



BSI Standards Publication

Eurocode 3 — Design of steel structures

Part 1-6: Strength and Stability of Shell Structures

National foreword

This British Standard is the UK implementation of EN 1993-1-6:2007+A1:2017, incorporating corrigendum April 2009. It supersedes BS EN 1993-1-6:2007, which is withdrawn.

The start and finish of text introduced or altered by corrigendum is indicated in the text by tags. Text altered by CEN corrigendum April 2009 is indicated in the text by **AC1** **AC1**.

The start and finish of text introduced or altered by amendment is indicated in the text by tags. Tags indicating changes to CEN text carry the number of the CEN amendment. For example, text altered by CEN amendment A1 is indicated by **A1** **A1**.

The structural Eurocodes are divided into packages by grouping Eurocodes for each of the main materials: concrete, steel, composite concrete and steel, timber, masonry and aluminium; this is to enable a common date of withdrawal (DOW) for all the relevant parts that are needed for a particular design. The conflicting national standards will be withdrawn at the end of the co-existence period, after all the EN Eurocodes of a package are available.

Following publication of the EN, there is a period allowed for national calibration during which the National Annex is issued, followed by a coexistence period of a maximum three years. During the co-existence period Member States are encouraged to adapt their national provisions. At the end of this co-existence period, the conflicting parts of national standard(s) will be withdrawn.

In the UK there are no conflicting national standards.

Where a normative part of this EN allows for a choice to be made at the national level, the range and possible choice will be given in the normative text as Recommended Values, and a note will qualify it as a Nationally Determined Parameter (NDP). NDPs can be a specific value for a factor, a specific level or class, a particular method or a particular application rule if several are proposed in the EN.

UK National Annex to BS EN 1993-1-6

To enable EN 1993-1-6 to be used in the UK, the committee has decided that no National Annex will be issued and recommend the following:

- all the Recommended Values should be used;
- all Informative Annexes may be used; and
- no NCCI have currently been identified.

The UK participation in its preparation was entrusted to Technical Committee CB/203, Design & execution of steel structures.

A list of organizations represented on this committee can be obtained on request to its committee manager.

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Date	Text affected
28 February 2010	Implementation of CEN corrigendum April 2009, and correction to national foreword
31 December 2017	Implementation of CEN amendment A1:2017
31 December 2023	Correction to formulae (8.14) and (8.27), and correction to national foreword

English Version

Eurocode 3 - Design of steel structures - Part 1-6: Strength and Stability of Shell Structures

Eurocode 3 - Calcul des structures en acier - Partie 1-6:
Résistance et stabilité des structures en coque

Eurocode 3 - Bemessung und Konstruktion von
Stahlbauten - Teil 1-6: Festigkeit und Stabilität von Schalen

This European Standard was approved by CEN on 12 June 2006.

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Foreword

This European Standard EN 1993-1-6, Eurocode 3: Design of steel structures: Part 1-6 Strength and stability of shell structures, has been prepared by Technical Committee CEN/TC250 «Structural Eurocodes», the Secretariat of which is held by BSI. CEN/TC250 is responsible for all Structural Eurocodes.

This European Standard shall be given the status of a National Standard, either by publication of an identical text or by endorsement, at the latest by August 2007, and conflicting National Standards shall be withdrawn at latest by March 2010.

This Eurocode supersedes ENV 1993-1-6.

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Foreword to amendment A1

This document (EN 1993-1-6:2007/A1:2017) has been prepared by Technical Committee CEN/TC 250 “Structural Eurocodes”, the secretariat of which is held by BSI.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by April 2018, and conflicting national standards shall be withdrawn at the latest by April 2018.

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National annex for EN 1993-1-6

This standard gives alternative procedures, values and recommendations with notes indicating where national choices may have to be made. Therefore the National Standard implementing EN 1993-1-6 should have a National Annex containing all Nationally Determined Parameters to be used for the design of steel structures to be constructed in the relevant country.

National choice is allowed in EN 1993-1-6 through:

- 3.1.(4)
- 4.1.4 (3)
- 5.2.4 (1)
- **A1** 6.2.1 (6) **A1**
- 6.3 (5)
- 7.3.1 (1)
- 7.3.2 (1)
- 8.4.2 (3)
- 8.4.3 (2)
- 8.4.3 (4)
- 8.4.4 (4)
- 8.4.5 (1)
- 8.5.2 (2)
- 8.5.2 (4)
- **A1** 8.6.3 (5) **A1**
- 8.8.2 (9)
- 8.8.2 (18)
- 8.8.2 (20) (2 times)
- 9.2.1 (2)P

1. General

1.1 Scope

- (1) EN 1993-1-6 gives basic design rules for plated steel structures that have the form of a shell of revolution.
- (2) This Standard is intended for use in conjunction with EN 1993-1-1, EN 1993-1-3, EN 1993-1-4, EN 1993-1-9 and the relevant application parts of EN 1993, which include:
 - Part 3.1 for towers and masts;
 - Part 3.2 for chimneys;
 - Part 4.1 for silos;
 - Part 4.2 for tanks;
 - Part 4.3 for pipelines.
- (3) This Standard defines the characteristic and design values of the resistance of the structure.
- (4) This Standard is concerned with the requirements for design against the ultimate limit states of:
 - plastic limit;
 - cyclic plasticity;
 - buckling;
 - fatigue.
- (5) Overall equilibrium of the structure (sliding, uplifting, overturning) is not included in this Standard, but is treated in EN 1993-1-1. Special considerations for specific applications are included in the relevant application parts of EN 1993.
- (6) The provisions in this Standard apply to axisymmetric shells and associated circular or annular plates and to beam section rings and stringer stiffeners where they form part of the complete structure. General procedures for computer calculations of all shell forms are covered. Detailed expressions for the hand calculation of unstiffened cylinders and cones are given in the Annexes.
- (7) Cylindrical and conical panels are not explicitly covered by this Standard. However, the provisions can be applicable if the appropriate boundary conditions are duly taken into account.
- (8) This Standard is intended for application to steel shell structures. Where no standard exists for shell structures made of other metals, the provisions of this standards may be applied provided that the appropriate material properties are duly taken into account.
- (9) The provisions of this Standard are intended to be applied within the temperature range defined in the relevant EN 1993 application parts. The maximum temperature is restricted so that the influence of creep can be neglected if high temperature creep effects are not covered by the relevant application part.
- (10) The provisions in this Standard apply to structures that satisfy the brittle fracture provisions given in EN 1993-1-10.
- (11) The provisions of this Standard apply to structural design under actions that can be treated as quasi-static in nature.
- (12) In this Standard, it is assumed that both wind loading and bulk solids flow can, in general, be treated as quasi-static actions.
- (13) Dynamic effects should be taken into account according to the relevant application part of EN 1993, including the consequences for fatigue. However, the stress resultants arising from dynamic behaviour are treated in this part as quasi-static.
- (14) The provisions in this Standard apply to structures that are constructed in accordance with EN 1090-2.
- (15) This Standard does not cover the aspects of leakage.

(16) This Standard is intended for application to structures within the following limits:

design metal temperatures within the range -50°C to $+300^{\circ}\text{C}$;
radius to thickness ratios within the range 20 to 5000.

NOTE: It should be noted that the stress design rules of this standard may be rather conservative if applied to some geometries and loading conditions for relatively thick-walled shells.

1.2 Normative references

(1) This European Standard incorporates, by dated or undated reference, provisions from other publications. These normative references are cited at the appropriate places in the text and the publications are listed hereafter. For dated references, subsequent amendments to or revisions of any of these publications apply to this European Standard only when incorporated in it by amendment or revision. For undated references the latest edition of the publication referred to applies.

