

Quantities and units

Part 2: Mathematical signs and symbols to be used in the natural sciences and technology

ICS 01.060

National foreword

This British Standard is the UK implementation of EN ISO 80000-2:2013. It supersedes BS ISO 80000-2:2009, which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee SS/7, General metrology, quantities, units and symbols.

A list of organizations represented on this committee can be obtained on request to its secretary.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Compliance with a British Standard cannot confer immunity from legal obligations.

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 31 January 2010

© The British Standards Institution 2013. Published by BSI Standards Limited 2013

ISBN 978 0 580 79018 8

Amendments/corrigenda issued since publication

Date	Comments
31 May 2013	This corrigendum rennumbers BS ISO 80000-2:2009 as BS EN ISO 80000-2:2013

English Version

Quantities and units - Part 2: Mathematical signs and symbols to
be used in the natural sciences and technology (ISO 80000-
2:2009)

Grandeurs et unités - Partie 2: Signes et symboles
mathématiques à employer dans les sciences de la nature
et dans la technique (ISO 80000-2:2009)

Größen und Einheiten - Teil 2: Mathematische Zeichen für
Naturwissenschaft und Technik (ISO 80000-2:2009)

This European Standard was approved by CEN on 14 March 2013.

CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration. Up-to-date lists and bibliographical references concerning such national standards may be obtained on application to the CEN-CENELEC Management Centre or to any CEN member.

This European Standard exists in three official versions (English, French, German). A version in any other language made by translation under the responsibility of a CEN member into its own language and notified to the CEN-CENELEC Management Centre has the same status as the official versions.

CEN members are the national standards bodies of Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, Former Yugoslav Republic of Macedonia, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and United Kingdom.



EUROPEAN COMMITTEE FOR STANDARDIZATION
COMITÉ EUROPÉEN DE NORMALISATION
EUROPÄISCHES KOMITEE FÜR NORMUNG

Management Centre: Avenue Marnix 17, B-1000 Brussels

Foreword

The text of ISO 80000-2:2009 has been prepared by Technical Committee ISO/TC 12 “Quantities and units” of the International Organization for Standardization (ISO) and has been taken over as EN ISO 80000-2:2013.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by October 2013, and conflicting national standards shall be withdrawn at the latest by October 2013.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

According to the CEN-CENELEC Internal Regulations, the national standards organizations of the following countries are bound to implement this European Standard: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, Former Yugoslav Republic of Macedonia, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and the United Kingdom.

Endorsement notice

The text of ISO 80000-2:2009 has been approved by CEN as EN ISO 80000-2:2013 without any modification.

Contents

Page

Foreword	iv
Introduction.....	vi
1 Scope	1
2 Normative references	1
3 Variables, functions, and operators	1
4 Mathematical logic	3
5 Sets	4
6 Standard number sets and intervals	6
7 Miscellaneous signs and symbols	8
8 Elementary geometry	10
9 Operations	11
10 Combinatorics	14
11 Functions.....	15
12 Exponential and logarithmic functions	18
13 Circular and hyperbolic functions	19
14 Complex numbers	21
15 Matrices	22
16 Coordinate systems	24
17 Scalars, vectors, and tensors	26
18 Transforms	30
19 Special functions	31
Annex A (normative) Clarification of the symbols used	36
Bibliography.....	40

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 80000-2 was prepared by Technical Committee ISO/TC 12, *Quantities and units*, in collaboration with IEC/TC 25, *Quantities and units*.

This first edition cancels and replaces ISO 31-11:1992, which has been technically revised. The major technical changes from the previous standard are the following:

- Four clauses have been added, i.e. “Standard number sets and intervals”, “Elementary geometry”, “Combinatorics” and “Transforms”.

