

BS EN 62059-31-1:2008



BSI British Standards

Electricity metering equipment — Dependability —

Part 31-1: Accelerated reliability testing — Elevated temperature and humidity

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National foreword

This British Standard is the UK implementation of EN 62059-31-1:2008. It is identical to IEC 62059-31-1:2008.

The UK participation in its preparation was entrusted to Technical Committee PEL/13, Electricity meters.

A list of organizations represented on this committee can be obtained on request to its secretary.

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**Electricity metering equipment -
Dependability -
Part 31-1: Accelerated reliability testing -
Elevated temperature and humidity
(IEC 62059-31-1:2008)**

Equipements de comptage de l'électricité -
Sûreté de fonctionnement -
Partie 31-1: Essais de fiabilité accélérés -
Température et humidité élevées
(CEI 62059-31-1:2008)

Elektrizitätszähler -
Zuverlässigkeit -
Teil 31-1: Zeitraffende
Zuverlässigkeitsprüfung -
Temperatur und Luftfeuchte erhöht
(IEC 62059-31-1:2008)

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CENELEC

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Foreword

The text of document 13/1437A/FDIS, future edition 1 of IEC 62059-31-1, prepared by IEC TC 13, Electrical energy measurement, tariff- and load control, was submitted to the IEC-CENELEC parallel vote and was approved by CENELEC as EN 62059-31-1 on 2008-11-01.

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Annex ZA has been added by CENELEC.

Endorsement notice

The text of the International Standard IEC 62059-31-1:2008 was approved by CENELEC as a European Standard without any modification.

In the official version, for Bibliography, the following notes have to be added for the standards indicated:

IEC 61124	NOTE Harmonized as EN 61124:2006 (not modified).
IEC 61163-1	NOTE Harmonized as EN 61163-1:2006 (not modified).
IEC 61164	NOTE Harmonized as EN 61164:2004 (not modified).
IEC 61709	NOTE Harmonized as EN 61709:1998 (not modified).

Annex ZA (normative)

Normative references to international publications with their corresponding European publications

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE When an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

<u>Publication</u>	<u>Year</u>	<u>Title</u>	<u>EN/HD</u>	<u>Year</u>
IEC 60050-191	1990	International Electrotechnical Vocabulary (IEV) - Chapter 191: Dependability and quality of service	-	-
IEC 60300-3-5	2001	Dependability management - Part 3-5: Application guide - Reliability test conditions and statistical test principles	-	-
IEC 61649	2008	Weibull analysis	EN 61649	2008
IEC 61703	2001	Mathematical expressions for reliability, availability, maintainability and maintenance support terms	EN 61703	2002
IEC/TR 62059-11	2002	Electricity metering equipment - Dependability - Part 11: General concepts	-	-
IEC/TR 62059-21	2002	Electricity metering equipment - Dependability - Part 21: Collection of meter dependability data from the field	-	-
IEC 62059-41	2006	Electricity metering equipment - Dependability - Part 41: Reliability prediction	EN 62059-41	2006
IEC 62308	2006	Equipment reliability - Reliability assessment methods	EN 62308	2006

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INTRODUCTION

Electricity metering equipment are products designed for high reliability and long life under normal operating conditions, operating continuously without supervision. To manage metering assets effectively, it is important to have tools for predicting and estimating life characteristics of various types.

IEC 62059-41 provides methods for predicting the failure rate – assumed to be constant – of metering equipment based on the parts stress method.

IEC 62059-31-1 provides a method for estimating life characteristics using temperature and humidity accelerated testing.

It is practically impossible to obtain data about life characteristics by testing under normal operating conditions. Therefore, accelerated reliability test methods have to be used.

During accelerated reliability testing, samples taken from a defined population are operated beyond their normal operating conditions, applying stresses to shorten the time to failure, but without introducing new failure mechanisms.

The estimation is performed by recording and analysing failures during such accelerated testing, establishing the failure distribution under the test conditions and, using life stress models, extrapolating failure distribution under accelerated conditions of use to normal conditions of use.

The method provides quantitative results with their confidence limits and may be used to compare life characteristics of products coming from different suppliers or different batches from the same supplier.

ELECTRICITY METERING EQUIPMENT – DEPENDABILITY –

Part 31-1: Accelerated reliability testing – Elevated temperature and humidity

1 Scope

This part of IEC 62059 provides one of several possible methods for estimating product life characteristics by accelerated reliability testing.

Acceleration can be achieved in a number of different ways. In this particular standard, elevated, constant temperature and humidity is applied to achieve acceleration. The method also takes into account the effect of voltage and current variation.

Of course, failures not (or not sufficiently) accelerated by temperature and humidity will not be detected by the application of the test method specified in this standard.

Other factors, like temperature variation, vibration, dust, voltage dips and short interruptions, static discharges, fast transient burst, surges, etc. – although they may affect the life characteristics of the meter – are not taken into account in this standard; they may be addressed in future parts of the IEC 62059 series.

This standard is applicable to all types of metering equipment for energy measurement, tariff- and load control in the scope of IEC TC 13. The method given in this standard may be used for estimating (with given confidence limits) product life characteristics of such equipment prior to and during serial production. This method may also be used to compare different designs.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 60300-3-5 Ed. 1.0:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 61649 Ed 2.0: 2008, *Goodness-of-fit tests, confidence intervals and lower confidence limits for Weibull distributed data*

IEC 61703 Ed. 1.0: 2001, *Mathematical expressions for reliability, availability, maintainability and maintenance support terms*

IEC/TR 62059-11 Ed 1.0:2002, *Electricity metering equipment – Dependability – Part 11: General concepts*

IEC/TR 62059-21 Ed. 1.0:2002, *Electricity metering equipment – Dependability – Part 21: Collection of meter dependability data from the field*

IEC 62059-41 Ed. 1.0: 2006, *Electricity metering equipment – Dependability – Part 41: Reliability prediction*

IEC 62308 Ed. 1.0:2006, *Equipment reliability – Reliability assessment methods*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

NOTE 1 Here only those terms relevant to the subject are included, which have not been already included in IEC 62059-11.

