



BSI Standards Publication

Eurocode 3 — Design of steel structures

Part 1-6: Strength and stability of shell structures

National foreword

This British Standard is the UK implementation of EN 1993-1-6:2025. It supersedes BS EN 1993-1-6:2007+A1:2017, which will be withdrawn on 30 March 2028.

The UK participation in its preparation was entrusted to Technical Committee CB/203, Design & execution of steel structures.

A list of organizations represented on this committee can be obtained on request to its committee manager.

National choice is allowed in this standard where explicitly stated within notes. The National Annex to this standard contains the national choices to be used for buildings and civil engineering works constructed in the UK.

The first generation of EN Eurocodes was published between 2002 and 2007, with conflicting British Standards withdrawn in 2010. This document forms part of the second generation of EN Eurocodes.

The second generation of EN Eurocodes is expected to be published between 2023 and 2026. These documents are being published as soon as they are available. This is being done to enable users to prepare for the transition from the first generation to second generation of EN Eurocodes.

UK adoptions of the first generation of EN Eurocodes will be withdrawn by BSI on 30 March 2028. Until that date, the first generation documents should be considered as the applicable standards for buildings and civil engineering works constructed in the UK unless otherwise specified by the relevant authority or in the specification for a particular project.

This standard is intended to be used with its National Annex and other referenced documents, including other second generation Eurocodes, as an interdependent suite of documents.

While the use of provisions in this standard in conjunction with first generation Eurocodes is not precluded, it should be undertaken with care and should only be done when users are satisfied that it will not result in a lower level of reliability than the minimum level set in the first generation Eurocodes and associated UK National Annexes.

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Amendments/corrigenda issued since publication

Date	Text affected
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English Version

Eurocode 3 - Design of steel structures - Part 1-6: Strength and stability of shell structures

Eurocode 3 - Calcul des structures en acier - Partie 1-6 :
Résistance et stabilité des structures en coque

Eurocode 3 - Bemessung und Konstruktion von
Stahlbauten - Teil 1-6: Festigkeit und Stabilität von
Schalen

This European Standard was approved by CEN on 29 December 2024.

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COMITÉ EUROPÉEN DE NORMALISATION
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European foreword

This document (EN 1993-1-6:2025) has been prepared by Technical Committee CEN/TC 250 “Structural Eurocodes”, the secretariat of which is held by BSI. CEN/TC 250 is responsible for all Structural Eurocodes and has been assigned responsibility for structural and geotechnical matters by CEN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by September 2027, and conflicting national standards shall be withdrawn at the latest by March 2028.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN shall not be held responsible for identifying any or all such patent rights.

This document supersedes EN 1993-1-6:2007 and all of its amendments and corrigenda (if any).

EN 1993-1-6:2025 includes the following significant technical changes with respect to EN 1993-1-6:2007:

- The text of the entire document has been reworked significantly to enhance clarity, ease of use and to incorporate new scientific content.
- The scope of the document has been restricted to fabricated steel shell structures: manufactured circular hollow sections are now clearly out of scope and are instead under the remit of the re-developed EN 1993-1-1. Manufactured tubular piles remain in scope, although with provisions for imperfections and tolerances to be taken from EN 1993-5.
- The relationships between tolerance measurements on a constructed shell and the imperfection amplitudes assumed in design have been made strictly algebraic for all the three basic uniform stress regimes: uniform axial compression, uniform circumferential compression and uniform torsion. A strict methodology to interpret relatively complicated stress fields in terms of resistances for these three basic stress regimes has been codified for the first time.
- The resistance formulae in Annex D have been reworked significantly to address missing situations. For the important reference case of cylindrical shells under uniform axial (meridional) compression, these formulae have been amended to make them more accurately associated with the commonest imperfection form and to correct a minor error in the 2017 Amendment. These have also been extended to cover stainless steels as well as carbon steels. Resistance formulae for uniform circumferential compression and uniform torsion have been developed and included in Annex D.
- Annex D now contains updated provisions for stepped wall construction, with an entirely new treatment of stepped wall shells under uniform external pressure or unsymmetrical wind loading.
- The treatment of design assessment using the LBA-MNA method has been clarified.
- The entirely new method of design using Reference Resistances has been added as a fully algebraic means of verifying the Buckling Ultimate Limit State (LS3) for appropriate shell geometries and loading conditions. An entirely new normative Annex E has been created to host resistance functions for cylindrical shells under uniform bending and spherical shells under uniform external pressure.
- The verification of buckling-relevant geometrical tolerances has been reworked and made less onerous. The overall Fabrication Tolerance Quality Class is now correlated with the dominant buckling-relevant design membrane stress state coupled with the measured tolerance amplitudes on

the completed shell structure. New provisions have been introduced to assist in the interpretation of continuous tolerance measurements, such as those obtained from laser scans.

- Where the LS3 buckling verification checks are undertaken using GMNIA analysis, a more comprehensive description is given of the applicable model imperfections which now also share the same nominal amplitudes as those assumed in hand design calculations.
- The design provisions for the Fatigue Ultimate Limit State (LS4) have been entirely reworked and are now compatible with the re-developed EN 1993-1-9.
- References have now been introduced to key parts of the entirely new EN 1993-1-14 concerning meshing, verification and validation of finite element models that are used in LA, LBA, MNA, GNIA and GMNIA analyses, including appropriate assumptions for material properties.

The first generation of EN Eurocodes was published between 2002 and 2007. This document forms part of the second generation of the Eurocodes, which have been prepared under Mandate M/515 issued to CEN by the European Commission and the European Free Trade Association.

The Eurocodes have been drafted to be used in conjunction with relevant execution, material, product and test standards, and to identify requirements for execution, materials, products and testing that are relied upon by the Eurocodes.

The Eurocodes recognize the responsibility of each Member State and have safeguarded their right to determine values related to regulatory safety matters at national level through the use of National Annexes.

Any feedback and questions on this document should be directed to the users' national standards body. A complete listing of these bodies can be found on the CEN website.

According to the CEN-CENELEC Internal Regulations, the national standards organisations of the following countries are bound to implement this European Standard: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Republic of North Macedonia, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Türkiye and the United Kingdom.

0 Introduction

0.1 Introduction to the Eurocodes

The Structural Eurocodes comprise the following standards generally consisting of a number of Parts:

- EN 1990 Eurocode — Basis of structural and geotechnical design
- EN 1991 Eurocode 1 — Actions on structures
- EN 1992 Eurocode 2 — Design of concrete structures
- EN 1993 Eurocode 3 — Design of steel structures
- EN 1994 Eurocode 4 — Design of composite steel and concrete structures
- EN 1995 Eurocode 5 — Design of timber structures
- EN 1996 Eurocode 6 — Design of masonry structures
- EN 1997 Eurocode 7 — Geotechnical design
- EN 1998 Eurocode 8 — Design of structures for earthquake resistance
- EN 1999 Eurocode 9 — Design of aluminium structures
- New parts are under development, e.g. Eurocode for design of structural glass

The Eurocodes are intended for use by designers, clients, manufacturers, constructors, relevant authorities (in exercising their duties in accordance with national or international regulations), educators, software developers, and committees drafting standards for related product, testing and execution standards.

NOTE Some aspects of design are most appropriately specified by relevant authorities or, where not specified, can be agreed on a project-specific basis between relevant parties such as designers and clients. The Eurocodes identify such aspects making explicit reference to relevant authorities and relevant parties.

0.2 Introduction to EN 1993 (all parts)

EN 1993 (all parts) applies to the design of buildings and civil engineering works in steel. It complies with the principles and requirements for the safety and serviceability of structures, the basis of their design and verification that are given in EN 1990 Eurocode – Basis of structural and geotechnical design.

EN 1993 (all parts) is concerned only with requirements for resistance, serviceability, durability and fire resistance of steel structures. Other requirements, e.g. concerning thermal or sound insulation, are not covered.

EN 1993 is subdivided in various parts:

EN 1993-1, *Design of Steel Structures — Part 1: General rules and rules for buildings*;

EN 1993-2, *Design of Steel Structures — Part 2: Bridges*;

EN 1993-3, *Design of Steel Structures — Part 3: Towers, masts and chimneys*;

EN 1993-4, *Design of Steel Structures — Part 4: Silos and tanks*;

EN 1993-5, *Design of Steel Structures — Part 5: Piling*;

EN 1993-6, *Design of Steel Structures — Part 6: Crane supporting structures*;

EN 1993-7¹, *Design of steel structures — Part 7: Sandwich panels*.

EN 1993-1 in itself does not exist as a physical document, but comprises the following 14 separate parts, the basic part being EN 1993-1-1:

EN 1993-1-1, *Design of Steel Structures — Part 1-1: General rules and rules for buildings*;

EN 1993-1-2, *Design of Steel Structures — Part 1-2: Structural fire design*;

EN 1993-1-3, *Design of Steel Structures — Part 1-3: Cold-formed members and sheeting*;

NOTE Cold formed hollow sections supplied according to EN 10219 are covered in EN 1993-1-1.

EN 1993-1-4, *Design of Steel Structures — Part 1-4: Stainless steel structures*;

EN 1993-1-5, *Design of Steel Structures — Part 1-5: Plated structural elements*;

EN 1993-1-6, *Design of Steel Structures — Part 1-6: Strength and stability of shell structures*;

EN 1993-1-7, *Design of Steel Structures — Part 1-7: Plate assemblies with elements under transverse loads*;

EN 1993-1-8, *Design of Steel Structures — Part 1-8: Joints*;

EN 1993-1-9, *Design of Steel Structures — Part 1-9: Fatigue*;

EN 1993-1-10, *Design of Steel Structures — Part 1-10: Material toughness and through-thickness properties*;

EN 1993-1-11, *Design of Steel Structures — Part 1-11: Tension components*;

EN 1993-1-12, *Design of Steel Structures — Part 1-12: Additional rules for steel grades up to S960*;

EN 1993-1-13, *Design of Steel Structures — Part 1-13: Beams with large web openings*;

EN 1993-1-14, *Design of Steel Structures — Part 1-14: Design assisted by finite element analysis*.

All subsequent parts EN 1993-1-2 to EN 1993-1-14 treat general topics that are independent from the structural type such as structural fire design, cold-formed members and sheeting, stainless steels, plated structural elements, etc.

All subsequent parts numbered EN 1993-2 to EN 1993-7 treat topics relevant for a specific structural type such as steel bridges, towers, masts and chimneys, silos and tanks, piling, crane supporting structures, etc. EN 1993-2 to EN 1993-7 refer to the generic rules in EN 1993-1 and supplement, modify or supersede them, where relevant.

0.3 Introduction to EN 1993-1-6

EN 1993-1-6 gives design requirements for steel shell structures that are subject to forces and pressures that induce membrane and bending stress resultants in the shell. It also covers annular plates and ring stiffeners. Its provisions can be used for a wide variety of stiffened and unstiffened curved structures through the application of computational methods. It is applicable to silos, tanks, chimneys, wind turbine towers, biodigesters and piles.

¹ Under preparation. Stage at the time of publication: prEN 1993-7.

0.4 Verbal forms used in the Eurocodes

The verb “shall” expresses a requirement strictly to be followed and from which no deviation is permitted in order to comply with the Eurocodes.

The verb “should” expresses a highly recommended choice or course of action. Subject to national regulation and/or any relevant contractual provisions, alternative approaches could be used/adopted where technically justified.

The verb “may” expresses a course of action permissible within the limits of the Eurocodes.

The verb “can” expresses possibility and capability; it is used for statements of fact and clarification of concepts.

