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Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application

Part 4: Guidelines to applications

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Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 4: Guidelines to applications

Détermination des limites caractéristiques (seuil de décision, limite de détection et limites de l'intervalle élargi) pour le mesurage des rayonnements ionisants — Principes fondamentaux et applications —

Partie 4: Lignes directrices relatives aux applications

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 85, *Nuclear energy, nuclear technologies, and radiological protection*, Subcommittee SC 2, *Radiological protection*.

This second edition of ISO 11929-4 together with ISO 11929-1:2019, ISO 11929-2:2019 and ISO 11929-3:2019, cancels and replaces ISO 11929:2010, which has been technically revised, specifically with reference to the type of statistical treatment of the data and extended with respect to the methodology of uncertainty assessment from the ISO/IEC Guide 98-3 and the ISO/IEC Guide 98-3:2008/Suppl.1.

A list of all parts of ISO 11929 can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

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Introduction

Measurement uncertainties and characteristic values, i.e. characteristic limits such as the decision threshold, the detection limit and limits of the coverage interval for measurements as well as the best estimate and its associated standard measurement uncertainty, are of importance in metrology, in general, and for radiological protection, in particular. The quantification of the uncertainty associated with a measurement result provides a basis for the trust an individual can have in a measurement result.

NOTE 1 Conformity with regulatory limits, constraints or reference values can only be demonstrated taking into account and quantifying all sources of uncertainty. Characteristic limits provide – in the end – the basis for deciding accepting results under uncertainty.

ISO 11929 (all parts) provides characteristic values of a non-negative measurand of ionizing radiation. It is applicable for a wide range of measuring methods extending beyond measurements of ionizing radiation.

The limits to be provided according to ISO 11929 (all parts) for specified probabilities of wrong decisions allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

— the "decision threshold" allows a decision to be made on whether or not the physical effect quantified by the measurand is present;

— the "detection limit" indicates the smallest true quantity value of the measurand that can still be detected with the applied measurement procedure; this gives and allows for a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose;

— the "limits of the coverage interval" enclose, in the case of the physical effect recognized as present, a coverage interval containing the true quantity value of the measurand with a specified probability.

Hereinafter, the limits mentioned are jointly called "characteristic limits".

NOTE 2 According to ISO/IEC Guide 99 updated by JCGM 200:2012, the term "coverage interval" is used here instead of "confidence interval" in order to distinguish the wording of Bayesian terminology from that of conventional statistics.

All the characteristic values are based on Bayesian statistics and on the ISO/IEC Guide 98-3 as well as on the ISO/IEC Guide 98-3:2008/Suppl.1 and ISO/IEC Guide 98-3:2008/Suppl.2. As explained in detail in ISO 11929-2, the characteristic values are mathematically defined by means of moments and quantiles of probability distributions of the possible measurand values.

Since measurement uncertainty plays an important role in all parts of ISO 11929, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to the ISO/IEC Guide 98-3 and to the ISO/IEC Guide 98-3:2008/Suppl.1; see also References [21 to 25]. This enables the strict separation of the evaluation of the measurements, on the one hand, and the provision and calculation of the characteristic values, on the other hand. ISO 11929 (all parts) makes use of a theory of uncertainty in measurement[26 to 28] based on Bayesian statistics (e.g. References [29 to 36]) in order to allow taking into account also those uncertainties that cannot be derived from repeated or counting measurements. The latter uncertainties cannot be handled by frequentist statistics.

Because of developments in metrology concerning measurement uncertainty, laid down in the ISO/IEC Guide 98-3, ISO 11929:2010 was drawn up on the basis of ISO/IEC Guide 98-3, but using Bayesian statistics and the Bayesian theory of measurement uncertainty. This theory provides a Bayesian foundation for the ISO/IEC Guide 98-3. Moreover, ISO 11929:2010 was based on the definitions of the characteristic values[21], the standard proposal[22], and the introducing article[23]. It unified and replaced all earlier parts of ISO 11929 and was applicable not only to a large variety of particular measurements of ionizing radiation but also, in analogy, to other measurement procedures. Some

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explanatory material about the basics of ISO 11929 (all parts), in general, and its application in has been published elsewhere[42][43].

Since the ISO/IEC Guide 98-3:2008/Suppl.1 has been published, the Monte Carlo method has been used to deal comprehensively with a more general treatment of measurement uncertainty in complex measurement evaluations. This development provided an incentive for writing a corresponding Monte Carlo supplement[24] to ISO 11929:2010. The revised ISO 11929 (all parts) is also essentially founded on Bayesian statistics and can serve as a bridge between documents ISO 11929:2010 and the ISO/IEC Guide 98-3:2008/Suppl.1. Moreover, more general definitions of the characteristic values (ISO 11929-2) and the Monte Carlo computation of the characteristic values make it possible to go a step beyond the present state of standardization laid down in ISO 11929:2010 since probability distributions rather than uncertainties can be propagated. It is thus more comprehensive and extending the range of applications.

The revised ISO 11929 (all parts), moreover, is more explicit on the calculation of the characteristic values. Reference [25] gives a survey on the basis of the revision. Further, in ISO 11929-3, it gives detailed advice how to calculate characteristic values in the case of multivariate measurements using unfolding methods. For such measurements, the ISO/IEC Guide 98-3:2008/Suppl.2 provides the basis of the uncertainty evaluation.

Formulae are provided for the calculation of the characteristic values of an ionizing radiation measurand via the “standard measurement uncertainty” of the measurand (hereinafter “standard uncertainty”) derived according to the ISO/IEC Guide 98-3 as well as via probability density functions (PDFs) of the measurand derived on the basis of the ISO/IEC Guide 98-3:2008/Suppl.1. The standard uncertainties or probability density functions take into account the uncertainties of the actual measurement as well as those of sample treatment, calibration of the measuring system and other influences. The latter uncertainties are assumed to be known from previous investigations.

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Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 4: Guidelines to applications

1 Scope

This document specifies a procedure, in the field of ionizing radiation metrology, for the calculation of the "decision threshold", the "detection limit" and the "limits of the coverage interval" for a non-negative ionizing radiation measurand when counting measurements with preselection of time or counts are carried out. The measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment and by other factors.

ISO 11929 has been divided into four parts covering elementary applications in ISO 11929-1, advanced applications on the basis of the ISO/IEC Guide 98-3:2008/Suppl.1 in ISO 11929-2, applications to unfolding methods in ISO 11929-3, and guidance to the application in ISO 11929-4.

ISO 11929-1 covers basic applications of counting measurements frequently used in the field of ionizing radiation metrology. It is restricted to applications for which the uncertainties can be evaluated on the basis of the ISO/IEC Guide 98-3 (JCGM 2008). In ISO 11929-1:2019, Annex A the special case of repeated counting measurements with random influences and in ISO 11929-1:2019, Annex B, measurements with linear analogous ratemeters are covered.

ISO 11929-2 extends ISO 11929-1 to the evaluation of measurement uncertainties according to the ISO/IEC Guide 98-3:2008/Suppl.1. ISO 11929-2 also presents some explanatory notes regarding general aspects of counting measurements and Bayesian statistics in measurements.

ISO 11929-3 deals with the evaluation of measurements using unfolding methods and counting spectrometric multi-channel measurements if evaluated by unfolding methods, in particular, alpha- and gamma-spectrometric measurements. Further, it provides some advice how to deal with correlations and covariances.

ISO 11929-4 gives guidance to the application of ISO 11929 (all parts), summarizing shortly the general procedure and then presenting a wide range of numerical examples. The examples cover elementary applications according to ISO 11929-1 and ISO 11929-2.

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The ISO 11929 (all parts) also applies analogously to other measurements of any kind if a similar model of the evaluation is involved. Further practical examples can be found in other International Standards, for example, see References [1 to 20].

