuring runs per track section is required.

The number and characteristics of track sections and the number of measuring runs considered in the study shall be reported.

In order to reach sufficient agreement it shall be ensured that the repeated rail profile measurements to be compared are carried out at the same location of the track with a reasonable tolerance. Therefore a synchronization of the measuring runs is necessary.

NOTE 1 For synchronization measured track gauge signals are often used. Also additional location markers are useful in order to improve the localization accuracy. Subdividing the track sections in parts of e.g. 200 m length each can also be useful for improving synchronization.

To estimate the reproducibility of the determination of contact parameters, the differences to the mean value of each considered parameter shall be calculated at every measuring point of the synchronized data sets. After that the 95th percentile of the absolute values of all differences shall be calculated as an estimate of the uncertainty of the complete process.

NOTE 2 Plotting the frequency distribution of the calculated differences gives a better understanding of the measuring system's characteristics.

NOTE 3 Typical values for uncertainty of the complete process for determination of equivalent conicity lie below 0,05.

13.3 Reproducibility of contact parameter determination based on wheel profile measurement

13.3.1 Manual wheel profile measuring devices

In order to determine the reproducibility of the wheel-rail contact parameter evaluations based on measurements of wheel profiles in normal operational conditions, the results (e.g. equivalent conicity, rolling radii coefficient) of several wheel profile measurements on the same wheelset shall be compared with each other. The selected wheelsets shall include a wide range of wheel wear conditions.

The following conditions shall be covered for each wheelset where applicable:

- measurement by different trained users (at least three users are recommended);
- both orientations of the measuring system (use of both measuring heads for twinhead systems).

Any conditions not covered by the investigation shall be justified in the report.

After each measurement the measuring system shall be completely removed, turned by 180° and placed again on the wheelset.

In order to get reliable results of the statistical analysis a total of at least 40 measurements is required including a minimum number of 3 wheelsets and 12 measurements per wheelset.

The number of wheelsets, the measurement conditions and the number of measurements considered in the study shall be reported.

To estimate the reproducibility of the determination of contact parameters, the differences to the mean value of each considered parameter shall be calculated for every wheelset. After that the 95th percentile of the absolute values of all differences shall be calculated as an estimate of the uncertainty of the complete process.

NOTE If the measuring conditions are significantly different from the conditions of the reproducibility test the achievable uncertainty values may be affected.

13.3.2 Ground based wheel profile measuring systems

In order to determine the reproducibility of the wheel-rail contact parameter evaluations based on in-service measurements of wheel profiles, the results (e.g. equivalent conicity, rolling radii coefficient) of several wheel profile measurements on the same wheelsets shall be compared with each other.

For ground based measuring systems the wheelsets shall cover a wide range of wheelset types (vehicle type, axle load and wheel diameter where applicable) and wheel wear conditions. A minimum of 10 wheelsets shall be used.

The following conditions shall be varied for each wheelset when applicable for the measuring system:

- measuring speed including at least the minimum and maximum possible measuring speeds;
- both measuring directions (forwards/backwards);
- both orientations of the vehicle (turned by 180°).

Any conditions not covered by the investigation shall be justified in the report.

In order to get reliable results of the statistical analysis a minimum number of 8 measuring runs per wheelset is required.

The number and characteristics of wheelsets and the number of measuring runs considered in the study shall be reported.

To estimate the reproducibility of the determination of contact parameters, the differences to the mean value of each considered parameter shall be calculated for every wheelset. After that the 95th percentile of the absolute values of all differences shall be calculated as an estimate of the uncertainty of the complete process.

NOTE 1 Plotting the frequency distribution of the calculated differences gives a better understanding of the measuring system's characteristics.

NOTE 2 Typical values for uncertainty of the complete process for determination of equivalent conicity lie below 0,05.