ISO 80000 consists of the following parts, under the general title *Quantities and units*:

- *Part 1: General*
- *Part 2: Mathematical signs and symbols to be used in the natural sciences and technology¹⁾*
- *Part 3: Space and time*
- *Part 4: Mechanics*
- *Part 5: Thermodynamics*
- *Part 7: Light*
- *Part 8: Acoustics*
- *Part 9: Physical chemistry and molecular physics*
- *Part 10: Atomic and nuclear physics*
- *Part 11: Characteristic numbers*
- *Part 12: Solid state physics*

1) Title to be shortened to read “Mathematics” in the second edition of ISO 80000-2.

IEC 80000 consists of the following parts, under the general title *Quantities and units*:

- *Part 6: Electromagnetism*
- *Part 13: Information science and technology*
- *Part 14: Telebiometrics related to human physiology*

Introduction

Arrangement of the tables

The first column “Item No.” of the tables contains the number of the item, followed by either the number of the corresponding item in ISO 31-11 in parentheses, or a dash when the item in question did not appear in ISO 31-11.

The second column “Sign, symbol, expression” gives the sign or symbol under consideration, usually in the context of a typical expression. If more than one sign, symbol or expression is given for the same item, they are on an equal footing. In some cases, e.g. for exponentiation, there is only a typical expression and no symbol.

The third column “Meaning, verbal equivalent” gives a hint on the meaning or how the expression may be read. This is for the identification of the concept and is not intended to be a complete mathematical definition.

The fourth column “Remarks and examples” gives further information. Definitions are given if they are short enough to fit into the column. Definitions need not be mathematically complete.

The arrangement of the table in Clause 16 “Coordinate systems” is somewhat different.

Quantities and units —

Part 2:

Mathematical signs and symbols to be used in the natural sciences and technology

1 Scope

ISO 80000-2 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in ISO 80000-2 are intended mainly for use in the natural sciences and technology, but also apply to other areas where mathematics is used.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 80000-1:—²⁾, *Quantities and units — Part 1: General*

3 Variables, functions and operators

Variables such as x , y , etc., and running numbers, such as i in $\sum_i x_i$ are printed in italic (sloping) type. Parameters, such as a , b , etc., which may be considered as constant in a particular context, are printed in italic (sloping) type. The same applies to functions in general, e.g. f , g .

An explicitly defined function not depending on the context is, however, printed in Roman (upright) type, e.g. $\sin$, $\exp$, $\ln$, Γ . Mathematical constants, the values of which never change, are printed in Roman (upright) type, e.g. $e = 2,718\ 218\ 8\dots$; $\pi = 3,141\ 592\dots$; $i^2 = -1$. Well-defined operators are also printed in Roman (upright) style, e.g. $\div$, δ in δx and each d in df/dx .

Numbers expressed in the form of digits are always printed in Roman (upright) style, e.g. 351 204; 1,32; 7/8.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\alpha + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation symbol, such as $+$, $-$, $\times$, $\cdot$ or $/$, the parentheses around the argument may be omitted. In these cases, there should be a thin space between the symbol for the function and the argument, e.g. $\int 2,4$; $\sin n\pi$; $\operatorname{arcosh} 2A$; $\operatorname{Ei} x$.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

2) To be published. (Revision of ISO 31-0:1992)

A comma, semicolon or other appropriate symbol can be used as a separator between numbers or expressions. The comma is generally preferred, except when numbers with a decimal comma are used.

If an expression or equation must be split into two or more lines, one of the following methods shall be used.

- a) Place the line breaks immediately after one of the symbols $=$, $+$, $-$, $\pm$ or $\mp$, or, if necessary, immediately after one of the symbols $\times$, $\cdot$, or $/$. In this case, the symbol indicates that the expression continues on the next line or next page.
- b) Place the line breaks immediately before one of the symbols $=$, $+$, $-$, $\pm$ or $\mp$, or, if necessary, immediately before one of the symbols $\times$, $\cdot$, or $/$. In this case, the symbol indicates that the expression is a continuation of the previous line or page.

