BRITISH STANDARD

BS ISO 11929:2010

# Determination of the characteristic limits (decision threshold, detection limit and limits of the confidence interval) for measurements of ionizing radiation — Fundamentals and application

ICS 17.240

NO COPYING WITHOUT BSI PERMISSION EXCEPT AS PERMITTED BY COPYRIGHT LAW

Document Sharing Hub
British Standards

Full document details: BS ISO 11929:2010

# National foreword

This British Standard is the UK implementation of ISO 11929:2010. It supersedes BS ISO 11929-1:2000, BS ISO 11929-2:2000, BS ISO 11929-3:2000, BS ISO 11929-4:2001, BS ISO 11929-5:2005, BS ISO 11929-6:2005, BS ISO 11929-7:2005 and BS ISO 11929-8:2005, which are withdrawn.

The UK participation in its preparation was entrusted to Technical Committee NCE/2, Radiation protection and measurement.

A list of organizations represented on this committee can be obtained on request to its secretary.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

**Compliance with a British Standard cannot confer immunity from legal obligations.**

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 31 March 2010

© BSI 2010

**ISBN 978 0 580 59044 3**

**Amendments/corrigenda issued since publication**

| Date | Comments |
|---|---|
| | |
| | |
| | |
| | |

Documents
Document Sharing Hub

Full document details: BS ISO 11929:2010

# INTERNATIONAL STANDARD

# ISO 11929

First edition
2010-03-01

# Determination of the characteristic limits (decision threshold, detection limit and limits of the confidence interval) for measurements of ionizing radiation — Fundamentals and application

*Détermination des limites caractéristiques (seuil de décision, limite de détection et extrémités de l'intervalle de confiance) pour mesurages de rayonnements ionisants — Principes fondamentaux et applications*

Reference number
ISO 11929:2010(E)

Documents
Document Sharing Hub

© ISO 2010

Full document details: BS ISO 11929:2010

**PDF disclaimer**

This PDF file may contain embedded typefaces. In accordance with Adobe's licensing policy, this file may be printed or viewed but shall not be edited unless the typefaces which are embedded are licensed to and installed on the computer performing the editing. In downloading this file, parties accept therein the responsibility of not infringing Adobe's licensing policy. The ISO Central Secretariat accepts no liability in this area.

Adobe is a trademark of Adobe Systems Incorporated.

Details of the software products used to create this PDF file can be found in the General Info relative to the file; the PDF-creation parameters were optimized for printing. Every care has been taken to ensure that the file is suitable for use by ISO member bodies. In the unlikely event that a problem relating to it is found, please inform the Central Secretariat at the address given below.

**COPYRIGHT PROTECTED DOCUMENT**

© ISO 2010

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland



© ISO 2010 – All rights reserved

Documents
Document Sharing Hub

Full document details: BS ISO 11929:2010

# Contents

Page

**Foreword** ..... iv
**Introduction** ..... v
**1 Scope** ..... 1
**2 Normative references** ..... 1
**3 Terms and definitions** ..... 2
**4 Quantities and symbols** ..... 4
**5 Fundamentals** ..... 5
**5.1 General aspects concerning the measurand** ..... 5
**5.2 Model** ..... 6
**5.3 Calculation of the standard uncertainty as a function of the measurand** ..... 8
**6 Characteristic limits and assessments** ..... 10
**6.1 Specifications** ..... 10
**6.2 Decision threshold** ..... 10
**6.3 Detection limit** ..... 10
**6.4 Limits of the confidence interval** ..... 12
**6.5 Assessment of a measurement result** ..... 13
**6.6 Assessment of a measurement procedure** ..... 14
**7 Documentation** ..... 15
**Annex A** (informative) **Overview of the general procedure** ..... 16
**Annex B** (normative) **Various applications** ..... 18
**Annex C** (normative) **Applications to counting spectrometric measurements** ..... 25
**Annex D** (informative) **Application examples** ..... 37
**Annex E** (informative) **Distribution function of the standardized normal distribution** ..... 49
**Annex F** (informative) **Explanatory notes** ..... 51
**Bibliography** ..... 59

© ISO 2010 – All rights reserved


Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 11929 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 2, *Radiation protection*.

This first edition of ISO 11929 cancels and replaces ISO 11929-1:2000, ISO 11929-2:2000, ISO 11929-3:2000, ISO 11929-4:2001, ISO 11929-5:2005, ISO 11929-6:2005, ISO 11929-7:2005 and ISO 11929-8:2005, which have been technically revised, specifically with reference to the type of statistical treatment of the data.


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

# Introduction

The limits to be provided according to this International Standard by means of statistical tests and specified probabilities allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

— the “decision threshold” gives a decision on whether or not the physical effect quantified by the measurand is present;

— the “detection limit” indicates the smallest true value of the measurand which can still be detected with the applied measurement procedure; this gives a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose;

— the “limits of the confidence interval” enclose, in the case of the physical effect recognized as present, a confidence interval containing the true value of the measurand with a specified probability.

Hereinafter, the limits mentioned are jointly called “characteristic limits”.

Since measurement uncertainty plays an important part in this International Standard, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to ISO/IEC Guide 98-3; see also References [1, 2]. This enables the strict separation of the evaluation of the measurements, on the one hand (Clause 5), and the provision and calculation of the characteristic limits, on the other hand (Clause 6). This International Standard is based on Bayesian statistics according to References [6 to 19], such that uncertain quantities and influences, which do not behave randomly in measurements repeated several times or in counting measurements, can also be taken into account.

Equations are provided for the calculation of the characteristic limits of an ionizing radiation measurand via the “standard measurement uncertainty” of the measurand (hereinafter “standard uncertainty”). The standard uncertainties of the measurement, as well as those of sample treatment, calibration of the measuring system and other influences are taken into account. However, the latter standard uncertainties are assumed to be known from previous investigations.

© ISO 2010 – All rights reserved


Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

Documents
Document Sharing Hub

Full document details: BS ISO 11929:2010

INTERNATIONAL STANDARD ISO 11929:2010(E)

# Determination of the characteristic limits (decision threshold, detection limit and limits of the confidence interval) for measurements of ionizing radiation — Fundamentals and application

## 1 Scope

This International Standard specifies a procedure, in the field of ionizing radiation metrology, for the calculation of the "decision threshold", the "detection limit" and the "limits of the confidence interval" for a non-negative ionizing radiation measurand, when counting measurements with preselection of time or counts are carried out, and the measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment and by other factors.