Annex A
(informative)

Example of presentation of contact geometry functions

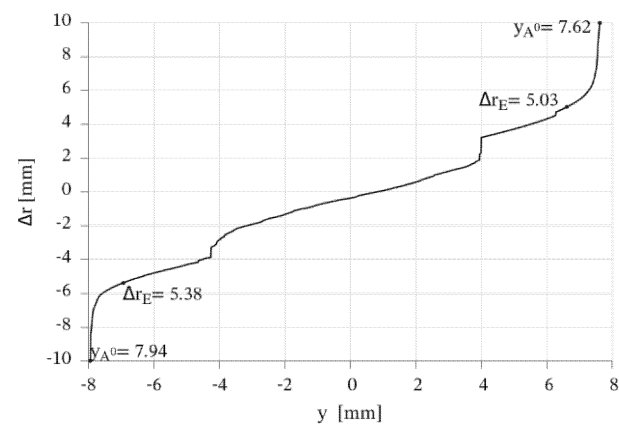


Figure A.1 — Rolling radius difference function $\Delta r = f(y)$

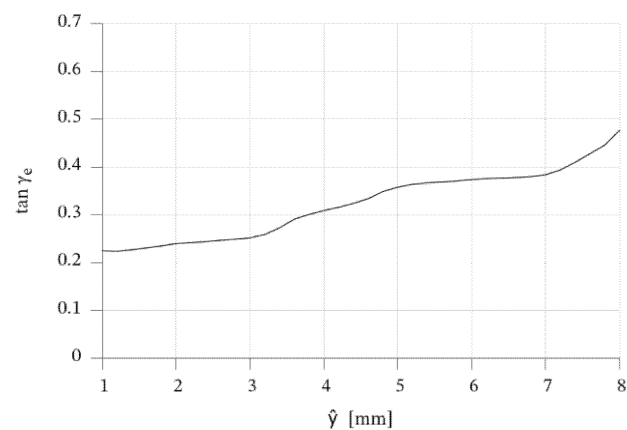


Figure A.2 — Equivalent conicity function $\tan \gamma_e = f(y)$

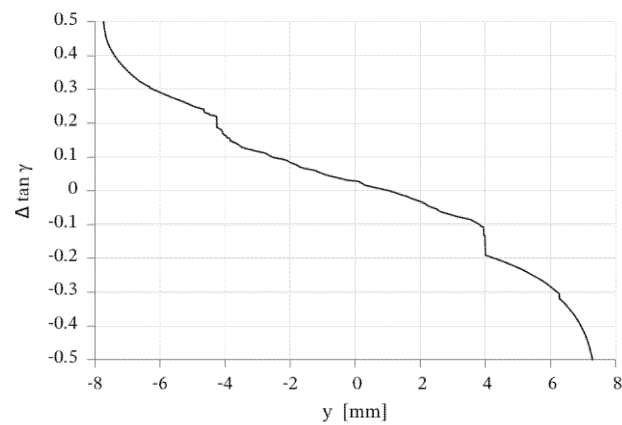


Figure A.3 — Function of difference of the tangents of the contact angles $\Delta \tan \gamma = f(y)$

Annex B (informative)

Derivation of the kinematic equation of wheelset motion

The movement of the wheelset's centre of gravity on the track (Figure B.1 and Figure B.2) can be formalized on the basis of the angle:

$$\Psi = \frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} \quad (\text{B.1})$$

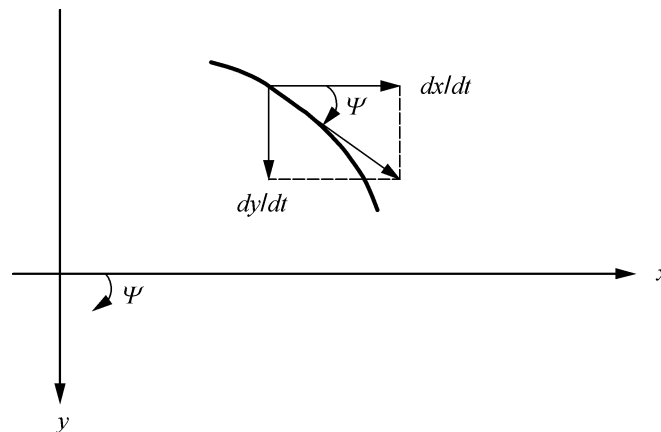


Figure B.1 — Representation of dx, dy

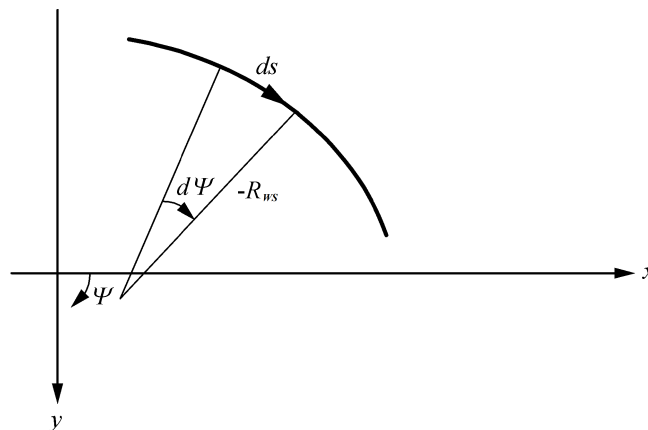


Figure B.2 — Representation of ds, dΨ

$$ds = -R_{ws} d\Psi$$

for a small angle Ψ , $ds \cong dx$

$$dx = -R_{ws} d\Psi \quad (\text{B.2})$$

and, taking into account Formula (B.1), this yields:

