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Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)

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National foreword

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The UK participation in its preparation was entrusted to Technical Committee MS/6, Methodologies for business process improvement using statistical methods.

A list of organizations represented on this committee can be obtained on request to its secretary.

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ISBN 978 0 580 75793 8

ICS 03.120.30

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This British Standard was published under the authority of the Standards Policy and Strategy Committee on 31 July 2014.

Amendments issued since publication

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INTERNATIONAL STANDARD

ISO 16336

First edition
2014-07-01

Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)

Application de méthodologies statistiques et connexes pour le développement de nouvelles technologies et de nouveaux produits — Modèle paramétrique robuste

Reference number
ISO 16336:2014(E)

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Foreword

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The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 8, *Application of statistical and related methodology for new technology and product development*.

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Introduction

Robust parameter design, also called parameter design, can be applied in product design stage to identify the optimum nominal values of design parameters based on the assessment of robustness of its function. Robustness assessment is performed as a consideration of overall loss during the product's life cycle. The overall loss is composed of costs and losses at each stage of the product's life. It includes all the costs incurred during not only its production stage, but also its disposal stages.

When a product is not robust, the product causes many environmental and social economic losses (including losses to the manufacturer and the users) due to its poor quality caused by functional variability throughout its usable lifetime from shipping to final disposal. Product suppliers have responsibilities and obligations to supply robust products to the market to avert losses and damages resulting from defects in the products.

The aim of applying parameter design in product design is to prevent defects, failures, and quality problems that can occur during the usage of the product. A robust product, an output of parameter design, is a product which is designed in such a way as to minimize user's quality losses caused by defects, failures, and quality problems. Note that defects, failures, and quality problems are caused by functional variability of a non-robust product. In parameter design, optimum nominal values of a product's design parameters can be selected by treating a product's design parameters as control factors and by assessing robustness under noise factors. The use of parameter design at development and design stages makes it possible to determine the optimum product design and specification so that the product is robust in the market.

At manufacturing stage, the product suppliers manufacture their products that meet the product specifications. One can optimize manufacturing processes to produce the products that meet the specifications. However, robustness against customer's environment and products' aging can be addressed only by product design.

Robust parameter design methodology provides effective methods for achieving robustness through its design of specification determination, and it is a preventive countermeasure against various losses in the market.

In practice, many product's defects and failures occur due to the product's response that deviates from or varies around the designed target values by the change in usage environment and deterioration, i.e. noise conditions. The variability of product's response due to noises can be used as a measure of robustness, because market losses increase in proportion to the magnitude of variability of product's response. SN ratio, corresponding to the inverse of the variability measure, is used as a measure of goodness in robustness. In other words, the higher the SN ratio is, the less the market losses are.

For the experimental plan of parameter design, direct product of inner array and outer arrays is proposed. Control factors are assigned to the inner array, and signal and noise factors are assigned to the outer array. By using a direct product plan, all the first level interactions between control factors and noise factors can be assessed and can be utilized to select the optimum level of control factors from the point of view of robustness.

Assessing robustness through SN ratio is a key of parameter design. The outer array is for evaluating SN ratio, robustness, for each combination of levels of control factors indicated by the inner array. The inner array is for comparing SN ratios and selecting the optimum combination of system's design parameters. As for the inner array, an orthogonal array L_{18}, is recommended as an efficient plan, and then only the applications of an orthogonal array L_{18} are discussed in this International Standard. Applications of experimental layout other than orthogonal array L_{18} can be found in the examples in references in the Bibliography. More detailed discussions on inner array and orthogonal arrays can be found in the references.

Robust parameter design (RPD), and thus this International Standard, is directly targeted at the losses incurred at the usage stage. Where possible, losses at other stages are also investigated so that the results of parameter design can be applied to perform the optimum product design for the whole stages of the product's life cycle.

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INTERNATIONAL STANDARD ISO 16336:2014(E)

Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)

1 Scope

This International Standard gives guidelines for applying the optimization method of robust parameter design, also called as parameter design, an effective methodology for optimization based on Taguchi Methods, to achieve robust products.

This International Standard prescribes signal-to-noise ratio (hereafter SN ratio) as a measure of robustness, and the procedures of parameter design to design robust products utilizing this measure. The word "robust" in this International Standard means minimized variability of product's function under various noise conditions, that is, insensitivity of the product's function to the changes in the levels of noises. For robust products, their responses are sensitive to signal and insensitive to noises.

The approach of this International Standard can be applied to any products that are designed and manufactured, including machines, chemical products, electronics, foods, consumer goods, software, new materials, and services. Manufacturing technologies are also regarded as products that are used by manufacturing processes.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-3, *Statistics — Vocabulary and symbols — Part 3: Design of experiments*

3 Terms and definitions and symbols

3.1 Term and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-3, and the following apply.

3.1.1
function
work which a system performs in order to fulfil its objective

Note 1 to entry: A function can be expressed by the mathematical form of input-output relation.

3.1.2
robustness
degree of smallness in variability of a system's function under various noise conditions

Note 1 to entry: System's performance can be assessed by robustness. SN ratio is a quantitative measure of robustness.

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3.1.3
signal-to-noise ratio
SN ratio
ratio of useful effects to harmful effects in response variations

Note 1 to entry: SN ratio is usually expressed in db value. The notation of db is used instead of dB for SN ratios of robustness measurements.

Note 2 to entry: The anti-logarithm value of an SN ratio, real number, is the inverse of a variation measure such as a variance or a coefficient of variation, and inversely proportional to monetary loss.

Note 3 to entry: The change in response caused by intentional change of input signal value is a useful effect. In case of the ideal function being zero point proportional, the linear slope forced through the zero point is a useful term.

Note 4 to entry: The change in response caused by noise factors is a harmful effect. Effects of noise factors and deviation from the ideal function are examples.

Note 5 to entry: SN ratio should contain the variability under noise factors and the discrepancy from the ideal function under average usage condition.

3.1.4
sensitivity
amount of change in response caused by unit change of input

Note 1 to entry: Sensitivity is usually expressed in db value.

Note 2 to entry: For dynamic characteristic cases, the sensitivity shows the magnitude of linear coefficient due to input signal, β^2, where β is a proportional constant.

Note 3 to entry: For the nominal-the-best response, the sensitivity shows the magnitude of mean, m^2, where m is an average of responses.

3.1.5
noise
variable which disturbs a system's function

Note 1 to entry: Any variable in the user's conditions for operating is either a signal or a noise.

Note 2 to entry: Noise is composed of internal noise and external noise. They are sometimes called as capacity and demand, respectively. Changes of internal constant of the system or its parts over time, such as deterioration, aging. and wear, and manufacturing variations are examples of internal noises. Usage conditions and environment conditions of the product are examples of external noises.

3.1.6
signal
input variable to the system, which is intentionally changed by the user to get an intended value of response in input-output relation

Note 1 to entry: Any variable in the user's conditions for operating is either a signal or a noise

Note 2 to entry: There are two kinds of signal: active signal and passive signal. Active signal is operated by user to get intended response, for example, rotating angle of a steering wheel to change the vehicle's direction. Passive signal is used by user to know the value of input from response reading, for example, temperature in thermal measurement. In both cases, output will change by changing the value of the signal but the user wants to get response value in the active case, and the user wants to know the value of signal in the passive case.

3.1.7
dynamic characteristics
output response which has multiple ideal target values depending on the value of a signal

Note 1 to entry: The relation between dynamic characteristics and a signal can be expressed by input-output functional form. The output of a system's function is dynamic characteristics in many cases.

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3.1.8
static characteristics
non-dynamic characteristics
output response which has a fixed target value

Note 1 to entry: Static characteristics can be categorized into three groups depending on the target value; nominal-the-best, smaller-the-better, and larger-the-better characteristics, where the target value is a finite value, zero, and infinity, respectively.

3.1.9
inner array
experimental plan where design parameters are assigned as control factors or indicative factors

Note 1 to entry: Each treatment run will be assessed for robustness using SN ratio and sensitivity.

Note 2 to entry: Orthogonal arrays are recommended for the inner array because many design parameters can be taken into consideration in one set of experiments as control factors.

Note 3 to entry: Experimental factors should be categorized by their roles and assigned separately to inner array or outer array based on their roles in parameter design. Control factors and indicative factors should be assigned to the inner array.

3.1.10
outer array
experimental plan where variables in users' conditions are assigned as noise factors or signal factors for evaluating SN ratio and sensitivity

Note 1 to entry: Any variable in user's conditions for operating is either a signal or a noise.

Note 2 to entry: Experimental factors should be categorized by their roles and assigned separately to inner array or outer array based on their roles in parameter design. Noise factors and signal factors should be assigned to the outer array.