EN 1090-2	<i>Execution of steel structures and aluminium structures – Part 2: Technical requirements for steel structures;</i>
EN 1990	<i>Basis of structural design;</i>
EN 1991	<i>Eurocode 1: Actions on structures;</i>
EN 1993	<i>Eurocode 3: Design of steel structures:</i>
[A1] Part 1.1:2005 [A1]	<i>General rules and rules for buildings;</i>
Part 1.3:	<i>Cold formed thin gauged members and sheeting;</i>
Part 1.4:	<i>Stainless steels;</i>
Part 1.5:	<i>Plated structural elements;</i>
Part 1.9:	<i>Fatigue strength of steel structures;</i>
Part 1.10:	<i>Selection of steel for fracture toughness and through-thickness properties;</i>
Part 1.12:	<i>Additional rules for the extension of EN 1993 up to steel grades S 700</i>
Part 2:	<i>Steel bridges;</i>
Part 3.1:	<i>Towers and masts;</i>
Part 3.2:	<i>Chimneys;</i>
Part 4.1:	<i>Silos;</i>
Part 4.2:	<i>Tanks;</i>
Part 4.3:	<i>Pipelines;</i>
Part 5:	<i>Piling.</i>

1.3 Terms and definitions

The terms that are defined in EN 1990 for common use in the Structural Eurocodes apply to this Standard. Unless otherwise stated, the definitions given in ISO 8930 also apply in this Standard. Supplementary to EN 1993-1-1, for the purposes of this Standard, the following definitions apply:

1.3.1 Structural forms and geometry

1.3.1.1 shell

A structure or a structural component formed from a curved thin plate.

1.3.1.2 shell of revolution

A shell whose geometric form is defined by a middle surface that is formed by rotating a meridional generator line around a single axis through 2π radians. The shell can be of any length.

1.3.1.3 complete axisymmetric shell

A shell composed of a number of parts, each of which is a shell of revolution.

1.3.1.4 shell segment

A shell of revolution in the form of a defined shell geometry with a constant wall thickness: a cylinder, conical frustum, spherical frustum, annular plate, toroidal knuckle or other form.

1.3.1.5 shell panel

An incomplete shell of revolution: the shell form is defined by a rotation of the generator about the axis through less than 2π radians.

1.3.1.6 middle surface

The surface that lies midway between the inside and outside surfaces of the shell at every point. Where the shell is stiffened on either one or both surfaces, the reference middle surface is still taken as the middle surface of the curved shell plate. The middle surface is the reference surface for analysis, and can be discontinuous at changes of thickness or at shell junctions, leading to eccentricities that may be important to the shell structural behaviour.

1.3.1.7 junction

The line at which two or more shell segments meet: it can include a stiffener. The circumferential line of attachment of a ring stiffener to the shell may be treated as a junction.

1.3.1.8 stringer stiffener

A local stiffening member that follows the meridian of the shell, representing a generator of the shell of revolution. It is provided to increase the stability, or to assist with the introduction of local loads. It is not intended to provide a primary resistance to bending effects caused by transverse loads.

1.3.1.9 rib

A local member that provides a primary load carrying path for bending down the meridian of the shell, representing a generator of the shell of revolution. It is used to transfer or distribute transverse loads by bending.

1.3.1.10 ring stiffener

A local stiffening member that passes around the circumference of the shell of revolution at a given point on the meridian. It is normally assumed to have no stiffness for deformations out of its own plane (meridional displacements of the shell) but is stiff for deformations in the plane of the ring. It is provided to increase the stability or to introduce local loads acting in the plane of the ring.

1.3.1.11 base ring

A structural member that passes around the circumference of the shell of revolution at the base and provides a means of attachment of the shell to a foundation or other structural member. It is needed to ensure that the assumed boundary conditions are achieved in practice.

1.3.1.12 ring beam or ring girder

A circumferential stiffener that has bending stiffness and strength both in the plane of the shell circular section and normal to that plane. It is a primary load carrying structural member, provided for the distribution of local loads into the shell.

1.3.2 Limit states**A1 1.3.2.1 plastic failure limit state (LS1)**

The ultimate limit state where the structure develops zones of yielding in a pattern such that its ability to resist increased loading is deemed to be exhausted. A1

1.3.2.2 tensile rupture

The ultimate limit state where the shell plate experiences gross section failure due to tension.

1.3.2.3 cyclic plasticity

The ultimate limit state where repeated yielding is caused by cycles of loading and unloading, leading to a low cycle fatigue failure where the energy absorption capacity of the material is exhausted.

1.3.2.4 buckling

The ultimate limit state where the structure suddenly loses its stability under membrane compression and/or shear. It leads either to large displacements or to the structure being unable to carry the applied loads.

1.3.2.5 fatigue

The ultimate limit state where many cycles of loading cause cracks to develop in the shell plate that by further load cycles may lead to rupture.

1.3.3 Actions

1.3.3.1 axial load

Externally applied loading acting in the axial direction.

1.3.3.2 radial load

Externally applied loading acting normal to the surface of a cylindrical shell.

1.3.3.3 internal pressure

Component of the surface loading acting normal to the shell in the outward direction. Its magnitude can vary in both the meridional and circumferential directions (e.g. under solids loading in a silo).

1.3.3.4 external pressure

Component of the surface loading acting normal to the shell in the inward direction. Its magnitude can vary in both the meridional and circumferential directions (e.g. under wind).

1.3.3.5 hydrostatic pressure

Pressure varying linearly with the axial coordinate of the shell of revolution.

1.3.3.6 wall friction load

Meridional component of the surface loading acting on the shell wall due to friction connected with internal pressure (e.g. when solids are contained within the shell).

1.3.3.7 local load

Point applied force or distributed load acting on a limited part of the circumference of the shell and over a limited height.

1.3.3.8 patch load

Local distributed load acting normal to the shell.

1.3.3.9 suction

Uniform net external pressure due to the reduced internal pressure in a shell with openings or vents under wind action.

1.3.3.10 partial vacuum

Uniform net external pressure due to the removal of stored liquids or solids from within a container that is inadequately vented.

1.3.3.11 thermal action

Temperature variation either down the shell meridian, or around the shell circumference or through the shell thickness.

1.3.4 Stress resultants and stresses in a shell**1.3.4.1 membrane stress resultants**

The membrane stress resultants are the forces per unit width of shell that arise as the integral of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell. Under elastic conditions, each of these stress resultants induces a stress state that is uniform through the shell thickness. There are three membrane stress resultants at any point (see figure 1.1(e)).

1.3.4.2 bending stress resultants

The bending stress resultants are the bending and twisting moments per unit width of shell that arise as the integral of the first moment of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell. Under elastic conditions, each of these stress resultants induces a stress state that varies linearly through the shell thickness, with value zero at the middle surface. There are two bending moments and one twisting moment at any point.

1.3.4.3 transverse shear stress resultants

The transverse stress resultants are the forces per unit width of shell that arise as the integral of the distribution of shear stresses acting normal to the shell middle surface through the thickness of the shell. Under elastic conditions, each of these stress resultants induces a stress state that varies parabolically through the shell thickness. There are two transverse shear stress resultants at any point (see figure 1.1(f)).

1.3.4.4 membrane stress

The membrane stress is defined as the membrane stress resultant divided by the shell thickness (see figure 1.1(e)).

1.3.4.5 bending stress

The bending stress is defined as the bending stress resultant multiplied by 6 and divided by the square of the shell thickness. It is only meaningful for conditions in which the shell is elastic.

1.3.5 Types of analysis**1.3.5.1 global analysis**

An analysis that includes the complete structure, rather than individual structural parts treated separately.

1.3.5.2 membrane theory analysis

An analysis that predicts the behaviour of a thin-walled shell structure under distributed loads by assuming that only membrane forces satisfy equilibrium with the external loads.

A1 1.3.5.3 semi-membrane theory analysis

An analysis that predicts the behaviour of an unsymmetrically loaded or supported thin-walled cylindrical shell structure by assuming that only membrane forces and circumferential bending moments satisfy equilibrium with the external loads". A1

1.3.5.4 linear elastic shell analysis (LA)

An analysis that predicts the behaviour of a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell.

1.3.5.5 linear elastic bifurcation (eigenvalue) analysis (LBA)

An analysis that evaluates the linear bifurcation eigenvalue for a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell. It should be noted that, where an eigenvalue is mentioned, this does not relate to vibration modes.

1.3.5.6 geometrically nonlinear elastic analysis (GNA)

An analysis based on the principles of shell bending theory applied to the perfect structure, using a linear elastic material law but including nonlinear large deflection theory for the displacements that accounts full for any change in geometry due to the actions on the shell. A bifurcation eigenvalue check is included at each load level.