0.5 National Annex for EN 1993-1-6

National choice is allowed in this document where explicitly stated within notes. National choice includes the selection of values for Nationally Determined Parameters (NDPs).

The national standard implementing EN 1993-1-6 can have a National Annex containing all national choices to be used for the design of buildings and civil engineering works to be constructed in the relevant country.

When no national choice is given, the default choice given in this document is to be used.

When no national choice is made and no default is given in this document, the choice can be specified by a relevant authority or, where not specified, agreed for a specific project by appropriate parties.

National choice is allowed in EN 1993-1-6 through notes to the following clauses:

4.4 (3)	6.3.2 (3)	6.3.4 (2)	9.8.2 (12)
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National choice is allowed in EN 1993-1-6 on the application of the following informative annexes:

Annex A	Annex B	Annex C
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The National Annex can contain, directly or by reference, non-contradictory complementary information for ease of implementation, provided it does not alter any provisions of the Eurocodes.

1 Scope

1.1 Scope of EN 1993-1-6

(1) EN 1993-1-6 provides rules for the structural design of plated steel structures that have the form of a shell of revolution (axisymmetric shell).

(2) This document is applicable to unstiffened fabricated axisymmetric shells formed from isotropic rolled plates using both algebraic and computational procedures, and to stiffened axisymmetric shells with different wall constructions using computational procedures. It also applies to associated circular or annular plates and to beam section rings and stringer stiffeners where they form part of the complete shell structure. The general computational procedures are applicable to all shell forms.

(3) This document does not apply to manufactured shells or to shell panels or to elliptical shell forms, except that its computational procedures are applicable to all shell structures. This document does not apply to structures under seismic or other dynamic loading. It does not cover the aspects of leakage of stored liquids or solids.

(4) Cylindrical and conical panels are not explicitly covered by this document. However, the provisions of 9.8 can be used provided that appropriate boundary conditions are taken into account.

(5) This document defines the characteristic and design values of the resistance of the structure.

(6) This document is concerned with the requirements for design against the ultimate limit states of:

- plastic failure;
- cyclic plasticity;
- buckling;
- fatigue.

(7) Overall equilibrium of the structure (sliding, uplifting, overturning) is not included in this document. Special considerations for specific applications are included in the relevant application parts of EN 1993.

(8) Detailed formulae for the simple calculation of unstiffened cylinders, cones and spherical domes are given in the Annexes.

(9) Provisions for simple calculations on specific stiffened shell types are given in EN 1993-4-1.

(10) This document is intended for application to steel shell structures. Where no standard exists for shell structures made of other metals, including high strength steels, the provisions of this document are applicable provided the appropriate material properties of the metal are taken into account.

(11) The provisions of this document are intended to be applied within the temperature ranges defined in the relevant EN 1993 application parts.

(12) Where no application part defines a different range, this document applies to structures within the following limits:

- design metal temperatures lie within the range -50 °C to $+100\text{ °C}$, except when using the special provisions given in 5.1;
- radius to thickness ratios (r/t) within the range 50 to 2 000;

- manufactured circular hollow sections according to EN 10210 and EN 10219 are outside the scope of this document and are covered by EN 1993-1-1. However, if no other provisions are available, the rules of this document are useful for manufactured circular hollow sections. In particular, this document is applicable to the design of manufactured piles (see EN 1993-5) provided the imperfections and tolerance requirements of EN 1993-5 are adopted in place of those specified in this document, and where no other standard covers the specific pile geometry.

NOTE 1 Experimental and theoretical data relating to manufactured circular hollow sections were not considered when this document was drafted. The application of this document to such structures therefore remains the responsibility of the user.

NOTE 2 The stress design rules of this document can be rather conservative if applied to some geometries and loading conditions for relatively thick-walled shells.

NOTE 3 Thinner shells than $r/t = 2\,000$ can be treated using these provisions but the provisions have not been verified for such thin shells.

NOTE 4 The maximum temperature is restricted so that the influence of creep can be ignored where high temperature creep effects are not covered by the relevant application part.

NOTE 5 Where temperatures outside the above range are involved and the long-term effects of creep are considered negligible, the thermally adjusted properties can be found in EN 1993-1-2 or other CEN standards as appropriate. Where no other standard is available, see EN 1993-1-2 which, though intended for the design of steel structures against fire, gives general temperature-dependent material properties that can be more widely used (see 5.1(10)).

NOTE 6 Where the effects of creep cannot be ignored, such as where a structure is subject to temperatures above 100 °C for prolonged periods of time, appropriate thermally adjusted properties can be found in other EN standards or established peer-reviewed technical literature.

1.2 Assumptions

(1) Unless specifically stated, the provisions of EN 1990, EN 1991 (all parts) and the other relevant parts of EN 1993-1 (all parts) apply.

(2) The design methods given in this document are applicable if:

- the execution quality is as specified in EN 1090-2, and
- the construction materials and products used are as specified in the relevant parts of EN 1993 (all parts), or in the relevant material and product specifications.

NOTE The buckling-related tolerance requirements of this document differ in some aspects from those of EN 1090-2 (see Clause 9).

(3) The provisions in this document apply to materials that satisfy the brittle fracture provisions given in EN 1993-1-4, EN 1993-1-10 and EN 1993-1-12.

(4) In this document, it is assumed that wind loading, seismic actions and bulk solids flow can, in general, be treated as quasi-static actions.

(5) Dynamic effects are outside the scope of this document, and are covered by the relevant application part of EN 1993 or EN 1998, including the consequences for fatigue. However, the stress resultants arising from dynamic behaviour are treated in this part as quasi-static.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE See the Bibliography for a list of other documents cited that are not normative references, including those referenced as recommendations (i.e. in 'should' clauses), permissions ('may' clauses), possibilities ('can' clauses), and in notes.

EN 1090-2, *Execution of steel structures and aluminium structures — Part 2: Technical requirements for steel structures*

EN 1990, *Eurocode — Basis of structural and geotechnical design*

EN 1991 (all parts), *Eurocode 1 — Actions on structures*

EN 1993 (all parts), *Eurocode 3 — Design of steel structures*

ISO 8930, *General principles on reliability for structures — Vocabulary*

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in EN 1990, EN 1993-1-1, ISO 8930 and the following apply.

3.1.1 Structural forms and geometry

3.1.1.1

base ring

structural member that passes around the circumference of the shell of revolution at the base and provides a means of attachment of the shell to a foundation or other structural member, needed to ensure that the assumed boundary conditions are achieved in practice

3.1.1.2

circumferential joint

joint that passes around the circumference of an axisymmetric shell

3.1.1.3

complete shell or shell assembly

shell composed of a number of shell segments

Note 1 to entry: In this document, each segment of the shell assembly is assumed to be a shell of revolution (e.g. cylindrical, conical, spherical, etc.).

3.1.1.4

constructional detail

part of a shell with a geometry that causes locally raised stresses relevant to the fatigue limit state (LS4)

Note 1 to entry: The geometric feature that causes the stress raising effect is also referred to as a "notch" in EN 1993-1-9 and can be a welded joint, bolted joint or other type of connection.

3.1.1.5**course**

set of rolled plates connected by vertical joints that make up a single layer of shell between horizontal joints

Note 1 to entry: Several courses of the same thickness can together become a strake.

3.1.1.6**fabricated shell**

shell structure that is constructed by rolling plates into curved cylindrical panel sections and then assembling them by welding or bolting into a complete shell form

3.1.1.7**joint**

line between two pieces of shell that are part of the same shell segment but fabricated from different pieces of shell plate

Note 1 to entry: A joint can be welded or bolted or connected in any other manner. The term “joint” is extensively used in shell structures, but it is used with a slightly different meaning from that found in EN 1993-1-8.

3.1.1.8**junction**

line at which two or more shell segments meet

Note 1 to entry: A junction can include a stiffener, which can be treated as a junction at the circumferential line of attachment of a ring stiffener to the shell.

3.1.1.9**lap joint**

joint in which the two shell plates overlap across the joint, increasing the total shell thickness locally

3.1.1.10**manufactured shell**

shell or tubular member that is made in a factory by controlled processes in which the complete circular or elliptical form is achieved through folding, rolling or similar processes and using longitudinal or spiral welding

Note 1 to entry: Manufactured shells or tubular members are typically manufactured to meet the specifications of EN 10210 or EN 10219. Manufactured shells are outside the scope of this document except where permitted by 1.1(3) and 1.1(12).

3.1.1.11**meridian and meridional direction**

line on a shell surface that lies in the plane through the axisymmetric shell axis

Note 1 to entry: The meridional direction is the tangent to the meridian at any point. In a cylinder, the meridian is parallel to the axis and the meridional direction is synonymous with the axial direction. In conical shells the meridian is straight but inclined to the axis. In other shell forms the meridional direction changes with axial position.

3.1.1.12**meridional joint**

joint that lies on the meridian of an axisymmetric shell

3.1.1.13

middle surface

surface that lies midway between the inside and outside surfaces of the shell at every point, which is the reference surface for analysis, and can be discontinuous at changes of thickness or at shell junctions, leading to eccentricities that can be important to the shell structural behaviour

Note 1 to entry: In a shell stiffened on either one or both surfaces, the reference middle surface is still taken as the middle surface of the curved shell plate.

3.1.1.14

notch

position in a constructional detail where locally raised stresses arise that are relevant to the fatigue limit state (LS4)

Note 1 to entry: The term “notch” is widely used in EN 1993-1-9.

3.1.1.15

rib

local member that provides a primary load carrying path for bending down the meridian of the shell, representing a generator of the shell of revolution, used to transfer or distribute transverse loads by bending

3.1.1.16

ring beam or ring girder

circumferential stiffener that has bending stiffness and strength both in the plane of the shell circular section and normal to that plane, acting as a primary load carrying structural member and provided for the distribution of local loads into the shell

3.1.1.17

ring stiffener

local stiffening member that passes around the circumference of the shell of revolution at a given point on the meridian, normally assumed to have no stiffness for deformations out of its own plane corresponding to meridional displacements of the shell but to be stiff for deformations in the plane of the ring, and provided to increase the stability or to introduce local loads acting in the plane of the ring

3.1.1.18

shell

structure or a structural component formed from a curved thin plate

Note 1 to entry: The curvature plays a vital role in its structural resistance and can be either in one direction (cylinder or cone) or two directions (spherical, ellipsoidal, toroidal, hyperboloid etc.).

3.1.1.19

shell of revolution

shell whose geometric form is defined by a middle surface that is formed by rotating a meridional generator line around a single axis through 2π radians

3.1.1.20

shell panel

incomplete shell of revolution

Note 1 to entry: The shell of revolution is termed incomplete if it has meridional boundaries that lie at circumferential locations less than 2π radians apart.

Note 2 to entry: Shell panels are outside the scope of this document except where permitted by 1.1(4).

3.1.1.21

shell segment

shell of revolution in the form of a defined shell geometry, usually with a constant wall thickness but sometimes consisting of multiple strakes

Note 1 to entry: A shell segment can be a cylinder, conical frustum, spherical frustum, annular plate, toroidal knuckle or any other form of shell of revolution.