The symbol shall not be given twice around the line break; two minus signs could for example give rise to sign errors. Only one of these methods should be used in one document. If possible, the line break should not be inside of an expression in parentheses.

It is customary to use different sorts of letters for different sorts of entities. This makes formulas more readable and helps in setting up an appropriate context. There are no strict rules for the use of letter fonts which should, however, be explained if necessary.

4 Mathematical logic

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-4.1 (11-3.1)	$p \wedge q$	conjunction of p and q , p and q	
2-4.2 (11-3.2)	$p \vee q$	disjunction of p and q , p or q	This “or” is inclusive, i.e. $p \vee q$ is true, if either p or q , or both are true.
2-4.3 (11-3.3)	$\neg p$	negation of p , not p	
2-4.4 (11-3.4)	$p \Rightarrow q$	p implies q , if p , then q	$q \Leftarrow p$ has the same meaning as $p \Rightarrow q$. $\Rightarrow$ is the implication symbol.
2-4.5 (11-3.5)	$p \Leftrightarrow q$	p is equivalent to q	$(p \Rightarrow q) \wedge (q \Rightarrow p)$ has the same meaning as $p \Leftrightarrow q$. $\Leftrightarrow$ is the equivalence symbol.
2-4.6 (11-3.6)	$\forall x \in A \ p(x)$	for every x belonging to A , the proposition $p(x)$ is true	If it is clear from the context which set A is being considered, the notation $\forall x \ p(x)$ can be used. $\forall$ is the universal quantifier. For $x \in A$, see 2-5.1.
2-4.7 (11-3.7)	$\exists x \in A \ p(x)$	there exists an x belonging to A for which $p(x)$ is true	If it is clear from the context which set A is being considered, the notation $\exists x \ p(x)$ can be used. $\exists$ is the existential quantifier. For $x \in A$, see 2-5.1. $\exists^1 x \ p(x)$ is used to indicate that there is exactly one element for which $p(x)$ is true. $\exists!$ is also used for $\exists^1$.