1.3.5.7 materially nonlinear analysis (MNA)

A1 An analysis based on shell bending theory applied to the perfect structure, using the assumption of small deflections, as in **AC1** 1.3.5.4 **AC1**, but adopting an ideal elastic plastic material law (idealised perfectly plastic response after yield). **A1**

1.3.5.8 geometrically and materially nonlinear analysis (GMNA)

A1 An analysis based on shell bending theory applied to the perfect structure, using the assumptions of nonlinear large deflection theory for the displacements and a fully nonlinear elastic-plastic-hardening material law, where appropriate, and in which a bifurcation eigenvalue check is included at each load level. **A1**

1.3.5.9 geometrically nonlinear elastic analysis with imperfections included (GNIA)

An analysis with imperfections explicitly included, similar to a GNA analysis as defined in **AC1** 1.3.5.5 **AC1**, but adopting a model for the geometry of the structure that includes the imperfect shape (i.e. the geometry of the middle surface includes unintended deviations from the ideal shape). The imperfection may also cover the effects of deviations in boundary conditions and / or the effects of residual stresses. A bifurcation eigenvalue check is included at each load level.

1.3.5.10 geometrically and materially nonlinear analysis with imperfections included (GMNIA)

A1 An analysis with imperfections explicitly included, based on the principles of shell bending theory applied to the imperfect structure (i.e. the geometry of the middle surface includes unintended deviations from the ideal shape), including nonlinear large deflection theory for the displacements that accounts fully for any change in geometry due to the actions on the shell and a fully nonlinear elastic-plastic-hardening material law, where appropriate.

NOTE 1: The imperfections may also include imperfections in boundary conditions and residual stresses. A bifurcation eigenvalue check is included at each load level. **A1**

1.3.6 Stress categories used in stress design

1.3.6.1 Primary stresses

The stress system required for equilibrium with the imposed loading. This consists primarily of membrane stresses, but in some conditions, bending stresses may also be required to achieve equilibrium.

1.3.6.2 Secondary stresses

Stresses induced by internal compatibility or by compatibility with the boundary conditions, associated with imposed loading or imposed displacements (temperature, prestressing, settlement, shrinkage). These stresses are not required to achieve equilibrium between an internal stress state and the external loading.

1.3.7 Special definitions for buckling calculations

1.3.7.1 critical buckling resistance

The smallest bifurcation or limit load determined assuming the idealised conditions of elastic material behaviour, perfect geometry, perfect load application, perfect support, material isotropy and absence of residual stresses (LBA analysis).

1.3.7.2 critical buckling stress

The membrane stress associated with the critical buckling resistance.

1.3.7.3 plastic reference resistance

The plastic limit load, determined assuming the idealised conditions of rigid-plastic material behaviour, perfect geometry, perfect load application, perfect support and material isotropy (modelled using MNA analysis).

1.3.7.4 characteristic buckling resistance

The load associated with buckling in the presence of inelastic material behaviour, the geometrical and structural imperfections that are inevitable in practical construction, and follower load effects.

1.3.7.5 characteristic buckling stress

The membrane stress associated with the characteristic buckling resistance.

1.3.7.6 design buckling resistance

The design value of the buckling load, obtained by dividing the characteristic buckling resistance by the partial factor for resistance.

1.3.7.7 design buckling stress

The membrane stress associated with the design buckling resistance.

1.3.7.8 key value of the stress

The value of stress in a non-uniform stress field that is used to characterise the stress magnitudes in a buckling limit state assessment.

1.3.7.9 fabrication tolerance quality class

The category of fabrication tolerance requirements that is assumed in design, see 8.4.

1.4 Symbols

- (1) In addition to those given in EN 1990 and EN 1993-1-1, the following symbols are used:
- (2) Coordinate system, see figure 1.1:
 - r radial coordinate, normal to the axis of revolution;
 - x meridional coordinate;
 - z axial coordinate;
 - θ circumferential coordinate;
 - ϕ meridional slope: angle between axis of revolution and normal to the meridian of the shell;
- (3) Pressures:
 - p_n normal to the shell;
 - p_x meridional surface loading parallel to the shell;
 - p_θ circumferential surface loading parallel to the shell;

(4) Line forces:

P_n	load per unit circumference normal to the shell;
P_x	load per unit circumference acting in the meridional direction;
P_θ	load per unit circumference acting circumferentially on the shell;

(5) Membrane stress resultants:

n_x	meridional membrane stress resultant;
n_θ	circumferential membrane stress resultant;
$n_{x\theta}$	membrane shear stress resultant;

(6) Bending stress resultants:

m_x	meridional bending moment per unit width;
m_θ	circumferential bending moment per unit width;
$m_{x\theta}$	twisting shear moment per unit width;
q_{xn}	transverse shear force associated with meridional bending;
$q_{\theta n}$	transverse shear force associated with circumferential bending;

(7) Stresses:

σ_x	meridional stress;
σ_θ	circumferential stress;
σ_{eq}	von Mises equivalent stress (can also take negative values during cyclic loading);
$\tau, \tau_{x\theta}$	in-plane shear stress;
$\tau_{xn}, \tau_{\theta n}$	meridional, circumferential transverse shear stresses associated with bending;

(8) Displacements:

u	meridional displacement;
v	circumferential displacement;
w	displacement normal to the shell surface;
β_ϕ	meridional rotation, see 5.2.2;

(9) Shell dimensions:

d	internal diameter of shell;
L	total length of the shell;
ℓ	length of shell segment;
ℓ^g	gauge length for measurement of imperfections;
$\ell_{g\theta}$	gauge length in circumferential direction for measurement of imperfections;
ℓ_{gw}	gauge length across welds for measurement of imperfections;
ℓ_{gx}	gauge length in meridional direction for measurement of imperfections;
ℓ_R	limited length of shell for buckling strength assessment;
r	radius of the middle surface, normal to the axis of revolution;
t	thickness of shell wall;
t_{max}	maximum thickness of shell wall at a joint;
t_{min}	minimum thickness of shell wall at a joint;
t_{ave}	average thickness of shell wall at a joint;
β	apex half angle of cone;

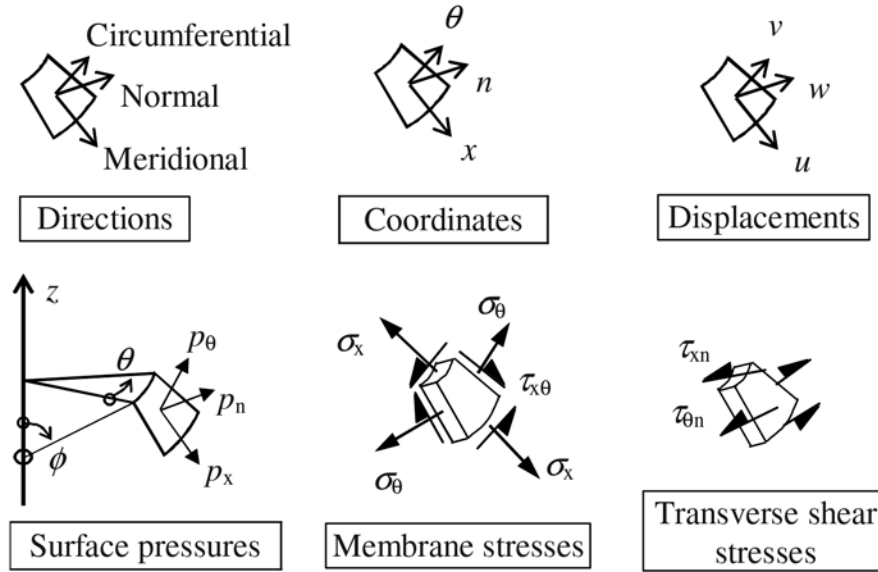


Figure 1.1: Symbols in shells of revolution

(10) Tolerances, see 8.4:

e	eccentricity between the middle surfaces of joined plates;
U_e	$\overline{AC1}$ non-intended $\overline{AC1}$ eccentricity tolerance parameter;
U_r	out-of-roundness tolerance parameter;
U_n	initial dimple imperfection amplitude parameter for numerical calculations;
U_0	initial dimple tolerance parameter;
Δw_0	tolerance normal to the shell surface;

(11) Properties of materials:

E	Young's modulus of elasticity;
f_{eq}	von Mises equivalent strength;
f_y	yield strength;
f_u	ultimate strength;
ν	Poisson's ratio;

(12) Parameters in strength assessment:

C	coefficient in buckling strength assessment;
D	cumulative damage in fatigue assessment;
F	generalised action;
F_{Ed}	action set on a complete structure corresponding to a design situation (design values);
F_{Rd}	calculated values of the action set at the maximum resistance condition of the structure (design values);
R_k	characteristic reference resistance ratio (used with subscripts to identify the basis): defined as the ratio (F_{Rk} / F_{Ed});
R_{pl}	plastic reference resistance ratio (defined as a load factor on design loads using MNA analysis);
R_{cr}	critical buckling resistance ratio (defined as a load factor on design loads using LBA analysis);
k	calibration factor for nonlinear analyses;
k	power of interaction expressions in buckling strength interaction expressions;
n	number of cycles of loading;

α	elastic buckling reduction factor in buckling strength assessment;
α_G	geometric reduction factor;
α_I	imperfection reduction factor; $\langle A_1 \rangle$
β	plastic range factor in buckling interaction;
γ	partial factor;
Δ	range of parameter when alternating or cyclic actions are involved;
ε_p	plastic strain;
η	interaction exponent for buckling;
$\bar{\lambda}$	relative slenderness of shell;
$\bar{\lambda}$	overall relative slenderness for the complete shell (multiple segments);
λ_0	squash limit relative slenderness (value of $\bar{\lambda}$ above which resistance reductions due to instability or change of geometry occur);
$\bar{\lambda}_p$	plastic limit relative slenderness (value of $\bar{\lambda}$ below which plasticity affects the stability);
ω	relative length parameter for shell;
χ	elastic-plastic buckling reduction factor for elastic-plastic effects in buckling strength assessment; $\langle A_1 \rangle$
χ_{ov}	overall elastic-plastic buckling reduction factor for a complete shell; $\langle A_1 \rangle$

(13) Subscripts:

E	value of stress or displacement (arising from design actions);
F	actions;
M	material;
R	resistance;
cr	critical buckling value;
d	design value;
int	internal;
k	characteristic value;
max	maximum value;
min	minimum value;
nom	nominal value;
pl	plastic value;
u	ultimate;
y	yield.

(14) Further symbols are defined where they first occur.

1.5 Sign conventions

(1) Outward direction positive: internal pressure positive, outward displacement positive, except as noted in (4).

(2) Tensile stresses positive, except as noted in (4).

NOTE: Compression is treated as positive in EN 1993-1-1.

(3) Shear stresses positive as shown in figures 1.1 and D.1.

(4) For simplicity, in section 8 and Annex D, compressive stresses are treated as positive. For these cases, both external pressures and internal pressures are treated as positive where they occur.

2 Basis of design and modelling

2.1 General

- (1)P The basis of design shall be in accordance with EN1990, as supplemented by the following.
- (2) In particular, the shell should be designed in such a way that it will sustain all actions and satisfy the following requirements:
- overall equilibrium;
 - equilibrium between actions and internal forces and moments, see sections 6 and 8;
 - limitation of cracks due to cyclic plastification, see section 7;
 - limitation of cracks due to fatigue, see section 9.
- (3) The design of the shell should satisfy the serviceability requirements set out in the appropriate application standard (EN1993 Parts 3.1, 3.2, 4.1, 4.2, 4.3).
- (4) The shell may be proportioned using design assisted by testing. Where appropriate, the requirements are set out in the appropriate application standard (EN1993 Parts 3.1, 3.2, 4.1, 4.2, 4.3).
- (5) All actions should be introduced using their design values according to EN 1991 and EN 1993 Parts 3.1, 3.2, 4.1, 4.2, 4.3 as appropriate.

2.2 Types of analysis

2.2.1 General

- (1) One or more of the following types of analysis should be used as detailed in section 4, depending on the limit state and other considerations:
- Global analysis, see 2.2.2;
 - Membrane theory analysis, see 2.2.3;
 - Linear elastic shell analysis, see 2.2.4;
 - Linear elastic bifurcation analysis, see 2.2.5;
 - Geometrically nonlinear elastic analysis, see 2.2.6;
 - Materially nonlinear analysis, see 2.2.7;
 - Geometrically and materially nonlinear analysis, see 2.2.8;
 - Geometrically nonlinear elastic analysis with imperfections included, see 2.2.9;
 - Geometrically and materially nonlinear analysis with imperfections included, see 2.2.10.

2.2.2 Global analysis

- (1) In a global analysis simplified treatments may be used for certain parts of the structure.

2.2.3 Membrane theory analysis

- (1) A membrane theory analysis should only be used provided that the following conditions are met:
- the boundary conditions are appropriate for transfer of the stresses in the shell into support reactions without causing significant bending effects;
 - the shell geometry varies smoothly in shape (without discontinuities);
 - the loads have a smooth distribution (without locally concentrated or point loads).
- (2) A membrane theory analysis does not necessarily fulfil the compatibility of deformations at boundaries or between shell segments of different shape or between shell segments subjected to different loading. However, the resulting field of membrane forces satisfies the requirements of primary stresses (LS1).

2.2.4 Linear elastic shell analysis (LA)

(1) The linearity of the theory results from the assumptions of a linear elastic material law and the linear small deflection theory. Small deflection theory implies that the assumed geometry remains that of the undeformed structure.

(2) An LA analysis satisfies compatibility in the deformations as well as equilibrium. The resulting field of membrane and bending stresses satisfy the requirements of primary plus secondary stresses $\overline{A_1}$ (LS1 to LS4) $\overline{A_1}$.

2.2.5 Linear elastic bifurcation analysis (LBA)

(1) The conditions of 2.2.4 concerning the material and geometric assumptions are met. However, this linear bifurcation analysis obtains the lowest eigenvalue at which the shell may buckle into a different deformation mode, assuming no change of geometry, no change in the direction of action of the loads, and no material degradation. Imperfections of all kinds are ignored. This analysis provides the elastic critical buckling resistance R_{cr} , see $\overline{A_1}$ 8.7 and 8.8 $\overline{A_1}$ (LS3).

2.2.6 Geometrically nonlinear elastic analysis (GNA)

(1) A GNA analysis satisfies both equilibrium and compatibility of the deflections under conditions in which the change in the geometry of the structure caused by loading is included. The resulting field of stresses matches the definition of primary plus secondary stresses $\overline{AC_1}$ (LS2 and LS4) $\overline{AC_1}$.

(2) Where compression or shear stresses are predominant in some part of the shell, a GNA analysis delivers the elastic buckling load of the perfect structure, including changes in geometry, that may be of assistance in checking the limit state LS3, see $\overline{A_1}$ 8.8 $\overline{A_1}$.

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system must be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

2.2.7 Materially nonlinear analysis (MNA)

(1) The result of an MNA analysis gives the plastic limit load, which can be interpreted as a load amplification factor R_{pl} on the design value of the loads F_{Ed} . This analysis provides the plastic reference resistance ratio R_{pl} used in $\overline{A_1}$ 8.7 and 8.8 $\overline{A_1}$.

(2) An MNA analysis may be used to verify $\overline{AC_1}$ limit states LS1 and LS3 $\overline{AC_1}$.

(3) An MNA analysis may be used to give the plastic strain increment $\Delta\epsilon$ during one cycle of cyclic loading that may be used to verify limit state LS2.

2.2.8 Geometrically and materially nonlinear analysis (GMNA)

$\overline{A_1}$ (1) The result of a GMNA analysis, analogously to 2.2.7, gives the geometrically nonlinear plastic failure load of the perfect structure and the plastic strain increment, that may be used for checking the limit states LS1 and LS2.

(2) Where compression or shear stresses are predominant in some part of the shell, a GMNA analysis gives the elasto-plastic buckling load of the perfect structure. This perfect shell buckling load should always be determined when the limit state LS3 is verified using GMNIA analysis, see 8.8. $\overline{A_1}$

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

2.2.9 Geometrically nonlinear elastic analysis with imperfections included (GNIA)

(1) A GNIA analysis is used in cases where compression or shear stresses dominate in the shell. It delivers elastic buckling loads of the imperfect structure, that may be of assistance in checking the limit state LS3, see $\overline{A_1}$ 8.8 $\overline{A_1}$.

(2) Where this analysis is used for a buckling load evaluation (LS3), the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path. Care must be taken to ensure that the local stresses do not exceed values at which material nonlinearity may affect the behaviour.

2.2.10 Geometrically and materially nonlinear analysis with imperfections included (GMNIA)

(1) A GMNIA analysis is used in cases where compression or shear stresses are dominant in the shell. It delivers elasto-plastic buckling loads for the “real” imperfect structure, that may be used for checking the limit state LS3, see [A1](#) 8.8 [A1](#).