3.1

accelerated life test

a test in which the applied stress level is chosen to exceed that stated in the reference conditions in order to shorten the time duration required to observe the stress response of the item, or to magnify the response in a given time duration

NOTE To be valid, an accelerated life test shall not alter the basic fault modes and failure mechanisms, or their relative prevalence.

[IEV 191-14-07, modified]

3.2

ageing failure, wear-out failure

a failure whose probability of occurrence increases with the passage of time, as a result of processes inherent in the item

[IEV 191-04-09]

3.3

burn-in (for repairable hardware)

a process of increasing the reliability performance of hardware employing functional operation of every item in a prescribed environment with successive corrective maintenance at every failure during the early failure period

[IEV 191-17-02]

3.4

burn-in (for a non-repairable item)

a type of screening test employing the functional operation of an item

[IEV 191-17-03]

3.5

censoring

termination of the test after either a certain number of failures or a certain time at which there are still items functioning

[IEC 60300-3-5, 3.1.2]

3.6

constant failure intensity period

that period, if any, in the life of a repaired item during which the failure intensity is approximately constant

[IEV 191-10-08]

3.7**constant failure rate period**

that period, if any, in the life of a non-repaired item during which the failure rate is approximately constant

[IEV 191-10-09]

3.8**equipment under prediction****EUP (abbreviation)**

the electricity metering equipment for which a reliability prediction is being made

3.9**estimated**

qualifies a value obtained as the result of the operation made for the purpose of assigning, from the observed values in a sample, numerical values to the parameters of the distribution chosen as the statistical model of the population from which this sample is taken

NOTE The result may be expressed either as a single numerical value (a point estimate) or as a confidence interval.

[IEV 191-18-04, modified]

3.10**extrapolated**

qualifies a predicted value based on observed or estimated values for one or a set of conditions, intended to apply to other conditions such as time, maintenance and environmental conditions

[IEV 191-18-03]

3.11**failure**

termination of the ability of an item to perform a required function

NOTE 1 After failure the item has a fault.

NOTE 2 "Failure" is an event, as distinguished from "fault", which is a state.

[IEV 191-04-01, modified]

3.12**failure cause**

the circumstances during design, manufacture or use which have led to a failure

NOTE The term "root cause of the failure" is used and described in IEC 62059-21 Clause 8.

[IEV 191-04-17, modified]

3.13**failure mechanism**

the physical, chemical or other process which has led to a failure

[IEV 191-04-18]

3.14**failure rate acceleration factor**

the ratio of the failure rate under accelerated testing conditions to the failure rate under stated reference test conditions

NOTE Both failure rates refer to the same time period in the life of the tested items.

[IEV 194-14-11]

3.15**fault**

the state of an item characterized by the inability to perform a required function, excluding the inability during preventive maintenance or other planned actions, or due to lack of external resources

NOTE A fault is often the result of a failure of the item itself, but may exist without prior failure.

[IEV 191-05-01]

3.16**fault mode**

one of the possible states of a faulty item, for a given required function

NOTE 1 The use of the term "failure mode" in this sense is now deprecated.

NOTE 2 A function-based fault mode classification is described in IEC 62059-21 Clause 7.

[IEV 191-05-22, modified]

3.17**(instantaneous) failure rate**

the limit, if it exists, of the quotient of the conditional probability that the instant of a failure of a non-repaired item falls within a given time interval ($t, t + \Delta t$) and the duration of this time interval, Δt , when Δt tends to zero, given that the item has not failed up to the beginning of the time interval

NOTE 1 The instantaneous failure rate is expressed by the formula:

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \frac{F(t + \Delta t) - F(t)}{R(t)} = \frac{f(t)}{R(t)}$$

where $F(t)$ and $f(t)$ are respectively the distribution function and the probability density of the failure instant, and where $R(t)$ is the reliability function, related to the reliability $R(t_1, t_2)$ by $R(t) = R(0, t)$.

NOTE 2 An estimated value of the instantaneous failure rate can be obtained by dividing the ratio of the number of items which have failed during a given time interval to the number of non-failed items at the beginning of the time interval, by the duration of the time interval.

NOTE 3 In English, the instantaneous failure rate is sometimes called "hazard function".

[IEV 191-12-02, modified]

3.18**item****entity**

any part, component, device, subsystem, functional unit, equipment or system that can be individually considered

NOTE 1 An item may consist of hardware, software or both, and may also in particular cases, include people.

NOTE 2 A number of items, e.g. a population of items or a sample, may itself be considered as an item.

[IEV 191-01-01]

3.19**life test**

test with the purpose of estimating, verifying or comparing the lifetime of the class of items being tested

NOTE The end of the useful life will often be defined as the time when a certain percentage of the items have failed for non-repaired items and as the time when the failure intensity has increased to a specified level for repaired items.

3.20**mean time to failure****MTTF (abbreviation)**

the expectation of the time to failure

NOTE The term “expectation” has statistical meaning.