3.1.1.22

stepped wall

shell with a fixed geometric shape in which different parts have different thicknesses to accommodate the variation of local resistance requirements

3.1.1.23

strake

zone of constant thickness within a shell constructed with a stepped wall

3.1.1.24

stringer stiffener

local stiffening member that follows the meridian of the shell, representing a generator of the shell of revolution, provided to increase the stability, or to assist with the introduction of local loads, but not intended to provide a primary resistance to bending effects caused by transverse loads

3.1.2 Limit states

3.1.2.1

buckling (LS3)

ultimate limit state where the shell structure suddenly loses its stability under membrane compression and/or shear, leading either to large displacements or to the shell being unable to support the applied loads

3.1.2.2

cyclic plasticity (LS2)

ultimate limit state where repeated yielding is caused by cycles of loading and unloading, leading to a low cycle fatigue failure where the local energy absorption capacity of the material is exhausted

3.1.2.3

fatigue (LS4)

ultimate limit state where more than N_f cycles of loading cause cracks to develop in any part of the structure, so that further load cycles can lead to rupture

Note 1 to entry: This limit state is termed “high cycle fatigue” in EN 1990. The value of N_f is defined in 6.3.4(2). EN 1993-1-9 has no provisions for numbers of cycles less than 10 000. For lower numbers of cycles involving high stresses, Clause 8 (LS2) is relevant.

3.1.2.4

plastic failure limit state (LS1)

ultimate limit state where the shell develops zones of yielding with combined membrane and bending deformations in a pattern such that its ability to resist increased loading of the same form is deemed to be exhausted

Note 1 to entry: Many ductile shell structures can continue to resist increased loading with extensive yielding and substantial changes of geometry. For these conditions, a limitation on deformation is used to define the plastic failure limit state.

3.1.2.5

tensile rupture (LS1)

ultimate limit state where the shell plate experiences gross section failure due to membrane tension

3.1.3 Actions

3.1.3.1

axial load

externally applied loading acting in the axial direction in an axisymmetric shell

3.1.3.2

axial compression

axial load inducing compressive membrane stresses in a cylindrical shell

3.1.3.3

external pressure

component of the surface loading acting normal to the shell in the inward direction q

Note 1 to entry: The magnitude of the external pressure can vary in both the meridional and circumferential directions (e.g. under snow, see EN 1991-1-3, or wind, see EN 1991-1-4).

3.1.3.4

global bending

actions causing a cylindrical or conical shell to bend as a complete structure about an axis normal to the axis of the shell

Note 1 to entry: This corresponds to a cosine variation (harmonic 1) of the axial stresses around the circumference of the shell and is equivalent to beam bending.

3.1.3.5

hydrostatic pressure

pressure varying linearly with the axial coordinate in an axisymmetric shell, which is deemed to have its axis vertical

3.1.3.6

internal pressure

component of the surface loading acting normal to the shell in the outward direction p

Note 1 to entry: The magnitude of the internal pressure can vary in both the meridional and circumferential directions (e.g. under solids loading in a silo, see EN 1991-4, or under sloshing pressures in a tank, see EN 1998-4, or tilt settlements under large diameter tanks).

3.1.3.7

local load

point applied force or distributed load acting on a limited part of the circumference of the shell and over a limited height

3.1.3.8

partial vacuum

uniform net external pressure, typically caused by the removal of stored liquids, solids or gas from within a container that is inadequately vented

Note 1 to entry: See EN 1991-4 for actions on silos and tanks.

3.1.3.9

patch load

local distributed load acting normal to the shell

3.1.3.10

radial load

externally applied loading acting normal to the surface of a cylindrical shell or normal to the axis in an axisymmetric shell

3.1.3.11

suction

uniform net external pressure q due to the reduced internal pressure in a shell

Note 1 to entry: The external pressure is given a separate notation q to simplify the provisions of this document. Otherwise many formulae would involve negative values of the outward pressure p . Suction can arise due to openings or vents under wind action, see EN 1991-1-4, or due to a partial vacuum.

3.1.3.12

thermal action

temperature variation either down the shell meridian, or around the shell circumference or through the shell thickness, or combinations of these spatial variations

3.1.3.13

wall friction load

meridional component of the surface loading acting on the shell wall due to friction connected with internal pressure

Note 1 to entry: When solids are contained within the shell, see EN 1991-4.

3.1.4 Stress resultants and stresses in a shell

3.1.4.1

bending stress

bending stress resultant multiplied by 6 and divided by the square of the shell thickness

Note 1 to entry: The above definition is only valid at locations in the shell where the stress distribution through the thickness has remained fully elastic. The subscript notation for a bending stress relates to the direction of the stress, not the axis about which the shell is deformed.

3.1.4.2

bending stress resultants

bending and twisting moments per unit width of shell that arise as the integral of the first moment of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell, such that under elastic conditions, each of these stress resultants induces a stress state that varies linearly through the shell thickness, with a value of zero at the middle surface, resulting in two bending moments and one twisting moment at any point

Note 1 to entry: See Figure 3.3 b).

Note 2 to entry: Under plastic and partially yielded conditions, the same stress resultants lead to different and often complex stress patterns through the thickness.

3.1.4.3

membrane stress

membrane stress resultant divided by the shell thickness

Note 1 to entry: See Figures 3.2 and 3.3 a).

3.1.4.4

membrane stress resultant

force per unit width of the shell wall that arises as the integral of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell

Note 1 to entry: Under elastic conditions, each of these stress resultants induces a stress state that is uniform through the shell thickness, resulting in three membrane stress resultants at any point (see Figures 3.2 and 3.3 a)).

3.1.4.5

transverse shear stress resultants

forces per unit width of shell that arise as the integral of the distribution of shear stresses acting normal to the shell middle surface through the thickness of the shell, such that under elastic conditions, each of these stress resultants induces a stress state that varies parabolically through the shell thickness, resulting in two transverse shear stress resultants at any point

Note 1 to entry: See Figure 3.2.

3.1.5 Types of analysis and their use

3.1.5.1

computational analysis

use of shell analysis software to produce a numerical analysis of the structure

Note 1 to entry: This can take different forms depending on the assumptions adopted in the numerical model, which is usually on the basis of the finite element method.

3.1.5.2

eigenvalue

multiplier on the applied actions that induces a bifurcation

Note 1 to entry: In a computational shell buckling analysis, it is necessary to detect bifurcations from the primary load path. Such possible bifurcations are found using an eigenvalue analysis. The mode of buckling corresponding to an eigenvalue is termed its eigenmode. The term eigenvalue in this document does not relate to a mode of vibration.

3.1.5.3

geometrically and materially nonlinear analysis

GMNA

computational analysis based on shell bending theory applied to the perfect structure, using the assumptions of nonlinear large deflection theory for the displacements and a fully nonlinear elastic-plastic hardening material law, where appropriate, and in which a bifurcation eigenvalue check is included at each load level

3.1.5.4**geometrically and materially nonlinear analysis with imperfections explicitly included GMNIA**

computational analysis with imperfections explicitly included, based on the principles of shell bending theory applied to the imperfect structure, including nonlinear large deflection theory for the displacements that accounts fully for any change in geometry due to the actions on the shell and a fully nonlinear elastic-plastic hardening material law and including a bifurcation eigenvalue check at each load level

Note 1 to entry: The definition of the computational model includes one or more of the following unintended features: deviations of the middle surface from the ideal shape, residual stresses, variations of thickness, misalignment of plates and imperfections in the boundary conditions.

3.1.5.5**geometrically nonlinear elastic analysis****GNA**

computational analysis based on the principles of shell bending theory applied to the perfect structure, using a linear elastic material law but including nonlinear large deflection theory for the displacements that fully accounts for any change in geometry due to the actions on the shell, including a bifurcation eigenvalue check at each load level

3.1.5.6**geometrically nonlinear elastic analysis with imperfections explicitly included****GNIA**

computational analysis with imperfections explicitly included, similar to a GNA analysis as defined in 3.1.5.5, but adopting a model for the geometry of the structure that includes one or more of the following unintended features: deviations of the middle surface from the ideal shape, residual stresses, variations of thickness, misalignment of plates and imperfections in the boundary conditions

Note 1 to entry: It includes a bifurcation eigenvalue check at each load level.

3.1.5.7**global analysis**

analysis that includes the complete structure, rather than individual structural parts treated separately

Note 1 to entry: This is usually a computational analysis.

3.1.5.8**linear elastic shell analysis****LA**

analysis that predicts the behaviour of a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell

Note 1 to entry: It can use standard formulae (see Annex C) or computational analysis.

3.1.5.9**linear elastic bifurcation (eigenvalue) analysis****LBA**

analysis that evaluates the linear bifurcation eigenvalue for a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell

Note 1 to entry: It can use standard formulae (see Annexes D and E) or computational analysis.

3.1.5.10

load level

loading condition achieved at the end of each increment in the progressive incrementation of actions until the limit state is reached in a nonlinear computational analysis

Note 1 to entry: Nonlinear computational analyses are specifically MNA, GNA, GNIA, GMNA or GMNIA.

3.1.5.11

materially nonlinear analysis

MNA

analysis based on shell bending theory applied to the perfect structure, using the assumption of small displacement theory, but adopting an ideal elastic-plastic material law and with no limitation on the plastic strain that can develop

Note 1 to entry: It can use standard formulae (see Annex B) or computational analysis. An ideal elastic-plastic material law assumes an idealized perfectly plastic response after yield with no strain hardening.

3.1.5.12

membrane theory analysis

analysis that predicts the behaviour of a thin-walled shell structure under distributed loads by assuming that only membrane forces satisfy equilibrium with the external loads

Note 1 to entry: Where global bending of a cylindrical or conical shell is involved without unsymmetrical normal pressures, membrane theory and beam theory lead to the same outcome. Where wind or other unsymmetrical normal pressures are involved, beam theory is no longer valid. Standard results for membrane theory are given in Annex A.

3.1.5.13

semi-membrane theory analysis

analysis that predicts the behaviour of an unsymmetrically loaded or supported thin-walled cylindrical shell structure by assuming that only membrane forces and circumferential bending moments satisfy equilibrium with the external loads

3.1.6 Stress categories used in stress design

3.1.6.1

primary stresses

stress system required for equilibrium with the imposed loading

Note 1 to entry: This stress system consists primarily of membrane stresses, but under some situations bending stresses can also be required to achieve equilibrium.

3.1.6.2

secondary stresses

stresses induced by internal compatibility or by compatibility with the boundary conditions, associated with imposed loading or imposed displacements (temperature, pre-stressing, settlement, shrinkage), and not required to achieve equilibrium between an internal stress state and the external loading

3.1.6.3

local stresses

stresses associated with the detailed geometry of constructional details and notches in the shell wall

Note 1 to entry: Relevant constructional details and notches refer to locations such as holes, welds, stepped walls, attachments, stiffener terminations, shell junctions and joints, connections and similar local conditions. These locations lead to local stresses that vary rapidly on a scale smaller than the local thickness of the shell. Such local stresses can generally be ignored in structural resistance evaluations, except those that concern fatigue.

3.1.7 Special definitions for buckling calculations

3.1.7.1

capacity curve

algebraic description of the resistances of all structural systems from elastic imperfect slender systems through elastic-plastic to fully plastic and hardening systems, characterised through the capacity parameters α_G , α_I , β , η_0 , η_p , λ_0 and χ_h

3.1.7.2

characteristic buckling resistance

load associated with buckling in the presence of the geometrical and structural imperfections that are inevitable in practical construction, inelastic material behaviour where appropriate, and follower load effects if relevant

Note 1 to entry: The characteristic buckling resistance is always defined in terms of the characteristic values of the elastic modulus and yield stress of the material.

3.1.7.3

characteristic buckling stress

membrane stress or membrane stress component associated with the characteristic buckling resistance

Note 1 to entry: Most stress states in a practical shell involve membrane stresses in different directions at any location, here termed membrane stress components. When buckling is being assessed, it is typical to take only the most significant component in assessing the buckling resistance, sometimes modified by weaker effects of other components.

