



BSI Standards Publication

Ophthalmic optics and instruments – Reporting aberrations of the human eye

National foreword

This British Standard is the UK implementation of EN ISO 24157:2008+A1:2020. It is identical to ISO 24157:2008, incorporating amendment 1:2020. It supersedes BS EN ISO 24157:2008, which is withdrawn.

The start and finish of text introduced or altered by amendment is indicated in the text by tags. Tags indicating changes to ISO text carry the number of the ISO amendment. For example, text altered by ISO amendment 1 is indicated by **A1** **A1**.

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Ophthalmic optics and instruments - Reporting aberrations of the human eye (ISO 24157:2008)

Optique et instruments ophtalmiques -
Méthodes de présentation des aberrations
de l'oeil humain (ISO 24157:2008)

Augenoptik und ophthalmische Instrumente -
Verfahren zur Darstellung von Abbildungsfehlern
des menschlichen Auges (ISO 24157:2008)

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European foreword

This document (EN ISO 24157:2008) has been prepared by Technical Committee ISO/TC 172 "Optics and optical instruments" in collaboration with Technical Committee CEN/TC 170 "Ophthalmic optics" the secretariat of which is held by DIN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by January 2009, and conflicting national standards shall be withdrawn at the latest by January 2009.

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Endorsement notice

The text of ISO 24157:2008 has been approved by CEN as a EN ISO 24157:2008 without any modification.

Foreword to amendment A1

This document (EN ISO 24157:2008/A1:2020) has been prepared by Technical Committee ISO/TC 172 "Optics and photonics" in collaboration with Technical Committee CEN/TC 170 "Ophthalmic optics" the secretariat of which is held by DIN.

This Amendment to the European Standard EN ISO 24157:2008 shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by August 2020, and conflicting national standards shall be withdrawn at the latest by August 2020.

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Endorsement notice

The text of ISO 24157:2008/Amd 1:2020 has been approved by CEN as EN ISO 24157:2008/A1:2020 without any modification.

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

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ISO 24157 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 7, *Ophthalmic optics and instruments*.

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Ophthalmic optics and instruments – Reporting aberrations of the human eye

1 Scope

This International Standard specifies standardized methods for reporting aberrations of the human eye.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

[ISO 8429](#), *Optics and optical instruments — Ophthalmology — Graduated dial scale*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply. Symbols used are summarized in [Table 1](#).

A1 3.1

line of sight

ray path from the point of interest (i.e. point of fixation) in object space to the centre of the entrance pupil of the eye and its continuation in image space from the centre of the exit pupil to the retinal point of fixation (generally the foveola)

Note 1 to entry: These two parts of the ray path are distinct and separate segments.

[SOURCE: ISO 13666:2019, 3.2.24 **A1**]

3.2

Zernike polynomial function

one of a complete set of functions defined and orthogonal over the unit circle, the product of three terms, a normalization term, a radial term and a meridional term, parameterized by a dimensionless radial parameter, ρ , and a dimensionless meridional parameter, θ , designated by a non-negative radial integer index, n , and a signed meridional index, m , and given by the equation

$$Z_n^m = N_n^m R_n^{|m|}(\rho) M(m\theta) \quad (1)$$

where

N_n^m is the normalization term;

$R_n^{|m|}$ is the radial term;

$M(m\theta)$ is the meridional term;

the parameter ρ is a real number continuous over its range of 0 to 1,0;

the parameter θ is a real number continuous over its range of 0 to 2π .

Note 1 to entry: For a given value of radial index n , the meridional index m may only take the values $-n$, $-n+2$, ..., $n-2$ and n .

3.2.1

radial term

Zernike polynomial function term with indices n and m given by the equation

$$R_n^{|m|}(\rho) = \sum_{s=0}^{0,5(n-|m|)} \frac{(-1)^s (n-s)!}{s! [0,5(n+|m|)-s]! [0,5(n-|m|)-s]!} \rho^{n-2s} \quad (2)$$

where s is an integer summation index incremented by one unit

3.2.2

radial parameter

ρ

dimensionless number taking values between 0 and 1, its value at any radial distance, r , from the aperture centre being given by the expression

$$\rho = \frac{r}{a} \quad (3)$$

where a is the value of the aperture radius

3.2.3

meridional term

Zernike polynomial function term with index m given by the equations

$$M(m\theta) = \cos(m\theta) \quad \text{if } m \geq 0 \quad (4)$$

$$M(m\theta) = \sin(|m|\theta) \quad \text{if } m < 0 \quad (5)$$

Note 1 to entry: The meridional term is also known as the *azimuthal* term.

3.2.4

meridional parameter

θ

angular value taking values between 0 and 2π (0° and 360°), expressed in the coordinate system defined in [Clause 4](#)

Note 1 to entry: This is also called the *azimuthal* angle.

3.2.5

normalization term

Zernike polynomial function term with indices n and m , equal to 1,0 for “un-normalized” functions ([3.2.7](#)) and for “normalized” functions ([3.2.6](#)) by the equation

$$N_n^m = \sqrt{(2 - \delta_{0,m})(n+1)} \quad (6)$$

where $\delta_{0,m} = 1$ if $m = 0$, $\delta_{0,m} = 0$ if $m \neq 0$.