3.2 Symbols

f	degree of freedom
k	number of levels of signal factor
L	linear form
L_i	linear form for level of i
M	signal factor/input signal
Mi	signal level of i
M_i	value of signal level of i
N	noise factor
n	number of levels of noise factor
Ni	noise level of i
p_0	standardized error rate
r	sum of squares of input signal levels/effective divisor
S	sensitivity
S_T	total sum of squares

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S_m sum of squares due to mean

S_β sum of squares due to linear slope β

$S_{N \times \beta}$ sum of squares due to the variation of linear slope β between noise levels

S_e sum of squares due to error

S_{opt} estimated value of sensitivity for optimum condition

S_{base} estimated value of sensitivity for baseline condition

S_{cur} estimated value of sensitivity for current condition

V_e variance due to error/error variance

V_N variance due to pooled error/variance due to error and noise

y output response

β sensitivity coefficient/linear slope

ΔS gain in sensitivity

$\Delta\eta$ gain in SN ratio

η SN ratio

η_{opt} estimated value of SN ratio for optimum condition

η_{base} estimated value of SN ratio for baseline condition

η_{cur} estimated value of SN ratio for current condition

ρ_0 standardized contribution ratio

4 Robust parameter design — Overview

4.1 Requirements

Robust parameter design is a rational and efficient assessment for discovering technical means to improve robustness in the designing process. It is, therefore, necessary to provide the following two procedures:

a) a procedure for accurate and simple evaluation of robustness;

b) a procedure for efficient assessment of multiple technical means.

This clause provides the approach to the goal of parameter design, and more detailed and specific steps of a robustness evaluation and a parameter design experiment are described in Clauses 5 and 6.

4.2 Assessing the robustness of a system

How can the robustness of a system be accurately assessed by the SN ratio? The robustness of a system is associated with many usage conditions of the system, so it cannot be assessed by a simple measurement. To clarify hidden factors associated with the robustness, the assessment should be approached from the following two viewpoints.

a) Use of an ideal function: The ideal function is a target function of the system. Actual function of the system should be measured and compared with the ideal function of the system in the robustness

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evaluation. It is important to avoid defects, failure modes, or quality problems for achieving the ideal function of the system.

b) Use of noise factors: Actual system in usage is working under various noise conditions. Noise effects should be intentionally introduced in the experiment by changing noise levels and the actual function of the system should be measured and evaluated under those predetermined noise conditions. Evaluation of the robustness strongly depends on the choice of noise factors and their levels. It is essential to apply effective noise strategies.

The function of a system is a work that it performs in order to fulfil its objective. For example, the function of an electric lamp is to transform electric energy into light energy, and the function of a wind turbine is to transform natural wind energy into rotating energy to perform a work such as water pumping. The function is normally expressed in a mathematical functional form of a relationship between input and output energies. The mathematical functional form can be expressed in many ways. Zero-point proportional formula is common in energy transformation of real physical systems. The details will be discussed in Clause 5.

Input and output characteristics are fixed based on the system's ideal function. Input characteristic is called as a signal in input-output relationship; this is because the changes in output are acquired by the user's intentional changing of the input in real usage and also in the experiment of parameter design. The signal is associated with energy or information necessary to perform its function. The signal factor is one of the user's conditions for changing the input when the users of the system try to control the output of the system. The signal factor has three or more levels in the experiment for dynamic characteristic so that the straightness of the actual input-output relationship could be evaluated. There is no signal factor for static characteristic because it has only one target output. Output characteristic is called as an output response or simply a response.

It is important to identify a suitable measurement method of output response. In time-dependent phenomena, for example, detection of output response is difficult in some cases. New measurement methods should be developed in those cases. The output response is associated with the purpose of a system. In the case of illumination, for example, the output response is magnitude of light, and in the case of a water pump, it is quantity of water.

Noise condition is a source that makes the system's actual function deviate from the ideal function. Examples include environmental conditions in actual working, such as temperature and humidity, an actual supplied voltage, electrical noise conditions, frequency of operations, and stress. They are called as external noises. On the other hand, there are noise conditions which are called internal noises, such as aging and wear. Examples include operating and/or idling time length after started, deterioration of system's parts after long operation, and manufacturing variability of a system and/or its parts. These noise conditions always reduce the system's functional performance to lower level than the level expected at the time of design. Since the purpose of robustness assessment is to clarify the performance by measuring the extent of this reduction, the variation of the system's function under noise conditions should be estimated in robustness evaluation. This is the reason why noise conditions should be taken into the parameter design experiment as noise factors. Three categories of noise factors are a) environment, b) aging and changes over time, and c) manufacturing variations. For the effective noise strategy, various types of noise should be examined in actual usage and environmental conditions.