NOTE A code system, named UncertRadio, is available allowing for calculations according to ISO 11929-1 to ISO 11929-3. UncertRadio[40][41] which can be downloaded for free from https://www.thuenen.de/en/fi/fields-of-activity/marine-environment/coordination-centre-of-radioactivity/uncertradio/. The download contains a setup installation file that copies all files and folders into a folder specified by the user. After installation one has to add information to the PATH of Windows as indicated by a pop-up window during installation. English language can be chosen and extensive "help" information is available. Another tool is the package 'metRology'[44] which is available for programming in R. It contains the two R functions 'uncert' and 'uncertMC' which perform the GUM-conform uncertainty propagation, either analytically or by the Monte Carlo method, respectively. Covariances/correlations of input quantities are included. Applying these two functions within iterations for decision threshold and the detection limit calculations simplifies the programming effort significantly. It is also possible to implement this document in a spreadsheet containing a Monte Carlo add-in or into other commercial mathematics software.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document including any amendments applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 11929-1, *Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application — Part 1: Elementary applications*

ISO 11929-2, *Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application — Part 2: Advanced applications*

ISO 80000-1, *Quantities and units — Part 1: General*

ISO 80000-10, *Quantities and units — Part 10: Atomic and nuclear physics*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 1: Guide to the expression of uncertainty in measurement, JCGM 100:2008*

ISO/IEC Guide 98-3:2008/Suppl.1, *Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — a Propagation of distributions using a Monte Carlo method, JCGM 101:2008*

ISO/IEC Guide 98-3:2008/Suppl.2, *Evaluation of measurement data — Supplement 2 to the "Guide to the expression of uncertainty in measurement" — Extension to any number of output quantities, JCGM 102:2011*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

IEC/TR 62461, *Radiation protection instrumentation — Determination of uncertainty in measurement, Ed. 2.0, IEC 23.1.2015*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 80000-1, ISO 80000-10, ISO/IEC Guide 98-3, ISO/IEC Guide 98-3:2008/Suppl.1, ISO/IEC Guide 98-3:2008/Suppl.1, ISO/IEC Guide 99, ISO 3534-1, ISO 11929-1, and ISO 11929-2 apply.

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ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform available at https://www.iso.org/obp

— IEC Electropedia available at http://www.electropedia.org/

4 Quantities and symbols

The quantities and symbols below are used throughout this document. Additional quantities and symbols are listed – if necessary – in the respective examples.

m	mass of the test sample
n_{M}	number of Monte Carlo trials performed
X_i	input quantity $(i=1,...,m)$
Y	measurand, quantity of interest
G	model of evaluation connecting the input quantities with the measurand: $Y=G(X_1,...,X_m)$
x_i	measured value of the input quantity X_i, estimate of the true value $\tilde{x}_i$ of X_i
$\tilde{x}_i$	possible or assumed true values of the input quantity X_i
$u(x_i)$	standard uncertainty of the input quantity X_i associated with the estimate x_i
Δx_i	width of the region of the possible values of the input quantity X_i
$u_{\mathrm{rel}}(w)$	relative standard uncertainty of a quantity W associated with the estimate w
$\tilde{y}$	possible or assumed true values of the measurand; if the physical effect of interest is not present, then $\tilde{y}=0$ otherwise, $\tilde{y}>0$
y	determined value of the measurand Y, estimate of the measurand, primary measurement result of the measurand
y_j	values y from different measurements $(j=0,1,2, ...)$
$u(y)$	standard uncertainty of the measurand associated with the primary measurement result y
$\tilde{u}(\tilde{y})$	standard uncertainty of an estimator of the measurand Y as a function of an assumed true value $\tilde{y}$ of the measurand
$\hat{y}$	best estimate of the measurand
$u(\hat{y})$	standard uncertainty of the measurand associated with the best estimate $\hat{y}$
y^*	decision threshold of the measurand
$y^{\#}$	detection limit of the measurand
y_{r}	guideline value of the measurand
$y^{\triangleleft}$, $y^{\triangleright}$	lower and upper limit of the symmetric coverage interval, respectively, of the measurand

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$y^{<}$, $y^{>}$	lower and upper limit of the shortest coverage interval, respectively, of the measurand
$f(\tilde{y}\|y)$	posterior probability density function (PDF) for a true value $\tilde{y}$ given the estimate y taking NOT into account the condition that the measurand Y is non-negative
$f(\tilde{y}\|y, Y \geq 0)$	posterior probability density function (PDF) for a true value $\tilde{y}$ given the estimate y taking into account the condition that the measurand Y is non-negative
$f(y\|\tilde{y}=0)$	predictive probability density function (PDF) to obtain a measured value y if a true value $\tilde{y}=0$ of the measurand Y is assumed
$f(y\|\tilde{y}=y^{\#})$	predictive probability density function (PDF) to obtain a measured value y if a true value $\tilde{y}=y^{\#}$ of the measurand Y equal to the detection limit $y^{\#}$ is assumed
$\mathrm{Ga}(\tilde{r};n,1/t)$	Gamma distribution as the probability density function (PDF) of the true value $\tilde{r}$ of a count rate R given n counts obtained during a counting time t; see ISO 11929-2:2019, Annex A, for details.
$\mathrm{N}(\tilde{x};x,u(x))$	Normal or Gaussian distribution as the probability density function (PDF) of the true value $\tilde{x}$ of a quantity X given an estimate x with its associated standard uncertainty $u(x)$
$\mathrm{R}(\tilde{x};x_{\mathrm{L}},x_{\mathrm{U}})$	Rectangular distribution as the probability density function (PDF) of the true value $\tilde{x}$ of a quantity X given the lower and upper limits x_{L} and x_{U}
$\mathrm{T}(\tilde{x};a,b)$	Triangular distribution as the probability density function (PDF) of the true value $\tilde{x}$ of a quantity X being the sum of two quantities, A and B being assigned rectangular probability distributions with the lower and upper limits a_{L} and b_{L} respectively a_{U} and b_{L} and with $a=a_{\mathrm{L}}+b_{\mathrm{L}}$ and $b=a_{\mathrm{U}}+b_{\mathrm{U}}$.
$\mathrm{H}(x)$	Heaviside step function: $\mathrm{H}(x)=\begin{cases}0 & \text{for } x<0\\ 1 & \text{for } x\geq 0\end{cases}$
r_{g}, r_0	estimate of the gross count rate and of the background count rate, respectively
r_{n}	estimate of the net count rate
n_i	number of counted pulses obtained from the measurement of the count rate R_i
n_{g}, n_0	number of counted pulses of the gross measurement and of the background measurement, respectively
t_i	measurement duration of the measurement of the count rate R_i
t_{g}, t_0	duration of the gross and the background measurement, respectively
r_i	estimate of the count rate R_i
A	activity
a	estimate of the activity A
w	calibration factor
a_{m}	activity per unit mass
a_{s}	activity per unit surface
α, β	probability of a false positive and false negative decision, respectively

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$1-\gamma$	probability for the coverage interval of the measurand
q_p	quantile of a distribution for the probability p
$\Phi(x)$	distribution function of the standard normal distribution
ω	$\omega=\Phi[y/u(y)]$, value of the distribution function of the standard normal distribution at $y/u(y)$

5 Summary of this document

5.1 Procedures according to ISO 11929 (all parts)

ISO 11929-1 standardizes the evaluation of measurements of ionizing radiation for a wide range of models of evaluation and the calculation of characteristic limits (decision threshold, detection limit and limits of coverage intervals) on the basis of the ISO/IEC Guide 98-3. However, important exceptions exist for which the procedures do not provide reliable results and other procedures need to be applied, such as those described in ISO/IEC Guide 98-3:2008/Suppl.1. Such procedures are dealt with in ISO 11929-2. Both the aspects of the procedure and the tools to ascertain whether ISO 11929-1 or ISO 11929-2 is suitable or not for the specific application are described in ISO 11929-1.