$$\psi = \frac{dy}{dx} = - \frac{dy}{R_{WS} d\psi}$$

hence

$$\psi d\psi = - \frac{dy}{R_{WS}} \quad (\text{B.3})$$

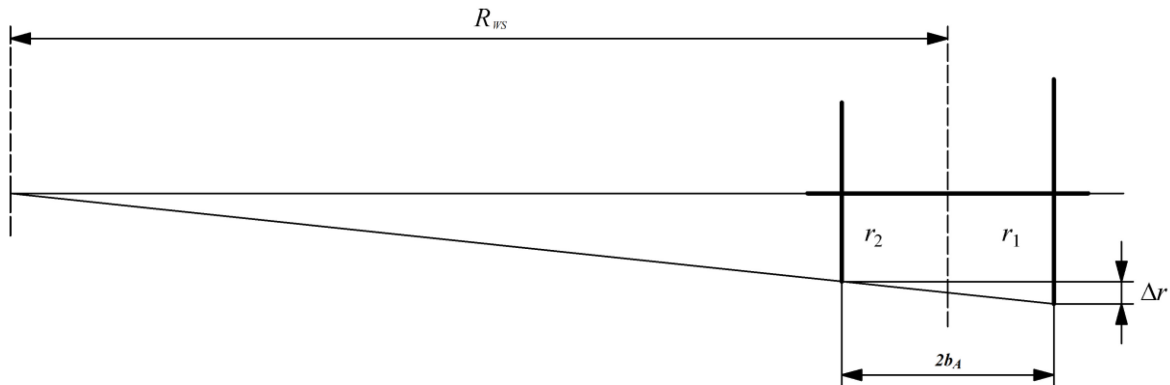


Figure B.3 — Representation of r_1, r_2, b_A

From Figure B.3 follows:

$$\frac{R_{WS}}{2b_A} = \frac{r_1 + r_2}{r_1 - r_2}$$

where

$$\frac{r_1 + r_2}{2} = r_0$$

$$r_1 - r_2 = \Delta r$$

Thus:

$$R_{WS} = \frac{2b_A r_0}{\Delta r} \quad (\text{B.4})$$

and, by replacing R_{WS} in (B.3), we get:

$$\psi d\psi = \frac{-\Delta r(y) dy}{2b_A r_0} \quad (\text{B.5})$$

$$\frac{\psi}{dy} d\psi + \frac{\Delta r(y)}{2b_A r_0} = 0$$

with:

$$\psi = \frac{dy}{dx}$$

and

$$\frac{d\Psi}{dx} = \frac{d^2 y}{dx^2}$$

the kinematic equation of the wheelset motion becomes

$$\frac{d^2 y}{dx^2} + \frac{\Delta r(y)}{2b_A r_0} = 0 \quad (\text{B.6})$$

Annex C (informative)

Determination of the lateral peak displacements

The solution of the equation of motion described in Annex B is a periodic movement with peak displacements at y_{emax} and y_{emin} and an amplitude $\hat{y} = (y_{emax} - y_{emin}) / 2$. Some methods for computing the equivalent conicity require y_{emax} and y_{emin} as functions of $\hat{y}$. This is straightforward for symmetric profiles, where $y_{emax} = -y_{emin} = \hat{y}$. In the case of asymmetric profiles, these functions can be computed with the help of Formula (B.5):

$$\Psi d\Psi = -\frac{1}{2b_A r_0} \Delta r(y) dy.$$

Integration from a lateral displacement y_1 to y_2 gives

$$\frac{1}{2}\Psi_2^2 - \frac{1}{2}\Psi_1^2 = -\frac{1}{2b_A r_0} \int_{y_1}^{y_2} \Delta r(y) dy.$$

A half wave between two peaks runs from $y_1 = y_{emin}$, $\Psi_1 = 0$ to $y_2 = y_{emax}$, $\Psi_2 = 0$. With this, y_{emax} can be determined as the value of y where the integral of Δr , starting at y_{emin} , becomes zero. It is also possible to compute the integral once over the whole available range of y , and then find corresponding pairs of y_{emin} and y_{emax} at identical values of the integral.