3.2.6

normalized Zernike polynomial function

Zernike polynomial function whose normalization term takes the form given in [3.2.5](#) for “normalized” functions defined as orthogonal in the sense that it satisfies the following equation

$$\int_0^1 \rho d\rho \int_0^{2\pi} Z_n^m Z_{n'}^{m'} d\theta = \pi \delta_{n,n'} \delta_{m,m'}$$

where

$\delta_{n,n'} = 1$ if $n = n'$, $\delta_{n,n'} = 0$ if $n \neq n'$;

$\delta_{m,m'} = 1$ if $m = m'$, $\delta_{m,m'} = 0$ if $m \neq m'$.

3.2.7

un-normalized Zernike polynomial function

Zernike polynomial function whose normalization term is equal to 1,0 and defined as orthogonal in the sense that it satisfies the equation

$$(2 - \delta_{0,m})(n+1) \int_0^1 \rho d\rho \int_0^{2\pi} Z_n^m Z_n^{m'} d\theta = \pi \delta_{n,n'} \delta_{m,m'} \quad (8)$$

where

$\delta_{n,n'} = 1$ if $n = n'$, $\delta_{n,n'} = 0$ if $n \neq n'$;

$\delta_{m,m'} = 1$ if $m = m'$, $\delta_{m,m'} = 0$ if $m \neq m'$;

$\delta_{0,m} = 1$ if $m = 0$, $\delta_{0,m} = 0$ if $m \neq 0$.

3.2.8

order

value of the radial index n of a Zernike polynomial function

3.3

Zernike coefficient

member of a set of real numbers,

, which is multiplied by its associated Zernike function to yield a term that is subsequently used in a sum of terms to give a value equal to the best estimate of the surface, $S(\rho, \theta)$, that has been fitted with Zernike terms, such a sum being represented by

$$S(\rho, \theta) = \sum_{\text{all } n \text{ and } m} c_n^m Z_n^m \quad (9)$$

Note 1 to entry: Each set of Zernike coefficients has associated with it the aperture diameter that was used to generate the set from surface elevation data. The set is incomplete without this aperture information.

Note 2 to entry: [Annex A](#) gives information on a method to find Zernike coefficients from wavefront slope (gradient) data.

3.3.1

normalized Zernike coefficient

Zernike coefficient generated using normalized Zernike functions and so designed to be used with them to reconstruct a surface

Note 1 to entry: Normalized Zernike coefficients have dimensional units of length.

3.3.2

un-normalized Zernike coefficient

Zernike coefficient generated using un-normalized Zernike functions and so designed to be used with them to reconstruct a surface

Note 1 to entry: Un-normalized Zernike coefficients have dimensional units of length.

3.4 wavefront error (of an eye) $W(x,y)$ or $W(r,\theta)$

optical path-length (i.e. physical distance times refractive index) between a plane wavefront in the eye's entrance pupil and the wavefront of light exiting the eye from a point source on the retina, and specified as a function (wavefront error function) of the (x,y) (or r,θ) coordinates of the entrance pupil

Note 1 to entry: Wavefront error is measured in an axial direction (i.e. parallel to the z -axis defined in [Clause 4](#)) from the pupil plane towards the wavefront.

Note 2 to entry: By convention, the wavefront error is set to zero at the pupil centre by subtracting the central value from values at all other pupil locations.

Note 3 to entry: Wavefront error has physical units of metres (typically reported in micrometres) and pertains to a specified wavelength.

3.5 optical path-length difference OPD

negative of the *wavefront error* ([3.4](#)) at each point in a wavefront representing the correction of the optical path-length needed to correct the wavefront error

3.6 root mean square wavefront error RMS wavefront error

(of an eye) quantity computed as the square root of the variance of the *wavefront error* ([3.4](#)) function and defined as

$$RMS_{WFE} = \sqrt{\frac{\iint_{\text{pupil}} [W(x,y)]^2 dx dy}{A}} \quad (10)$$

where A is the area of the pupil, if the wavefront error function is expressed in terms of normalized Zernike coefficients, a quantity equal to the square root of the sum of the squares of the coefficients with radial indices 2 or greater

$$RMS_{WFE} = \sqrt{\sum_{n>1, \text{all } m} (c_n^m)^2} \quad (11)$$

Note 1 to entry: Piston and average tilt should be excluded from this calculation because they correspond to lateral displacements of the image rather than image degradation *per se*.

Note 2 to entry: The RMS error can also be found using the discrete set of wavefront error values that were used to generate the Zernike coefficients and standard statistical methods. When this is done it might be found that this RMS value does not exactly match the value found using the formula given above. This is more likely to happen in cases where the locations in the pupil used to sample the wavefront error form a non-uniformly spaced grid. Then the data set does not lead to the formation of discrete, orthogonal Zernike functions.

3.7 higher-order aberrations

those aberrations experienced by the eye in addition to sphero-cylindrical refractive errors and prismatic error and thus, if the wavefront error is expressed in terms of Zernike polynomial function coefficients, those of order 3 and higher