5 Sets

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.1 (11-4.1)	$x \in A$	x belongs to A , x is an element of the set A	$A \ni x$ has the same meaning as $x \in A$.
2-5.2 (11-4.2)	$y \notin A$	y does not belong to A , y is not an element of the set A	$A \not\ni y$ has the same meaning as $y \notin A$. The negating stroke may also be vertical.
2-5.3 (11-4.5)	$\{x_1, x_2, \dots, x_n\}$	set with elements $x_1, x_2, \dots, x_n$	Also $\{x_i \mid i \in I\}$, where I denotes a set of subscripts.
2-5.4 (11-4.6)	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbf{R} \mid x \leq 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used (for example $\{x \mid x \leq 5\}$, if it is clear that x is a variable for real numbers).
2-5.5 (11-4.7)	card A $ A $	number of elements in A , cardinality of A	The cardinality can be a transfinite number. See also 2-9.16.
2-5.6 (11-4.8)	$\emptyset$	the empty set	
2-5.7 (11-4.18)	$B \subseteq A$	B is included in A , B is a subset of A	Every element of B belongs to A . $\subset$ is also used, but see remark to 2-5.8. $A \supseteq B$ has the same meaning as $B \subseteq A$.
2-5.8 (11-4.19)	$B \subset A$	B is properly included in A , B is a proper subset of A	Every element of B belongs to A , but at least one element of A does not belong to B . If $\subset$ is used for 2-5.7, then $\subsetneq$ shall be used for 2-5.8. $A \supset B$ has the same meaning as $B \subset A$.
2-5.9 (11-4.24)	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B . $A \cup B = \{x \mid x \in A \vee x \in B\}$
2-5.10 (11-4.26)	$A \cap B$	intersection of A and B	The set of elements which belong to both A and B . $A \cap B = \{x \mid x \in A \wedge x \in B\}$
2-5.11 (11-4.25)	$\bigcup_{i=1}^n A_i$ $A_1 \cup A_2 \cup \dots \cup A_n$	union of the sets $A_1, A_2, \dots, A_n$	The set of elements belonging to at least one of the sets $A_1, A_2, \dots, A_n$ $\bigcup_{i=1}^n$, $\bigcup$ and $\bigcup_{i \in I}$ are also used, where I denotes a set of subscripts.
2-5.12 (11-4.27)	$\bigcap_{i=1}^n A_i$ $A_1 \cap A_2 \cap \dots \cap A_n$	intersection of the sets $A_1, \dots, A_n$	The set of elements belonging to all sets $A_1, A_2, \dots, A_n$ $\bigcap_{i=1}^n$, $\bigcap$ and $\bigcap_{i \in I}$ are also used, where I denotes a set of subscripts.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.13 (11-4.28)	$A \setminus B$	difference of A and B , A minus B	The set of elements which belong to A but not to B . $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used. $\complement_A B$ is also used. $\complement_A B$ is mainly used when B is a subset of A , and the symbol A may be omitted if it is clear from the context which set A is being considered.
2-5.14 (11-4.30)	(a, b)	ordered pair a, b , couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. If the comma can be mistaken as the decimal sign, then the semicolon (;) or a stroke () may be used as separator.
2-5.15 (11-4.31)	$(a_1, a_2, \dots, a_n)$	ordered n -tuple	See remark to 2-5.14.
2-5.16 (11-4.32)	$A \times B$	Cartesian product of the sets A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
2-5.17 (—)	$\prod_{i=1}^n A_i$ $A_1 \times A_2 \times \dots \times A_n$	Cartesian product of the sets $A_1, A_2, \dots, A_n$	The set of ordered n -tuples $(x_1, x_2, \dots, x_n)$ such that $x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n$. $A \times A \times \dots \times A$ is denoted by A^n , where n is the number of factors in the product.
2-5.18 (11-4.33)	id_A	identity relation on A , diagonal of $A \times A$	id_A is the set of all pairs (x, x) where $x \in A$. If the set A is clear from the context, the subscript A can be omitted.

6 Standard number sets and intervals

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.1 (11.4.9)	N	the set of natural numbers, the set of positive integers and zero	$\mathbf{N} = \{0, 1, 2, 3, \dots\}$ $\mathbf{N}^* = \{1, 2, 3, \dots\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{N}_{>5} = \{n \in \mathbf{N} \mid n > 5\}$ The symbols $\mathbf{N}$ and $\mathbb{N}$ are also used.
2-6.2 (11.4.10)	Z	the set of integers	$\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ $\mathbf{Z}^* = \{n \in \mathbf{Z} \mid n \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Z}_{\geq -3} = \{n \in \mathbf{Z} \mid n \geq -3\}$ The symbol $\mathbb{Z}$ is also used.
2-6.3 (11.4.11)	Q	the set of rational numbers	$\mathbf{Q}^* = \{r \in \mathbf{Q} \mid r \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Q}_{<0} = \{r \in \mathbf{Q} \mid r < 0\}$ The symbols $\mathbb{Q}$ and $\mathbb{Q}$ are also used.
2-6.4 (11.4.12)	R	the set of real numbers	$\mathbf{R}^* = \{x \in \mathbf{R} \mid x \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{R}_{\geq 0} = \{x \in \mathbf{R} \mid x \geq 0\}$ The symbols $\mathbb{R}$ and $\mathbb{R}$ are also used.
2-6.5 (11.4.13)	C	the set of complex numbers	$\mathbf{C}^* = \{z \in \mathbf{C} \mid z \neq 0\}$ The symbols $\mathbb{C}$ and $\mathbb{C}$ are also used.
2-6.6 (—)	P	the set of prime numbers	$\mathbf{P} = \{2, 3, 5, 7, 11, 13, 17, \dots\}$ The symbols $\mathbb{P}$ and $\mathbb{P}$ are also used.
2-6.7 (11.4.14)	$[a, b]$	closed interval from a included to b included	$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$
2-6.8 (11.4.15)	$(a, b]$	left half-open interval from a excluded to b included	$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$ The notation $]a, b]$ is also used.
2-6.9 (11.4.16)	$[a, b)$	right half-open interval from a included to b excluded	$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$ The notation $[a, b[$ is also used.
2-6.10 (11.4.17)	(a, b)	open interval from a excluded to b excluded	$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$ The notation $]a, b[$ is also used.
2-6.11 (—)	$(-\infty, b]$	closed unbounded interval up to b included	$(-\infty, b] = \{x \in \mathbf{R} \mid x \leq b\}$ The notation $] -\infty, b]$ is also used.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.12 (—)	$(-\infty, b)$	open unbounded interval up to b excluded	$(-\infty, b) = \{x \in \mathbf{R} \mid x < b\}$ The notation $]-\infty, b[$ is also used.
2-6.13 (—)	$[a, +\infty)$	closed unbounded interval onward from a included	$[a, +\infty) = \{x \in \mathbf{R} \mid a \leq x\}$ The notations $[a, \infty [$, $[a, +\infty [$ and $[a, \infty)$ are also used.
2-6.14 (—)	$(a, +\infty)$	open unbounded interval onward from a excluded	$(a, +\infty) = \{x \in \mathbf{R} \mid a < x\}$ The notations $]a, +\infty[$, $]a, \infty [$ and (a, ∞) are also used.