This process is illustrated in Figure C.1 for benchmark case “Modified Wheel A (–2 mm on left wheel diameter) / Rail A”. The topmost diagram plots the Δr function against the lateral displacement y . The central diagram shows the integral of the Δr function, shifted along the ordinate in order to be zero at the minimum, which corresponds to $\Delta r = 0$. The horizontal lines connect corresponding pairs of y_{emin} and y_{emax} . The midpoints of these lines are indicated by dots. The same lines are plotted again in the lowermost diagram, but with the amplitude $\hat{y} = (y_{emax} - y_{emin}) / 2$ on the ordinate. This leads to the desired relation between $\hat{y}$ and y_{emin} and y_{emax} . The output values of y_{emin} and y_{emax} are typically interpolated at $\hat{y}$ -steps of 0,1 mm.

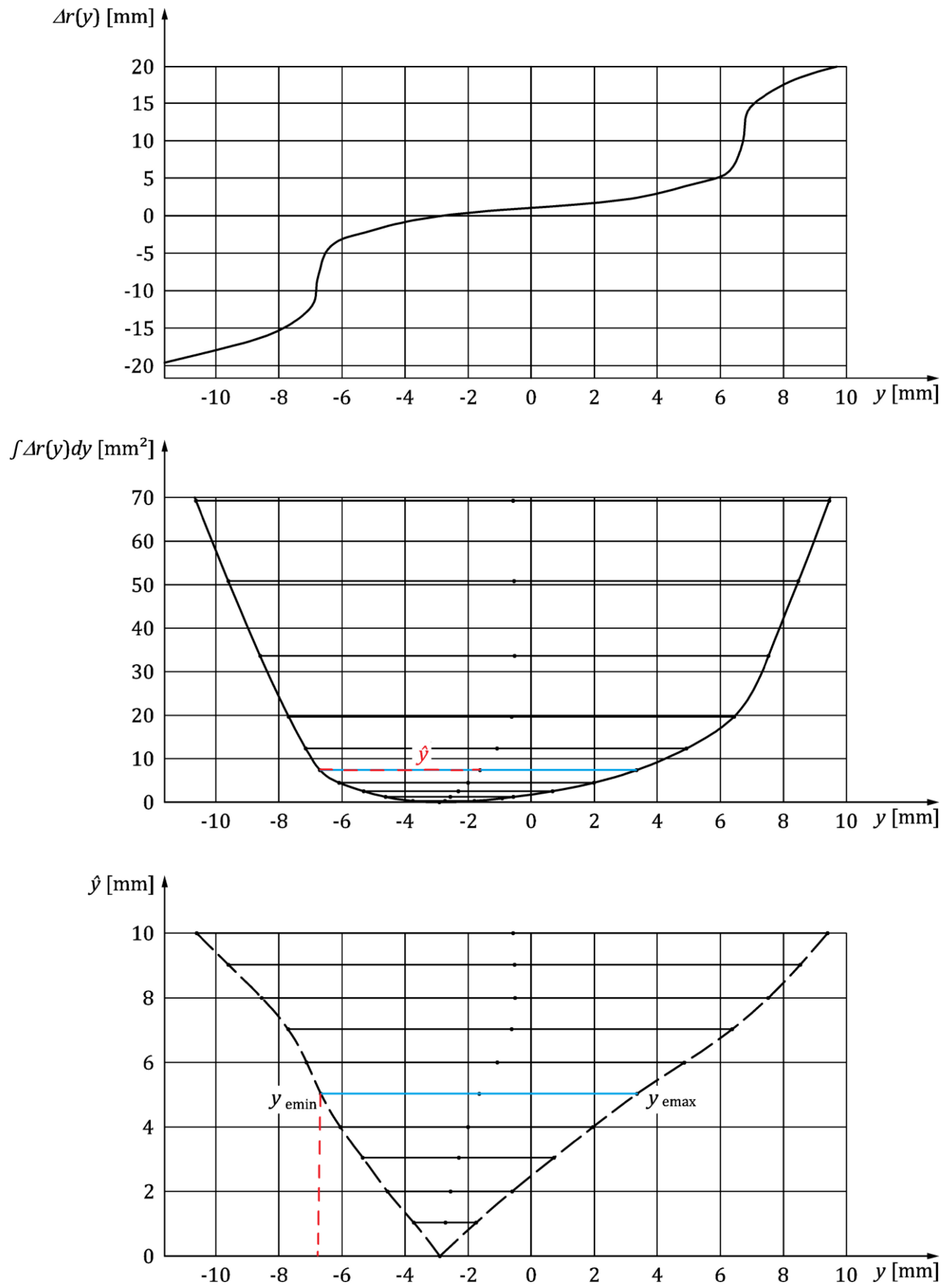


Figure C.1 — Illustration of determination of amplitudes

Annex D (informative)