3.1.7.4

characteristic plastic resistance

load associated with the formation of a complete plastic mechanism generally involving large displacements and extensive strain hardening which can be limited by achievement of an acceptable displacement criterion

Note 1 to entry: The characteristic plastic resistance is usually much larger than the reference plastic resistance since the complete plastic mechanism in a shell involves a complex and spatially varying interaction between membrane and bending stress resultants, together with significant change of geometry. The characteristic plastic resistance is always defined in terms of the characteristic value of the yield stress of the material.

3.1.7.5

critical buckling resistance

smallest bifurcation load determined assuming the idealized conditions of elastic material behaviour, small deflection theory with no changes of geometry, perfect geometry, perfect load application, perfect support, material isotropy and absence of residual stresses

Note 1 to entry: The term “critical” is strictly limited to this meaning alone, and only an LBA analysis can be used to compute this resistance.

3.1.7.6

critical buckling stress

membrane stress associated with the reference critical buckling resistance

3.1.7.7

design buckling resistance

design value of the buckling load, obtained by dividing the characteristic buckling resistance by the partial factor for resistance

3.1.7.8

design buckling stress

membrane stress or membrane stress component associated with the design buckling resistance

3.1.7.9

design plastic resistance

design value of the plastic resistance, obtained by dividing the characteristic plastic resistance by the partial factor for resistance

3.1.7.10

fabrication tolerance quality class

category of fabrication tolerance requirements that is assumed in design

Note 1 to entry: Buckling-relevant geometrical tolerances are defined in 9.4.

3.1.7.11

key value of the stress

value of stress in a non-uniform stress field that is used to characterise the complete pattern of varying stresses in a buckling limit state assessment

3.1.7.12

reference critical buckling resistance

critical buckling resistance used as a reference resistance in the context of the interactions between elastic and plastic behaviour in defining the characteristic resistance of a shell

Note 1 to entry: Only an LBA analysis can be used to compute this resistance.

3.1.7.13

reference plastic resistance

plastic limit load, determined assuming the idealized conditions of rigid-plastic material behaviour, small deflection theory with no changes of geometry, perfect geometry, perfect load application, perfect support and material isotropy

Note 1 to entry: Only an MNA analysis can be used to compute this resistance.

3.2 Symbols

For the purposes of this document, the following symbols apply.

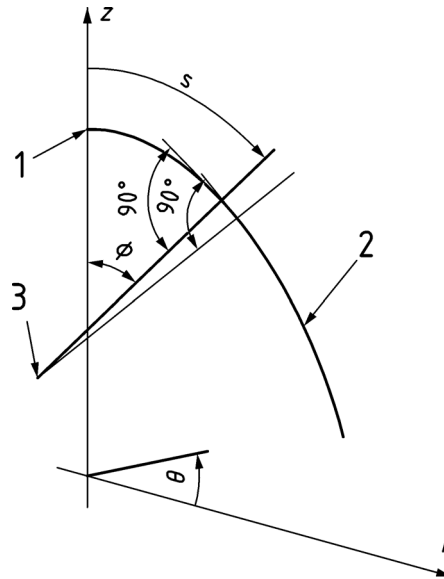
3.2.1 Coordinate system

For coordinate system, see Figure 3.1:

- r radial coordinate of the shell middle surface, normal to the axis of revolution;
- s curvilinear meridional coordinate on general axisymmetric shell;
- x direction tangential to the meridian of a shell;

NOTE This definition of x aligns with the axial direction in a cylindrical shell.

- z axial coordinate of a point on the shell middle surface;
- θ circumferential coordinate of a point on the shell middle surface;
- ϕ meridional slope: angle between axis of revolution and normal to the meridian of the shell.

**Key**

- 1 pole
- 2 shell meridian
- 3 instantaneous centre of meridional curvature

Figure 3.1 — Coordinate system for a shell of revolution**3.2.2 Shell dimensions**

- d diameter of the middle surface of a cylindrical shell;
- h_i height of shell extending from the upper boundary to the base of course i ;
- h_{cr} height of the critical buckle in a stepped wall cylinder under external pressure;
- L cylinder or cone length between defined boundaries;
- ℓ_s length of a shell segment between boundaries that are either BC1 or BC2;
- ℓ_g gauge length for measurement of geometric imperfections;
- $\ell_{g\theta}$ gauge length in circumferential direction for measurement of geometric imperfections;
- ℓ_{gw} gauge length across welds for measurement of geometric imperfections;
- ℓ_{gx} gauge length in meridional direction for measurement of geometric imperfections;
- ℓ_R boundary zone length in which buckling strength assessment may be omitted (see D.4.3);
- r radius of the shell middle surface, normal to the axis in an axisymmetric shell;
- r_1, r_2 simple radii of the top and bottom of a conical shell;
- r_s radius of a spherical shell;
- t thickness of shell wall;
- t_{max} maximum thickness of shell wall at a joint;
- t_{min} minimum thickness of shell wall at a joint;
- t_{av} average thickness of shell wall at a joint;
- x_e exclusion distance for stress locations involving a tangent modulus reduction (see 5.1);

- β apex half angle of cone;
 ϕ_0 meridional slope at the support of a spherical shell.

3.2.3 Distributed surface loads and pressures

- p_n pressure normal to the shell (outward);
 p_x meridional surface load parallel to the shell;
 p_θ circumferential surface load parallel to the shell;
 q pressure normal to the shell (inward).

3.2.4 Line forces

- P_n load per unit circumference normal to the shell (outward);
 P_x load per unit circumference acting in the meridional direction;
 P_θ load per unit circumference acting circumferentially on the shell.

3.2.5 Membrane stress resultants

- n_x meridional (axial in a cylinder) membrane stress resultant;
 n_θ circumferential membrane stress resultant;
 $n_{x\theta}$ membrane shear stress resultant.

3.2.6 Bending stress resultants

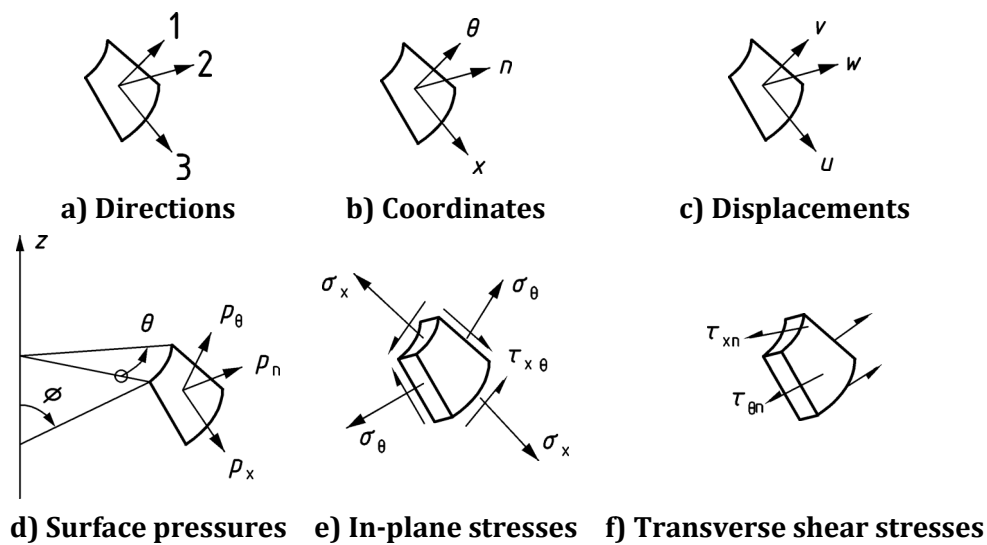
- m_x meridional (axial in a cylinder) bending moment per unit width;
 m_θ circumferential bending moment per unit width;
 $m_{x\theta}$ twisting shear moment per unit width;
 q_{xn} transverse shear force associated with meridional bending;
 $q_{\theta n}$ transverse shear force associated with circumferential bending.

3.2.7 Stresses

- σ_x meridional (axial in a cylinder) stress;
 σ_θ circumferential stress;
 σ_{eq} von Mises equivalent stress (can also take negative values during cyclic loading);
 $\tau, \tau_{x\theta}$ in-plane shear stress;
 $\tau_{xn}, \tau_{\theta n}$ meridional, circumferential transverse shear stresses associated with bending.

3.2.8 Displacements relative to the shell middle surface

- u meridional displacement;
 v circumferential displacement;
 w displacement normal to the shell surface;
 β_ϕ meridional rotation, see 4.3 (4) and 6.2.2.



- Key
- 1 circumferential
 - 2 normal
 - 3 meridional

Figure 3.2 — Symbols in a shell of revolution

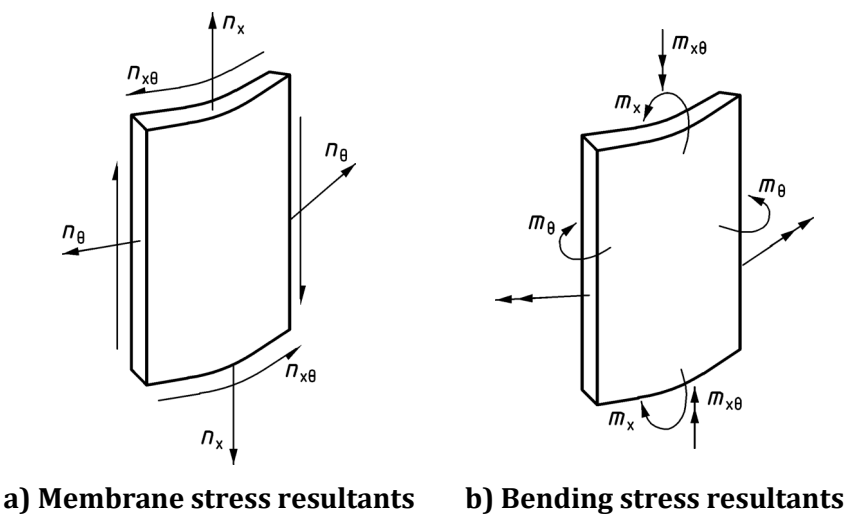


Figure 3.3 — Membrane and bending stress resultants in a cylindrical shell

3.2.9 Tolerances

- e_a eccentricity between the middle surfaces of joined plates;
- U_e unintended eccentricity tolerance parameter;
- U_r out-of-roundness tolerance parameter;
- U_n initial dimple imperfection amplitude parameter for computational calculations;
- U_0 initial dimple tolerance parameter;
- δ_a calculated amplitude in a computational treatment (Figure 9.4);
- δ_0 assumed geometric imperfection amplitude in design and, equivalently, the measured tolerance normal to the shell surface;
- δ_{u0} assumed interface flatness amplitude in design and, equivalently, the measured meridional interface flatness tolerance between a shell and its support;
- δ_m measured amplitude using a tolerance measurement (Figure 9.4).

3.2.10 Properties of materials

- E Young's modulus of elasticity;
- E_{red} reduced elastic modulus to account for stress-strain nonlinearity or thermal effects;
- E_{sh} tangent strain hardening modulus;
- f_{eq} von Mises equivalent strength;
- $f_{p,\theta}$ temperature-dependent stress-strain proportionality limit;
- f_y yield strength;
- f_u ultimate strength;
- ν Poisson's ratio.