(2) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(3) Where this analysis is used for a buckling load evaluation, an additional GMNA analysis of the perfect shell should always be conducted to ensure that the degree of imperfection sensitivity of the structural system is identified.

2.3 Shell boundary conditions

(1) The boundary conditions assumed in the design calculation should be chosen in such a way as to ensure that they achieve a realistic or conservative model of the real construction. Special attention should be given not only to the constraint of displacements normal to the shell wall (deflections), but also to the constraint of the displacements in the plane of the shell wall (meridional and circumferential) because of the significant effect these have on shell strength and buckling resistance.

(2) In shell buckling (eigenvalue) calculations (limit state LS3), the definition of the boundary conditions should refer to the incremental displacements during the buckling process, and not to total displacements induced by the applied actions before buckling.

(3) The boundary conditions at a continuously supported lower edge of a shell should take into account whether local uplifting of the shell is prevented or not.

(4) The shell edge rotation β_ϕ should be particularly considered in short shells and in the calculation of secondary stresses in longer shells (according to the limit states LS2 and LS4).

(5) The boundary conditions set out in 5.2.2 should be used in computer analyses and in selecting expressions from Annexes A to D.

(6) The structural connections between shell segments at a junction should be such as to ensure that the boundary condition assumptions used in the design of the individual shell segments are satisfied.

3 Materials and geometry

3.1 Material properties

(1) The material properties of steels should be obtained from the relevant application standard.

(2) Where materials with nonlinear stress-strain curves are involved and a buckling analysis is carried out under stress design (see 8.5), the initial tangent value of Young's modulus E should be replaced by a reduced value. If no better method is available, the secant modulus at the 0,2% proof stress should be used when assessing the elastic critical load or elastic critical stress.

(3) In a global numerical analysis using material nonlinearity, the 0,2% proof stress should be used to represent the yield stress f_y in all relevant expressions. The stress-strain curve should be obtained from EN1993-1-5 Annex C for carbon steels and EN1993-1-4 Annex C for stainless steels.

- (4) The material properties apply to temperatures not exceeding 150°C.

NOTE: The national annex may give information about material properties at temperatures exceeding 150°C.

3.2 Design values of geometrical data

- (1) The thickness t of the shell should be taken as defined in the relevant application standard. If no application standard is relevant, the nominal thickness of the wall, reduced by the prescribed value of the corrosion loss, should be used.
- (2) The thickness ranges within which the rules of this Standard may be applied are defined in the relevant EN1993 application parts.
- (3) The middle surface of the shell should be taken as the reference surface for loads.
- (4) The radius r of the shell should be taken as the nominal radius of the middle surface of the shell, measured normal to the axis of revolution.
- (5) The buckling design rules of this Standard should not be applied outside the ranges of the r/t ratio set out in section 8 or Annex D or in the relevant EN1993 application parts.

3.3 Geometrical tolerances and geometrical imperfections

- (1) Tolerance values for the deviations of the geometry of the shell surface from the nominal values are defined in the execution standards due to the requirements of serviceability. Relevant items are:

out-of-roundness (deviation from circularity),
eccentricities (deviations from a continuous middle surface in the direction normal to the shell across the junctions between plates),
local dimples (local normal deviations from the nominal middle surface).

NOTE: The requirements for execution are set out in EN 1090, but a fuller description of these tolerances is given here because of the critical relationship between the form of the tolerance measure, its amplitude and the evaluated resistance of the shell structure.

- (2) If the limit state of buckling (LS3, as described in 4.1.3) is one of the ultimate limit states to be considered, additional buckling-relevant geometrical tolerances have to be observed in order to keep the geometrical imperfections within specified limits. These buckling-relevant geometrical tolerances are quantified in section 8 or in the relevant EN1993 application parts.
- (3) Calculation values for the deviations of the shell surface geometry from the nominal geometry, as required for geometrical imperfection assumptions (overall imperfections or local imperfections) for the buckling design by global GMNIA analysis (see $\square_{A1}$ 8.8 $\square_{A1}$), should be derived from the specified geometrical tolerances. Relevant rules are given in $\square_{A1}$ 8.8 $\square_{A1}$ or in relevant EN1993 application parts.

4 Ultimate limit states in steel shells

4.1 Ultimate limit states to be considered

4.1.1 $\square_{A1}$ LS1: Plastic failure limit state $\square_{A1}$

- $\square_{A1}$ (1) The limit state of the plastic failure should be taken as the condition in which the capacity of the structure to resist the actions on it is exhausted by plasticity in the material.

The plastic failure resistance should be distinguished from the plastic reference resistance which is derived as the plastic collapse load obtained from a mechanism based on small displacement theory using an ideal elastic-plastic material law. $\square_{A1}$

(2) The limit state of tensile rupture should be taken as the condition in which the shell wall experiences gross section tensile failure, leading to separation of the two parts of the shell.

Ⓐ (3) In the absence of fastener holes, verification at the limit state of tensile rupture may be assumed to be covered by the check for the plastic failure limit state. However, where holes for fasteners occur, a supplementary check in accordance with EN 1993-1-1:2005, 6.2 should be carried out. Ⓐ

Ⓐ (4) In verifying the plastic failure limit state, plastic or partially plastic behaviour of the structure may be assumed (i.e. elastic compatibility considerations may be neglected).

NOTE Since the plastic failure limit state includes change of geometry, it may be noted that this limit state may also capture snap-through buckling, which may occur in the elastic state. The plastic reference resistance does not include change of geometry, so this apparent anomaly does not occur. Ⓐ

(5) All relevant load combinations should be accounted for when checking LS1.

(6) One or more of the following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS1:

- membrane theory;
- expressions in Annexes A and B;
- linear elastic analysis (LA);
- materially nonlinear analysis (MNA);
- geometrically and materially nonlinear analysis (GMNA).

4.1.2 LS2: Cyclic plasticity

(1) The limit state of cyclic plasticity should be taken as the condition in which repeated cycles of loading and unloading produce yielding in tension and in compression at the same point, thus causing plastic work to be repeatedly done on the structure, eventually leading to local cracking by exhaustion of the energy absorption capacity of the material.

NOTE: The stresses that are associated with this limit state develop under a combination of all actions and the compatibility conditions for the structure.

(2) All variable actions (such as imposed loads and temperature variations) that can lead to yielding, and which might be applied with more than three cycles in the life of the structure, should be accounted for when checking LS2.

(3) In the verification of this limit state, compatibility of the deformations under elastic or elasticplastic conditions should be considered.

(4) One or more of the following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS2:

- expressions in Annex C;
- elastic analysis (LA or GNA);
- MNA or GMNA to determine the plastic strain range.

(5) Low cycle fatigue failure may be assumed to be prevented if the procedures set out in this standard are adopted.

4.1.3 LS3: Buckling

(1) The limit state of buckling should be taken as the condition in which all or part of the structure suddenly develops large displacements normal to the shell surface, caused by loss of stability under compressive membrane or shear membrane stresses in the shell wall, leading to inability to sustain any increase in the stress resultants, possibly causing total collapse of the structure.

(2) One or more of the following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS3:

- membrane theory for axisymmetric conditions only (for exceptions, see relevant application parts of EN 1993)
- expressions in Annex A;
- linear elastic analysis (LA), which is a minimum requirement for stress analysis under general loading conditions (unless the load case is given in Annex A);
- linear elastic bifurcation analysis (LBA), which is required for shells under general loading conditions if the critical buckling resistance is to be used;
- materially nonlinear analysis (MNA), which is required for shells under general loading conditions if the reference plastic resistance is to be used;
- GMNIA, coupled with MNA, LBA and GMNA, using appropriate imperfections and calculated calibration factors.

(3) All relevant load combinations causing compressive membrane or shear membrane stresses in the shell should be accounted for when checking LS3.

(4) Because the strength under limit state LS3 depends strongly on the quality of construction, the strength assessment should take account of the associated requirements for execution tolerances.

NOTE: For this purpose, three classes of geometrical tolerances, termed “fabrication quality classes” are given in section 8.

4.1.4 LS4: Fatigue

(1) The limit state of fatigue should be taken as the condition in which repeated cycles of increasing and decreasing stress lead to the development of a fatigue crack.

(2) The following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS4:

- expressions in Annex C, using stress concentration factors;
- elastic analysis (LA or GNA), using stress concentration factors.