This International Standard also applies, in the same way to:

— counting measurements on moving objects (see B.2);

— measurements with linear-scale analogue count rate measuring instruments (hereinafter called ratemeters, see B.3);

— repeated counting measurements with random influences (see B.4);

— counting measurements on filters during accumulation of radioactive material (see B.5);

— counting spectrometric multi-channel measurements, if particular lines in the spectrum are to be considered and no adjustment calculations, for instance, an unfolding, have to be carried out (see C.2 to C.4);

— counting spectrometric multi-channel measurements if evaluated by unfolding methods (see C.5), in particular, alpha- and gamma-spectrometric measurements (see C.5.5 and C.5.6, respectively).

This International Standard also applies analogously to other measurements of any kind if the same model of the evaluation is involved. In this sense, it is also applicable to measurements with albedo dosimeters [18]. Further practical examples can be found in other International Standards, for example ISO 18589[21], ISO 9696[22], ISO 9697[23], ISO 9698[24], ISO 9699[25], ISO 10703[26], ISO 7503[27] and ISO 28218[28].

## 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 31-0, *Quantities and units — Part 0: General principles*

ISO 31-9, *Quantities and units — Part 9: Atomic and nuclear physics*

© ISO 2010 – All rights reserved



Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 31-0, ISO 31-9, ISO/IEC Guide 98-3, ISO/IEC Guide 99 and ISO 3534-1 and the following apply.

**3.1**
**measurement procedure**
set of operations, described specifically, used in the performance of particular measurements according to a given method

[ISO/IEC Guide 99:2007, 2.6]

**3.2**
**measurand**
particular quantity subject to measurement

[ISO/IEC Guide 99:2007, 2.3]

NOTE In this International Standard, a measurand is non-negative and quantifies a nuclear radiation effect. The effect is not present if the true value of the measurand is zero. An example of a measurand is the intensity of an energy line in a spectrum above the background in a spectrometric measurement.

**3.3**
**result of a measurement**
value attributed to a measurand, obtained by measurement

[ISO/IEC Guide 99:2007, 2.9]

**3.4**
**uncertainty of measurement**
**uncertainty**
non-negative parameter, which characterizes the dispersion of the values which could reasonably be attributed to the measurand

[ISO/IEC Guide 99:2007, 2.26]

See also ISO/IEC Guide 98-3.

NOTE The uncertainty of a measurement derived according to ISO/IEC Guide 98-3 comprises, in general, many components. Some of these components can be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which can also be characterized by standard deviations, are evaluated from assumed or known probability distributions based on experience and other information.

**3.5**
**model of evaluation**
set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

NOTE The model of evaluation does not need to be an explicit function; it can also be an algorithm realized by a computer code.


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

**3.6**
**decision threshold**
value of the estimator of the measurand, which when exceeded by the result of an actual measurement using a given measurement procedure of a measurand quantifying a physical effect, one decides that the physical effect is present

NOTE 1 The decision threshold is defined such that in cases where the measurement result, $y$, exceeds the decision threshold, $y^*$, the probability that the true value of the measurand is zero is less or equal to a chosen probability, $\alpha$.

NOTE 2 If the result, $y$, is below the decision threshold, $y^*$, the result cannot be attributed to the physical effect; nevertheless it cannot be concluded that it is absent.

**3.7**
**detection limit**
smallest true value of the measurand which ensures a specified probability of being detectable by the measurement procedure

NOTE With the decision threshold according to 3.6, the detection limit is the smallest true value of the measurand for which the probability of wrongly deciding that the true value of the measurand is zero is equal to a specified value, $\beta$, when, in fact, the true value of the measurand is not zero.

**3.8**
**limits of the confidence interval**
values which define a confidence interval containing the true value of the measurand with a specified probability

NOTE A confidence interval is sometimes known as a credible interval or a Bayesian interval. It is characterized in this International Standard by a specified probability $(1-\gamma)$.

**3.9**
**best estimate of the true value of the measurand**
expectation value of the probability distribution of the true value of the measurand, given the experimental result and all prior information on the measurand

NOTE The best estimate, among all possible estimates of the measurand on the basis of given information, which is associated with the minimum uncertainty.

**3.10**
**guideline value**
value which corresponds to scientific, legal or other requirements and which is intended to be assessed by the measurement procedure

NOTE 1 The guideline value can be given, for example, as an activity, a specific activity or an activity concentration, a surface activity or a dose rate.

NOTE 2 The comparison of the detection limit with a guideline value allows a decision on whether or not the measurement procedure satisfies the requirements set forth by the guideline value and is therefore suitable for the intended measurement purpose. The measurement procedure satisfies the requirement if the detection limit is smaller than the guideline value.

**3.11**
**background effect**
measurement effect caused by radiation other than that caused by the object of the measurement itself

EXAMPLE Natural radiation sources.

**3.12**
**net effect**
contribution of the possible radiation of a measurement object (for instance, of a radiation source or radiation field) to the measurement effect

© ISO 2010 – All rights reserved

Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

**3.13**
**gross effect**
measurement effect caused by the background effect and the net effect

**3.14**
**shielding factor**
factor describing the reduction of the background count rate by the effect of shielding caused by the measurement object

**3.15**
**relaxation time constant**
duration in which the output signal of a linear-scale ratemeter decreases to 1/e times the starting value after stopping the sequence of the input pulses

**3.16**
**background**
⟨spectrometric measurements⟩ number of events of no interest in the region of a specific line in the spectrum

NOTE The events can be due to the background effect by the environmental radiation and also to the sample itself (for instance, from other lines).

## 4 Quantities and symbols

The symbols for auxiliary quantities and the symbols only used in the annexes are not listed. Physical quantities are denoted by upper-case letters but shall be carefully distinguished from their values, denoted by the corresponding lower-case letters. In addition, the special quantities and symbols for unfolding in spectrometric measurements given in C.5.1 and for Bayesian statistics given in F.2.1 are used.

$m$ number of input quantities

$X_i$ input quantity $(i = 1, \ldots, m)$

$x_i$ estimate of the input quantity $X_i$

$u(x_i)$ standard uncertainty of the input quantity $X_i$ associated with the estimate $x_i$

$h_1(x_1)$ standard uncertainty $u(x_1)$ as a function of the estimate $x_1$

$\Delta x_i$ width of the region of the possible values of the input quantity $X_i$

$u_{\mathrm{rel}}(w)$ relative standard uncertainty of a quantity $Y_k$ associated with the estimate $w$

$G$ model function

$Y$ random variable as an estimator of the measurand; also used as the symbol for the non-negative measurand itself, which quantifies the physical effect of interest

$\tilde{y}$ true value of the measurand; if the physical effect of interest is not present, then $\tilde{y} = 0$; otherwise, $\tilde{y} > 0$

$y$ determined value of the estimator $Y$, estimate of the measurand, primary measurement result of the measurand

$y_j$ values $y$ from different measurements $(j = 0, 1, 2, \ldots)$

$u(y)$ standard uncertainty of the measurand associated with the primary measurement result $y$


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

| | |
|---|---|
| $\tilde{u}(\tilde{y})$ | standard uncertainty of the estimator $Y$ as a function of the true value $\tilde{y}$ of the measurand |
| $\hat{y}$ | best estimate of the measurand |
| $u(\hat{y})$ | standard uncertainty of the measurand associated with the best estimate $\hat{y}$ |
| $y^*$ | decision threshold of the measurand |
| $y^\#$ | detection limit of the measurand |
| $\tilde{y}_i$ | approximations of the detection limit $y^\#$ |
| $y_\mathrm{r}$ | guideline value of the measurand |
| $y^\triangleleft$, $y^\triangleright$ | lower and upper limit of the confidence interval, respectively, of the measurand |
| $\rho_i$ | count rate as an input quantity $X_i$ |
| $\rho_\mathrm{n}$ | count rate of the net effect (net count rate) |
| $\rho_\mathrm{g}$ | count rate of the gross effect (gross count rate) |
| $\rho_0$ | count rate of the background effect (background count rate) |
| $n_i$ | number of counted pulses obtained from the measurement of the count rate $\rho_i$ |
| $n_\mathrm{g}$, $n_0$ | number of counted pulses of the gross effect and of the background effect, respectively |
| $t_i$ | measurement duration of the measurement of the count rate $\rho_i$ |
| $t_\mathrm{g}$, $t_0$ | measurement duration of the measurement of the gross effect and of the background effect, respectively |
| $r_i$ | estimate of the count rate $\rho_i$ |
| $r_\mathrm{g}$, $r_0$ | estimate of the gross count rate and of the background count rate, respectively |
| $\tau_\mathrm{g}$, $\tau_0$ | relaxation time constant of a ratemeter used for the measurement of the gross effect and of the background effect, respectively |
| $\alpha$, $\beta$ | probability of the error of the first and second kind, respectively |
| $1-\gamma$ | probability for the confidence interval of the measurand |
| $k_p$, $k_q$ | quantiles of the standardized normal distribution for the probabilities $p$ and $q$, respectively (for instance $p = 1-\alpha$, $1-\beta$ or $1-\gamma/2$) |
| $\Phi(t)$ | distribution function of the standardized normal distribution; $\Phi(k_p) = p$ applies |

# 5 Fundamentals

## 5.1 General aspects concerning the measurand

A non-negative measurand shall be assigned to the physical effect to be investigated according to a given measurement task. The measurand shall quantify the effect. It assumes the true value $\tilde{y} = 0$ if the effect is not present in a particular case.

© ISO 2010 – All rights reserved

Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

Then, a random variable $Y$, an estimator, shall be assigned to the measurand. The symbol $Y$ is also used in this clause for the measurand itself. A value $y$ of the estimator $Y$, determined from measurements, is an estimate of the measurand. It shall be calculated as the primary measurement result together with the primary standard uncertainty $u(y)$, of the measurand associated with $y$. These two values form the primary complete measurement result for the measurand and are obtained in accordance with ISO/IEC Guide 98-3 (see also References [1, 2]) by evaluation of the measurement data and other information by means of a model (of the evaluation), which mathematically connects all the quantities involved (see 5.2). In general, the fact that the measurand is non-negative is not explicitly taken into account in the evaluation. Therefore, $y$ may be negative, especially when the measurand nearly assumes the true value $\tilde{y} = 0$. The primary measurement result, $y$, differs from the best estimate, $\hat{y}$, of the measurand calculated in 6.5. With $\hat{y}$, the knowledge that the measurand is non-negative is taken into account. The standard uncertainty, $u(\hat{y})$, associated with $\hat{y}$ is smaller than $u(y)$.

## 5.2 Model

### 5.2.1 General model

In many cases, the measurand, $Y$, is a function of several input quantities, $X_i$, in the form of Equation (1):

$$Y = G(X_1, \ldots, X_m) \tag{1}$$

Equation (1) is the model of the evaluation. Substituting given estimates, $x_i$, of the input quantities, $X_i$, in the model function, $G$, Equation (1) yields the primary measurement result $y$ of the measurand as:

$$y = G(x_1, \ldots, x_m) \tag{2}$$

The standard uncertainty, $u(y)$, of the measurand associated with the primary measurement result, $y$, is calculated according to Equation (3), if the input quantities, $X_i$, are independently measured and standard uncertainties, $u(x_i)$, associated with the estimates, $x_i$, are given, from the relation:

$$u^2(y) = \sum_{i=1}^{m} \left( \frac{\partial G}{\partial X_i} \right)^2 u^2(x_i) \tag{3}$$

In Equation (3), the estimates, $x_i$, shall be substituted for the input quantities, $X_i$, in the partial derivatives of $G$. The determination of the estimates, $x_i$, and the associated standard uncertainties, $u(x_i)$, and also the numerical or experimental determination of the partial derivatives are in accordance with ISO/IEC Guide 98-3 or References [1, 2]. For a count rate, $X_i = \rho_i$, with the given counting result, $n_i$, recorded during the measurement of duration, $t_i$, the specifications $x_i = r_i = n_i / t_i$ and $u^2(x_i) = n_i / t_i^2 = r_i / t_i$ apply (see also F.1). If the input quantities are not independently measured and for more complicated measurement evaluations such as unfolding, see C.5.2.

In 5.2.2, the input quantity, $X_1$, for instance the gross count rate, is taken as that quantity whose value, $x_1$, is not given when a true value, $\tilde{y}$, of the measurand, $Y$, is specified within the framework of the calculation of the decision threshold and the detection limit. Analogously, the input quantity, $X_2$, is assigned in a suitable way to the background effect. The data of the other input quantities are taken as given from independent previous investigations.

### 5.2.2 Model in ionizing radiation measurements

In this International Standard, the measurand, $Y$, with its true value, $\tilde{y}$, relates to a sample of radioactive material and is determined from counting the gross effect and the background effect with preselection of time or counts. In particular, $Y$ can be the net count rate, $\rho_n$, or the net activity, $A$, of the sample. The symbols belonging to the counting of the gross effect and of the background effect are marked in the following by the subscripts g and 0, respectively.


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

In this International Standard, the model is specified as follows:

$$Y = G(X_1, ..., X_m) = (X_1 - X_2 X_3 - X_4) \cdot \frac{X_6 X_8 \cdots}{X_5 X_7 \cdots} = (X_1 - X_2 X_3 - X_4) \cdot W \tag{4}$$

with

$$W = \frac{X_6 X_8 \cdots}{X_5 X_7 \cdots} \tag{5}$$

$X_1 = \rho_g$ is the gross count rate and $X_2 = \rho_0$ is the background count rate. The other input quantities, $X_i$, are calibration, correction or influence quantities, or conversion factors, for instance the emission or response probability or, in particular, $X_3$ is a shielding factor and $X_4$ an additional background correction quantity. If some of the input quantities are not involved, $x_i = 1$ $(i = 3; i > 4)$, $x_4 = 0$ and $u(x_i) = 0$ shall be set for them. For the count rates, $x_1 = r_g = n_g / t_g$ and $u^2(x_1) = n_g / t_g^2 = r_g / t_g$ as well as $x_2 = r_0 = n_0 / t_0$ and $u^2(x_2) = n_0 / t_0^2 = r_0 / t_0$ apply.

By substituting the estimates, $x_i$, in Equation (4), the primary estimate, $y$, of the measurand, $Y$, gives the result:

$$y = G(x_1, ..., x_m) = (x_1 - x_2 x_3 - x_4) \cdot w = (r_g - r_0 x_3 - x_4) \cdot w = \left( \frac{n_g}{t_g} - \frac{n_0}{t_0} x_3 - x_4 \right) \cdot w \tag{6}$$

with

$$w = \frac{x_6 x_8 \cdots}{x_5 x_7 \cdots} \tag{7}$$

With the partial derivatives:

$$\frac{\partial G}{\partial X_1} = W\,;\ \frac{\partial G}{\partial X_2} = -X_3 W\,;\ \frac{\partial G}{\partial X_3} = -X_2 W\,;\ \frac{\partial G}{\partial X_4} = -W\,;\ \frac{\partial G}{\partial X_i} = \pm \frac{Y}{X_i} \quad (i \geqslant 5) \tag{8}$$

and by substituting the estimates $x_i$, $w$ and $y$, Equation (3) yields the standard uncertainty $u(y)$ of the measurand associated with $y$:

$$\begin{aligned} u(y) &= \sqrt{w^2 \cdot \left[ u^2(x_1) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3) + u^2(x_4) \right] + y^2 u_{\text{rel}}^2(w)} \\ &= \sqrt{w^2 \cdot \left[ r_g / t_g + x_3^2 r_0 / t_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + y^2 u_{\text{rel}}^2(w)} \end{aligned} \tag{9}$$

where

$$u_{\text{rel}}^2(w) = \sum_{i=5}^{m} \frac{u^2(x_i)}{x_i^2} \tag{10}$$

is the sum of the squared relative standard uncertainties of the quantities $X_5$ to $X_m$. For $m < 5$, the values $w = 1$ and $u_{\text{rel}}^2(w) = 0$ apply.

The estimate $x_i$ and the standard uncertainty $u(x_i)$ of $X_i$ $(i = 3, ..., m)$ are taken as known from previous investigations or as values of experience according to other information. In the previous investigations, $x_i$ can be determined as an arithmetic mean value and $u^2(x_i)$ as an empirical variance (see B.4.1). If necessary,

© ISO 2010 – All rights reserved

Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

$u^2(x_i)$ can also be calculated as the variance of a rectangular distribution over the region of the possible values of $X_i$ with the width $\Delta x_i$. This yields $u^2(x_i) = (\Delta x_i)^2/12$.

For the application of the procedure to particular measurements, including spectrometric measurements, see Annexes B and C.

## 5.3 Calculation of the standard uncertainty as a function of the measurand

### 5.3.1 General aspects

For the provision and numerical calculation of the decision threshold in 6.2 and of the detection limit in 6.3, the standard uncertainty of the measurand is needed as a function $\tilde{u}(\tilde{y})$ of the true value $\tilde{y} \geqslant 0$ of the measurand. This function shall be determined in a way similar to $u(y)$ within the framework of the evaluation of the measurements by application of ISO/IEC Guide 98-3; see also References [1, 2]. In most cases, $\tilde{u}(\tilde{y})$ shall be formed as a positive square root of a variance function $\tilde{u}^2(\tilde{y})$ calculated first. This function shall be defined, unique and continuous for all $\tilde{y} \geqslant 0$ and shall not assume negative values.

In most cases, $\tilde{u}(\tilde{y})$ can be explicitly specified, provided that $u(x_1)$ is given as a function $h_1(x_1)$ of $x_1$. In such cases, $y$ shall be formally replaced by $\tilde{y}$ and Equation (2) shall be solved for $x_1$. With a specified $\tilde{y}$, the value $x_1$ can also be calculated numerically from Equation (2); for instance, by means of an iteration procedure, which results in $x_1$ as a function of $\tilde{y}$ and $x_2$, ..., $x_m$. This function shall replace $x_1$ in Equation (3) and in $u(x_1) = h_1(x_1)$, which finally yields $\tilde{u}(\tilde{y})$ instead of $u(y)$. In the case of the model according to Equation (6) and 5.3.2, one shall proceed in this way. Otherwise, 5.3.3 shall be applied, where $\tilde{u}(\tilde{y})$ follows as an approximation by interpolation from the data $y_j$ and $u(y_j)$ of several measurements.

### 5.3.2 Explicit calculation

When, in the case of the model according to Equation (6), the standard uncertainty, $u(x_1)$, of the gross count rate $X_1 = \rho_g$, is given as a function $h_1(x_1)$ of the estimate, $x_1 = r_g$, either $h_1(x_1) = \sqrt{x_1/t_g}$ or $h_1(x_1) = x_1/\sqrt{n_g}$ applies if the measurement duration, $t_g$ (time preselection), or, respectively, the number, $n_g$, of recorded pulses (preselection of counts) is specified.

The value $y$ shall be formally replaced by $\tilde{y}$. This allows the elimination of $x_1$ in the general case and, in particular, of $n_g$ with time preselection and of $t_g$ with preselection of counts in Equation (9) by means of Equation (6). These values are not available when $\tilde{y}$ is specified. This yields in the general case according to Equation (6)

$$x_1 = \tilde{y}/w + x_2 x_3 + x_4 \tag{11}$$

By substituting $x_1$ according to Equation (11) in the given function $h_1(x_1)$, i.e. with $u^2(x_1) = h_1^2(\tilde{y}/w + x_2 x_3 + x_4)$, the following results from Equation (9):

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[ h_1^2(\tilde{y}/w + x_2 x_3 + x_4) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{\mathrm{rel}}^2(w)} \tag{12}$$

With time preselection and because of $x_1 = n_g/t_g$ and $x_2 = r_0$,

$$n_g = t_g \cdot (\tilde{y}/w + r_0 x_3 + x_4) \tag{13}$$


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

is obtained from Equation (11). Then, with $h_1^2(x_1) = x_1/t_g = n_g/t_g^2$ and by substituting $n_g$ according to Equation (13) and with $u^2(x_2) = r_0/t_0$, Equation (12) leads to

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[ (\tilde{y}/w + r_0 x_3 + x_4)/t_g + x_3^2 r_0/t_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{\text{rel}}^2(w)} \tag{14}$$

With preselection of counts,

$$t_g = \frac{n_g}{\tilde{y}/w + r_0 x_3 + x_4} \tag{15}$$

is analogously obtained. Then, with $h_1^2(x_1) = x_1/t_g = n_g/t_g^2$ and by substituting $t_g$ according to Equation (15) and with $u^2(x_2) = r_0^2/n_0$, Equation (12) leads to

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[ (\tilde{y}/w + r_0 x_3 + x_4)^2/n_g + x_3^2 r_0^2/n_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{\text{rel}}^2(w)} \tag{16}$$

Equation (22) has a solution, which is the detection limit, $y^{\#}$, if, with time preselection, the following condition is satisfied:

$$k_{1-\beta} u_{\text{rel}}(w) < 1 \tag{17}$$

or with preselection of counts, the following condition is satisfied:

$$k_{1-\beta} \cdot \sqrt{\frac{1}{n_g} + u_{\text{rel}}^2(w)} < 1 \tag{18}$$

Otherwise, it can happen that a detection limit does not exist because of too great an uncertainty of the quantities $X_5$ to $X_m$, summarily expressed by $u_{\text{rel}}(w)$. The condition according to Equation (17) also applies in the case of Equation (12) if $h_1(x_1)$ increases for growing $x_1$ more slowly than $x_1$, i.e. if $h_1(x_1)/x_1 \to 0$ for $x_1 \to \infty$.

### 5.3.3 Approximations

It is often sufficient to use the following approximations for the function $\tilde{u}(\tilde{y})$, in particular, if the standard uncertainty, $u(x_1)$, is not known as a function $h_1(x_1)$. A prerequisite is that measurement result, $y_j$, and associated standard uncertainties, $u(y_j)$, calculated according to 5.1 and 5.2 from previous measurements of the same kind, are already available $(j = 0,1,2,...)$. The measurements shall be carried out on different samples with differing activities, but in other respects as far as possible under similar conditions. One of the measurements can be a background effect measurement or a blank measurement with $\tilde{y} = 0$ and, for instance, $j = 0$. Then, $y_0 = 0$ shall be set and $\tilde{u}(0) = u(y_0)$. The measurement currently carried out can be taken as a further measurement with $j = 1$.

The function $\tilde{u}(\tilde{y})$ often shows a rather slow increase. Therefore, the approximation $\tilde{u}(\tilde{y}) = u(y_1)$ is sufficient in some of these cases, especially if the primary measurement result, $y_1$, of the measurand is not much larger than the associated standard uncertainty $u(y_1)$.

If only $\tilde{u}(0) = u(y_0)$, $y_1 > 0$ and $u(y_1)$ are known, the following linear interpolation often suffices:

$$\tilde{u}^2(\tilde{y}) = \tilde{u}^2(0)(1 - \tilde{y}/y_1) + u^2(y_1)\tilde{y}/y_1 \tag{19}$$

© ISO 2010 – All rights reserved

Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

If the results $y_0$, $y_1$ and $y_2$ as well as the associated standard uncertainties $u(y_0)$, $u(y_1)$, and $u(y_2)$ from three measurements are available, the following bilinear interpolation can be used:

$$\tilde{u}^2(\tilde{y}) = u^2(y_0)\cdot\frac{(\tilde{y}-y_1)(\tilde{y}-y_2)}{(y_0-y_1)(y_0-y_2)} + u^2(y_1)\cdot\frac{(\tilde{y}-y_0)(\tilde{y}-y_2)}{(y_1-y_0)(y_1-y_2)} + u^2(y_2)\cdot\frac{(\tilde{y}-y_0)(\tilde{y}-y_1)}{(y_2-y_0)(y_2-y_1)} \quad (20)$$

If results from many similar measurements are given, the parabolic shape of the function $\tilde{u}^2(\tilde{y})$ can also be determined by an adjustment calculation.

## 6 Characteristic limits and assessments

### 6.1 Specifications

The probability, $\alpha$, of the error of the first kind, the probability, $\beta$, of the error of the second kind and the probability, $1-\gamma$, for the confidence interval shall be specified. The choice depends on the application. A frequently cited choice is $\alpha = \beta$ and the value 0,05 for $\alpha$ and $\beta$. Then, $k_{1-\alpha} = k_{1-\beta} = 1{,}65$. If the value of 0,05 is chosen for $\gamma$, then $k_{1-\gamma/2} = 1{,}96$ (see Annex E).

If it is to be assessed whether or not a measurement procedure for the measurand satisfies the requirements to be fulfilled for scientific, legal or other reasons (see 6.6), a guideline value, $y_r$, as a value of the measurand, for instance, an activity, shall also be specified.

### 6.2 Decision threshold

The decision threshold, $y^*$, of the non-negative measurand according to 5.1, quantifying the physical effect of interest, is the value of the estimator, $Y$, which allows the conclusion that the physical effect is present, if the primary measurement result, $y$, exceeds the decision threshold, $y^*$. If the result, $y$, is below the decision threshold, $y^*$, the result cannot be attributed to the physical effect, nevertheless it cannot be concluded that it is absent. If the physical effect is really absent, the probability of taking the wrong decision, that the effect is present, is equal to the specified probability, $\alpha$ (error of the first kind; see 6.1 and 6.5).

A determined primary measurement result, $y$, for the non-negative measurand is only significant for the true value of the measurand to differ from zero ($\tilde{y} > 0$), if it is larger than the decision threshold

$$y^* = k_{1-\alpha}\tilde{u}(0) \quad (21)$$

### 6.3 Detection limit

The detection limit, $y^\#$, is the smallest true value of the measurand, for which, by applying the decision rule according to 6.2, the probability of the wrong assumption that the physical effect is absent (error of the second kind) does not exceed the specified probability, $\beta$ (see 6.1).

In order to find out whether a measurement procedure is suitable for the measurement purpose, the detection limit, $y^\#$, is compared with the specified guideline value, $y_r$, of the measurand (see 6.1 and 6.6). The detection limit, $y^\#$, is the smallest true value of the measurand which can be detected with the measurement procedure used. It is high enough compared to the decision threshold, $y^*$, that the probability of the error of the second kind does not exceed $\beta$. The detection limit, $y^\#$, is obtained as the smallest solution of Equation (22):

$$y^\# = y^* + k_{1-\beta}\tilde{u}(y^\#) \quad (22)$$

where $y^\# \geqslant y^*$ always applies.


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

Equation (22) is an implicit equation; its right-hand side also depends on $y^{\#}$. The detection limit can be calculated by solving Equation (22) for $y^{\#}$ or, more simply, by iteration (see Figure 1). Thus, an improved approximation $\tilde{y}_{i+1}$ is obtained by repeatedly substituting an approximation $\tilde{y}_i$ for $y^{\#}$ on the right-hand side of Equation (22). This leads to Equation (23):

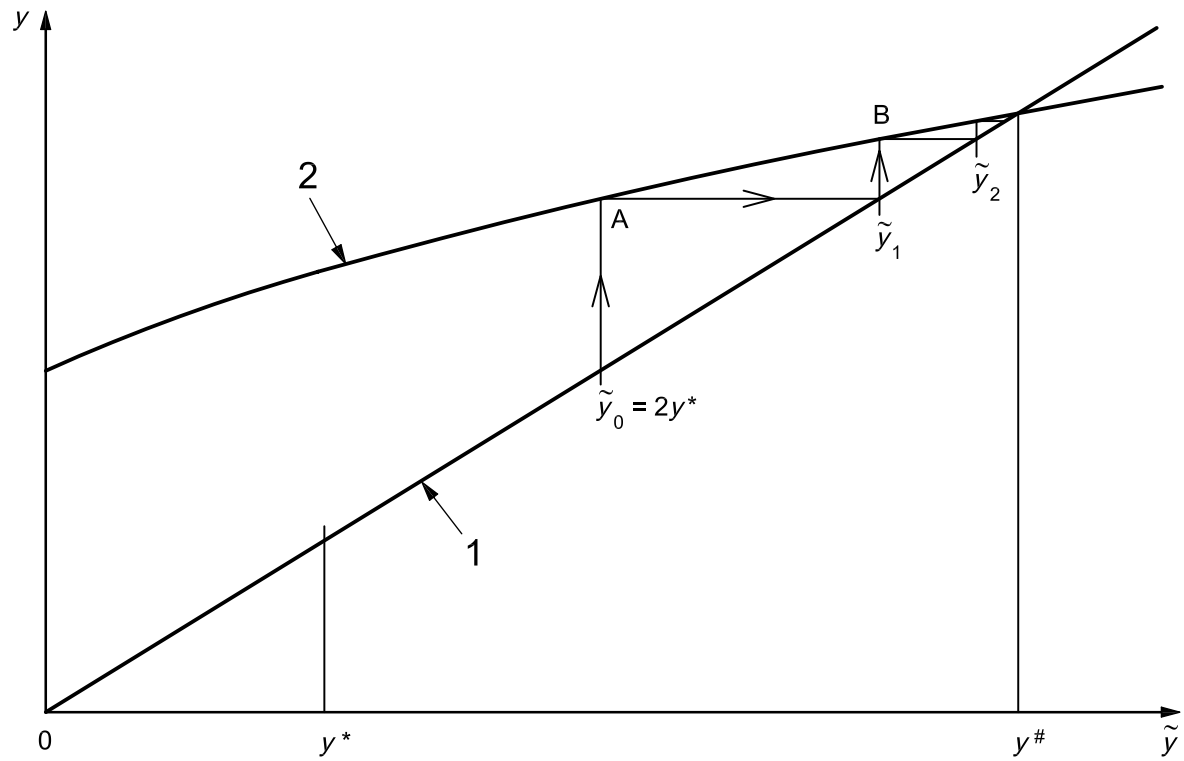
$$\tilde{y}_{i+1} = y^* + k_{1-\beta}\tilde{u}(\tilde{y}_i) \tag{23}$$

As a starting approximation, for instance, $\tilde{y}_0 = 2y^*$ can be chosen. The iteration converges in most cases, but not if Equation (22) does not have a solution, $y^{\#}$. In the latter case or if $y^{\#} < y^*$ results, a detection limit cannot be established for this measurement procedure (see 5.3.2 and 6.6). In this situation enlarging $\beta$ can result in a solution to Equation (22).