It is a characteristic of measurements of ionizing radiation that they have to be performed in the presence of a radiation background, which has to be subtracted from a gross measurement quantity. However, the procedures described in this document likewise are applicable to any measurements where a background or blank contribution has to be subtracted from a gross quantity.

5.2 Survey on the examples

This document gives numerical examples of elementary applications of ISO 11929-1 and ISO 11929-2. The data in the tables are often given with more digits than are meaningful, so that the calculations can also be reconsidered and verified with higher accuracy, in particular for testing computer programs under development.

NOTE 1 The Monte Carlo calculations were performed in this guide with 1 000 000 trials. As a consequence, the results for the characteristic values remain uncertain in the third significant digit. In addition, the user has to be aware that the necessary number of trials depends significantly on the model of evaluation and on the PDFs assigned to the input quantities. In practical applications, a number of 10 000 trials is often sufficient.

A criterion whether ISO 11929-1 or ISO 11929-2 is to be preferred in practice is given in IEC/TR 62461. It recommends “the results of both methods should be given in order to display their difference. When the 95 % coverage intervals of the Monte Carlo method and of the analytical method do not deviate by more than 10 %, then the analytical one may be used for the uncertainty determination in similar cases, i.e. a similar model function and similar or smaller values of the uncertainty of the input quantities”. See also ISO 11929-1:2019, 5.3.

The examples, the models, and the data are exemplary and not normative.

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In ISO 11929-1, a quite general model is specified as follows:

$$y=(x_1-x_2\cdot x_3-x_4)\cdot\frac{x_6\cdot x_8\cdots}{x_5\cdot x_7\cdots}=(x_1-x_2x_3-x_4)\cdot w \tag{1}$$

with

$$w=\frac{x_6\cdot x_8\cdots}{x_5\cdot x_7\cdots} \tag{2}$$

where $x_1=r_g$ is the gross count rate and $x_2=r_0$ is the background count rate. The other input quantities, x_i, are calibration, correction or influence quantities, or conversion factors, for instance the emission or response probability. In particular, x_3 is a shielding factor and x_4 an additional background correction quantity. If some of the input quantities are not involved, $x_i=1$ $(i=3;i>4)$, $x_4=0$ and $u(x_i)=0$ shall be set for them.

NOTE 2 Generally, physical quantities are denoted by uppercase letters and have to be carefully distinguished from their values, denoted by the corresponding lowercase letters. However, for sake of simplicity no distinction is made between uppercase letters for quantities and lowercase letters for values throughout this document. Only lowercase letters are used because the examples deal with values rather than with quantities.

In this document, the simplest form of Formula (1) used for most examples for ease of understanding is:

$$y=(x_1-x_2)\cdot w=(r_g-r_0)\cdot w \tag{3}$$

This simple model according to Formula (3) of a **counting measurement with small or moderate relative uncertainties** below 25 % such as in the majority of laboratory experiments is dealt with in **Clause 6**.

Small count numbers provide a particular problem in counting measurements since the information about the associated count rates is scarce. An example describing the measurement of alpha-particles with low count numbers according to ISO 11929-2:2019, A.1 is given in **Clause 7**.

Clause 8 gives an example of a counting measurement with **uncertain counting geometry** where the uncertainties of the calibration factor are generally large and where, in consequence, a dominating uncertainty exists for a quantity in the numerator of the calibration factor of Formula (2).

Clause 9 gives an example of a counting measurement in form of a **wipe test** where the uncertainties are generally large and where, in particular, a dominating uncertainty exists for a quantity in the denominator of the calibration factor of Formula (2).

Clause 10 is an example where **large uncertainties occur due to shielding of the background** as in the case of measurements with a portal monitor.

Clause 11 exemplifies also the **general model** according to Formula (1) by dealing with the clearance measurement of bulk samples of concrete with a special counting device and with consideration of natural radioactivity in the background.

Clause 12 exemplifies the **general model** according to Formula (1) by dealing with the γ-spectrometry of ^{235}U with an interference from ^{226}Ra.

Clause 13 deals with the case of measurements of ambient dose rates with a measuring device of unknown functionality and algorithm, i.e. a so-called **black box measurement** according to A.4 of ISO 11929-1 where no technical details about the measuring procedure are known and only readings of a device are available.

The particular cases of **counting measurements with unknown random influences** from the sample treatment according to ISO 11929-1:2019, A.2 are dealt with in **Clause 14**.

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The case of **counting measurements with known random influences** from the sample treatment according to of ISO 11929-1:2019, A.3 is dealt with in **Clause 15**.

The examples in Clause 16 and Clause 18 demonstrate the applicability of ISO 11929-1 and ISO 11929-2 to non-counting measurements. The case of an **active personal dosemeter** is treated according ISO 11929-1 and ISO 11929-2 in **Clause 16**. The particlular feature of this example is that the standard uncertainty associated with the gross quantity is constant, thereby allowing to calculate the standard uncertainty as a function of an assumed true value of the measurand .

The case of a **dose rate measurement using a neutron area monitor** treated according to ISO 11929-1 and ISO 11929-2 is dealt with in **Clause 17**. In this example it is known that the measurement is a counting measurement. However, the actual gross and background count numbers are unknown. In spite of that, the standard uncertainty as a function of an assumed true value of the measurand can be calculated using a general feature of the relative uncertainty of the gross indication.

A simple example of the calculation of the calibration factor w is given in Annex A.

The examples are generic and are not intended to standardize measurement procedures for the respective applications. They shall only serve as explanations for the application of ISO 11929-1 and ISO 11929-2.

Calculations are performed according to ISO 11929-1 and ISO 11929-2 for each example in order to allow the reader to judge the appropriateness of both methods. In all examples, the probabilistically symmetric coverage intervals as well as the shortest coverage intervals are calculated. The limits of the coverage intervals and the best estimate and its associated standard uncertainty are also calculated in cases where the primary measurement result is below the decision threshold, in spite of the fact that ISO 11929 (all parts) requires only to calculate the coverage intervals if the primary measurement result exceeds the decision threshold.

Small $y/u(y)$-values, i.e. large relative uncertainties of the primary measurement result, can be due to small count numbers (example in Clause 7) or to large relative uncertainties of the calibration factor. The latter ones can be due to relatively large uncertainty of a component in the numerator of the calibration factor (example in Clause 8) or due to such a component in the denominator of the calibration factor (example in Clause 9).

A general assessment regarding the relevance of the choice of the coverage interval in the case of small $y/u(y)$-values given the decision threshold is not possible. Therefore, all characteristic limits, i.e. the decision threshold, the detection limit and the limits of the probabilistically symmetric and the shortest coverage interval, are calculated in this document for each example in order to allow discussing and judging about the relevance of the different coverage intervals given the actual data.

In models of evaluation with $x_3 = 1$, $u(x_3) = 0$ and $x_4 = 0$, $u(x_4) = 0$, the decision threshold does not depend on the uncertainty of the calibration factor. The decision threshold depends on the uncertainties of x_3 and x_4 in case of $x_3 \neq 1$ or $x_4 > 0$. The example of a portal monitor is one of the cases where the decision threshold depends on the uncertainty of the shielding factor. Also, the example for the general model (gamma-spectrometry of ^{235}U with interference by ^{226}Ra) is such a case as well as the example regarding the clearance measurement with consideration of natural radioactivity.

From the didactic point of view one has to distinguish four cases that often lead to misunderstandings and, therefore, need clarification.

This case is characterized by $y^* < y$ and $y^\# < y_r$. A primary result above the decision threshold was obtained. One decides to conclude that an effect of the sample was recognized. The detection limit is lower than the guideline value. One concludes that the measurement method is suitable for the measurement purpose.