(3) All variable actions that will be applied with more than N_f cycles in the design life time of the structure according to the relevant action spectrum in EN 1991 in accordance with the appropriate application part of EN1993-3 or EN1993-4, should be accounted for when checking LS4.

NOTE: The National Annex may choose the value of N_f . The value $N_f = 10\,000$ is recommended.

4.2 Design concepts for the limit states design of shells

4.2.1 General

(1) The limit state verification should be carried out using one of the following:

- stress design;
- direct design by application of standard expressions;
- design by global numerical analysis (for example, by means of computer programs such as those based on the finite element method).

(2) Account should be taken of the fact that elasto-plastic material responses induced by different stress components in the shell have different effects on the failure modes and the ultimate limit states. The stress components should therefore be placed in stress categories with different limits. Stresses that develop to meet equilibrium requirements should be treated as more significant than stresses that are induced by the compatibility of deformations normal to the shell. Local stresses caused by notch effects in construction details may be assumed to have a negligibly small influence on the resistance to static loading.

- (3) The categories distinguished in the stress design should be primary, secondary and local stresses. Primary and secondary stress states may be replaced by stress resultants where appropriate.
- (4) In a global analysis, the primary and secondary stress states should be replaced by the limit load and the strain range for cyclic loading.
- (5) In general, it may be assumed that primary stress states control LS1, LS3 depends strongly on primary stress states but may be affected by secondary stress states, LS2 depends on the combination of primary and secondary stress states, and local stresses govern LS4.

4.2.2 Stress design

4.2.2.1 General

- (1) Where the stress design approach is used, the limit states should be assessed in terms of three categories of stress: primary, secondary and local. The categorisation is performed, in general, on the von Mises equivalent stress at a point, but buckling stresses cannot be assessed using this value.

4.2.2.2 Primary stresses

A1 (1) The primary stresses should be taken as the stress system required for equilibrium with the imposed loading. They may be calculated from any realistic statically admissible determinate system. The plastic failure limit state (LS1) should be deemed to be reached when the primary stress reaches the yield strength throughout the full thickness of the wall at a sufficient number of points, such that only the strain hardening reserve or a change of geometry would lead to an increase in the resistance of the structure.

(2) The calculation of primary stresses should be based on any system of stress resultants, consistent with the requirements of equilibrium of the structure. It may also take into account the benefits of plasticity theory. Alternatively, since linear elastic analysis satisfies equilibrium requirements, its predictions may also be used as a safe representation of the plastic failure limit state (LS1). Any of the analysis methods given in 5.3 may be applied. **A1**

(3) Because limit state design for LS1 allows for full plastification of the cross-section, the primary stresses due to bending moments may be calculated on the basis of the plastic section modulus, see 6.2.1. Where there is interaction between stress resultants in the cross-section, interaction rules based on the von Mises yield criterion may be applied.

(4) The primary stresses should be limited to the design value of the yield strength, see section 6 (LS1).

4.2.2.3 Secondary stresses

(1) In statically indeterminate structures, account should be taken of the secondary stresses, induced by internal compatibility and compatibility with the boundary conditions that are caused by imposed loading or imposed displacements (temperature, prestressing, settlement, shrinkage).

NOTE: As the von Mises yield condition is approached, the displacements of the structure increase without further increase in the stress state.

(2) Where cyclic loading causes plasticity, and several loading cycles occur, consideration should be given to the possible reduction of resistance caused by the secondary stresses. Where the cyclic loading is of such a magnitude that yielding occurs both at the maximum load and again on unloading, account should be taken of a possible failure by cyclic plasticity associated with the secondary stresses.

(3) If the stress calculation is carried out using a linear elastic analysis that allows for all relevant compatibility conditions (effects at boundaries, junctions, variations in wall thickness etc.), the stresses that vary linearly through the thickness may be taken as the sum of the primary and secondary stresses and used in an assessment involving the von Mises yield criterion, see 6.2.

NOTE: The secondary stresses are never needed separately from the primary stresses.

- (4) The secondary stresses should be limited as follows:

The sum of the primary and secondary stresses (including bending stresses) should be limited to $2f_{yd}$ for the condition of cyclic plasticity (LS2: see section 7);
The membrane component of the sum of the primary and secondary stresses should be limited by the design buckling resistance (LS3: see section 8).
The sum of the primary and secondary stresses (including bending stresses) should be limited to the fatigue resistance (LS4: see section 9).

4.2.2.4 Local stresses

- (1) The highly localised stresses associated with stress raisers in the shell wall due to notch effects (holes, welds, stepped walls, attachments, and joints) should be taken into account in a fatigue assessment (LS4).
- (2) For construction details given in EN 1993-1-9, the fatigue design may be based on the nominal linear elastic stresses (sum of the primary and secondary stresses) at the relevant point. For all other details, the local stresses may be calculated by applying stress concentration factors (notch factors) to the stresses calculated using a linear elastic stress analysis.
- (3) The local stresses should be limited according to the requirements for fatigue (LS4) set out in section 9.

4.2.3 Direct design

- (1) Where direct design is used, the limit states may be represented by standard expressions that have been derived from either membrane theory, plastic mechanism theory or linear elastic analysis.
- (2) The membrane theory expressions given in Annex A may be used to determine the primary stresses needed for assessing LS1 and LS3.
- (3) The expressions for plastic design given in Annex B may be used to determine the plastic limit loads needed for assessing LS1.
- (4) The expressions for linear elastic analysis given in Annex C may be used to determine stresses of the primary plus secondary stress type needed for assessing LS2 and LS4. An LS3 assessment may be based on the membrane part of these expressions.

4.2.4 Design by global numerical analysis

- (1) Where a global numerical analysis is used, the assessment of the limit states should be carried out using one of the alternative types of analysis specified in 2.2 (but not membrane theory analysis) applied to the complete structure.
- (2) Linear elastic analysis (LA) may be used to determine stresses or stress resultants, for use in assessing LS2 and LS4. The membrane parts of the stresses found by LA may be used in assessing LS3. LS1 may be assessed using LA, but LA only gives an approximate estimate and its results should be interpreted as set out in section 6.
- (3) Linear elastic bifurcation analysis (LBA) may be used to determine the critical buckling resistance of the structure, for use in assessing LS3.
- (4) A materially nonlinear analysis (MNA) may be used to determine the plastic reference resistance, and this may be used for assessing LS1. Under a cyclic loading history, an MNA analysis may be used to determine plastic strain incremental changes, for use in assessing LS2. The plastic reference resistance is also required as part of the assessment of LS3, and this may be found from an MNA analysis.
- (5) Geometrically nonlinear elastic analyses (GNA and GNIA) include consideration of the deformations of the structure, but none of the design methodologies of section 8 permit these to be used without a GMNIA analysis. A GNA analysis may be used to determine the elastic buckling load of the

perfect structure. A GNIA analysis may be used to determine the elastic buckling load of the imperfect structure.

(6) Geometrically and materially nonlinear analysis (GMNA and GMNIA) may be used to determine collapse loads for the perfect (GMNA) and the imperfect structure (GMNIA). The GMNA analysis may be used in assessing LS1, as detailed in 6.3. The GMNIA collapse load may be used, with additional consideration of the GMNA collapse load, for assessing LS3 as detailed in [\[A1\] 8.8 \[A1\]](#). Under a cyclic loading history, the plastic strain incremental changes taken from a GMNA analysis may be used for assessing LS2.

5 Stress resultants and stresses in shells

5.1 Stress resultants in the shell

(1) In principle, the eight stress resultants in the shell wall at any point should be calculated and the assessment of the shell with respect to each limit state should take all of them into account. However, the shear stresses τ_{xn} , $\tau_{\theta n}$ due to the transverse shear forces q_{xn} , $q_{\theta n}$ are insignificant compared with the other components of stress in almost all practical cases, so they may usually be neglected in design.

(2) Accordingly, for most design purposes, the evaluation of the limit states may be made using only the six stress resultants in the shell wall n_x , n_θ , $n_{x\theta}$, m_x , m_θ , $m_{x\theta}$. Where the structure is axisymmetric and subject only to axisymmetric loading and support, only n_x , n_θ , m_x and m_θ need be used.

(3) If any uncertainty arises concerning the stress to be used in any of the limit state verifications, the von Mises equivalent stress on the shell surface should be used.

5.2 Modelling of the shell for analysis

5.2.1 Geometry

(1) The shell should be represented by its middle surface.