[IEV 191-12-07, modified]

3.21**mean time to first failure****MTTFF (abbreviation)**

the expectation of the time to first failure

NOTE The term “expectation” has statistical meaning.

[IEV 191-12-06, modified]

3.22**measure (in the probabilistic treatment of dependability)**

a function or a quantity used to describe a random variable or a random process

NOTE For a random variable, examples of measures are the distribution function and the mean.

[IEV 191-01-11]

3.23**non-relevant failure**

a failure that should be excluded in interpreting test or operational results or in calculating the value of a reliability performance measure

NOTE The criteria for the exclusion should be stated.

[IEV 191-04-14]

3.24**non-repaired item**

item which is not repaired after failure

[IEV 191-01-03]

3.25**operating time**

time interval during which an item is in an operating state

[IEV 191-09-01]

3.26**population**

the totality of items under consideration

3.27**prediction**

the process of computation used to obtain the predicted value(s) of a quantity

NOTE The term “prediction” may also be used to denote the predicted value(s) of a quantity.

[IEV 191-16-01]

3.28**relevant failure**

a failure that should be included in interpreting test or operational results or in calculating the value of a reliability performance measure

NOTE The criteria for the inclusion should be stated.

[IEV 191-04-13]

3.29**reliability test**

experiment carried out in order to measure, quantify or classify a reliability measure or property of an item

NOTE 1 Reliability testing is different from environmental testing where the aim is to prove that the items under test can survive extreme conditions of storage, transportation and use.

NOTE 2 Reliability test may include environmental testing.

3.30**stress condition**

set of conditions to which the metering equipment is exposed during accelerated reliability testing

3.31**stress model**

a mathematical model used to describe the influence of relevant applied stresses on a reliability performance measure or any other property of an item

[IEV 191-16-10]

3.32**time acceleration factor**

the ratio between the time durations necessary to obtain the same stated number of failures or degradations in two equal size samples, under two different sets of stress conditions involving the same failure mechanisms and fault modes and their relative prevalence

NOTE One of the two sets of stress conditions should be a reference set.

[IEV 191-14-10]

3.33**time between failures**

time duration between two consecutive failures of a repaired item

[IEV 191-10-03]

3.34**time to failure**

cumulative operating time of an item, from the instant it is first put in an up state, until failure or, from the instant of restoration until next failure

[IEV 191-10-02, modified]

3.35 time to suspension

cumulative operating time of a non-failed item, from the instant it is first put in an up state or from the instant of restoration, until the test is terminated (censored)

3.36**use condition**

set of conditions to which the metering equipment is exposed during normal use



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4 Symbols, acronyms and abbreviations

Symbol / Acronym / Abbreviation	Meaning
A	Constant used in the life stress model (e.g. in Arrhenius model, Eyring model or Peck's temperature-humidity model)
$AccThr$	Acceptance threshold
AF	Acceleration factor
CL	Confidence level
E_a	Activation energy in electron volts
$f(t)$	Probability density function (<i>pdf</i>) of the (operating) time to failure
$F(t)$	Unreliability function, i.e. the probability of failure until time t or fraction of items that have failed up to time t
k	Boltzmann constant ($8,617 \times 10^{-5}$ eV/K)
MRR	Median rank regression
n	Exponent characteristic of the product (in Peck's temperature-humidity model)
N	Number of items put on a reliability test
p	Number of items which failed by the end of the reliability test
pdf	Probability density function
q	Number of items which have not failed by the end of the reliability test
r	Reaction rate (in Arrhenius model)
r_0	Constant (in Arrhenius model)
$R(t)$	Reliability function, i.e. the probability of survival until time t or fraction of items that have not failed up to time t
R	Correlation coefficient
RH	Percent relative humidity
RH_s	Percent relative humidity at stress condition
RH_u	Percent relative humidity at normal use condition
S	Applied stress (in Eyring model)
t	Operating time to failure in hours
t_s	Time to failure at stress temperature T_s
t_u	Time to failure at normal use temperature T_u
T	Reaction temperature in K
T_s	Stress temperature
TTF_i	Observed time to failure of the i^{th} failed item
TTS_j	Observed time to suspension of the j^{th} non failed item
T_u	Normal use temperature
$U5_i$	Unreliability at rank i with a confidence level of 5 % on a sample of N items
$TTF5_i$	Time to failure corresponding to $U5_i$
$U50_i$	Median rank of the i^{th} failure, or unreliability estimate of the i^{th} failure (at rank i) on a sample of N items with a confidence level of 50 %
$U95_i$	Unreliability at rank i with a confidence level of 95 % on a sample of N items
$TTF95_i$	Time to failure corresponding to $U95_i$
β	Weibull shape parameter
η	Weibull characteristic life or scale parameter
γ	Location parameter in hours
$\lambda(t)$	Instantaneous failure rate function, also referred to as the hazard rate function

5 Description of quantitative accelerated life tests

5.1 Introduction

Quantitative accelerated life testing may be achieved either by *usage rate acceleration* or *overstress acceleration*.

For equipment that do not operate continuously, the acceleration can be obtained by continuous operation. This is *usage rate acceleration*. It is usually not applicable for electricity metering equipment because they work and measure continuously in normal use conditions. Therefore usage rate acceleration is not considered in this standard.

The second form of acceleration can be obtained by stressing the equipment; this is *overstress acceleration*. This involves applying stresses that exceed the normal use conditions. The time to failure data obtained under such stresses are then used to extrapolate to use conditions. Accelerated life tests can be performed at high or low temperature, humidity, current and voltage, in order to accelerate or stimulate the failure mechanisms. They can also be performed using a combination of these stresses.