See Clauses 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, and 17 for examples.

A frequent misunderstanding results from comparison of the primary measurement result with the detection limit. Such a comparison is not according to the stipulations of ISO 11929 (all parts). The only

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characteristic limit to be compared with the primary measurement result is the decision threshold, not the detection limit. If the primary measurement result exceeds the decision threshold one decides to conclude that an effect of the sample was recognized.

This decision to be made does not depend on whether the primary measurement result is below or above the detection limit. The only purpose of the detection limit is to characterize the quality of the measurement procedure by comparison with a guideline value. If the detection limit is lower than the guideline value, one concludes that the measurement method is suitable for the measurement purpose.

This case is characterized by $y < y^*$ and $y^\# < y_r$. A result below the decision threshold was obtained. One decides to conclude that no effect of the sample was recognized, though it might be present. The detection limit is lower than the guideline value. One concludes that the measurement method is suitable for the measurement purpose.

See Clause 7 for an example.

NOTE 3 Also in case that the primary measurement result is smaller than the decision threshold all characteristic values are calculated and presented in the examples.

This case is characterized by $y^* < y$ and $y_r < y^\#$. A result above the decision threshold was obtained. One decides to conclude that an effect of the sample was recognized. However, the detection limit is higher than the guideline value. Consequently, the measurement method is not suitable for the measurement purpose in principle.

Nevertheless, $y > y^*$ is a primary measurement result with its associated standard uncertainty, which needs no justification regarding to the suitability of the procedure.

See Clause 14 for an example.

This case is characterized by $y < y^*$ and $y_r < y^\#$. A primary result below the decision threshold was obtained. One decides to conclude that no effect of the sample was recognized. The detection limit is higher than the guideline value. One concludes that the measurement method is not suitable for the measurement purpose and does not satisfy the measurement purpose. In total, the procedure is not suitable and the measurement was not successful.

There is no example of such a pathological case in this document.

NOTE 4 In this document, the guideline value, y_r, in comparison with the detection limit, $y^\#$, simply serves to demonstrate the decision whether or not the measuring procedure is suitable for the measurement purpose. This document does not provide stipulations whether or not a measurement result, y, conforms to requirement from legal or other sources. Such assessments are beyond the scope of this document. An application of ISO 11929-1 to such conformity assessments can be found elsewhere[45].

5.3 General stipulations

The measurand and the characteristic values are calculated according to ISO 11929-1 and ISO 11929-2. First, the model of evaluation is given; then the input data as well as their PDFs are specified. The evaluation of the measurement and the calculation of the characteristic limits according ISO 11929-1 are presented in detail. The standard uncertainties are calculated according to the mathematical formalism of the ISO/IEC Guide 98-3.

The Monte Carlo calculations according to ISO 11929-2 were performed with the code UncertRadio[40][41]. The results of both evaluations (according to ISO 11929-1 and ISO 11929-2) are summarized. Finally, a comparison of the results obtained by application of ISO 11929-1 and ISO 11929-2 is given together with a documentation of the results and an assessment of the measurement.

For the purpose of comparison, both the probabilistically symmetric coverage interval and the shortest coverage interval are calculated. In a real case the regulator or customer has to decide which coverage interval has to be calculated.

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Unless otherwise stated the following stipulations are valid throughout this document.

— The probabilities α and β for the calculation of the decision threshold and the detection limit are chosen $\alpha = \beta = 5$ %. A coverage probability $1 - \gamma = 95$ % is assumed for the coverage interval.

— The standard uncertainties of the durations of the measurements is neglected.

— The nuclear data used in the examples are in Reference [38].

— Before the measurement of a sample the background effect has to be measured. It is assumed that the background effect as well as the calibration factor and its standard uncertainty were determined by independent experiments. The standard uncertainty associated with the calibration factor is evaluated by applying the ISO/IEC Guide 98-3. See Annex A for an example of the determination of a calibration factor.

6 Counting measurements with small or moderate uncertainties

6.1 Definition of the task and general aspects

The activity of a radionuclide in a sample is investigated by a counting measurement. This is a simplified example according to the general model of ISO 11929-1.

The comparison between the primary measurement result and the decision threshold provides information on whether or not an activity of the sample is observed. If the primary measurement result is larger than the decision threshold, it is concluded that an activity of the sample has been observed.

A guideline value $a_r = 3\,\mathrm{Bq}$ is to be compared with the detection limit. If the detection limit is smaller than the guide value, it is concluded that the measuring method is suitable for the measurement purpose.

6.2 Model of evaluation and standard uncertainty

The measurand is the activity in the sample. The primary result a of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation using preselection of the counting times

$$a = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0} \right) \cdot w = \left(r_g - r_0 \right) \cdot w = r_n \cdot w \tag{4}$$

The standard uncertainty $u(a)$ associated with the primary result a is calculated by

$$u^2(a) = r_n^2 \cdot u^2(w) + w^2 \cdot u^2(r_n) = w^2 \cdot \left(\frac{n_g}{t_g^2} + \frac{n_0}{t_0^2} \right) + a^2 \cdot u_{rel}^2(w) \tag{5}$$

6.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 1. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 1.

Table 1 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	21 670	—	—	1
Count time of the gross measurement	t_g	1 200	—	—	s

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Table 1 *(continued)*

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the background effect	n_0	73 150	—	—	1
Count time of the background effect	t_0	12 000	—	—	s
Calibration factor	w	4,1	0,6	$\mathrm{N}(\tilde{w};w,u(w))$	Bq s
Intermediate values					
Gross count rate	r_g	18,06	0,123	$\mathrm{Ga}(\tilde{r}_g;n_g,1/t_g)$	s^{-1}
Count rate of the background effect	r_0	6,10	0,022 5	$\mathrm{Ga}(\tilde{r}_0;n_0,1/t_0)$	s^{-1}

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description.

6.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

6.4.1 Background effect

The background effect of the sample has been measured. The number of the counted pulses is $n_0 = 73\,150$. This gives a count rate of

$$r_0 = \frac{n_0}{t_0} = 6{,}096\,\mathrm{s}^{-1} \tag{6}$$

The standard uncertainty of the count rate is given by

$$u(r_0) = \sqrt{\frac{n_0}{t_0^2}} = 0{,}022\,5\,\mathrm{s}^{-1} \tag{7}$$

6.4.2 Primary result and its associated standard uncertainty

The activity of the sample is calculated by using the Formula (4). This relation depends on the calibration factor *w*, which needs to be evaluated first, as well as its standard uncertainty $u(w)$.

$$w = 4{,}1\,\mathrm{Bq\,s}\ ,\ u(w) = 0{,}6\,\mathrm{Bq\,s}\ ,$$
$$u_{\mathrm{rel}}(w) = 0{,}146 \tag{8}$$

The activity *a* of the sample is given by

$$a = r_{\mathrm{n}} \cdot w = \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}} - r_0 \right) \cdot w = 49{,}05\,\mathrm{Bq} \tag{9}$$

with the standard uncertainty

$$u(a) = \sqrt{r_{\mathrm{n}}^2 \cdot u^2(w) + w^2 \cdot \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}^2} + \frac{n_0}{t_0^2} \right)} = 7{,}20\,\mathrm{Bq} \tag{10}$$

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6.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a}=\tilde{r}_{\mathrm{n}}w=\left(\tilde{r}_{\mathrm{g}}-\frac{n_0}{t_0}\right)w$ of the measurand, one expects a gross count rate $\tilde{r}_{\mathrm{g}}=\frac{\tilde{a}}{w}+\frac{n_0}{t_0}$ and with Formula (5) one obtains

$$\tilde{u}^2(\tilde{a})=\tilde{r}_{\mathrm{n}}^{\,2}\cdot u^2(w)+w^2\cdot u^2(\tilde{r}_{\mathrm{n}})=w^2\cdot\left(\frac{\tilde{a}}{t_{\mathrm{g}}w}+\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)+\tilde{a}^2\cdot u_{\mathrm{rel}}^2(w) \tag{11}$$

6.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha})=1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by

$$a^*=k_{1-\alpha}\cdot\tilde{u}(0)=k_{1-\alpha}\cdot\sqrt{w^2\cdot\left(\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)}=0{,}504\,\mathrm{Bq} \tag{12}$$

The measurement result a exceeds the decision threshold a^*.