(2) The radius of curvature should be taken as the nominal radius of curvature. Imperfections should be neglected, except as set out in section 8 (LS3 buckling limit state).

(3) An assembly of shell segments should not be subdivided into separate segments for analysis unless the boundary conditions for each segment are chosen in such a way as to represent interactions between them in a conservative manner.

(4) A base ring intended to transfer local support forces into the shell should not be separated from the shell it supports in an assessment of limit state LS3.

(5) Eccentricities and steps in the shell middle surface should be included in the analysis model if they induce significant bending effects as a result of the membrane stress resultants following an eccentric path.

(6) At junctions between shell segments, any eccentricity between the middle surfaces of the shell segments should be considered in the modelling.

(7) A ring stiffener should be treated as a separate structural component of the shell, except where the spacing of the rings is closer than $1,5 \sqrt{rt}$.

(8) A shell that has discrete stringer stiffeners attached to it may be treated as an orthotropic uniform shell, provided that the stringer stiffeners are no further apart than $5 \sqrt{rt}$.

(9) A shell that is corrugated (vertically or horizontally) may be treated as an orthotropic uniform shell provided that the corrugation wavelength is less than $0,5 \sqrt{rt}$.

(10) A hole in the shell may be neglected in the modelling provided its largest dimension is smaller than $0,5 \sqrt{rt}$.

(11) The overall stability of the complete structure should be verified as detailed in EN 1993 Parts 3.1, 3.2, 4.1, 4.2 or 4.3 as appropriate.

5.2.2 Boundary conditions

(1) The appropriate boundary conditions should be used in analyses for the assessment of limit states according to the conditions shown in table 5.1. For the special conditions needed for buckling calculations, reference should be made to 8.3.

(2) Rotational restraints at shell boundaries may be neglected in modelling for limit state LS1, but should be included in modelling for limit states LS2 and LS4. For short shells (see Annex D), the rotational restraint should be included for limit state LS3.

(3) Support boundary conditions should be checked to ensure that they do not cause excessive non-uniformity of transmitted forces or introduced forces that are eccentric to the shell middle surface. Reference should be made to the relevant EN 1993 application parts for the detailed application of this rule to silos and tanks.

(4) When a global numerical analysis is used, the boundary condition for the normal displacement w should also be used for the circumferential displacement v , except where special circumstances make this inappropriate.

Table 5.1: Boundary conditions for shells

Boundary condition code	Simple term	Description	Normal displacement s	Meridional displacements	Meridional rotation
BC1r	Clamped	radially restrained meridionally restrained rotation restrained	$w = 0$	$u = 0$	$\beta_{\varphi} = 0$
BC1f		radially restrained meridionally restrained rotation free	$w = 0$	$u = 0$	$\beta_{\varphi} \neq 0$
BC2r		radially restrained meridionally free rotation restrained	$w = 0$	$u \neq 0$	$\beta_{\varphi} = 0$
BC2f	Pinned	radially restrained meridionally free rotation free	$w = 0$	$u \neq 0$	$\beta_{\varphi} \neq 0$
BC3	Free edge	radially free meridionally free rotation free	$w \neq 0$	$u \neq 0$	$\beta_{\varphi} \neq 0$
NOTE: The circumferential displacement v is closely linked to the displacement w normal to the surface, so separate boundary conditions are not identified for these two parameters (see (4)) but the values in column 4 should be adopted for displacement v .					

5.2.3 Actions and environmental influences

- (1) Actions should all be assumed to act at the shell middle surface. Eccentricities of load should be represented by static equivalent forces and moments at the shell middle surface.
- (2) Local actions and local patches of action should not be represented by equivalent uniform loads except as detailed in section 8 for buckling (LS3).
- (3) The modelling should account for whichever of the following are relevant:
 - local settlement under shell walls;
 - local settlement under discrete supports;
 - uniformity / non-uniformity of support of structure;
 - thermal differentials from one side of the structure to the other;
 - thermal differentials from inside to outside the structure;
 - wind effects on openings and penetrations;
 - interaction of wind effects on groups of structures;
 - connections to other structures;
 - conditions during erection.

5.2.4 Stress resultants and stresses

- (1) Provided that the radius to thickness ratio is greater than $(r/t)_{\min}$, the curvature of the shell may be ignored when calculating the stress resultants from the stresses in the shell wall.

NOTE: The National Annex may choose the value of $(r/t)_{\min}$. The value $(r/t)_{\min} = 25$ is recommended.

5.3 Types of analysis

- (1) The design should be based on one or more of the types of analysis given in table 5.2. Reference should be made to 2.2 for the conditions governing the use of each type of analysis.

Table 5.2: Types of shell analysis

Type of analysis	Shell theory	Material law	Shell geometry
Membrane theory of shells	membrane equilibrium	not applicable	perfect
Linear elastic shell analysis (LA)	linear bending and stretching	linear	perfect
Linear elastic bifurcation analysis (LBA)	linear bending and stretching	linear	perfect
Geometrically non-linear elastic analysis (GNA)	non-linear	linear	perfect
Materially non-linear analysis (MNA)	linear	$\overline{A_1}$ ideal elastic-plastic A_1	perfect
Geometrically and materially non-linear analysis (GMNA)	non-linear	non-linear	perfect
Geometrically non-linear elastic analysis with imperfections (GNIA)	non-linear	linear	imperfect
Geometrically and materially non-linear analysis with imperfections (GMNIA)	non-linear	non-linear	imperfect

6 A1 Plastic failure limit state (LS1) A1

6.1 Design values of actions

- (1)P The design values of the actions shall be based on the most adverse relevant load combination (including the relevant γ_F and Ψ factors).
- (2) Only those actions that represent loads affecting the equilibrium of the structure need be included.

6.2 Stress design

6.2.1 Design values of stresses

A1 (1) Although stress design is based on an elastic analysis and therefore cannot accurately predict the plastic failure limit state, it may be used, on the basis of the lower bound theorem, to provide a conservative assessment of the plastic collapse resistance which is used to represent the plastic failure limit state, see 4.1.1. A1

(2) The Ilyushin yield criterion may be used, as detailed in (6), that comes closer to the true plastic collapse state than a simple elastic surface stress evaluation.

(3) At each point in the structure the design value of the stress $\sigma_{eq,Ed}$ should be taken as the highest primary stress determined in a structural analysis that considers the laws of equilibrium between imposed design load and internal forces and moments.

(4) The primary stress may be taken as the maximum value of the stresses required for equilibrium with the applied loads at a point or along an axisymmetric line in the shell structure.

A1 (5) Where a membrane theory analysis is used, or where a linear bending theory analysis (LA) is used subject to the conditions defined in (6), the resulting two-dimensional field of stress resultants $n_{x,Ed}$, $n_{\theta,Ed}$ and $n_{x\theta,Ed}$ may be represented by the equivalent design stress $\sigma_{eq,Ed}$ obtained from:

$$\sigma_{eq,Ed} = \frac{1}{t} \sqrt{n_{x,Ed}^2 + n_{\theta,Ed}^2 - n_{x,Ed} \cdot n_{\theta,Ed} + 3n_{x\theta,Ed}^2} \quad \dots (6.1)$$

(6) Where an LA or GNA analysis is used, and the magnitude of the largest von Mises surface stress found using Formulae (6.2) to (6.4) exceeds j times the von Mises membrane stress found using Formula (6.1) at the same location, the equivalent stress should be taken as the value determined using Formulae (6.2) to (6.4).

$$\sigma_{eq,Ed} = \sqrt{\sigma_{x,Ed}^2 + \sigma_{\theta,Ed}^2 - \sigma_{x,Ed} \cdot \sigma_{\theta,Ed} + 3\tau_{x\theta,Ed}^2} \quad \dots (6.2)$$

in which:

$$\sigma_{x,Ed} = \frac{n_{x,Ed}}{t} \pm \frac{m_{x,Ed}}{(t^2/4)} \quad \sigma_{\theta,Ed} = \frac{n_{\theta,Ed}}{t} \pm \frac{m_{\theta,Ed}}{(t^2/4)} \quad \dots (6.3)$$

$$\tau_{x\theta,Ed} = \frac{n_{x\theta,Ed}}{t} \pm \frac{m_{x\theta,Ed}}{(t^2/4)} \quad \dots (6.4)$$

NOTE 1 Formulae (6.2) to (6.4) give a simplified conservative equivalent stress for design purposes.