After the calculation of $\tilde{y}_1$ or, for instance, with a suitable choice of e.g. $\tilde{y}_1 = 3y^*$, it is more advantageous for $i \geqslant 1$ to apply the *regula falsi*, which in general converges more rapidly. For this purpose, Equation (23) shall be replaced by

$$\tilde{y}_{i+1} = \frac{y^* + k_{1-\beta}\cdot\left[\tilde{y}_i\tilde{u}(\tilde{y}_j) - \tilde{y}_j\tilde{u}(\tilde{y}_i)\right]\Big/(\tilde{y}_i - \tilde{y}_j)}{1 - k_{1-\beta}\cdot\left[\tilde{u}(\tilde{y}_i) - \tilde{u}(\tilde{y}_j)\right]\Big/(\tilde{y}_i - \tilde{y}_j)} \tag{24}$$

with $j < i$. Therefore, $j = 0$ should be set or $j$ be fixed after several iteration steps (see Figure 1).




**Key**

1 straight line $y = \tilde{y}$

2 curve $y = y^* + k_{1-\beta}\,\tilde{u}(\tilde{y})$

$\tilde{y}$ true value of the measurand

$y$ estimate of the measurand

For the other symbols, see text.

**Figure 1 — Calculation of the detection limit by iteration**

© ISO 2010 – All rights reserved

Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

With the iteration according to Equations (23) or (24) and beginning with a starting approximation, $\tilde{y}_0$, for instance, $\tilde{y}_0 = 2y^*$ as shown, the sequences of the improved approximations $\tilde{y}_i$ $(j = 1, 2, ...)$ converge to the detection limit $y^{\#}$, which is the abscissa of the intersection point of the straight line (key item 1 in Figure 1) and curve (key item 2 in Figure 1). $y^*$ is the decision threshold. With the alternative application of the *regula falsi* according to Equation (24), the sequence, $\tilde{y}_i$, is generated by means of secants of curve 2, for instance, through points A and B. The shown hyperbolic shape of the curve (key item 2 in Figure 1) is typical of many applications, for instance, those with Equations (14) or (16). The detection limit does not exist if the curve does not intersect the straight line (key item 1 in Figure 1) at any abscissa $\tilde{y} \geqslant y^*$.

Any iteration shall be stopped if a specified accuracy of $\nu$ digits is attained, i.e. if the first digits, $\nu$, of the successive approximations no longer change. But, if too high an accuracy is demanded, then, even with an iteration converging in principle, the successive approximations in general permanently fluctuate around and close to the exact solution but never attain it. A smaller $\nu$ shall then be chosen.

With the approximation $\tilde{u}(\tilde{y}) = u(y)$ (see 5.3.3), $y^{\#} = (k_{1-\alpha} + k_{1-\beta})u(y)$ applies.

The linear interpolation according to Equation (19) leads to the approximation:

$$y^{\#} = a + \sqrt{a^2 + (k_{1-\beta}^2 - k_{1-\alpha}^2)\,\tilde{u}^2(0)} \tag{25}$$

with

$$a = k_{1-\alpha}\,\tilde{u}(0) + \frac{1}{2}\left\{\left(k_{1-\beta}^2 / y_1\right)\left[(u^2(y_1) - \tilde{u}^2(0)\right]\right\} \tag{26}$$

If $\alpha = \beta$, $y^{\#} = 2a$ follows.

If $\alpha = \beta$ is chosen and $\tilde{u}^2(\tilde{y})$ is obtained or approximated by a second-order polynomial $\tilde{u}^2(\tilde{y}) = c_0 + c_1\tilde{y} + c_2\tilde{y}^2$ as in Equations (14), (16) and (20), then with $k = k_{1-\alpha} = k_{1-\beta}$

$$y^* = k\sqrt{c_0} \tag{27}$$

and

$$y^{\#} = \frac{2y^* + k^2 c_1}{1 - k^2 c_2} \tag{28}$$

## 6.4 Limits of the confidence interval

The limits of the confidence interval are provided for a physical effect, recognized as present according to 6.2, in such a way that the confidence interval contains the true value of the measurand with the specified probability $1-\gamma$ (see 6.1). The limits of the confidence interval take into account the fact that the measurand is non-negative.

With a primary measurement result, $y$, of the measurand and the standard uncertainty, $u(y)$, associated with $y$ (see 5.2), the lower limit of the confidence interval, $y^{\triangleleft}$, and the upper limit of the confidence interval, $y^{\triangleright}$, are provided by:

$$y^{\triangleleft} = y - k_p\,u(y) \text{ with } p = \omega \cdot (1 - \gamma/2) \tag{29}$$

$$y^{\triangleright} = y + k_q\,u(y) \text{ with } q = 1 - \omega\gamma/2 \tag{30}$$


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010

where

$$\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y/u(y)} \exp(-\frac{v^2}{2}) \mathrm{d}v = \Phi\left[y/u(y)\right] \tag{31}$$

For the distribution function, $\Phi(t)$, of the standardized normal distribution and for its inversion, $k_p = t$ for $\Phi(t) = p$, see Table E.1. For methods for its calculation, see Annex E or, for instance, Reference [29].

In general, the limits of the confidence interval are located neither symmetrical to $y$, nor to the best estimate, $\hat{y}$ (see 6.5 and Figure 2), but the probabilities of the measurand being smaller than $y^{\triangleleft}$ or larger than $y^{\triangleright}$ both equal $\gamma/2$. The relations $0 < y^{\triangleleft} < y^{\triangleright}$ apply.

$\omega = 1$ may be set if $y \geqslant 4u(y)$. In this case, the following approximations symmetrical to $y$ apply:

$$y^{\triangleleft} = y - k_{1-\gamma/2}\, u(y) \text{ and } y^{\triangleright} = y + k_{1-\gamma/2}\, u(y) \tag{32}$$

## 6.5 Assessment of a measurement result

The determined primary measurement result, $y$, of the measurand shall be compared with the decision threshold, $y^*$. If $y > y^*$, the physical effect quantified by the measurand is recognized as present. Otherwise, it is decided that the effect is absent.