Special attention must be paid when defining stress(es) and stress levels: these should not reveal fault modes that would never appear under normal conditions. Please refer to 12.2.2.

Accelerated reliability testing is based on two main models: The *life distribution* of the product, which describes the product at each stress level, and the *life-stress model*.

5.2 The life distribution

The life distribution is a statistical distribution describing the time to failure of a product. The goal of accelerated life testing is to obtain this life distribution under normal use conditions; this life distribution is the use level probability density function, or *pdf*, of the time to failure of the product. Annex A presents this statistical concept of *pdf* and provides a basic statistical background as it applies to life data analysis.

Once this use level *pdf* of the time to failure of the product is obtained, all other desired reliability characteristics can be easily determined. In typical data analysis, this use level *pdf* of the time to failure can be easily determined using regular time to failure data and an underlying distribution such as Weibull distribution. See clause 6.

In accelerated life testing, the challenge is to determine the *pdf* at normal use conditions from accelerated life test data rather than from time to failure data obtained under use conditions. For this, a method of extrapolation is used to extrapolate from data collected at accelerated conditions to provide an estimation of characteristics at normal use conditions.

5.3 The life-stress model

The life-stress model quantifies the manner in which the life distribution changes with different stress levels.

The combination of both an underlying life distribution and a life-stress model with time to failure data obtained at different stress levels, will provide an estimation of the characteristics at normal use conditions.

The most commonly used life stress models are:

- the Arrhenius temperature acceleration model (see C.1);
- the Eyring model (see C.2).

6 The Weibull distribution

6.1 Introduction

This clause presents numerical and graphical methods to be used for plotting data, to make a goodness of fit test, to estimate the parameters of the life distribution and to plot confidence limits.

The Weibull distribution is one of the most commonly used distribution types in reliability engineering. It can be used to model material strength, time to failure data of electronic and mechanical components, equipment or systems.

The main characteristics of the Weibull distribution are presented in Annex B.

6.2 Graphical representation

To allow a linear representation, the Weibull unreliability function has to be transformed first into a linear form. Starting from the unreliability function:

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}$$

we obtain:

$$\ln(1 - F(t)) = -\left(\frac{t-\gamma}{\eta}\right)^\beta$$

$$\ln\{-\ln(1 - F(t))\} = \beta \ln\left(\frac{t-\gamma}{\eta}\right) = -\beta \ln(\eta) + \beta \ln(t - \gamma)$$

This can be expressed as:

$$y = A + Bx \text{ with } y = \ln\{-\ln(1 - F(t))\}, A = -\beta \ln(\eta), B = \beta, \text{ and } x = \ln(t - \gamma).$$

This equation shows that the unreliability function should be a straight line if it is represented on a Weibull probability plotting paper, where the unreliability is plotted on a log log reciprocal scale against $(t - \gamma)$ on a log scale. In other words, if unreliability data are plotted on a Weibull probability paper, and if they conform to a straight line, that supports the contention that the distribution is Weibull.

β , the shape parameter, gives the slope of the unreliability function, when it is represented on a Weibull probability paper.

As shown in Table 1 to Table 4, a Weibull probability paper can be constructed as follows:

Table 1 – Construction of ordinate (Y)

R(t)	F(t)	$\ln(-\ln(1-F(t)))$	Ordinate Y ($\ln(-\ln(1-F(t)))+4,60$)	R(t)	F(t)	$\ln(-\ln(1-F(t)))$	Ordinate Y ($\ln(-\ln(1-F(t)))+4,60$)
0,99	0,01	-4,60	0,00	0,8	0,2	-1,50	3,10
0,98	0,02	-3,90	0,70	0,7	0,3	-1,03	3,57
0,97	0,03	-3,49	1,11	0,6	0,4	-0,67	3,93
0,96	0,04	-3,20	1,40	0,5	0,5	-0,37	4,23
0,95	0,05	-2,97	1,63	0,4	0,6	-0,09	4,51
0,94	0,06	-2,78	1,82	0,3	0,7	0,19	4,79
0,93	0,07	-2,62	1,98	0,2	0,8	0,48	5,08
0,92	0,08	-2,48	2,12	0,1	0,9	0,83	5,43
0,91	0,09	-2,36	2,24	0,01	0,99	1,53	6,13
0,9	0,1	-2,25	2,35				

Table 2 – Construction of abscissa (t-γ)

t - γ (h)	$\ln(t-\gamma)$	Abscissa $\ln(t-\gamma) - 2,30$	t - γ (h)	$\ln(t-\gamma)$	Abscissa $\ln(t-\gamma) - 2,30$	t - γ (h)	$\ln(t-\gamma)$	Abscissa $\ln(t-\gamma) - 2,30$
10	2,30	0,00	200	5,30	3,00	3000	8,01	5,71
20	3,00	0,70	300	5,70	3,40	4000	8,29	5,99
30	3,40	1,10	400	5,99	3,69	5000	8,52	6,22
40	3,69	1,39	500	6,21	3,91	6000	8,70	6,40
50	3,91	1,61	600	6,40	4,10	7000	8,85	6,55
60	4,09	1,79	700	6,55	4,25	8000	8,99	6,69
70	4,25	1,95	800	6,68	4,38	9000	9,10	6,80
80	4,38	2,08	900	6,80	4,50	10000	9,21	6,91
90	4,50	2,20	1000	6,91	4,61	20000	9,90	7,60
100	4,61	2,31	2000	7,60	5,30	30000	10,31	8,01