NOTE 2 The National Annex may choose the value of j . The recommended value is 3. A1

6.2.2 Design values of resistance

(1) The von Mises design strength should be taken from:

$$f_{\text{eq,Rd}} = f_{\text{yd}} = f_{\text{yk}} / \gamma_{\text{M0}} \quad \dots (6.5)$$

(2) The partial factor for resistance γ_{M0} should be taken from the relevant application standard.

(3) Where no application standard exists for the form of construction involved, or the application standard does not define the relevant values of γ_{M0} , the value of γ_{M0} should be taken from EN1993-1-1.

(4) Where the material has a nonlinear stress strain curve, the value of the characteristic yield strength f_{yk} should be taken as the 0,2% proof stress.

(5) The effect of fastener holes should be taken into account in accordance with 6.2.3 of EN1993-1-1 for tension and 6.2.4 of EN1993-1-1 for compression. For the tension check, the resistance should be based on the design value of the ultimate strength f_{ud} .

6.2.3 Stress limitation

(1)P In every verification of this limit state, the design stresses shall satisfy the condition:

$$\sigma_{\text{eq,Ed}} \leq f_{\text{eq,Rd}} \quad \dots (6.6)$$

6.3 Design by global numerical MNA or GMNA analysis

[A1] (1)P The design plastic failure resistance shall be determined as a load factor R_{pl} applied to the design values F_{Ed} of the combination of actions for the relevant load case. **[A1]**

(2) The design values of the actions F_{Ed} should be determined as detailed in 6.1. The relevant load cases should be formed according to the required load combinations.

[A1] (3) In an MNA or GMNA analysis based on the design yield strength f_{yd} , the shell should be subject to the design values of the load cases detailed in (2), progressively increased by the load ratio R until the plastic failure condition at the load ratio R_{pl} is reached. **[A1]**

(4) Where an MNA analysis is used, the load ratio R_{MNA} may be taken as the largest value attained in the analysis, ignoring the effect of strain hardening. This load ratio is identified as the plastic reference resistance ratio R_{pl} in **[A1]** 8.8 **[A1]**.

[A1] (5) Where a GMNA analysis is used, if the analysis predicts a maximum load followed by a descending path, the maximum value should be used to determine the load ratio R_{GMNA} . Where a GMNA analysis does not predict a maximum load, but produces a progressively rising action-displacement relationship without strain hardening of the material, the load ratio R_{GMNA} should be taken as no larger than the value at which the maximum von Mises equivalent plastic strain in the structure attains the value $\varepsilon_{\text{mps}} = n_{\text{mps}} \cdot (f_{\text{yd}}/E)$.

NOTE The National Annex may choose the value of n_{mps} . The value $n_{\text{mps}} = (66 \cdot f_{\text{yd}}/15)$, where f_{yd} is in MPa, is recommended.

(6) A GMNA analysis may not be used to establish the plastic reference resistance R_{pl} , which is used in Clause 8 as part of the LBA-MNA design method.

(7) The characteristic plastic failure resistance $R_{\text{pl,k}}$ should be taken as either R_{MNA} or R_{GMNA} according to the analysis that has been used."

(8)P The design plastic failure resistance F_{Rd} shall be obtained from:

$$F_{Rd} = \frac{F_{Rk}}{\gamma_{M0}} = \frac{R_k \cdot F_{Ed}}{\gamma_{M0}} = R_d \cdot F_{Ed} \quad \dots (6.7) \quad \text{A1}$$

(9)P It shall be verified that:

$$F_{Ed} \leq F_{Rd} = r_{Rd} \cdot F_{Ed} \quad \text{or} \quad r_{Rd} \geq 1 \quad \dots (6.8)$$

6.4 Direct design

(1) For each shell segment in the structure represented by a basic loading case as given by Annex A, the highest von Mises membrane stress $\sigma_{eq,Ed}$ determined under the design values of the actions F_{Ed} should be limited to the stress resistance according to 6.2.2.

(2) For each shell or plate segment in the structure represented by a basic load case as given in Annex B, the design value of the actions F_{Ed} should not exceed the resistance F_{Rd} based on the design yield strength f_{yd} .

(3) Where net section failure at a bolted joint is a design criterion, the design value of the actions F_{Ed} should be determined for each joint. Where the stress can be represented by a basic load case as given in Annex A, and where the resulting stress state involves only membrane stresses, F_{Ed} should not exceed the resistance F_{Rd} based on the design ultimate strength f_{ud} , see 6.2.2(5).

7 Cyclic plasticity limit state (LS2)

7.1 Design values of actions

(1) Unless an improved definition is used, the design values of the actions for each load case should be chosen as the characteristic values of those parts of the total actions that are expected to be applied and removed more than three times in the design life of the structure.

(2) Where an elastic analysis or the expressions from Annex C are used, only the varying part of the actions between the extreme upper and lower values should be taken into account.

(3) Where a materially nonlinear computer analysis is used, the varying part of the actions between the extreme upper and lower values should be considered to act in the presence of coexistent permanent parts of the load.

7.2 Stress design

7.2.1 Design values of stress range

(1) The shell should be analysed using an LA or GNA analysis of the structure subject to the two extreme design values of the actions F_{Ed} . For each extreme load condition in the cyclic process, the stress components should be evaluated. From adjacent extremes in the cyclic process, the design values of the change in each stress component $\Delta\sigma_{x,Ed,i}$, $\Delta\sigma_{\theta,Ed,i}$, $\Delta\tau_{x\theta,Ed,i}$ on each shell surface (represented as $i=1,2$ for the inner and outer surfaces of the shell) and at any point in the structure should be determined. From these changes in stress, the design value of the von Mises equivalent stress change on the inner and outer surfaces should be found from:

$$\Delta\sigma_{eq,Ed,i} = \sqrt{\Delta\sigma_{x,Ed,i}^2 - \Delta\sigma_{x,Ed,i} \cdot \Delta\sigma_{\theta,Ed,i} + \Delta\sigma_{\theta,Ed,i}^2 + 3\Delta\tau_{x\theta,Ed,i}^2} \quad \dots (7.1)$$

(2) The design value of the stress range $\Delta\sigma_{eq,Ed}$ should be taken as the largest change in the von Mises equivalent stress changes $\Delta\sigma_{eq,Ed,i}$, considering each shell surface in turn ($i=1$ and $i=2$ considered separately).

(3) At a junction between shell segments, where the analysis models the intersection of the middle surfaces and ignores the finite size of the junction, the stress range may be taken at the first physical point in the shell segment (as opposed to the value calculated at the intersection of the two middle surfaces).

NOTE: This allowance is relevant where the stress changes very rapidly close to the junction.

7.2.2 Design values of resistance

(1) The von Mises equivalent stress range resistance $\Delta f_{eq,Rd}$ should be determined from:

$$\Delta f_{eq,Rd} = 2f_{yd} \quad \dots (7.2)$$

7.2.3 Stress range limitation

(1)P In every verification of this limit state, the design stress range shall satisfy:

$$\Delta\sigma_{eq,Ed} \leq \Delta f_{eq,Rd} \quad \dots (7.3)$$

7.3 Design by global numerical MNA or GMNA analysis

7.3.1 Design values of total accumulated plastic strain

(1) Where a materially nonlinear global numerical analysis (MNA or GMNA) is used, the shell should be subject to the design values of the varying and permanent actions detailed in 7.1.

NOTE 1: It is usual to use an MNA analysis for this purpose.

NOTE 2: The National Annex may give recommendations for a more refined analysis.

(2) The total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ at the end of the design life of the structure should be assessed.

(3) The total accumulated von Mises equivalent plastic strain may be determined using an analysis that models all cycles of loading during the design life.

(4) Unless a more refined analysis is carried out, the total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ may be determined from:

$$\varepsilon_{p,eq,Ed} = n\Delta\varepsilon_{p,eq,Ed} \quad \dots (7.4)$$

where:

n	is	the number of cycles of loading in the design life of the structure;
$\Delta\varepsilon_{p,eq,Ed}$	is	the largest increment in the von Mises equivalent plastic strain during one complete load cycle at any point in the structure, occurring after the third cycle.

(5) It may be assumed that “at any point in the structure” means at any point not closer to a notch or local discontinuity than the thickest adjacent plate thickness.

7.3.2 Total accumulated plastic strain limitation

(1) Unless a more sophisticated low cycle fatigue assessment is undertaken, the design value of the total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ should satisfy the condition: