

Equipment reliability testing —

Part 4: Statistical procedures for exponential distribution — Point estimates, confidence intervals, prediction intervals and tolerance intervals

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National foreword

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The UK participation in its preparation was entrusted to Technical Committee DS/1, Dependability and terotechnology, which has the responsibility to:

- aid enquirers to understand the text;
- present to the responsible international/European committee any enquiries on the interpretation, or proposals for change, and keep the UK interests informed;
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Equipment reliability testing –

Part 4:

**Statistical procedures for exponential
distribution – Point estimates, confidence
intervals, prediction intervals and
tolerance intervals**



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INTERNATIONAL ELECTROTECHNICAL COMMISSION

EQUIPMENT RELIABILITY TESTING –

**Part 4: Statistical procedures for exponential distribution –
Point estimates, confidence intervals, prediction intervals
and tolerance intervals**

FOREWORD

- 1) The IEC (International Electrotechnical Commission) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committees). The object of the IEC is to promote international co-operation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, the IEC publishes International Standards. Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. The IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
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International Standard IEC 60605-4 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition published in 1986 and its amendment 1 (1989), of which it constitutes a technical revision.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/737/FDIS	56/756/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 3.

Annexes A and B are for information only.

Annexes C, D, E and F form an integral part of this standard.

The committee has decided that the contents of this publication will remain unchanged until 2006. At this date, the publication will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition, or
- amended.

EQUIPMENT RELIABILITY TESTING –

Part 4: Statistical procedures for exponential distribution – Point estimates, confidence intervals, prediction intervals and tolerance intervals

1 Scope

This International Standard provides statistical methods for evaluating point estimates, confidence intervals, prediction intervals and tolerance intervals for the failure rate of items, whose time to failure follows an exponential distribution. This implies that the failure rate (see IEC 191-12-02) is constant with respect to time. It should be noted that although reference is made above to failure rate, the numerical methods to be described are equally applicable to other event rates provided the times to the occurrence of the event follow an exponential distribution. Thus, with this qualification, the numerical methods will apply, for example, to constant failure intensities and constant repair rates (see IEC 191-12-04 and IEC 191-13-02 respectively). For convenience, however, and to avoid unnecessary repetition, reference will be made in what follows only to failures and failure rates.

Use of the procedures in this standard should be supported by tests to confirm the validity of the assumption of constant failure rate or failure intensity (see IEC 60605-6).

This standard is also applicable whenever a random sample of items is subjected to a test of times to failure for the purpose of estimating measures of reliability performance.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of IEC 60605. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of IEC 60605 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of IEC and ISO maintain registers of currently valid International Standards.

IEC 60050(191):1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 60605-6:1997, *Equipment reliability testing – Part 6: Tests for the validity of the constant failure rate or constant failure intensity assumptions*

ISO 3534-1:1993, *Statistics – Vocabulary and symbols – Part 1: Probability and general statistical terms*

ISO 3534-2:1993, *Statistics – Vocabulary and symbols – Part 2: Statistical quality control*

3 Definitions and symbols

For the purposes of this International Standard, the terms and definitions of IEC 60050(191), ISO 3534-1 and ISO 3534-2 apply, together with the following definitions and symbols. Additional constants and functions are defined in the text.

3.1 Definitions

3.1.1

two-sided confidence interval (see ISO 3534-1, 2.57)

when m_{L2} and m_{U2} are two functions of the observed values such that, m being a population parameter to be estimated, the probability $\Pr(m_{L2} \leq m \leq m_{U2})$ is at least equal to $(1-\alpha)$ (where $(1-\alpha)$ is a fixed number, positive and less than 1), the interval between m_{L2} and m_{U2} is a two-sided $(1-\alpha)$ confidence interval for m

3.1.2

one-sided confidence interval (see ISO 3534-1, 2.58)

when m_{U1} (or m_{L1}) is a function of the observed values such that, m being a population parameter to be estimated, the probability $\Pr(m_{U1} \geq m)$ (or the probability $\Pr(m_{L1} \leq m)$) is at least equal to $(1-\alpha)$ (where $(1-\alpha)$ is a fixed number, positive and less than 1), the interval from the smallest possible value of m up to m_{U1} (or the interval from m_{L1} up to the largest possible value of m) is a one-sided $(1-\alpha)$ confidence interval for m

NOTE See annex B for further discussion.

3.1.3

tolerance limits (see ISO 3534-2, 1.4.3)

two limits ℓ_{L2} and ℓ_{U2} on the observed number of failures between which at least a proportion P of the population may be asserted (with probability $1-\alpha$) to lie

NOTE The interval ℓ_{L2} to ℓ_{U2} is known as the tolerance interval.

3.1.4 (see ISO 3534-2, 1.4.4)

tolerance

difference between the upper and lower tolerance limits.

NOTE More descriptive definitions of the above quantities and their relationship to one another are given in annex B.