Method for determining the wavelength of the wheelset motion by two-step integration of the nonlinear differential equation

D.1 General

This annex presents a method to compute the wavelength of the lateral wheelset motion by two steps of numerical processing:

- 1) Compute $\psi(y)$ using Formula (B.5);
- 2) Compute $x(y)$ by inserting $\psi(y)$ into Formula (B.1) and determine the wavelength of the wheelset motion.

These steps are described in more detail below.

D.2 Step 1

This step is similar to the computation of the amplitudes in Annex C. Integrating Formula (B.5).

$$\psi d\psi = -\frac{1}{2b_A r_0} \Delta r(y) dy$$

from y_1 to y gives:

$$\frac{1}{2}\psi^2 - \frac{1}{2}\psi_1^2 = -\frac{1}{2b_A r_0} \int_{y_1}^y \Delta r(y) dy.$$

and with initial conditions $y_1 = y_{emin}$, $\psi_1 = 0$:

$$\psi(y) = \sqrt{-\frac{2}{2b_A r_0} \int_{y_{emin}}^y \Delta r(y) dy}$$

D.3 Step 2

From Formula (B.1) it follows:

$$dx = \frac{1}{\psi(y)} dy$$

and

$$x_2 - x_1 = \int_{y_1}^{y_2} \frac{1}{\psi(y)} dy$$

A half wave between two peaks runs from $y_1 = y_{emin}$, $x_1 = 0$ to $y_2 = y_{emax}$, $x_2 = \lambda / 2$. With this, the wavelength λ can be determined as twice the integral of $1 / \psi(y)$ over the range of y within half a cycle:

$$\lambda = 2 \int_{y_{emin}}^{y_{emax}} \frac{1}{\psi(y)} dy$$

Since $\psi(y) = 0$ at y_{emin} and y_{emax} , the integrand becomes infinite at the end points of the integration interval. This is called an improper integral. It can be solved by numerical integration (quadrature) as pointed out for a single wave in the following code written in MATLAB¹ language.

```
function lambda = integral_invpsi(y,psi2)
% Solution of integral of 1/psi for computation of wavelength
%
% Input:
%   y      ... vector of y from yemin to yemax
%   psi2 ... vector of psi^2 corresponding to y (result of step 1)
% Output:
%   lambda ... wavelength corresponding to yemin and yemax
% Intrinsic MATLAB functions used:
%   integral ... numerical quadrature
%   interp1 ... interpolation (here set to linear interpolation)
%   sqrt ... square root
function_to_call = @invpsi_y;
lambda = 2 * integral(function_to_call, y(1), y(end));
function invpsi = invpsi_y(yi)
invpsi = 1 ./ sqrt(interp1(y, psi2, yi, 'linear')) ;
end
end
```

While improper integrals with an integrand of $1/x$ are diverging at $x = 0$, they are converging with an integrand of $1/\sqrt{x}$. Hence, the square of ψ (denoted psi2), which can be directly obtained in step 1, is being used in the code. The square root is taken after a linear interpolation in order to obtain the converging form at the end points.

Alternatively, instead of using numerical quadrature as above with the integral function, a trapezoidal integration may also be used. In this case, care shall be taken that the integrand does not become infinite. This can be achieved through the approximation of $1/\psi(y_{emin})$ by $1/\psi(y_{emin} + \varepsilon)$ and $1/\psi(y_{emax})$ by $1/\psi(y_{emax} - \varepsilon)$ with $\varepsilon \ll 0,01$ mm. However, since the result obtained in this way will depend on the value of ε , this method may be unreliable, and it may be less efficient than the numerical quadrature described above.

The integral shall be solved for each amplitude of the lateral wheelset motion in order to find the respective wavelengths of the wheelset's kinematic motion.