3.2.11 Parameters in resistance assessment

- $a_{p,eq}$ cyclic plasticity assessment factor;
- a_i coefficient in buckling strength interaction (see D.4.3);
- C_x coefficient in axial compression critical buckling resistance;
- $C_\theta, C_{\theta s}$ coefficients in external pressure critical buckling resistance;
- $C_\tau, C_{\tau s}, C_{\tau L}$ coefficients in shear critical buckling resistance;
- D accumulated fatigue damage (see EN 1993-1-9);
- f_{lim} limiting stress for fatigue check (see 6.3.4 (2));
- F generalised action;
- F_{Ed} action set on a complete structure corresponding to a design situation (design values);
- F_{Rd} calculated values of the action set at the maximum resistance condition of the structure (design values);
- j_i joint efficiency factor, where $i = 1$ or 2 ;
- k_f stress concentration factor in fatigue assessment based on linear analysis;

$k_{f,imp}$	stress concentration factor in fatigue assessment accounting for local imperfection;
k_{GMNIA}	calibration factor on resistance when using nonlinear analyses;
$k_x, k_\theta, k_{x\theta}, k_i$	ratio of i^{th} design stress component to its uniform design buckling stress (see 9.4.2);
$k_{ix}, k_{i\theta}, k_{it}$	power of interactions in buckling strength interaction (see D.4.3);
k_{eq}	ratio of von Mises equivalent surface stress to equivalent membrane stress at a point;
N_f	number of cycles of loading in a fatigue assessment (LS4);
N_{cp}	number of cycles of loading in a cyclic plasticity assessment (LS2);
Q	fabrication tolerance quality parameter;
R	generalised resistance;
R_{cr}	reference elastic critical buckling resistance ratio (defined as a load factor on design loads using LBA analysis);
R_k	characteristic reference resistance ratio (used with subscripts to identify the basis): defined as a load factor on design loads using the ratio (F_{Rk} / F_{Ed});
R_{pl}	reference plastic resistance ratio (defined as a load factor on design loads using MNA analysis);
R_{plf}	plastic failure resistance ratio (defined as a load factor on design loads using GMNA analysis);
R_{GNA}	buckling resistance ratio determined in a GNA analysis;
R_{GMNA}	buckling resistance ratio determined in a GMNA analysis;
R_{GMNIA}	buckling resistance ratio determined in a GMNIA analysis (normally as R_k);
R_{MNA}	plastic resistance ratio determined in an MNA analysis;
S_{irat}	ratio of i^{th} stress state in determining the dominant stress component in buckling;
α	elastic buckling reduction factor in buckling strength assessment;
α_G	geometric reduction factor;
α_i	imperfection reduction factor;
α_{SLM}	elastic buckling reduction factor for a segment in the LBA-MNA procedure;
α_S	elastic buckling reduction factor for a segment or complete structure;
β	plastic range factor in buckling interaction;
γ_F	partial factor for actions and action effects;
γ_M	partial factor for resistance;
γ_{M0}	partial factor for plastic resistance;
γ_{M1}	partial factor for resistance to stability (buckling);
γ_{M2}	partial factor for resistance to tensile rupture, including the net section in bolted construction;
γ_{M4}	partial factor for resistance to cyclic plasticity;
γ_{Mf}	partial factor for resistance to fatigue;
γ_{Ff}	partial factor for fatigue loads and load effects;

Δ	range of parameter when alternating or cyclic actions are involved;
ε_{mps}	maximum permitted von Mises equivalent plastic true strain;
ε_p	plastic strain;
$\varepsilon_{p,eq,Ed}$	total accumulated von Mises equivalent plastic strain under cyclic plasticity;
η	interaction exponent for buckling;
η_0	value of interaction exponent at $\bar{\lambda} = \bar{\lambda}_0$;
η_p	value of interaction exponent at $\bar{\lambda} = \bar{\lambda}_p$;
$\bar{\lambda}$	relative slenderness of a shell;
$\bar{\lambda}_s$	complete shell relative slenderness for a complete shell or shell assembly (multiple segments);
$\bar{\lambda}_{SLM}$	shell segment relative slenderness in the LBA-MNA procedure;
$\bar{\lambda}_0$	squash limit relative slenderness (value of $\bar{\lambda}$ above which resistance reductions due to instability or change of geometry occur);
$\bar{\lambda}_p$	plastic limit relative slenderness (value of $\bar{\lambda}$ below which plasticity affects the stability);
μ	hardening exponent for buckling;
μ_0	value of hardening exponent at $\bar{\lambda} = \bar{\lambda}_0$;
μ_h	value of hardening exponent at $\bar{\lambda} = 0$;
ψ	load combination factor;
ω	first relative length parameter for a cylindrical shell;
ω_s	upper limit of relative length for short cylindrical shells under external pressure;
Ω	second relative length parameter for a cylindrical shell;
ξ	third relative length parameter for a cylindrical shell;
χ	buckling reduction factor including elastic-plastic effects in buckling strength assessment;
χ_h	buckling reduction factor in the hardening zone at $\bar{\lambda} = 0$;
χ_s	complete shell buckling reduction factor including elastic-plastic effects in a shell assembly.

3.2.12 Subscripts

E	value of stress or displacement (arising from design actions);
F	actions;
M	material;
R	resistance;
cr	critical buckling value (see 3.1.7.5);
d	design value;
eq	von Mises equivalent;

f	fatigue;
int	internal;
k	characteristic value;
max	maximum value;
min	minimum value;
nom	nominal value;
pl	plastic value;
plf	plastic failure value (LS1);
s	for a complete shell, potentially with multiple segments making a shell assembly;
u	ultimate;
y	yield.

Further symbols are defined where they first occur.

3.3 Sign conventions

(1) Outward direction positive: internal pressure positive, outward displacement positive, except as noted in (5).

(2) Tensile stresses positive, except as noted in (5).

NOTE Compression is treated as positive in EN 1993-1-1.

(3) Shear stresses positive as shown in Figures 3.2 and D.1.

NOTE Although the directions of direct stresses differ between Figures 3.2 and D.1, the direction of in-plane shear is retained unchanged.

(4) Bending moments are defined as positive when they induce tensile stresses on the outer surface of the shell.

(5) For simplicity, in Clause 9 and Annexes D and E, compressive stresses are treated as positive. For these cases, both external pressures and internal pressures are treated as positive where they occur, though the notations p and q are used to identify the direction.

4 Basis of design

4.1 General rules

4.1.1 Basic requirements

(1) The design of shell structures shall be in accordance with the general rules given in EN 1990 and EN 1991 (all parts), and the specific design provisions for steel structures given in the other relevant parts of EN 1993 (all parts).

(2) Steel structures designed according to this document shall be executed according to EN 1090-2 with construction materials and products used as specified in the relevant parts of EN 1993, or in the relevant material and product specifications.

(3) This document is intended for use in conjunction with EN 1993-1-1, EN 1993-1-2, EN 1993-1-3, EN 1993-1-4, EN 1993-1-7, EN 1993-1-9, EN 1993-1-14 and the relevant application parts of EN 1993, which include:

- Part 3 for towers, masts and chimneys;
- Part 4-1 for silos;
- Part 4-2 for tanks;
- Part 5 for piles.

4.1.2 Specific requirements

(1) The shell should be designed in such a way that it will sustain all actions and satisfy the following requirements:

- overall equilibrium;
- equilibrium between actions and internal forces and moments, see Clause 7 and Clause 9;
- limitation of cracks due to cyclic plastification, see Clause 8;
- limitation of cracks due to fatigue, see Clause 10.

(2) The design of the shell should satisfy the serviceability requirements in accordance with its intended use and as set out in appropriate application standards (EN 1993-4-1 and EN 1993-4-2).

(3) The shell may be proportioned using design assisted by testing. Where appropriate, the requirements are set out in the appropriate application standard (EN 1993-3, EN 1993-4-1 and EN 1993-4-2).

(4) All actions should be introduced using their design values according to EN 1990, EN 1991 (all parts) and EN 1993-4-1 and EN 1993-4-2, as appropriate.

4.2 Types of analysis

4.2.1 General

(1) One or more of the following types of analysis should be used, depending on the limit state and other considerations:

- Global analysis, see 4.2.2;
- Membrane theory analysis, see 4.2.3;
- Semi-membrane theory analysis, see 4.2.4;
- Linear elastic shell analysis (LA), see 4.2.5;
- Linear elastic bifurcation analysis (LBA), see 4.2.6;
- Geometrically nonlinear elastic analysis (GNA), see 4.2.7;
- Materially nonlinear analysis (MNA), see 4.2.8;
- Geometrically and materially nonlinear analysis (GMNA), see 4.2.9;
- Geometrically nonlinear elastic analysis with imperfections explicitly included (GNIA), see 4.2.10;
- Geometrically and materially nonlinear analysis with imperfections explicitly included (GMNIA), see 4.2.11.

4.2.2 Global analysis

(1) In a global analysis, simplified treatments may be used for discrete parts of the structure consisting of a cylinder, cone, sphere or other structural form, provided that the connections between different parts are appropriately modelled.

4.2.3 Membrane theory analysis

(1) A membrane theory analysis may be used provided that the following conditions are met:

- the boundary conditions are appropriate for transfer of the stresses in the shell into support reactions without causing unacceptable bending effects;
- the shell geometry varies smoothly in shape (without discontinuities);
- the loads have a smooth distribution (without locally concentrated or point loads).

(2) A membrane theory analysis does not meet the requirements of compatibility of deformations at boundaries or between shell segments of different shape or between shell segments subjected to different loading. However, the resulting field of membrane forces satisfies the requirements of equilibrium of the primary stresses (useful for LS1).

NOTE A membrane theory analysis using harmonic series of the form $\cos n\theta$ (for integer $n \geq 0$) can be useful for unsymmetrical loads with smooth variations (up to $\cos 4\theta$) where axial displacements of the boundary are not involved (e.g. see EN 1991-1-4:—², C.6.3).

4.2.4 Semi-membrane theory analysis

(1) A semi-membrane theory analysis may be used when a long cylindrical shell is subject to a circumferentially varying load with a variation more rapid than a single full cosine around the circumference (i.e. $\cos \theta$ or 'harmonic 1') and also subject to axial displacements at a boundary (e.g. wind loading with periodic anchors or discretely supported shells).

4.2.5 Linear elastic shell analysis (LA)

(1) The linearity of the theory results from the assumptions of a linear elastic material law and small deformation theory. Small deformation theory implies that the assumed geometry remains that of the undeformed structure. It satisfies compatibility in the deformations as well as equilibrium. The resulting field of membrane and bending stresses satisfy the requirements of primary plus secondary stresses (useful for LS1, LS2, LS3 and LS4).

(2) This analysis is normally undertaken as a computational analysis but in limited cases algebraic formulae may also be used.

NOTE Appropriate algebraic formulae are given in Annex C.

(3) Where a computational analysis is undertaken, the modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

4.2.6 Linear elastic bifurcation analysis (LBA)

(1) A linear bifurcation analysis (LBA) satisfies the conditions of 4.2.5 concerning the material and geometric assumptions. However, this linear bifurcation analysis obtains the lowest eigenvalue at which the shell can buckle into a different deformation mode, assuming no change of geometry, no change in the direction of action of the loads, and no material degradation. Imperfections of all kinds are ignored.

² Under preparation. Stage at the time of publication: prEN 1991-1-4:2024.

This analysis provides the reference elastic critical buckling resistance R_{cr} (see 9.6, 9.7 and 9.8; useful for LS3), which can be interpreted as a load amplification factor R_{cr} on the design value of the loads F_{Ed} .

(2) This analysis is normally undertaken as a computational analysis but in limited cases algebraic solutions may also be used (see Annexes D and E).

(3) Where a computational analysis is undertaken, the modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

(4) This perfect shell elastic critical load should always be determined when the limit state LS3 is verified using GMNIA analysis (see 9.8).

(5) Where this analysis is used as the basis for an LBA-MNA design procedure, multiple eigenvalues (not only the lowest eigenvalue) and their corresponding eigenmodes should be explored to ensure that the eigenmode whose imperfection sensitivity can lead to the lowest elastic buckling prediction is found (see 9.7).