Figure 1 shows an overview of the evaluation of robustness using a noise factor. Here, multiple data from X_1 to X_n should be obtained for the objective system under noise levels, N1 to Nn, and SN ratio η should be calculated using the data from X_1 to X_n as a robustness measure. Formulations of SN ratio are shown in Clause 5. When more than two systems are compared, the same levels of the same noise factor should be applied for all objective systems

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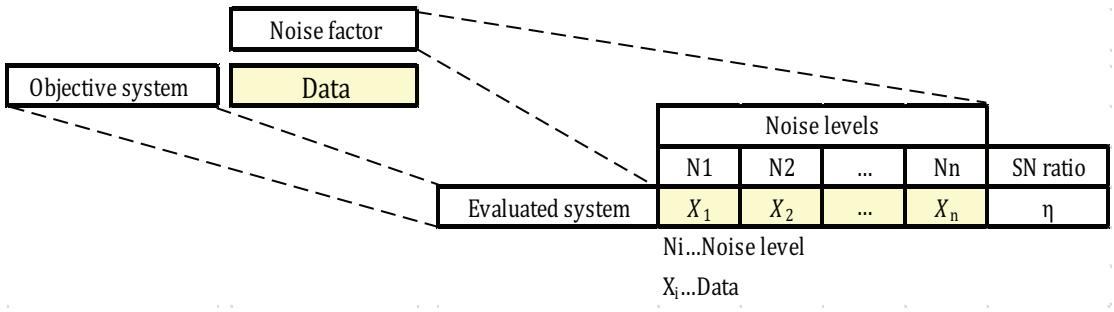

Figure 1 — Robustness assessment with a noise factor

When multiple different types of noise factors are applied in an experiment, an orthogonal array can be applied for determining the noise levels. In Figure 2, noise levels, N1 to Nn, are determined by the combination of levels of noise factors, such as A, B, and C, indicated by the orthogonal array. Experimental layout other than orthogonal arrays can be applicable for determining the noise levels.

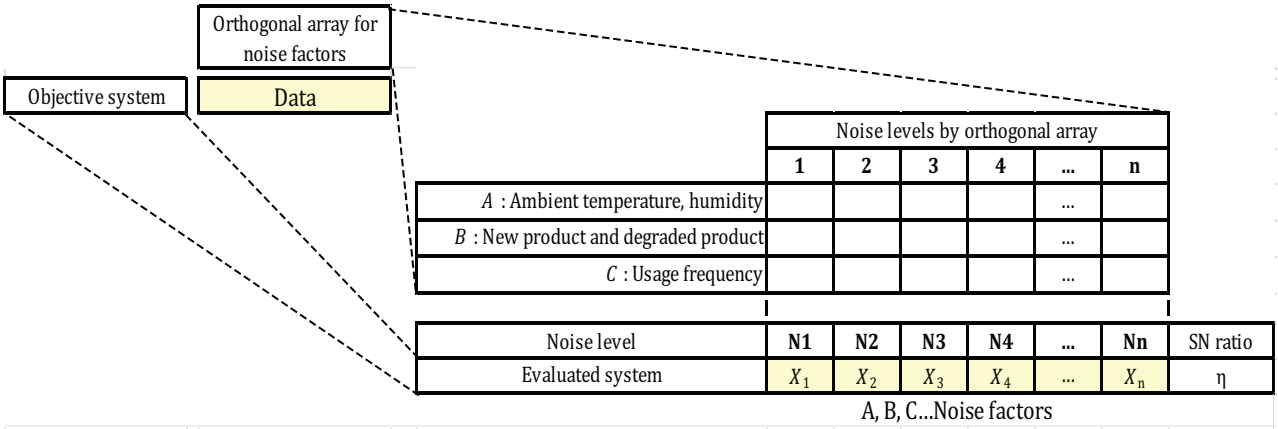

Figure 2 — Robustness assessment with noise levels assigned by an orthogonal array

4.3 Robustness assessment through SN ratio

The ideal function and the noise strategy are the key issues for the robustness assessment of robust parameter design. It is essential to measure and evaluate the variability and efficiency of the actual function of the system. The intent of doing this is that the evaluation covers the whole technical issues of the system's operations to prevent technical problems. Robustness evaluation also involves assessing the effects of noise factors that inhibit the required function. The results are expressed by SN ratio and sensitivity.