If $y \geqslant y^*$ and with $\omega$ according to Equation (31), the best estimate $\hat{y}$ of the measurand is given by (see 5.1 and Figure 2):

$$\hat{y} = y + \frac{u(y)\exp\left\{-y^2 \Big/ \left[2u^2(y)\right]\right\}}{\omega\sqrt{2\pi}} \tag{33}$$

The standard uncertainty associated with $\hat{y}$ reads

$$u(\hat{y}) = \sqrt{u^2(y) - (\hat{y} - y)\hat{y}} \tag{34}$$

NOTE 1 If the best estimate, $\hat{y}$, and its standard uncertainty, $u(\hat{y})$, are calculated, the recording of the primary measurement result, $y$, and its standard uncertainty, $u(y)$, may be omitted.

NOTE 2 If the decision rule defined by the decision threshold is not used and if $y < y^*$, the best estimate, $\hat{y}$, and its standard uncertainty, $u(\hat{y})$, can also be calculated.

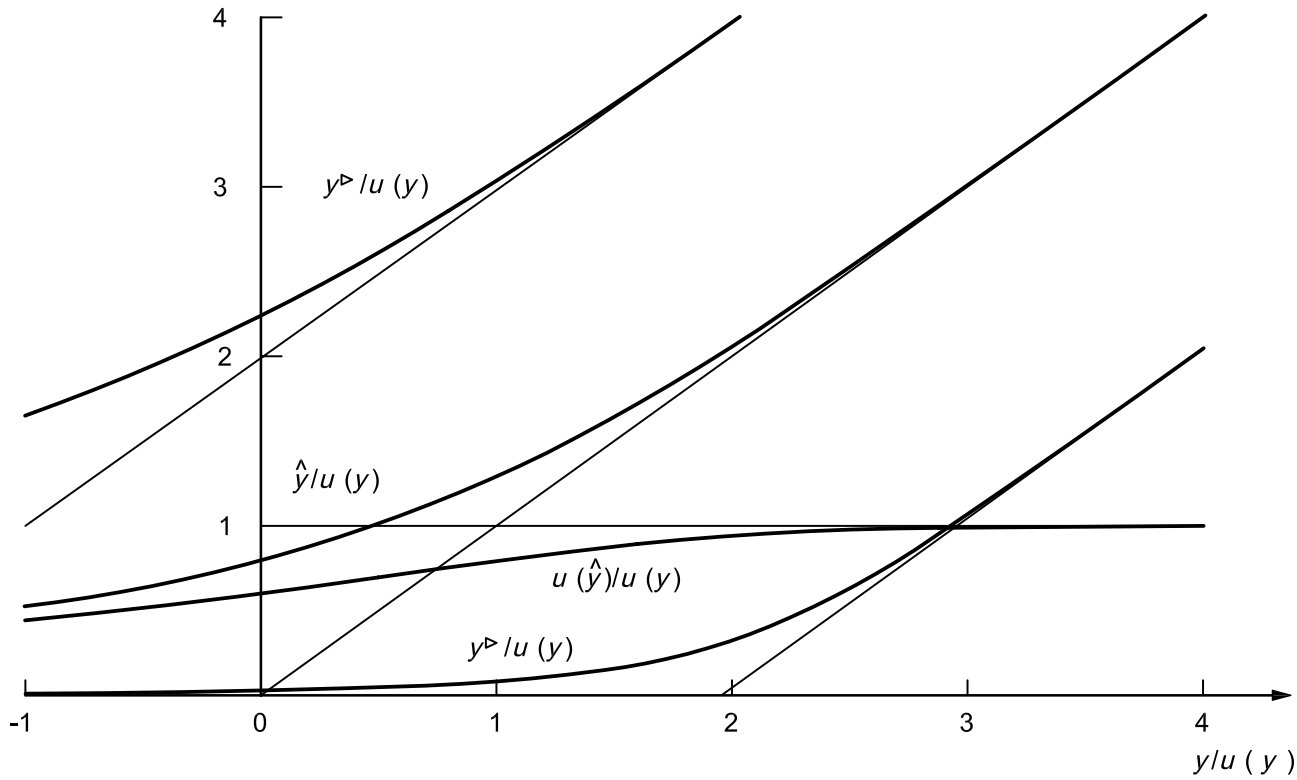
The relations $y < \hat{y}$, $0 < \hat{y}$ and $y^{\triangleleft} < \hat{y} < y^{\triangleright}$ as well as $u(\hat{y}) < u(y)$ and $u(\hat{y}) < \hat{y}$ apply; moreover, for $y \geqslant 4u(y)$, the approximations

$$\hat{y} = y \;;\; u(\hat{y}) = u(y) \tag{35}$$

hold true.

© ISO 2010 – All rights reserved

Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010




**Key**

$y/u(y)$ quotient of the primary measurement result of the measurand and the standard uncertainty of the measurand associated with the best estimate

For the curves, see the text.

**Figure 2 — Best estimate and limits of the confidence interval**

The best estimate, $\hat{y}$, of the measurand, the associated standard uncertainty, $u(\hat{y})$, the lower limit, $y^{\triangleleft}$, and the upper limit, $y^{\triangleright}$, of the confidence interval are given as functions of the primary measurement result, $y$. All these values are scaled with the standard uncertainty, $u(y)$, and $\gamma = 0{,}05$ is chosen. The ascending straight lines and the horizontal straight line with ordinate 1 are asymptotes.

## 6.6 Assessment of a measurement procedure

The decision on whether or not a measurement procedure to be applied sufficiently satisfies the requirements regarding the detection of the physical effect quantified by the measurand is made by comparing the detection limit, $y^{\#}$, with the specified guideline value, $y_{\mathrm{r}}$. If $y^{\#} > y_{\mathrm{r}}$ or if Equation (22) has no solution, $y^{\#}$, the measurement procedure is not suitable for the intended measurement purpose with respect to the requirements.

To improve the situation in the case of $y^{\#} > y_{\mathrm{r}}$, it can often be sufficient to choose longer measurement durations or to preselect more counts of the measurement procedure. This reduces the detection limit.


© ISO 2010 – All rights reserved
Documents
Document Sharing Hub
Full document details: BS ISO 11929:2010