NOTE The imperfection sensitivity of the shell depends on the size and form of the potential buckling mode, so a lower eigenvalue corresponding to an insensitive mode can fail to detect the mode with the lowest imperfect shell buckling resistance.

4.2.7 Geometrically nonlinear elastic analysis (GNA)

(1) A GNA analysis satisfies both equilibrium and compatibility of the deformations under conditions in which the change in the geometry of the structure caused by loading is included. The resulting field of stresses matches the definition of primary plus secondary stresses (useful for LS2 and LS4). This is normally undertaken as a computational analysis.

(2) Where membrane compression or shear stresses are predominant in some part of the shell, a GNA analysis delivers the elastic buckling load of the perfect structure, including changes in geometry, that can be of assistance towards a check of the limit state LS3 (see 9.8).

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system shall be checked throughout the loading path to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(4) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

4.2.8 Materially nonlinear analysis (MNA)

(1) The result of an MNA analysis leads to the reference plastic limit load, which can be interpreted as a load amplification factor R_{pl} on the design value of the loads F_{Ed} . This analysis provides the reference plastic resistance ratio R_{pl} used in 9.6, 9.7 and 9.8.

(2) This analysis may be undertaken using a computational analysis or algebraic formulae.

NOTE Appropriate algebraic formulae are given in Annex B.

(3) Where a computational analysis is undertaken, the modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

(4) An MNA analysis may be used to verify limit state LS1.

(5) An MNA analysis may be used to give the plastic strain increment $\Delta\varepsilon_p$ during one cycle of cyclic loading that may be used to verify limit state LS2.

(6) This perfect shell plastic limit load should always be determined when the limit state LS3 is verified using GMNIA analysis (see 9.8).

4.2.9 Geometrically and materially nonlinear analysis (GMNA)

(1) The result of a GMNA analysis, analogously to 4.2.7, gives the geometrically nonlinear plastic failure load of the perfect structure and the plastic strain increment that can be used for checking the limit states LS1 and LS2. This is strictly a computational analysis.

NOTE Where no yielding or plasticity is involved, a GMNA analysis produces the same result as a GNA analysis.

(2) Where compression or shear stresses are predominant in some part of the shell, a GMNA analysis gives the elastic-plastic buckling load of the perfect structure. This perfect shell buckling load should always be determined when the limit state LS3 is verified using GMNIA analysis (see 9.8).

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(4) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met.

4.2.10 Geometrically nonlinear elastic analysis with imperfections explicitly included (GNIA)

(1) A GNIA analysis is used in cases where compression or shear stresses dominate in the shell. It delivers elastic buckling loads of the imperfect structure that can be of assistance in checking the limit states LS3 and LS4 (see 9.8). This is strictly a computational analysis.

(2) Where this analysis is used for a buckling load evaluation (LS3), the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path. Care should be taken to ensure that the local stresses do not exceed values at which material nonlinearity can affect the behaviour.

NOTE 1 GNIA analysis is often useful for very thin shells where plasticity plays no role in the ultimate limit state.

NOTE 2 Where a GNIA analysis produces a buckling resistance that is very similar (or higher) to that of a GNA analysis, the imperfection modelled in the GNIA analysis is not one to which the structure is detrimentally sensitive.

(3) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met, but the imperfections should be defined according to this document.

4.2.11 Geometrically and materially nonlinear analysis with imperfections explicitly included (GMNIA)

(1) A GMNIA analysis is used in cases where compression or shear stresses are dominant in the shell. It delivers elastic-plastic buckling loads for the imperfect structure, that may be used for checking the limit state LS3 (see 9.8). This is strictly a computational analysis.

(2) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(3) Where this analysis is used for a buckling load evaluation, additional LBA, MNA and GMNA analyses of the perfect shell should always be conducted to ensure that the slenderness is properly recognised and that the degree of imperfection sensitivity of the structural system is identified.

NOTE Whilst not required, it is desirable that an additional GNA analysis is also undertaken to provide the maximum insight into the structural behaviour under nonlinear conditions.

(4) The modelling, mesh, validation and verification criteria of EN 1993-1-14 should be met, but the imperfections should be defined according to this document.

4.3 Shell boundary conditions

(1) The boundary conditions assumed in the design calculation should be chosen in such a way as to ensure that they achieve a realistic or conservative model of the real construction. Special attention

should be given not only to the constraint of displacements normal to the shell wall (deflections), but also to whether the displacements in the plane of the shell wall (meridional and circumferential) are adequately constrained because of the significant effect these displacements can have on the shell strength and buckling resistance.

NOTE The buckling resistance of a shell is often sensitive to any minor flexibility in the boundary conditions, making the modelling of realistic boundary conditions more critical than for a simple load-deformation analysis. Boundary conditions relating to membrane displacements in the shell often have a strong influence on buckling resistances.

(2) In shell buckling (eigenvalue) calculations (limit state LS3), the definition of the boundary conditions should refer to the incremental displacements during the buckling process, and not to total displacements induced by the applied actions before buckling.

(3) The boundary conditions at a continuously supported lower edge of a shell should take into account whether local uplifting of the shell is fully prevented or not.

(4) The shell edge rotation β_ϕ should be particularly considered in short shells and in the calculation of secondary stresses in longer shells (according to the limit states LS2 and LS4).

(5) The boundary conditions set out in 6.2.2.2 should be used in computer analyses and in selecting formulae from Annexes A to E.

(6) The structural connections between shell segments at a junction should be such as to ensure that the boundary condition assumptions used in the design of the individual shell segments are satisfied.

4.4 Verification by the partial factor method

(1) Where structural properties are determined by testing, the requirements and procedures of EN 1990 should be adopted.

(2) The partial factors γ_{Mi} for different limit states should be taken from Table 4.1.

Table 4.1 — Partial factors for resistance

Resistance to failure mode	Relevant γ
Resistance of welded or bolted shell wall to plastic limit state	γ_{M0}
Resistance of shell to stability	γ_{M1}
Resistance of welded or bolted shell wall to rupture	γ_{M2}
Resistance of shell to cyclic plasticity	γ_{M4}
Resistance of shell to fatigue	γ_{Ff}

(3) The numerical values in Table 4.2 are recommended for shell structures that are not covered by the provisions of EN 1993-3, EN 1993-4-1 and EN 1993-4-2.

Table 4.2 (NDP) — Numerical values for partial factors for resistance for shell structures outside the scope of EN 1993-3, EN 1993-4-1 and EN 1993-4-2

$\gamma_{M0} = 1,00$	$\gamma_{M1} = 1,10$	$\gamma_{M2} = 1,25$
$\gamma_{M4} = 1,00$		For γ_{Mf} see EN 1993-1-9

NOTE 1 The values of the partial factors γ_{Mi} are given in Table 4.2 (NDP) unless the National Annex gives different values.

NOTE 2 When a reliability analysis is used to determine the appropriate partial factor for shell buckling γ_{M1} , it is found to depend very strongly on the structural form, the slenderness of the shell, the load case and the buckling

mode, since the imperfection sensitivity and the consequent variability of the buckling resistance varies considerably with these factors. Due to lack of reliable data relevant to practical construction, the drafting committee chose to retain the historically accepted value of γ_{M1} at this time.

5 Materials and geometry

5.1 Material properties

(1) The relevant material properties of carbon steels should be obtained from the relevant application standard.

(2) For the mechanical properties of the structural carbon steels S235, S275, S355, S420 and S460 and also for weathering steel grades S235, S275 and S355, see EN 1993-1-1, except as defined in (3).

(3) For all the steels covered by this document, the design value of Poisson's ratio should be taken as $\nu = 0,3$. The characteristic value of the elastic modulus for structural steel should always be taken as $E = 200\,000\text{ N/mm}^2$, in accordance with the value defined for stability calculations in EN 1993-1-14.

NOTE Most of the design rules in this document have been derived from GMNIA calculations and not from experiments. The value of elastic modulus defined in EN 1993-1-14 provides a safe choice where it is required to define the elastic buckling resistance, which is the controlling resistance for most thin shells. This value is lower than that given in EN 1993-1-1 and is used here as a conservative choice. The same choice is also relevant to stainless steels.

(4) For stainless steels covered by this document, the characteristic value of the elastic modulus should always be taken as $E = 191\,000\text{ N/mm}^2$, which represents an initial tangent value of Young's modulus E and is in accordance with the value defined for stability calculations in EN 1993-1-14.

(5) Other relevant material properties of stainless steels should be obtained from EN 1993-1-4. For the yield stress and ultimate strength, the values given in EN 10088 for $R_{p,0.2}$ and R_m respectively should be used.

(6) In a computational analysis using materials with a nonlinear stress-strain relationship, the 0,2 % proof stress should be used to represent the yield stress f_y in all relevant formulae. The stress-strain curve should be modelled in accordance with the requirements of EN 1993-1-14:—³, 5.3.

(7) Where a material with a nonlinear stress-strain curve is involved and a buckling analysis is carried out under stress design (see 9.5) and the special provisions for stainless steel do not apply, the initial tangent value of Young's modulus E should be replaced by a reduced value E_{red} . If no better method is available, the linear elastic stress state should be examined. The peak value of the von Mises equivalent stress derived from the membrane stress components alone at any point in the structure that is more distant than x_e from any boundary should be found. The tangent modulus (from a tensile test) corresponding to this stress should then be taken as E_{red} to replace the elastic modulus E and thus to obtain an estimate of the quasi-elastic critical load or quasi-elastic critical stress.

(8) The exclusion distance x_e may be taken as equal to the boundary zone distance ℓ_R defined in D.4.3.

(9) The provisions of this document are relevant to material properties at temperatures not exceeding 100 °C, except as defined in (10).

(10) The variation of properties of steels at temperatures above 100 °C are given in EN 1993-1-2. Where these are adopted, the reduced yield stress may be conservatively taken as the temperature-dependent proportionality limit $f_{p,\theta}$ defined therein. The mechanical properties of steel grades not represented in EN 1993-1-2 should be based on reliable information.

³ Under preparation. Stage at the time of publication: FprEN 1993-1-14:2025.

(11) High strength steels, as defined by EN 1993-1-12, should only be used with a GMNA or GMNIA analysis for the purposes of verifying LS1, LS2 and LS3.

5.2 Design values of geometrical data

(1) The thickness t of the shell should be taken as defined in the relevant application standard. If no application standard is relevant, the nominal thickness of the wall, reduced by the prescribed value of the corrosion loss and ignoring any coatings, should be used.

(2) The thickness ranges within which the rules of this document apply are defined in the relevant EN 1993 application parts.

(3) The middle surface of the shell should be taken as the reference surface for loads.

(4) The simple radius r of the shell should be taken as the nominal radius of the middle surface of the shell, measured normal to the axis of revolution.

NOTE This radius varies with position on the axis in all shells that are not simply cylindrical.

(5) The buckling design rules of this document should not be applied outside the ranges of the r/t ratio set out in 1.1 (12), or where stricter restrictions apply as defined in Clause 9, Clause 10, Annex D or Annex E, or in the relevant EN 1993 application parts.

5.3 Geometrical tolerances and geometrical imperfections

(1) Tolerance values for the deviations of the geometry of the shell surface from the nominal values are defined in the execution standard EN 1090-2. Relevant categories for the design of shells for the ultimate limit state of buckling (LS3) (see 9.4) are:

- local dimples (local normal deviations from the nominal middle surface);
- out-of-roundness (deviation from circularity);
- eccentricities (deviations from a continuous middle surface in the direction normal to the shell across the junctions between plates);
- deviations of the base of a shell from full contact with the support.