The SN ratio can discriminate the true difference in robustness between designs. Only relative comparison of SN ratios is meaningful to perform, because absolute values of SN ratios are affected by setting levels of noise factors. Thus, it is preferable to perform benchmarking in assessing robustness.

A feature of this approach is that the only information needed to evaluate SN ratios is just that on the knowledge of the function of the system and the noise conditions. No detailed technical information about the objective system is needed. SN ratios can be calculated in the same way for the objective systems as long as they have the same function, that is, the same input-output relationship, even if they have different technical constituents. Since the robustness of systems can be accurately evaluated through SN ratios, then the robustness of various systems with different design concepts can be assessed and compared.

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The comparison of various systems based on different technologies or different design concepts can be performed in the same way through SN ratio. Systems, such as conventional systems and newly developed systems, one's own systems and one's competitor's systems, can be evaluated and assessed in the same way through SN ratio, when they have the same function. This is the idea to conduct benchmarking on various designs in the robustness assessment through SN ratio.

4.4 An efficient method for assessing technical ideas — Parameter design

Basic technologies and mechanisms should be selected as a design concept first to start designing a system of industrial products. When there are multiple system design concepts to be benchmarked, the robustness assessment introduced in the previous subclause can be applied to select the best design concept.

After selecting the best design concept, the next step is to perform a detailed design by selecting values of system's design parameters. In this detailed design step, designers can optimize the system by selecting the optimum nominal values so that the function of the designed system becomes the most robust and efficient. The system design optimization method performed at this step is called parameter design, because design optimization is performed by setting design parameters to the optimum nominal values.

Consider what sort of states might be significant. When a system is in the optimum state, it achieves the best overall performance in all conceivable usage conditions. More specifically, an industrial system can stably perform its intended function anytime, even when, for example, it is working under a wide range of temperature and humidity, and when it is used in many different ways and in different environments. The optimum design conditions are taken as a combination of design parameters' values that maximize the robustness of the product. Since optimization by parameter design implies optimizing for maximized robustness, that is, minimized variability and maximized efficiency, judgments should be made by using robustness measure, SN ratio, and sensitivity.

The basis of the system design optimization by robustness assessment through SN ratio is a criterion for optimization in parameter design. The robustness assessment should be performed with regard to all the possible designs in design space, but in practice this is impossible. This is because a vast number of tests would have to be performed to take all possible combinations of design parameters into consideration.

As a more practical method for applying in development and design stages, an experiment using an orthogonal array is recommended where the combinations of many design parameters can be tested under a limited number of experimental runs. An orthogonal array plan is recommended not only because it can reduce the number of experimental runs comparing with a full factorial plan with the same number of control factors, but also because it can assign maximum number of control factors in a plan under the situation of same number of experimental runs. Reliability of experimental results should be confirmed in the confirmation experimental run for reproducibility check. Clause 6 describes a specific method for performing the confirmation experiment to check the reproducibility.

Procedure of a parameter design experiment should be as follows:

— (Step 1) Clarify the system's ideal function;

— (Step 2) Select signal factor and its range;

— (Step 3) Select measurement method of output response;

— (Step 4) Develop noise strategy, and select noise factors and their levels;

— (Step 5) Select control factors and their levels from design parameters

— (Step 6) Assign experimental factors to inner or outer array;

— (Step 7) Conduct experiment and collect data;

— (Step 8) Calculate SN ratio, η, and sensitivity, S;

— (Step 9) Generate factorial effect diagrams on SN ratio and sensitivity;

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— (Step 10) Select the optimum condition;

— (Step 11) Estimate the improvement in robustness by the gain;

— (Step 12) Conduct a confirmation experiment and check the gain and “reproducibility”.