Table 3 – Equations format entered into a spreadsheet

	A	B	C	D	E
1	Gamma =				
2	Beta =				
3	Eta =				
4	t	(t- γ) abscissa	1-F(t)	$\ln(-\ln(1-F(t)))$	Ordinate Y
5	(h)	$(\ln(t-\text{Gamma})-2,3)$			
6	100	$\text{LN}(\text{A6}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A6}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C6}))$	D6+4,6
7	330	$\text{LN}(\text{A7}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A7}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C7}))$	D7+4,6
8	1000	$\text{LN}(\text{A8}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A8}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C8}))$	D8+4,6
9	2000	$\text{LN}(\text{A9}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A9}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C9}))$	D9+4,6
10	3000	$\text{LN}(\text{A10}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A10}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C10}))$	D10+4,6
11	4000	$\text{LN}(\text{A11}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A11}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C11}))$	D11+4,6
12	5000	$\text{LN}(\text{A12}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A12}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C12}))$	D12+4,6
13	6000	$\text{LN}(\text{A13}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A13}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C13}))$	D13+4,6
14	7000	$\text{LN}(\text{A14}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A14}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C14}))$	D14+4,6
15	8000	$\text{LN}(\text{A15}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A15}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C15}))$	D15+4,6
16	9000	$\text{LN}(\text{A16}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A16}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C16}))$	D16+4,6
17	10000	$\text{LN}(\text{A17}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A17}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C17}))$	D17+4,6
18	11000	$\text{LN}(\text{A18}-\text{Gamma})-2,3$	$\text{EXP}(-\text{POWER}((\text{A18}-\text{Gamma})/\text{Eta};\text{Beta}))$	$\text{LN}(-\text{LN}(\text{C18}))$	D18+4,6

Table 4 – Example with $\gamma = 3\,000$, $\beta = 1,1$, $\eta = 10\,000$

Gamma =	3 000				
Beta =	1,1				
Eta =	10 000				
t (h)	(t-γ) abscissa	R(t)	F(t)	$\ln(-\ln(1-F(t)))$	Ordinate Y
3 300	3,40	0,98	0,02	-3,86	0,74
4 000	4,61	0,92	0,08	-2,53	2,07
6 000	5,71	0,77	0,23	-1,32	3,28
8 000	6,22	0,63	0,37	-0,76	3,84
10 000	6,55	0,51	0,49	-0,39	4,21
12 000	6,80	0,41	0,59	-0,12	4,48
14 000	7,01	0,33	0,67	0,10	4,70
16 000	7,17	0,26	0,74	0,29	4,89
18 000	7,32	0,21	0,79	0,45	5,05
20 000	7,44	0,17	0,83	0,58	5,18
22 000	7,55	0,13	0,87	0,71	5,31
24 000	7,65	0,10	0,90	0,82	5,42
28 000	7,83	0,06	0,94	1,01	5,61

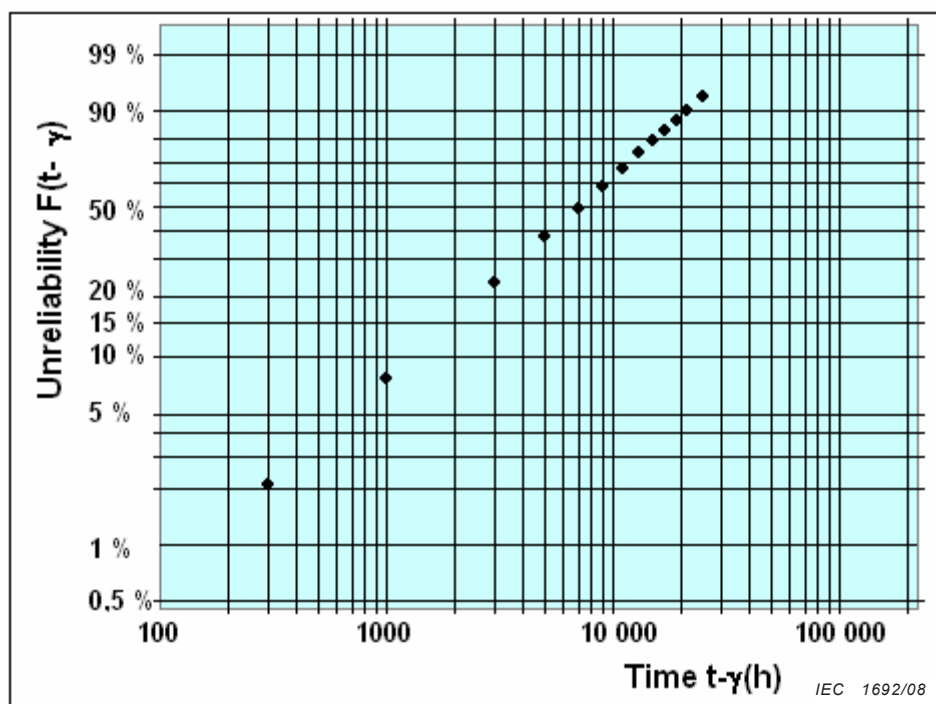


Figure 1 – Weibull unreliability representation example with $\gamma = 3\,000$, $\beta = 1.1$, $\eta = 10\,000$

6.3 Calculation of the distribution parameters

6.3.1 Input data to be used

When analysing life data from an accelerated reliability test, it is necessary to include data on the items that have failed, but also data on the items that have not failed. Data on items that have not failed are referred to as censored data (see IEC 60300-3-5, 8.3).