6.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$\begin{aligned} a^{\#}&=a^*+k_{1-\beta}\cdot\tilde{u}\left(a^{\#}\right)\\ &=a^*+k_{1-\beta}\cdot\sqrt{a^{\#2}\cdot u_{\mathrm{rel}}^2(w)+w^2\cdot\left(\frac{a^{\#}}{t_{\mathrm{g}}w}+\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)}=1{,}08\,\mathrm{Bq}\end{aligned} \tag{13}$$

Since $\alpha=\beta$, $k_{1-\alpha}=k_{1-\beta}=k$. The detection limit can also be calculated by the explicit formula

$$a^{\#}=a^*+k_{1-\beta}\cdot\tilde{u}\left(a^{\#}\right)=\frac{2\cdot a^*+(k^2\cdot w)/t_{\mathrm{g}}}{1-k^2\cdot u_{\mathrm{rel}}^2(w)}=1{,}08\,\mathrm{Bq} \tag{14}$$

The guideline value a_{r} exceeds the detection limit $a^{\#}$.

6.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^*. With $\omega=\Phi[a/u(a)]=1{,}00$ and $\gamma=0{,}05$ one obtains the probabilities by $p=\omega\cdot(1-\gamma/2)=0{,}975$ and $q=1-\omega\cdot\gamma/2=0{,}975$, and hence the quantiles k_p and k_q are equal to 1,960. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by

$$a^{\triangleleft}=a-k_p\cdot u(a)=34{,}94\,\mathrm{Bq}\ \text{ and }\ a^{\triangleright}=a+k_q\cdot u(a)=63{,}15\,\mathrm{Bq} \tag{15}$$

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Alternatively, the lower and upper limits of the shortest coverage interval are calculated according to Formula (16). With $\omega = \Phi[a/u(a)] = 1{,}00$, one obtains $p = [1+\omega\cdot(1-\gamma)]/2 = 0{,}975$, and hence the quantile k_p is equal to 1,960. With this, the limits of the shortest coverage interval are

$$a^{<} = a - k_p \cdot u(a) = 34{,}94\,\text{Bq} \text{ and } a^{>} = a + k_p \cdot u(a) = 63{,}15\,\text{Bq} \tag{16}$$

6.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample and its associated standard uncertainty $u(\hat{a})$ are given by

$$\hat{a} = a + \frac{u(a)\cdot \exp\left\{-a^2 \Big/ \left[2u^2(a)\right]\right\}}{\omega\sqrt{2\pi}} = 49{,}05\ \text{Bq} \tag{17}$$

and its associated standard uncertainty

$$u(\hat{a}) = \sqrt{u^2(a) - (\hat{a} - a)\cdot\hat{a}} = 7{,}20\ \text{Bq} \tag{18}$$

6.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The available information is $\boldsymbol{a} = \{n_g, t_g, n_0, t_0, w, u(w)\}$; see 6.3 and Table 1. The results and characteristic limits are summarized in Table 2.

Applying ISO 11929-2 the marginal posterior for the true value of the measurand taking into account that the measurand is non-negative is given by

$$f_A(\tilde{a}|\boldsymbol{a}) = C\cdot \mathrm{H}(\tilde{a})\cdot \int_{-\infty}^{+\infty} \mathrm{Ga}(\tilde{r}_g; n_g, 1/t_g)\cdot \mathrm{Ga}(\tilde{r}_0; n_0, 1/t_0)\cdot \mathrm{N}[\tilde{w}; w, u(w)]\cdot \delta\left[\tilde{a} - (\tilde{r}_g - \tilde{r}_0)\cdot\tilde{w}\right] \mathrm{d}\tilde{r}_g\, \mathrm{d}\tilde{r}_0\, \mathrm{d}\tilde{w} \tag{19}$$

Analogous formulae are valid for the marginal posteriors of the other examples in this document if the PDFs under the integrals are replaced by those indicated in the tables in the clauses “Available information, input data, and specifications.” Therefore, the respective posteriors are not repeated for each example.

The integral is solved using Monte Carlo methods using the code UncertRadio[40]. Annex B gives an exemplary description of the procedure following ISO 11929-2. The best estimate and its associated standard uncertainty are calculated as the mean and the square root of the variance of the PDF according to Formula (19); the limits of the coverage intervals are calculated as suitable quantiles of this PDF. The PDFs required for the calculation of the primary measurement result and its associated standard uncertainty as well as for the decision threshold and the detection limit are also given in Annex B.

The PDFs obtained by the Monte Carlo simulations are presented in Figure 1.

Table 2 — Results and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	49,05	49,05
Standard uncertainty associated with the primary result	$u(a)$	Bq	7,20	7,19
Relative standard uncertainty associated with the primary result	$u_{\text{rel}}(a)$	1	0,147	0,147
Decision threshold	a^*	Bq	0,504	0,510

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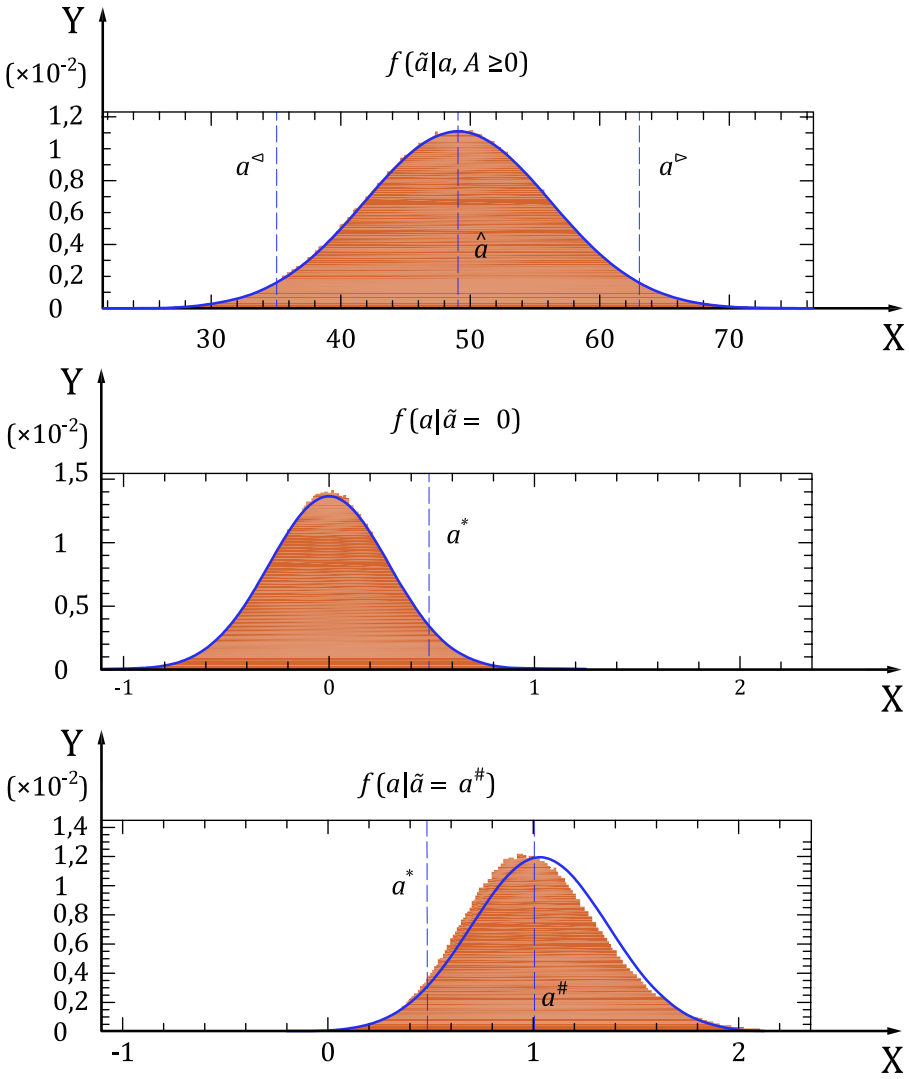
Table 2 *(continued)*