7 Miscellaneous signs and symbols

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.1 (11-5.1)	$a = b$	a is equal to b	The symbol $\equiv$ may be used to emphasize that a particular equality is an identity. See also 2-7.18.
2-7.2 (11-5.2)	$a \neq b$	a is not equal to b	The negating stroke may also be vertical.
2-7.3 (11-5.3)	$a := b$	a is by definition equal to b	EXAMPLE $p := mv$, where p is momentum, m is mass and v is velocity. The symbols $=_{\text{def}}$ and $\stackrel{\text{def}}{=}$ are also used.
2-7.4 (11-5.4)	$a \triangle b$	a corresponds to b	EXAMPLES When $E = kT$, then $1 \text{ eV} \triangle 11\,604,5 \text{ K}$ When 1 cm on a map corresponds to a length of 10 km, one may write $1 \text{ cm} \triangle 10 \text{ km}$. The correspondence is not symmetric.
2-7.5 (11-5.5)	$a \approx b$	a is approximately equal to b	It depends on the user whether an approximation is sufficiently good. Equality is not excluded.
2-7.6 (11-7.7)	$a \simeq b$	a is asymptotically equal to b	EXAMPLE $\frac{1}{\sin(x-a)} \simeq \frac{1}{x-a}$ as $x \rightarrow a$ (For $x \rightarrow a$, see 2-7.16.)
2-7.7 (11-5.6)	$a \sim b$	a is proportional to b	The symbol $\sim$ is also used for equivalence relations. The notation $a \propto b$ is also used.
2-7.8 (—)	$M \cong N$	M is congruent to N , M is isomorphic to N	M and N are point sets (geometrical figures). This symbol is also used for isomorphisms of mathematical structures.
2-7.9 (11-5.7)	$a < b$	a is less than b	
2-7.10 (11-5.8)	$b > a$	b is greater than a	
2-7.11 (11-5.9)	$a \leq b$	a is less than or equal to b	
2-7.12 (11-5.10)	$b \geq a$	b is greater than or equal to a	
2-7.13 (11-5.11)	$a \ll b$	a is much less than b	It depends on the user whether a is sufficiently small as compared to b .
2-7.14 (11-5.12)	$b \gg a$	b is much greater than a	It depends on the user whether b is sufficiently great as compared to a .