¹ MATLAB[®] is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by CEN or CENELEC of this product.

Annex E (informative)

Method for determining the wavelength of the wheelset motion by direct integration of the nonlinear differential equation

This method is based on the numerical integration of the differential equation of kinematic wheelset motion in the distance domain, as given in Formula (B.6). This second order differential equation shall be transformed into a system of first order differential equations and solved by means of numerical methods such as Runge-Kutta. In any case, it is recommended to use adaptive step-size control in order to reduce computation effort and rounding errors.

The initial conditions for the integration are normally taken as $y = y_{\text{emax}}$ and $\dot{y} = 0$. In order to determine the wavelength of the wheelset movement, the integration interval shall cover at least half of a cycle until $\dot{y} = 0$ again. If other initial conditions are taken it may be necessary to compute the full cycle. A good check of the selected error tolerances is to compute the full cycle and compare the initial lateral position with the final one.

The result of the numerical integration is the distance history of the lateral wheelset motion $y = f(x)$ and the wavelength λ can be derived from this.

For this method it is not essential to compute the range of lateral displacements beforehand as the amplitude is an output of the calculation. However, if the equivalent conicity is required for distinct amplitudes it may increase the efficiency of the calculation if the related lateral displacement ranges are determined according to Annex C.

Annex F (informative)

Method for determining the equivalent conicity by linear regression of the Δr function

F.1 General

For a linear Δr function the slope of the Δr function is equal to $2\tan\gamma_e$.

For a nonlinear Δr function the slope of a linear regression gives an approximation of $2\tan\gamma_e$.

For a given amplitude $\hat{y}$ the linear regression of the Δr function is calculated between the minimum and maximum lateral displacements y_{emin} and y_{emax} determined as described in Annex C. The equivalent conicity is derived from:

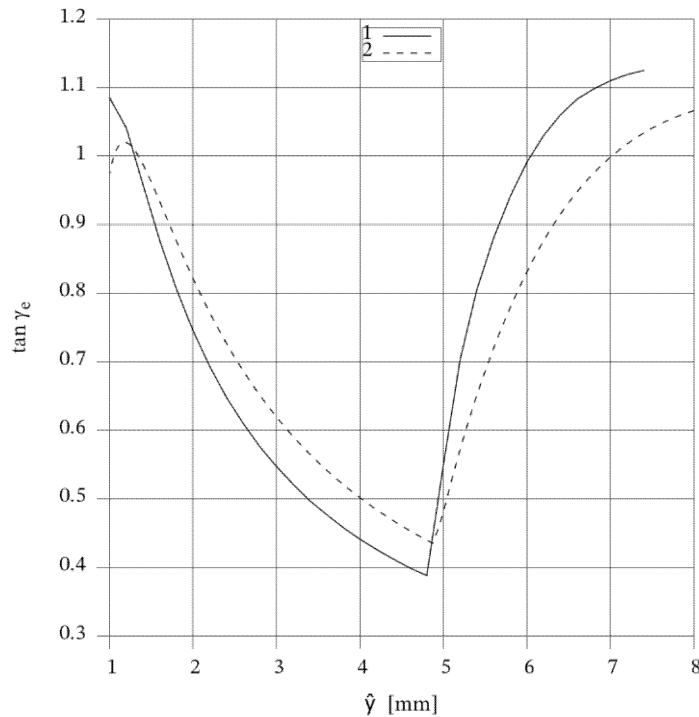
$$\tan\gamma_e(\hat{y}) = \frac{B}{2}$$

where

B is the slope of the regression.

F.2 Concerns regarding the method

The approximation by linear regression may give inaccurate results if the Δr function is strongly nonlinear. Figure F.1 presents such an example. In these cases, the use of this method is not recommended.

**Key**

- 1 analytical solution
- 2 solution of linear regression

Figure F.1 — Comparison of conicity functions

For particular types of asymmetric Δr functions it may be possible to adapt the method to improve the accuracy. For example if the $\Delta r = f(\gamma)$ characteristic presents a substantial asymmetry which is similar to two straight line segments either side of the midpoint, it is possible to apply a regression to each segment to obtain two conicities $\tan\gamma_{e,p}$ and $\tan\gamma_{e,n}$. The equivalent conicity can then be obtained by:

$$\tan\gamma_e = \left[\frac{2}{\frac{1}{\sqrt{\tan\gamma_{e,p}}} + \frac{1}{\sqrt{\tan\gamma_{e,n}}}} \right]^2$$