When the times to failure of all the items under test are observed, the data are said to be complete. In that case, the data logged during the test are all the times to failure of the items. If, however, items remain non-failed at the end of the test, then the observations are said to be censored:

- when the test is terminated after a time t , then for those items that have not failed the data are said to be time censored. The data logged is t ;
- when the test is terminated after a specified number of failures, then for these items the data are said to be failure censored. The data logged is the time to failure of the last item which failed plus one time unit (to differentiate the items not failed from the last one failed).

During an accelerated reliability test:

- if the test of the status (failed/non-failed) of the items under test is not done continuously, but intermittently with an interval of time between inspections noted IT ;
- and if p items fail during the n^{th} interval of time;
- then the values logged for the times to failure are: $(n \times IT) - (p \times IT / (p + 1))$, $(n \times IT) - ((p - 1) \times IT / (p + 1))$, ..., $(n \times IT) - (2 \times IT / (p + 1))$, $(n \times IT) - (IT / (p + 1))$.

6.3.2 Ranking of the time to failure

Let us assume that a reliability test has been done on a sample of N items. At the end of the test:

- p items failed: All the times to failure of these items were logged. These times to failure are noted: $TTF_1, TTF_2, \dots, TTF_i, \dots, TTF_p$;
- q items did not fail: These items were suspended at times $TTS_1, TTS_2, \dots, TTS_j, \dots, TTS_q$.

The ranking process of the time to failure data consists of arranging all time to failure data TTF_i , and all time to suspension TTS_j , in an ascending order, and calculate the adjusted ranks of all failed items in order to take into account the effects of non-failed items.

The adjusted rank for each failed item is calculated from the following formula (see IEC 61649):

$$\text{Adjusted.rank} = \frac{((\text{Reverse.rank}) \times (\text{Previous.adjusted.rank})) + (N + 1)}{(\text{Reverse.rank}) + 1}$$

Table 5 below gives an example of this ranking process: 8 items failed successively at 500, 1 200, 1 500, 2 300, 4 500, 5 600, 6 300 and 8 400 h. 2 items were suspended after 700 and 4 200 h.

Table 5 – Example of ranking process of times to failure

Rank	Time	Reverse rank	Adjusted rank
1	500F	10	1
2	700C	9	Suspended
3	1 200F	8	$[(8 \times 1) + (10 + 1)] / (8 + 1) = 2,111$
4	1 500F	7	$[(7 \times 2,111) + (10 + 1)] / (7 + 1) = 3,222$
5	2 300F	6	$[(6 \times 3,222) + (10 + 1)] / (6 + 1) = 4,333$
6	4 200C	5	Suspended
7	4 500F	4	$[(4 \times 4,333) + (10 + 1)] / (4 + 1) = 5,666$
8	5 600F	3	$[(3 \times 5,666) + (10 + 1)] / (3 + 1) = 7$
9	6 300F	2	$[(2 \times 7) + (10 + 1)] / (2 + 1) = 8,333$
10	8 400F	1	$[(1 \times 8,333) + (10 + 1)] / (1 + 1) = 9,666$

6.3.3 Reliability / unreliability estimates

The next step is to estimate the unreliability corresponding to each time to failure by calculating the corresponding median rank.

The Median Rank noted $U50_i$ (unreliability at the i^{th} failure with a confidence level of 50 %) is the true probability of failure $F(t_i)$ or unreliability estimate at the i^{th} failure on a sample of N items with a confidence level of 50 %. In other words, $U50_i$ is the estimate of the cumulative fraction of items that will fail at time TTF_i , where TTF_i is the time to failure of the i^{th} failure.

This value is obtained by solving the cumulative binomial distribution for X :

$$CL = \sum_{j=i}^N C_j^N X^j (1 - X)^{N-j}$$

where CL is the confidence level ($0 < CL < 1$), N is the sample size, and i is the order number (or adjusted rank as described in 6.3.2).

For Median Rank, $CL = 0,5$. In other words, $CL = 0,5$ means that half the population makes more (or less) than the median rank.

Rank tables are available in Annex D.

An example of unreliability estimates using median ranks is given in Table 6 below. For adjusted ranks which are not a multiple of 0,5, a linear interpolation is done between the two closest values of the rank tables.

Table 6 – Unreliability estimates by median rank

Rank	Time	Adjusted rank	Unreliability Estimate (Median rank for 10 samples)	Reliability Estimate
1	500F	1	0,067	0,933
2	700C	Suspended		
3	1 200F	2,111	$0,1623+(0,111 \times (0,2104-0,1623)/0,5)=0,1729$	0,8271
4	1 500F	3,222	$0,2586+(0,222 \times (0,3068-0,2586)/0,5)=0,28$	0,72
5	2 300F	4,333	$0,3551+(0,333 \times (0,4034-0,3551)/0,5)=0,3872$	0,6128
6	4 200C	Suspended		
7	4 500F	5,666	$0,50+(0,166 \times (0,5483-0,5)/0,5)=0,516$	0,484
8	5 600F	7	0,6449	0,3551
9	6 300F	8,333	$0,7414+(0,333 \times (0,7896-0,7414)/0,5)=0,7735$	0,2265
10	8 400F	9,666	$0,8857+(0,166 \times (0,933-0,8857)/0,5)=0,9014$	0,0986

6.3.4 Calculation of the parameters

6.3.4.1 General

Once times to failure have been ranked, and reliability/unreliability has been estimated for each time to failure, all data are ready to construct the graphical representation and to calculate the parameters of the distribution, following the procedure described in 6.2.

Parameters A and B of the equation $y = A + Bx$ can be estimated by performing a least squares/rank regression on y_i and x_i data, where:

- $x_i = \ln(TTF_i)$;
- $y_i = \ln(-\ln(1 - F(TTF_i)))$.