NOTE The requirements for execution are set out in EN 1090-2, but a fuller description of these tolerances is given here because there is a critical relationship between the form of the tolerance measure, its amplitude and the evaluated buckling resistance of a shell. The buckling-relevant tolerances defined here can differ from those defined in EN 1090-2.

(2) If the limit state of buckling (LS3, as described in 6.3.3) is one of the ultimate limit states to be considered, the buckling-relevant geometrical tolerances should be carefully observed in order to keep the geometrical imperfections within specified limits. These buckling-relevant geometrical tolerances and the conditions to which they are relevant are identified and quantified in Clause 9 or in the relevant EN 1993 application parts.

(3) Calculation values for the deviations of the shell surface geometry from the nominal geometry, as required for geometrical imperfection assumptions (complete shell imperfections or local imperfections) for buckling design by computational GMNIA analysis, should be derived from the specified geometrical tolerances (see 9.4). Relevant rules are given in 9.8 or in relevant EN 1993 application parts.

(4) If the limit state of fatigue (LS4, as described in 6.3.4) is one of the ultimate limit states to be considered, consideration should be given to adopting an appropriate choice of the buckling-relevant geometrical tolerances as imposed imperfections that can exacerbate fatigue failure at a specific location (see 9.4).

6 Structural analysis

6.1 Types of design

6.1.1 Stress design

6.1.1.1 General

(1) Where the stress design approach is used, the limit states should be assessed in terms of three categories of stress: primary, secondary and local. The categorisation is performed, in general, on the von Mises equivalent stress at a point, but buckling stresses cannot be assessed reliably using this value.

6.1.1.2 Primary stresses

(1) The primary stresses should be taken as the stress system required for equilibrium with the imposed loading. They may be calculated from any realistic statically admissible determinate system. The plastic failure limit state (LS1) should be deemed to be reached when the primary stress reaches the yield strength throughout the full thickness of the wall at a sufficient number of points, such that only the strain hardening reserve or a change of geometry would lead to an increase in the resistance of the structure.

(2) The calculation of primary stresses should be based on any system of stress resultants, consistent with the requirements of equilibrium of the structure. It may also take into account the benefits of plasticity theory. Alternatively, since linear elastic analysis satisfies equilibrium requirements, its predictions may also be used as a conservative representation of the plastic failure limit state (LS1). Any of the analysis methods given in 6.2.3 may be applied.

(3) Because limit state design for LS1 allows for full plastification of the cross-section, the primary stresses due to bending moments may be calculated on the basis of the plastic section modulus, see 7.2.1. Where there is interaction between stress resultants in the cross-section, interaction rules based on the Ilyushin yield criterion may be applied.

(4) The primary stresses should be limited to the design value of the yield strength, see Clause 7 (LS1).

6.1.1.3 Secondary stresses

(1) In statically indeterminate structures, account should be taken of the secondary stresses, induced by internal compatibility and compatibility with the boundary conditions, that are caused by imposed loading or imposed displacements (temperature, prestressing, settlement, shrinkage).

NOTE As the von Mises yield condition is approached, the local strains in the structure increase without further increase in the stress state.

(2) Where cyclic loading causes plasticity, and several loading cycles occur, consideration should be given to the possible reduction of resistance caused by the secondary stresses. Where the cyclic loading is of such a magnitude that yielding occurs both at the maximum load and again on unloading, account should be taken of a possible failure by cyclic plasticity associated with the secondary stresses (LS2).

(3) If the stress calculation is carried out using a linear elastic analysis that allows for all relevant compatibility conditions (effects at boundaries, junctions, variations in wall thickness, misalignment of the middle surface etc.), the stresses that vary linearly through the thickness may be taken as the sum of the primary and secondary stresses and used in an assessment involving the von Mises yield criterion, see 7.2.

NOTE The secondary stresses are never needed in an evaluation without inclusion of the primary stresses.

(4) The secondary stresses should be limited as follows:

- The sum of the cyclic change in the von Mises equivalent surface stress derived from the sum of the primary and secondary stresses (including bending stresses) should be limited to $2f_{yd}$ for the condition of cyclic plasticity, see Clause 8 (LS2);
- The membrane component of the sum of the primary and secondary stresses should be limited by the design buckling resistance, see Clause 9 (LS3).
- The sum of the cyclic change in the surface maximum principal stress derived from the primary and secondary stresses (including bending stresses) should be limited to the nominal fatigue resistance, see Clause 10 (LS4) and EN 1993-1-9.

6.1.1.4 Local stresses

(1) The highly localised stresses associated with stress raisers in the shell wall due to notch effects (holes, welds, stepped walls, attachments, stiffener terminations, shell junctions and joints) should be taken into account in a fatigue assessment (LS4) using the provisions for modified nominal stresses defined in EN 1993-1-9.

(2) For construction details given in EN 1993-1-9, the fatigue design may be based on the nominal linear elastic stresses (sum of the primary and secondary stresses) at the relevant point. For all other details, the local stresses may be calculated by applying stress concentration factors (notch factors) according to EN 1993-1-9 to the stresses calculated using a linear elastic analysis.

(3) The local stresses should be limited according to the requirements for fatigue (LS4) set out in Clause 10 and EN 1993-1-9.

6.1.2 Design using standard formulae

(1) Where this concept is used, the limit states may be represented by standard formulae that have been derived from either membrane theory, plastic mechanism theory, linear elastic analysis or geometrically and materially nonlinear analysis with explicit imperfections.

(2) The membrane theory formulae given in Annex A may be used to determine the primary stresses needed for assessing LS1 and LS3.

(3) The formulae for plastic design given in Annex B may be used to determine the reference plastic resistances needed for assessing LS1.

(4) The formulae for linear elastic analysis given in Annex C may be used to determine stresses of the primary plus secondary stress type needed for assessing LS2 and LS4. An LS3 assessment may be based on the membrane part of these formulae.

(5) Use the formulae given in Annex D (normative) to give a direct assessment of the design buckling resistance of cylindrical shells under uniform loads according to LS3.

(6) Use the formulae for reference resistance design given in Annex E (normative). To give a direct assessment of the design buckling resistance for assessing LS3. Where the formulae of Annex E are used with the imperfection amplitude δ_0 assigned to be zero, Annex E may also be used to assess LS1.

6.1.3 Design by computational analysis

(1) Where a computational analysis is used, the assessment of the limit states should be carried out using one of the alternative types of analysis specified in 4.2 (but not membrane or semi-membrane theory analysis) applied to the complete structure.

(2) Linear elastic analysis (LA) may be used to determine stresses or stress resultants, for use in assessing LS2 and LS4. The membrane parts of the stresses found by LA may be used in assessing LS3. LS1 may be

assessed using LA, but LA only gives an approximate and safe estimate and its results should be interpreted as set out in Clause 7.

(3) Linear elastic bifurcation analysis (LBA) may be used to determine the reference elastic critical buckling resistance of the structure, for use in assessing LS3.

(4) A materially nonlinear analysis (MNA) may be used to determine the reference plastic resistance, and this may be used for assessing LS1. Under a cyclic loading history, an MNA analysis may be used to determine plastic strain incremental changes, for use in assessing LS2. The reference plastic resistance is also required as part of the assessment of LS3, and this may be found from an MNA analysis.

(5) Geometrically nonlinear elastic analyses (GNA and GNIA) include consideration of the deformations of the structure, but none of the design methodologies of Clause 9 (LS3) permit these to be used without a GMNIA analysis. A GNA analysis may be used to determine the elastic buckling load of the perfect structure. A GNIA analysis may be used to determine the elastic buckling load of the imperfect structure. A GNIA analysis, with an appropriate choice of geometrical imperfections, may also be used to determine the stresses relevant to a fatigue assessment (LS4).

(6) Geometrically and materially nonlinear analysis (GMNA and GMNIA) may be used to determine collapse loads for the perfect (GMNA) and the imperfect structure (GMNIA). The GMNA analysis may be used in assessing LS1, as detailed in 7.3. The GMNIA collapse load may be used, with additional consideration of the results of LBA, MNA and GMNA analyses to assess LS3 as detailed in 9.8. Under a cyclic loading history, the plastic strain incremental changes taken from a GMNA analysis may be used for assessing LS2, as detailed in 8.3.

6.2 Stress resultants and stresses in shells

6.2.1 Stress resultants in the shell

(1) In principle, the eight stress resultants in the shell wall at any point should be calculated and the assessment of the shell with respect to each limit state should take all of them into account. However, the through thickness transverse shear stresses τ_{xn} , $\tau_{\theta n}$ due to the transverse shear forces q_{xn} , $q_{\theta n}$ are insignificant compared with the other components of stress in almost all practical cases, so they can usually be neglected.

(2) Accordingly, for most purposes, the evaluation of the limit states may be made using only the six stress resultants in the shell wall n_x , n_θ , $n_{x\theta}$, m_x , m_θ , $m_{x\theta}$. Where the structure is axisymmetric and subject only to axisymmetric loading and support, only n_x , n_θ , m_x and m_θ need be used.

(3) If any uncertainty arises concerning the stress to be used in any of the limit state verifications, the von Mises equivalent stress on the shell surface may be used as a safe estimate.

6.2.2 Modelling of the shell for analysis

6.2.2.1 Geometry

(1) The shell should be represented by its middle surface.

(2) The radius of curvature should be taken as the nominal radius of curvature. Imperfections should be neglected, except as set out in Clause 9 (LS3 buckling limit state) and Clause 10 (LS4 fatigue limit state).

(3) An assembly of shell segments should not be subdivided into separate segments for analysis unless the boundary conditions for each segment are chosen in such a way as to represent interactions between them in a conservative manner.

(4) A base ring intended to transfer local support forces into the shell should not be separated from the shell it supports in an assessment of limit state LS3.

(5) Eccentricities and steps in the shell middle surface should be included in the analysis model if they induce significant bending effects as a result of the membrane stress resultants following an eccentric path.

(6) At junctions between shell segments, any eccentricity between the middle surfaces of the shell segments should be considered in the modelling.

(7) A ring stiffener should be treated as a separate discrete structural component of the shell, except where the spacing of the rings is closer than $1,5\sqrt{rt}$ in which case a smeared orthotropic shell model may be used.

(8) A shell that has discrete stringer stiffeners attached to it may be treated as an orthotropic uniform shell, provided that the stringer stiffeners are no further apart than $5\sqrt{rt}$.

NOTE For a fuller treatment of discrete stringer stiffeners, see EN 1993-4-1.

(9) A shell that is corrugated (vertically or horizontally) may be treated as an orthotropic uniform shell provided that the corrugation full wavelength is less than $0,5\sqrt{rt}$ where t is the local plate thickness.

NOTE For a fuller treatment of corrugated shells, see EN 1993-4-1.

(10) A hole in the shell may be neglected in the modelling provided its largest dimension is smaller than $0,6\sqrt{rt}$.

(11) The overall stability of the complete structure should be verified as detailed in EN 1993-3, EN 1993-4-1 and EN 1993-4-2, as appropriate.

6.2.2.2 Boundary conditions

(1) The appropriate boundary conditions should be used in analyses for the assessment of limit states in shell segments using the boundary conditions defined in Table 6.1. For the special conditions needed for buckling calculations, see 9.3.

(2) Rotational restraints at shell boundaries may be neglected in modelling for limit state LS1, but should be included in modelling for limit states LS2 and LS4. For short shells, as classified in Annexes D and E, any boundary rotational restraint should be included for limit state LS3.