3.2 Symbols

cl	Lower or upper one-sided confidence limit on a dependability parameter, such as MTTF (mean time to failure), failure rate, reliability, etc.
α	Significance level: $100(1-\alpha)$ % is the confidence level at which confidence intervals and limits are calculated
ℓ_{L1}, ℓ_{U1}	Lower and upper one-sided values of a $100(1-\alpha)$ % tolerance interval
ℓ_{L2}, ℓ_{U2}	Lower and upper two-sided values of a $100(1-\alpha/2)$ % tolerance interval
λ	True constant failure rate

$\hat{\lambda}$	Estimated constant failure rate
$\lambda_{L1}, \lambda_{U1}$	Lower and upper one-sided confidence limit on true failure rate
$\lambda_{L2}, \lambda_{U2}$	Lower and upper two-sided confidence limit on true failure rate
m	True mean time to failure
$\hat{m}$	Estimated mean time to failure
m_{L1}, m_{U1}	Lower and upper one-sided confidence limit on true mean time to failure
m_{L2}, m_{U2}	Lower and upper two-sided confidence limit on true mean time to failure
n	Total number of items under test
P	Proportion of a future population (used for the purpose of assigning tolerance intervals)
r	Number of failures observed
r_{L1}, r_{U1}	Lower and upper one-sided values of a $100(1-\alpha)\%$ prediction interval
r_{L2}, r_{U2}	Lower and upper two-sided values of a $100(1-\alpha/2)\%$ prediction interval
t	Calendar time at which the test was terminated
T^*	Accumulated test time: the total number of item-time units, e.g. item-hours, obtained at the termination of the test (see annex C)
w_f	Length of future periods (used for the purpose of assigning prediction intervals)
w_p	Length of past periods (used for the purpose of assigning prediction intervals)
$F_\alpha(\nu_1, \nu_2)$	α fractile of the cumulative F distribution with ν_1 and ν_2 degrees of freedom
$\text{Poiss}(J : w_f \lambda)$	Sum of the first $(J + 1)$ terms of the Poisson distribution with mean equal to $w_f \lambda$
$R(t)$	Reliability function of the exponential distribution: $R(t) = \exp(-\lambda t)$ for a mission time t
$\chi_\alpha^2(\nu)$	α fractile of the cumulative χ^2 distribution with ν degrees of freedom

4 Assumptions and requirements

The statistical procedures described in this International Standard are valid only when the failure rate (see clause 1) of the items concerned, is constant with respect to time. When this is so, the mean time to failure of the population from which the items are drawn is the reciprocal of the failure rate. If the constant failure rate hypothesis needs to be tested, refer to IEC 60605-6.

4.1 Assumptions and information required for point estimates and confidence intervals

It is assumed that a sample of n items is drawn at random from a population of items whose failure rate is constant, and is put on test either together or at different instances of time. The testing environment shall be the same for all items being subjected to the test and failed items may or may not be replaced. The test may be stopped either when a pre-determined amount of test time, T^* , has been accumulated (time terminated test), or when a pre-determined number of failures r , have occurred (failure terminated test). The time to failure of each failed item is used in the calculations, together with the time accumulated by items still operating when the test is terminated.

NOTE Time and failure terminated tests are often referred to in the literature as type I and type II censored tests respectively.

The data to be analysed consists of the times to failure of items which are put on test and which are either replaced or not replaced when they fail. It is not necessary for the test to be continued until all the items have failed; the test may be stopped earlier.

The information required (see annex A and annex B for details) is as follows:

- the confidence level associated with the interpretation of the data $(1 - \alpha)$;
- the accumulated test time, T^* , observed during the test (see annex C);
- the number of failures r , observed during the test;
- a knowledge of whether the test was failure terminated or time terminated.

4.2 Assumptions and requirements for prediction intervals

When dealing with prediction intervals, slightly different assumptions to those above are made. Included in these is the assumption that a constant number of items have been “at risk” over a time period w_p and that it is desired to establish a prediction interval for a future time period w_f .

Thus, the following information is required:

- the confidence level associated with the interpretation of the data;
- the number of failures, r , occurring during the period w_p ;
- the values of the periods w_f and w_p .

4.3 Assumptions and requirements for tolerance intervals

When dealing with tolerance intervals, the assumptions made are similar to those for prediction intervals, so that the information needed is:

- the confidence level at which the tolerance interval will be calculated;
- the proportion P of items from the population that is predicted to fail;
- the accumulated test time, T^* : the total number of item-hours obtained at the termination of the test (see annex C)¹⁾;

¹⁾ Under the exponential model of times to failure, it is irrelevant whether T^* is obtained using one item on test for a long time, or many items on test for a relatively short time. However, in practice it is better to avoid both of these extremes, as the exponential model may no longer be true under such conditions. These statements apply to clause 4 in general.