6.3.4.2 Calculation of parameters A , B and the coefficient of determination

According to the least squares/rank regression principle, which minimizes the vertical distance between the data points and the straight line fitted to the data, the best fitting straight line to these data is the straight line $y = A + Bx$ such that F is minimum, where

$$F = \sum_{i=1}^p (A + Bx_i - y_i)^2$$

and p is the number of items which failed during the test.

By solving the equations $\frac{dF}{dA} = 0$ and $\frac{dF}{dB} = 0$, we obtain:

Estimation of B :

$$B = \frac{\sum_{i=1}^p x_i y_i - \frac{\sum_{i=1}^N x_i \sum_{i=1}^N y_i}{p}}{\sum_{i=1}^p x_i^2 - \frac{(\sum_{i=1}^p x_i)^2}{p}}$$

Estimation of A:

$$A = \frac{\sum_{i=1}^p y_i}{p} - B \frac{\sum_{i=1}^p x_i}{p}$$

Estimation of the Coefficient of determination R^2 :

$$R^2 = \frac{(\sum_{i=1}^p x_i y_i - \frac{\sum_{i=1}^p x_i \sum_{i=1}^p y_i}{p})^2}{(\sum_{i=1}^p x_i^2 - \frac{(\sum_{i=1}^p x_i)^2}{p})(\sum_{i=1}^p y_i^2 - \frac{(\sum_{i=1}^p y_i)^2}{p})}$$

R^2 gives an indication on the quality of the rank regression.

The goodness of fit test consists in verifying that R^2 is higher or equal to the acceptance threshold, *AccThr*.

According to the paper written by Carl D. Tarum “Determination of the critical correlation coefficient to establish a good fit for Weibull and Log-Normal Failure Distributions” (see in the Bibliography), *AccThr* depends on the number of failures observed (p):

- $AccThr = (1 - e^{-(\frac{p}{0.0399})^{0.177}})^2$ for a 2 parameter Weibull (β, η);
- $AccThr = (1 - e^{-(\frac{p}{0.00239})^{0.146}})^2$ for a 3 parameter Weibull (γ, β, η).

If R^2 is higher or equal to *AccThr* for a 2 parameter Weibull, it is an evidence that the data came from a 2 parameter Weibull distribution. If R^2 is lower than *AccThr* for a 2 parameter Weibull, the following analysis shall be conducted:

- if the plot shows a curvature, as if time doesn't start at zero, then introduce the location parameter γ into the process. By simulations, determine the value of γ which gives the highest value of R^2 . If R^2 becomes higher than *AccThr* for a 3 parameter Weibull, then it is an evidence that the data came from a 3 parameter Weibull distribution. Then there should be a good physical explanation of why failures cannot occur before a time equal to γ ;
- check whether the graphical representation contains more than one fault mode. If so, each fault mode should be plotted separately and then consolidated together (see mixture of fault modes in IEC 61649);

- if some points are far from the best fit straight line, a more detailed analysis of the failures corresponding to these points shall be conducted.

6.3.4.3 Calculation of the Weibull distribution parameters

For a confidence level of 50 %:

Parameters β and η can be calculated from the following equations obtained from 6.2:

$$\beta = B \text{ and } \eta = e^{-\frac{A}{B}}.$$

90 % confidence interval bounds:

90 % confidence interval bounds define the limits that contain 90 % of the expected variation of unreliability. In other words, these limits will contain the true reliability with frequency of 90 %.

If we note:

- N the number of items under test;
- $U50_i$ the unreliability at rank i with a confidence level of 50 % on a sample of N items (obtained from Annex D, sample size N , column 50 %, line i);
- $U5_i$ the unreliability at rank i with a confidence level of 5 % on a sample of N items (obtained from Annex D, sample size N , column 5 %, line i);
- $U95_i$ the unreliability at rank i with a confidence level of 95 % on a sample of N items (obtained from Annex D, sample size N , column 95 %, line i).

For each unreliability estimate $U50_i$ the corresponding time to failure for a confidence level of 95 %, $TTF95_i$, and the corresponding time to failure for a confidence level of 5 %, $TTF5_i$, are calculated from the following equations:

$$TTF95_i - \gamma = \eta(-\ln(1 - U95_i))^{\frac{1}{\beta}} \text{ and } TTF5_i - \gamma = \eta(-\ln(1 - U5_i))^{\frac{1}{\beta}}$$

Then all the couples $(TTF95_i, U50_i)$ and $(TTF5_i, U50_i)$ shall be plotted to represent the 90 % confidence interval bounds.

Example of the calculation of the parameters of a Weibull distribution:

A sample of 10 items was put under a reliability test until all items failed. The measured times to failure were: 475, 510, 550, 690, 850, 1 010, 1 090, 1 190, 2 100 and 2 800 h.

Table 7 gives the result of unreliability estimation process.

Table 7 – Example of unreliability estimation for Weibull distribution

Rank	Time	Unreliability
1	475F	0,067
2	510F	0,1623
3	550F	0,2586
4	690F	0,3551
5	850F	0,4517
6	1 010F	0,5483
7	1 090F	0,6449
8	1 190F	0,7414
9	2 100F	0,8377
10	2 800F	0,9330

The calculation by least squares/rank regression of R^2 gives $R^2 = 0,8577$. For a 2 parameter Weibull with 10 failures observed, the acceptance threshold *AccThr* is 0,8647. The result of goodness of fit test is “Rejected”.