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Detection limit	$a^{\#}$	Bq	1,08	1,05
Best estimate	$\hat{a}$	Bq	49,05	49,05
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	7,20	7,19
Relative uncertainty associated with the best estimate	$u_{\mathrm{rel}}(\hat{a})$	1	0,147	0,147
Lower limit of the probabilistically symmetric coverage interval	$a^{\triangleleft}$	Bq	34,94	34,98
Upper limit of the probabilistically symmetric coverage interval	$a^{\triangleright}$	Bq	63,15	63,17
Lower limit of the shortest coverage interval	$a^{<}$	Bq	34,94	34,94
Upper limit of the shortest coverage interval	$a^{>}$	Bq	63,15	63,13



NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 1 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

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6.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 49{,}05\,\text{Bq}$ exceeds the decision threshold $a^{*} = 0{,}504\,\text{Bq}$. It is decided to conclude that an effect from the sample was recognized.

— The detection limit $a^{\#} = 1{,}08\,\text{Bq}$ is below the guideline values $a_{\text{r}} = 3\,\text{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^{\triangleleft} = 34{,}94\,\text{Bq}$ and $a^{\triangleright} = 63{,}15\,\text{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^{<} = 34{,}94\,\text{Bq}$ and $a^{>} = 63{,}15\,\text{Bq}$.

— The best estimate is $\hat{a} = 49{,}05\ \text{Bq}$ with an associated standard uncertainty $u(\hat{a}) = 7{,}20\ \text{Bq}$.

Because the model of evaluation is linear and the relative uncertainty of the primary measurement result is small, the results obtained by application of ISO 11929-2 are practically identical with those obtained by ISO 11929-1:2019, Table 2. The ISO/IEC Guide 98-3, approximations assuming normal PDFs and using a Taylor expansion truncated after the linear term, hold.

The probabilistically symmetric coverage interval is equal to the shortest coverage interval. The best estimate and its associated standard uncertainty equal the primary measurement result and its associated standard uncertainty. According to Formula (B.3) in ISO 11929-2:2019, Annex B, this holds of for all results of measurements with $y/u(y) \geq 4$, i.e. for measurement procedures with relative uncertainties $u_{\text{rel}}(y) \leq 0{,}25$.

For larger relative uncertainties of the primary measurement result deviations between the limits of the two coverage intervals and between the primary measurement result and the best estimate and their respective standard uncertainties occur. This can be due to small count numbers (see Clause 7) or due to large uncertainties of the calibration factor. With respect to uncertainties of the calibration factor according to Formula (2), one has to distinguish whether they result from quantities in the numerator or the denominator of Formula (2). These cases are dealt with in Clause 9 and Clause 10, respectively. A further origin of large uncertainties can be due to those of the shielding factor; see Clause 11 for an example.

7 Counting measurement with small count numbers

7.1 Definition of the task and general aspects

This is an example for the application of ISO 11929-2:2019, Annex A. The activity of a radionuclide in a sample is investigated by a counting measurement in which only very low count numbers are obtained as e.g. in some applications of counting of α-particles. Small count numbers provide a particular problem in counting measurements since the approximation of the PDFs of the count rates by Gaussian distributions does no longer hold.

The comparison between the primary measurement result and the decision threshold provides information on whether or not an activity of the sample is observed. If the primary measurement result is larger than the decision threshold, it is concluded that an activity of the sample has been observed.

A guideline value $a_{\text{r}} = 0{,}1\,\text{Bq}$ is to be compared with the detection limit. If the detection limit is smaller than the guideline value, it is concluded that the measuring method is suitable for the measurement purpose.

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7.2 Model of evaluation and standard uncertainty

The measurand is the activity in the sample. The primary result a of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation

$$a=\left(\frac{n_g}{t_g}-\frac{n_0}{t_0}\right)\cdot w=\left(r_g-r_0\right)\cdot w=r_n\cdot w \tag{20}$$

The associated standard uncertainty is calculated by

$$u^2(a)=w^2\cdot u^2(r_n)+r_n^2\cdot u^2(w)=w^2\cdot\left(\frac{n_g}{t_g^2}+\frac{n_0}{t_0^2}\right)+a^2\cdot u_{rel}^2(w) \tag{21}$$

7.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 3. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 3.

Table 3 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	6	—	—	1
Count time of the gross measurement	t_g	1 200	—	—	s
Number of counts of the background effect	n_0	3	—	—	1
Count time of the background effect	t_0	1 200	—	—	s
Calibration factor	w	4,1	0,6	$N(\tilde{w};w,u(w))$	Bq s
Intermediate values					
Gross count rate	r_g	5,00E-3	2,04E-3	$Ga(\tilde{r}_g;n_g,1/t_g)$	s^{-1}
Count rate of the background effect	r_0	2,50E-3	1,44E-3	$Ga(\tilde{r}_0;n_0,1/t_0)$	s^{-1}

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also of ISO 11929-2:2019, 6.3 for a detailed description.

7.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

7.4.1 Background effect

The background effect of the sample has been measured. The number of the counted pulses is $n_0=3$. This gives a count rate of

$$r_0=\frac{n_0}{t_0}=0{,}00250\,s^{-1} \tag{22}$$

The standard uncertainty of the count rate is given by

$$u(r_0)=\sqrt{\frac{n_0}{t_0^2}}=0{,}001\,44\,s^{-1} \tag{23}$$

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7.4.2 Primary result and its associated standard uncertainty

The activity of the sample is calculated by using the Formula (20). This relation depends on the calibration factor *w,* which needs to be evaluated first, as well as its standard uncertainty $u(w)$.

The calibration factor is given by

$$w=4{,}1 \text{ with } u(w)=0{,}6 \text{ and } u_{\mathrm{rel}}(w)=0{,}146 \tag{24}$$

The activity a of the sample is given by

$$a=r_{\mathrm{n}}\cdot w=\left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}}-r_0\right)\cdot w=0{,}010\,3\,\mathrm{Bq} \tag{25}$$

with the standard uncertainty

$$u(a)=\sqrt{r_{\mathrm{n}}^2\cdot u^2(w)+w^2\cdot\left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}^2}+\frac{n_0}{t_0^2}\right)}=0{,}010\,4\,\mathrm{Bq} \tag{26}$$

and

$$u_{\mathrm{rel}}(a)=u(a)/a=1{,}01 \tag{27}$$

7.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a}=\tilde{r}_{\mathrm{n}}\cdot w$ of the measurand, one expects a gross count rate $\tilde{r}_{\mathrm{g}}=\frac{\tilde{a}}{w}+\frac{n_0}{t_0}$ and with Formula (21) one obtains

$$\tilde{u}^2(\tilde{a})=\tilde{r}_{\mathrm{n}}^{\,2}\cdot u^2(w)+w^2\cdot u^2(\tilde{r}_{\mathrm{n}})=w^2\cdot\left(\frac{\tilde{a}}{t_{\mathrm{g}}w}+\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)+\tilde{a}^2\cdot u_{\mathrm{rel}}^2(w) \tag{28}$$

7.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha})=1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by

$$a^*=k_{1-\alpha}\,\tilde{u}(0)=k_{1-\alpha}\cdot\sqrt{w^2\cdot\left(\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)}=0{,}013\,8\,\mathrm{Bq} \tag{29}$$

The measurement result a is below the decision threshold a^*.

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7.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$a^{\#}=a^{*}+k_{1-\beta}\cdot\sqrt{a^{\#2}\cdot u_{\mathrm{rel}}^{2}(w)+w^{2}\cdot\left(\frac{a^{\#}}{t_{\mathrm{g}}w}+\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)}=0{,}039\,\mathrm{Bq} \tag{30}$$

Since $\alpha=\beta$, $k_{1-\alpha}=k_{1-\beta}=k$. The detection limit can also be calculated by the explicit Formula (31)

$$a^{\#}=a^{*}+k_{1-\beta}\cdot\tilde{u}\left(a^{\#}\right)=\frac{2\cdot a^{*}+(k^{2}\cdot w)/t_{\mathrm{g}}}{1-k^{2}\cdot u_{\mathrm{rel}}^{2}(w)}=0{,}039\,\mathrm{Bq} \tag{31}$$

The guideline value a_{r} exceeds the detection limit $a^{\#}$.

7.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated exemplarily though the measurement result a is below the decision threshold a^{*}. With $\omega=\Phi(a/u(a))=0{,}837$ and $\gamma=0{,}05$ one obtains the probabilities $p=\omega\cdot(1-\gamma/2)=0{,}816$ and $q=1-\omega\cdot\gamma/2=0{,}979$, and hence the quantiles k_p and k_q are equal to 0,908 and 2,034, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by

$$a^{\triangleleft}=a-k_p\cdot u(a)=0{,}000\,854\,\mathrm{Bq}\ \text{and}\ a^{\triangleright}=a+k_q\cdot u(a)=0{,}031\,3\,\mathrm{Bq} \tag{32}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega=\Phi(a/u(a))=0{,}837$, one obtains $p=(1+\omega\cdot(1-\gamma))/2=0{,}897$ and hence the quantile k_p is equal to 1,27. With this, the limits of the shortest coverage interval according to Formula (33) are

$$a^{<}=-0{,}00291\,\mathrm{Bq}\ \text{and}\ a^{>}=a+k_p\cdot u(a)=0{,}023\,4\,\mathrm{Bq} \tag{33}$$

Since the lower limit of the shortest coverage interval according to Formula (33) is negative, one has to proceed according to Formula (34) and obtains with $q=1-\omega\cdot\gamma=0{,}958$ the quantile $k_q=1{,}728$

$$a^{<}=0\,\mathrm{Bq}\ \text{and}\ a^{>}=a+k_q\cdot u(a)=0{,}028\,2\,\mathrm{Bq} \tag{34}$$

7.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample is given by

$$\hat{a}=a+\frac{u(a)\cdot\exp\left(-a^{2}/\left(2u^{2}(a)\right)\right)}{\omega\sqrt{2\pi}}=0{,}013\,3\ \mathrm{Bq} \tag{35}$$

with its associated standard uncertainty $u(\hat{a})$

$$u(\hat{a})=\sqrt{u^{2}(a)-(\hat{a}-a)\cdot\hat{a}}=0{,}008\,20\ \mathrm{Bq} \tag{36}$$

7.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 4 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 2.

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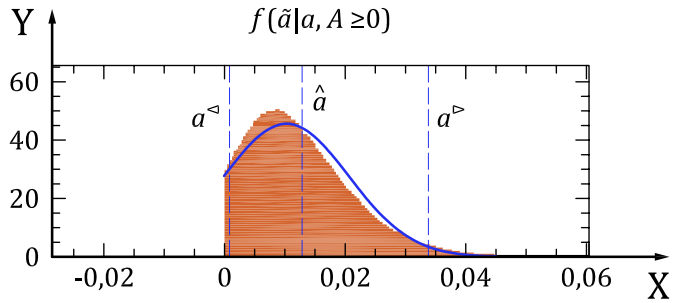
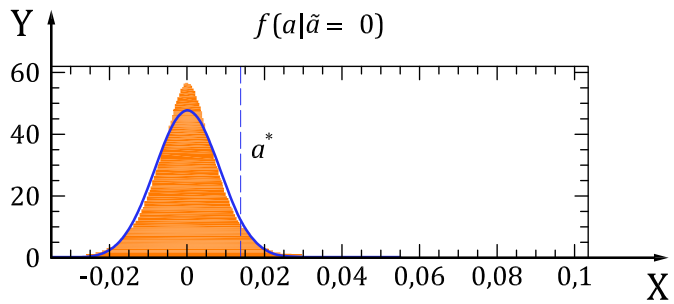
Table 4 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	1,03E-2	1,02E-2
Standard uncertainty associated with the primary result	$u(a)$	Bq	1,04E-2	1,05E-2
Relative standard uncertainty associated with the primary result	$u_{\mathrm{rel}}(a)$	1	1,01	1,03
Decision threshold	a^{*}	Bq	1,38E-2	1,38E-2
Detection limit	$a^{\#}$	Bq	3,90E-2	3,58E-2
Best estimate	$\hat{a}$	Bq	1,33E-2	1,29E-2
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	8,20E-3	8,75E-3
Relative uncertainty associated with the best estimate	$u_{\mathrm{rel}}(\hat{a})$	1	0,62	0,68
Lower limit of the probabilistically symmetric coverage interval	$a^{\triangleleft}$	Bq	8,54E-4	8,24E-4
Upper limit of the probabilistically symmetric coverage interval	$a^{\triangleright}$	Bq	3,13E-2	3,38E-2
Lower limit of the shortest coverage interval	$a^{<}$	Bq	0	0
Upper limit of the shortest coverage interval	$a^{>}$	Bq	2,82E-2	2,93E-2





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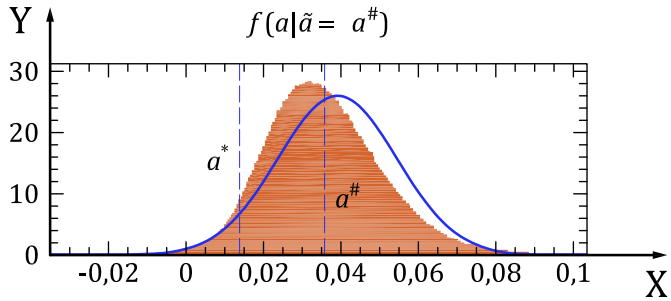

NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 2 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

7.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 0{,}010\,3\,\text{Bq}$ is below the decision threshold $a^* = 0{,}013\,8\,\text{Bq}$. It is decided to conclude that no effect of the sample was recognized. In spite of that all characteristic values were calculated and documented in this example.

— The detection limit $a^{\#} = 0{,}039\,\text{Bq}$ is below the guideline value $a_r = 0{,}1\,\text{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^{\triangleleft} = 0{,}000\,854\,\text{Bq}$ and $a^{\triangleright} = 0{,}031\,3\,\text{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^{<} = 0\,\text{Bq}$ and $a^{>} = 0{,}028\,2\,\text{Bq}$.

— The best estimate is $\hat{a} = 0{,}013\,3\ \text{Bq}$ with an associated standard uncertainty $u(\hat{a}) = 0{,}008\ 2\ \text{Bq}$.

There are just minor differences between the results obtained by applying ISO 11929-1 and ISO 11929-2, which are due to the deviation between the Gaussian distributions and the actual posterior distributions calculated according to ISO 11929-2; see Figure 2.