(3) Support boundary conditions should be checked to ensure that they do not cause excessive non-uniformity of transmitted forces or introduced forces that are eccentric to the shell middle surface. The provisions of the relevant EN 1993 application parts should be adopted when applying this rule to silos, tanks, chimneys and towers.

(4) When a computational analysis is used, the boundary condition for the normal displacement w should also be used for the circumferential displacement v , except where special circumstances make this inappropriate.

Table 6.1 — Boundary conditions for shell segments

Boundary condition code	Simple term	Displacements	Displacements normal to the shell surface	Meridional or axial displacements	Meridional rotation
BC1r	Clamped	radial fixed meridional fixed rotation fixed	$w = 0$	$u = 0$	$\beta_\phi = 0$
BC1f		radial fixed meridional fixed rotation free	$w = 0$	$u = 0$	$\beta_\phi \neq 0$
BC2r		radial fixed meridional free rotation fixed	$w = 0$	$u \neq 0$	$\beta_\phi = 0$
BC2f	Pinned	radial fixed meridional free rotation free	$w = 0$	$u \neq 0$	$\beta_\phi \neq 0$
BC2s		radial elastically restrained meridional free rotation free	w elastically restrained by a “stiff member” or “spring”	$u \neq 0$	$\beta_\phi \neq 0$
BC2u		radial fixed meridional elastically restrained rotation free	$w = 0$	u elastically restrained by a “stiff support” or “spring”	$\beta_\phi \neq 0$
BC3r		radially free meridionally free rotation fixed	$w \neq 0$	$u \neq 0$	$\beta_\phi = 0$
BC3f	Free edge	radial free meridional free rotation free	$w \neq 0$	$u \neq 0$	$\beta_\phi \neq 0$
The circumferential displacement v is closely linked to the displacement w normal to the surface, so separate boundary conditions are not identified for these two parameters. Instead, the values in the column “Displacements normal to the shell surface” should be adopted for displacement v. NOTE The required stiffness of the boundary ring to achieve a BC2f condition (rather than an enhanced BC3) under external pressure is defined in D.5.3.4.					

6.2.2.3 Actions and environmental influences

(1) Actions should all be assumed to act at the shell middle surface. Eccentricities of load should be represented by static equivalent forces and moments at the shell middle surface.

(2) Local actions and local patches of action should not be represented by equivalent uniform loads.

(3) The modelling should account for whichever of the following are relevant:

- residual stresses or deformations arising from the construction process;
- local settlement under shell walls;
- local settlement under discrete supports;
- uniformity / non-uniformity of support of the structure;
- thermal differentials from one side of the structure to the other;
- thermal differentials from inside to outside of the structure;
- wind effects on openings and penetrations;
- interaction of wind effects on groups of structures;
- connections to other structures;
- conditions during erection.

NOTE Significant residual forces can arise due to progressive construction processes (e.g. on rolled plate assemblies used in on-site construction).

6.2.3 Types of analysis

(1) The design should be based on one or more of the types of analysis given in Table 6.2. The conditions defined in 4.2 should be adopted when using each type of analysis.

Table 6.2 — Types of shell analysis

Type of analysis	Shell theory	Material law	Shell geometry
Membrane theory of shells	membrane equilibrium	not applicable	perfect
Semi-membrane theory of shells	linear circumferential bending, membrane shear and axial stretching	linear	perfect
Linear elastic shell analysis (LA)	linear bending and stretching	linear	perfect
Linear elastic bifurcation analysis (LBA)	linear bending and stretching	linear	perfect
Geometrically nonlinear elastic analysis (GNA)	nonlinear	linear	perfect
Materially nonlinear analysis (MNA)	linear	ideal elastic-plastic ($E_{sh} < 10^{-3}E$)	perfect
Geometrically and materially nonlinear analysis (GMNA)	nonlinear	nonlinear	perfect
Geometrically nonlinear elastic analysis with imperfections explicitly included (GNIA)	nonlinear	linear	imperfect
Geometrically and materially nonlinear analysis with imperfections explicitly included (GMNIA)	nonlinear	nonlinear	imperfect

6.3 Ultimate limit states to be considered

6.3.1 LS1: Plastic failure

(1) The limit state of the plastic failure should be taken as the condition in which the capacity of the structure to resist the actions on it is exhausted by plasticity in the material.

NOTE The plastic failure resistance differs from the reference plastic resistance. The reference plastic resistance is strictly found as the plastic collapse load of the perfect structure obtained from a mechanism based on small displacement theory using an ideal elastic-plastic material law (MNA).

(2) The limit state of tensile rupture should be taken as the condition in which the shell wall experiences gross section tensile failure, leading to separation of the two parts of the shell.

(3) In the absence of fastener holes, verification at the limit state of tensile rupture may be assumed to be covered by the check for the plastic failure limit state. However, where holes for fasteners occur, a supplementary check in accordance with EN 1993-1-1 or EN 1993-1-8 should be carried out.

(4) In verifying the plastic failure limit state, plastic or partially plastic behaviour of the structure may be assumed (i.e. elastic compatibility considerations may be neglected).

NOTE 1 Since the plastic failure limit state includes change of geometry, this limit state can also capture snap-through buckling, which can occur in the elastic state. The reference plastic resistance does not include changes of geometry, so this apparent anomaly does not occur.

NOTE 2 The plastic failure limit state does not include considerations of bifurcation, so no checks for bifurcation are required when a GMNA analysis is used to assess the plastic failure limit state LS1.

(5) All relevant load combinations should be accounted for when checking LS1.

(6) One or more of the following methods of analysis (see 4.2) should be used for the calculation of the design stresses and stress resultants when checking LS1:

- membrane theory;
- formulae in Annexes A and B;
- linear elastic analysis (LA);
- materially nonlinear analysis (MNA);
- geometrically and materially nonlinear analysis (GMNA).

6.3.2 LS2: Cyclic plasticity

(1) The limit state of cyclic plasticity should be taken as the condition in which repeated cycles of loading and unloading produce repeated yielding in tension and in compression at the same point, thus causing plastic work to be repeatedly done on the structure, eventually leading to local cracking by exhaustion of the energy absorption capacity of the material.

NOTE The stresses that are associated with this limit state develop under a combination of all actions and the compatibility conditions for the structure.

(2) All variable actions (such as imposed loads and temperature variations) that can lead to yielding, and which might be applied with more than three cycles in the life of the structure, should be considered when checking LS2.

(3) Where the number of cycles involving cyclic plasticity is greater than N_{cp} , the provisions of LS2 should be used.

NOTE The value of N_{cp} is taken as $N_{cp} = 20$, unless the National Annex gives a different value.

(4) In the verification of this limit state, compatibility of the deformations under elastic or elastic-plastic conditions should be considered.

(5) One or more of the following methods of analysis (see 4.2) should be used for the calculation of the design stresses and stress resultants when checking LS2:

- formulae in Annex C;
- elastic analysis (LA or GNA);
- MNA or GMNA to determine the plastic strain range.

(6) Low cycle fatigue failure may be assumed to be prevented if the procedures set out in this document are adopted.

6.3.3 LS3: Buckling

(1) The limit state of buckling should be taken as the condition in which all or part of the structure suddenly develops large displacements normal to the shell surface, caused by loss of stability under compressive membrane or shear membrane stresses in the shell wall, leading to inability to sustain any increase in the stress resultants, and possibly causing total collapse of the structure.

(2) One or more of the following methods of analysis (see 4.2) and buckling resistance assessment should be used for the calculation of the design stresses and stress resultants when checking LS3:

- membrane theory for axisymmetric loading and global bending conditions only (for exceptions, see the relevant application parts EN 1993-3, EN 1993-4-1 and EN 1993-4-2);
- formulae in Annexes A, D and E;
- reference resistance design, where the formulae in Annex E refer to the specific geometry, loading and boundary conditions of the structure;
- linear elastic analysis (LA), which is a minimum requirement for stress analysis under general loading conditions with formulae in Annex D (except where the stress analysis of the load case is given in Annex A, or where the buckling condition is treated as a special case in Annex D);
- linear elastic bifurcation analysis (LBA), which is required for shells under general loading conditions if the critical buckling resistance is to be used in an LBA-MNA assessment, or a GMNIA assessment;
- materially nonlinear analysis (MNA), which is required for shells under general loading conditions if the true reference plastic resistance (rather than a lower bound estimate taken from an LA analysis) is to be used in an LBA-MNA assessment;
- GMNIA, together with supporting MNA, LBA and GMNA analyses, and using appropriate imperfections and calculated calibration factors.

(3) All relevant load combinations causing compressive membrane or shear membrane stresses in the shell should be accounted for when checking LS3.

(4) Because the strength under limit state LS3 depends strongly on the quality of construction, the strength assessment should take account of the associated requirements for execution tolerances. For this purpose, three classes of geometrical tolerances, termed “fabrication quality classes” are given in 9.4.

6.3.4 LS4: Fatigue

(1) The limit state of high cycle fatigue should be taken as the condition in which repeated cycles of increasing and decreasing stress caused by variable actions lead to the development and propagation of a fatigue crack.

(2) A fatigue verification according to Clause 10 and EN 1993-1-9 should be carried out for shell structures exposed to high cycle variable actions. However, this verification may be omitted provided that, in the design life of the structure according to the relevant action spectrum defined in EN 1991 and the appropriate application parts of EN 1993-3, the following two criteria are both met:

- the design value of the peak von Mises equivalent surface stress at all points in the structure calculated using an LA analysis is less than f_{lim} ;
- no variable load is applied with more than N_f cycles.

NOTE The value of the peak stress f_{lim} is taken as 150 N/mm² and N_f is taken as 10 000 unless the National Annex gives different values.

(3) The appropriate method of analysis (see 4.2) among the following should be used for the calculation of the design stresses and stress resultants for use with the provisions of EN 1993-1-9 when checking LS4:

- analysis using membrane theory (beam theory) for long, thick-walled cylinders;
- formulae in Annex C;
- linear elastic analysis (LA);
- geometrically nonlinear elastic analysis (GNA)
- geometrically nonlinear imperfect elastic analysis (GNIA).

NOTE Additional stress concentration factors k_f and $k_{f,imp}$ could be needed to account for eccentricities, imperfections and other global stress-raising effects that are not included in the analysis calculation and are not specifically addressed by the fatigue classes of EN 1993-1-9. Further information on the relationship between the choice of analysis method, the applicable stress concentration factors k_f and $k_{f,imp}$ and the verification procedure of EN 1993-1-9 are given in Clause 10.

(4) Where the number of cycles involved in the assessment is less than N_f , the provisions for LS2 (Clause 8) should be adopted.

6.4 Concepts for the limit state verifications

6.4.1 General

(1) The limit state verification should be carried out using one of the following:

- stress design;
- standard formulae;
- computational analysis.

(2) Account should be taken of the fact that elastic-plastic material responses induced by different stress components in the shell have different effects on the failure modes and the ultimate limit states. The stress components should therefore be placed in stress categories with different limits. Stresses that develop to meet equilibrium requirements should be treated as more significant than stresses that are induced by the compatibility of deformations normal to the shell. Local stresses caused by notch effects in construction details may be assumed to have a negligibly small influence on the resistance to static loading.

(3) The categories distinguished in stress design should be primary, secondary and local stresses. Primary and secondary stress states may be replaced by stress resultants where appropriate.

(4) When using a computational analysis, the primary and secondary stress states should be replaced by the limit load and the strain range for cyclic loading.

(5) In general, it may be assumed that primary stress states control LS1, LS3 depends strongly on primary stress states but can be affected by yielding caused by secondary stress states, LS2 depends on the combination of primary and secondary stress states, and local surface stresses govern LS4.