By increasing γ from 0, it is observed that R^2 increases, and goes to a maximum for $\gamma = 461$. With $\gamma = 461$, the calculation by least squares/rank regression of β , η , and R^2 gives:

- $\beta = 0,693$;
- $\eta = 631$;
- $R^2 = 0,9874$. For a 3 parameter Weibull with 10 failures observed, the acceptance threshold *AccThr* is 0,9329. The result of goodness of fit test is “Accepted” (to accept a value of γ different from 0, there should be a good physical explanation of why failures cannot occur before a time equal to γ).

Table 8 below gives the result of the 90 % confidence interval bounds calculation for a Weibull distribution defined by $\beta = 0,693$, $\eta = 631$ and $\gamma = 461$.

Table 8 – Example of 90 % confidence bounds calculation for Weibull distribution

Order number	$U5_i$	$TTF5_i-\gamma$	$U95_i$	$TTF95_i-\gamma$	$U50_i$
0,5	0,0002	0	0,1708	56	0,0219
1,0	0,0051	0	0,2589	111	0,0670
1,5	0,0179	2	0,3306	169	0,1143
2,0	0,0368	6	0,3942	233	0,1623
2,5	0,0602	11	0,4525	304	0,2104
3,0	0,0873	20	0,5069	383	0,2586
3,5	0,1173	31	0,5581	471	0,3068
4,0	0,1500	46	0,6066	571	0,3551
4,5	0,1851	64	0,6527	684	0,4034
5,0	0,2224	86	0,6965	813	0,4517
5,5	0,2619	113	0,7381	962	0,5000
6,0	0,3085	150	0,7776	1 136	0,5483
6,5	0,3473	185	0,8149	1 341	0,5966
7,0	0,3934	232	0,8500	1 588	0,6449
7,5	0,4419	290	0,8827	1 893	0,6932

Order number	$U5_i$	$TTF5_i-\gamma$	$U95_i$	$TTF95_i-\gamma$	$U50_i$
8,0	0,4931	362	0,9127	2 280	0,7414
8,5	0,5475	452	0,9398	2 798	0,7896
9,0	0,6058	569	0,9632	3 531	0,8377
9,5	0,6694	730	0,9821	4 693	0,8857
10,0	0,7411	974	0,9949	6 942	0,9330

The resulting graphical representation of the corresponding unreliability function is shown below on Figure 2.

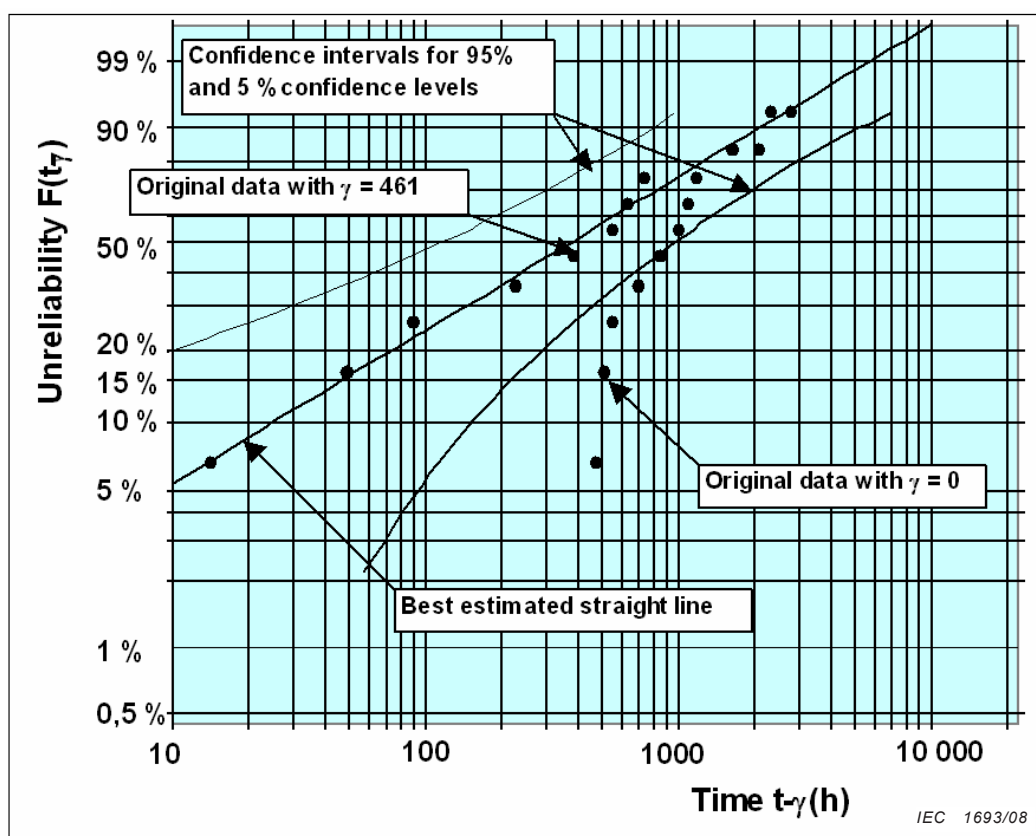


Figure 2 – Example of graphical representation of $F(t)$ in the case of Weibull distribution

7 The life-stress model

7.1 General

Life-stress models are described in Annex C. For temperature and humidity accelerated life tests, the model used is the Peck's temperature-humidity model.

The Peck's acceleration factor is:

$$AF = \left(\frac{RH_u}{RH_s} \right)^{-n} e^{\frac{E_a}{k} \left(\frac{1}{T_u} - \frac{1}{T_s} \right)}$$

where