If large relative uncertainties of the primary result are due to the uncertainties of the count rates, this does not affect the existence of the detection limit calculated according to ISO 11929-1. Also in this extreme case of low count numbers, the application of ISO 11929-1 is justified.

7.7 An alternative example of a measurement with small count numbers

7.7.1 General

The data of an alternative example are given in Table 5. The input data differ from the example of Table 3 just with respect of the count numbers, which are now $n_0 = 1$ and $n_g = 4$.

Table 5 — Input quantities and data

Quantity	symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	4	—	—	1

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Table 5 *(continued)*

Quantity	symbol	x_i	$u(x_i)$	PDF	Unit
Count time of the gross measurement	t_g	1 200	—	—	s
Number of counts of the background effect	n_0	1	—	—	1
Count time of the background effect	t_0	1 200	—	—	s
Calibration factor	w	4,1	0,6	$N(\tilde{w};w,u^2(w))$	s Bq
Intermediate values					
Gross count rate	r_g	3,33E-3	1,67E-3	$Ga(\tilde{r}_g;n_g,1/t_g)$	s^{-1}
Count rate of the background effect	r_0	8,33E-4	8,33E-4	$Ga(\tilde{r}_0;n_0,1/t_0)$	s^{-1}

NOTE This case with just one background count is regarded as a limiting case. In any reasonable single-channel counting measurement it is expected that at least one count is obtained in both, the gross and the background measurement. The case of a single-channel counting measurement with zero background counts is considered as a pathological case where the measurement or even the measurement method is insufficient.

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also of ISO 11929-2:2019, 6.3 for a detailed description.

7.7.2 Background effect

The background effect of the sample has been measured. The number of the counted pulses is $n_0 = 1$. This gives a count rate of

$$r_0 = \frac{n_0}{t_0} = 0{,}000\,833\,\mathrm{s}^{-1} \tag{37}$$

The standard uncertainty of the count rate is given by

$$u(r_0) = \sqrt{\frac{n_0}{t_0^2}} = 0{,}000\,833\,\mathrm{s}^{-1} \tag{38}$$

7.7.3 Primary result and its associated standard uncertainty

The activity of the sample is calculated by using the Formula (20). This relation depends on the calibration factor *w*, which needs to be evaluated first, as well as its standard uncertainty *u*(*w*).

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The calibration factor is given by

$$w=4{,}1 \text{ with } u(w)=0{,}6 \text{ and } u_{\mathrm{rel}}(w)=0{,}146 \tag{39}$$

The activity a of the sample is given by

$$a=r_{\mathrm{n}}\cdot w=\left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}}-r_0\right)\cdot w=0{,}0103\,\mathrm{Bq} \tag{40}$$

with the standard uncertainty

$$u(a)=\sqrt{r_{\mathrm{n}}^2\cdot u^2(w)+w^2\cdot\left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}^2}+\frac{n_0}{t_0^2}\right)}=0{,}00779\,\mathrm{Bq} \tag{41}$$

and

$$u_{\mathrm{rel}}(a)=u(a)/a=0{,}756 \tag{42}$$

7.7.4 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a}=\tilde{r}_{\mathrm{n}}\cdot w$ of the measurand one expects a gross count rate $r_{\mathrm{g}}=\frac{\tilde{a}}{w}+\frac{n_0}{t_0}$ and with Formula (21) one obtains

$$\tilde{u}^2(\tilde{a})=\tilde{r}_{\mathrm{n}}{}^2\cdot u^2(w)+w^2\cdot u^2(\tilde{r}_{\mathrm{n}})=\tilde{a}^2\cdot u_{\mathrm{rel}}^2(w)+w^2\cdot\left(\frac{\tilde{a}}{t_{\mathrm{g}}w}+\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right) \tag{43}$$

7.7.5 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha})=1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by

$$a^*=k_{1-\alpha}\,\tilde{u}(0)=k_{1-\alpha}\cdot\sqrt{w^2\cdot\left(\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)}=0{,}00795\,\mathrm{Bq} \tag{44}$$

The measurement result a is above the decision threshold a^*.

7.7.6 Detection limit

For β = 5 %, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$a^{\#}=a^*+k_{1-\beta}\cdot\sqrt{a^{\#2}\cdot u_{\mathrm{rel}}^2(w)+w^2\cdot\left(\frac{a^{\#}}{t_{\mathrm{g}}w}+\frac{n_0}{t_{\mathrm{g}}t_0}+\frac{n_0}{t_0^2}\right)}=0{,}0267\,\mathrm{Bq} \tag{45}$$

Since $\alpha=\beta$, $k_{1-\alpha}=k_{1-\beta}=k$. The detection limit can also be calculated by the explicit formula

$$a^{\#}=a^*+k_{1-\beta}\cdot\tilde{u}\left(a^{\#}\right)=\frac{2\cdot a^*+(k^2\cdot w)/t_{\mathrm{g}}}{1-k^2\cdot u_{\mathrm{rel}}^2(w)}=0{,}0267\,\mathrm{Bq} \tag{46}$$

The guideline value a_{r} exceeds the detection limit $a^{\#}$.

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7.7.7 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^*. With $\omega=\Phi[a/u(a)]=0{,}907$ and $\gamma=0{,}05$ one obtains the probabilities by $p=\omega\cdot(1-\gamma/2)=0{,}884$ and $q=1-\omega\cdot\gamma/2=0{,}977$, and hence the quantiles k_p and k_q are equal to 1,20 and 2,00, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by

$$a^{\triangleleft}=a-k_p\cdot u(a)=0{,}000\,970\,\text{Bq} \text{ and } a^{\triangleright}=a+k_q\cdot u(a)=0{,}025\,8\,\text{Bq} \tag{47}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega=\Phi[a/u(a)]=0{,}907$, one obtains $p=[1+\omega\cdot(1-\gamma)]/2=0{,}931$, and hence the quantile k_p is equal to 1,48. With this the limits of the shortest coverage interval according to Formula (48) were

$$a^{<}=a-k_p\cdot u(a)=-0{,}001\,24\,\text{Bq} \text{ and } a^{>}=a+k_p\cdot u(a)=0{,}021\,8\,\text{kBq} \tag{48}$$

Since the lower limit of the shortest coverage interval according to Formula (48) is negative, one has to proceed according to Formula (49) and obtains with $q=1-\omega\cdot\gamma=0{,}955$ the quantile $k_q=1{,}692$

$$a^{<}=0\,\text{Bq} \text{ and } a^{>}=a+k_q\cdot u(a)=0{,}0234\,\text{Bq} \tag{49}$$

7.7.8 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample is given by

$$\hat{a}=a+\frac{u(a)\cdot\exp\left\{-a^2\Big/\left[2u^2(a)\right]\right\}}{\omega\sqrt{2\pi}}=0{,}011\,7\ \text{Bq} \tag{50}$$

with its associated standard uncertainty $u(\hat{a})$

$$u(\hat{a})=\sqrt{u^2(a)-(\hat{a}-a)\cdot\hat{a}}=0{,}006\,62\ \text{Bq} \tag{51}$$

7.8 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 6 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 3.

Table 6 — Results and characteristic limits

Results	symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	1,03E-2	1,02E-2
Standard uncertainty associated with the primary result	$u(a)$	Bq	7,79E-3	7,86E-3
Relative standard uncertainty associated with the primary result	$u_{\text{rel}}(a)$	1	0,756	0,771
Decision threshold	a^*	Bq	7,95E-3	7,91E-3
Detection limit	$a^{\#}$	Bq	2,67E-2	2,30E-2
Best estimate	$\hat{a}$	Bq	1,17E-2	1,12E-2
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	6,62E-3	7,20E-3

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