4.5 Two-step optimization (Strategy of parameter design)

Figure 3 presents an overview of parameter design as described above. The experiment in this figure includes two orthogonal arrays; one orthogonal array for control factors, that is, for design parameters (inner array) and the other orthogonal array for noise factors (outer array). This layout is called a direct product plan. The number of experimental data corresponds to the product of the numbers of runs respectively specified in two orthogonal arrays. For example, in the case of the combination of inner array, L_{18}, and outer array, L_{12}, each array has runs of m = 18 and n = 12, and the number of total runs comes to 18 × 12 = 216.

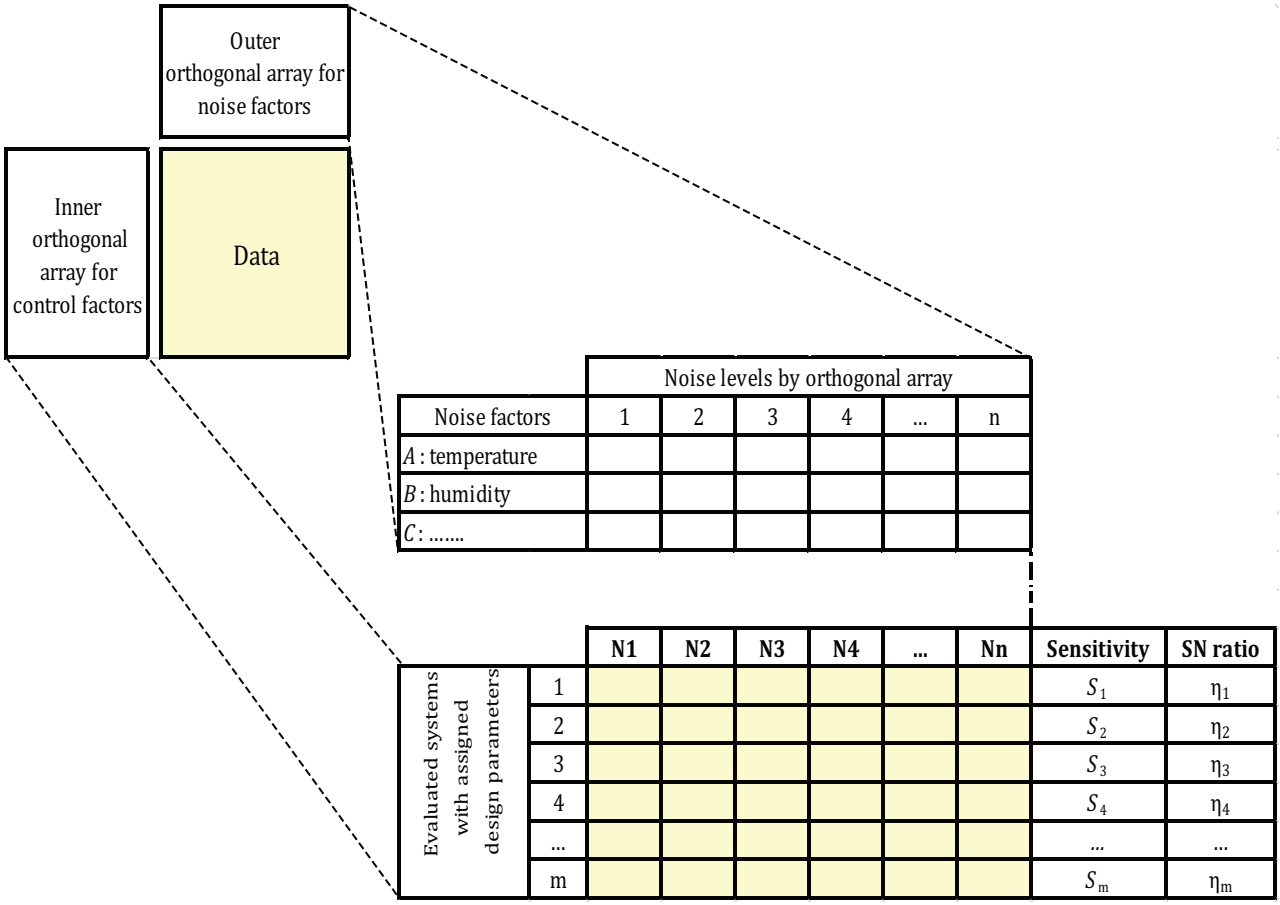

Figure 3 — Direct product plan for parameter design

Full factorial plan can be used as an outer array plan for noise and signal factors instead of an orthogonal array plan in some cases. In the case of physical tests, it is recommended to compound many noise factors into one compounded noise factor. However, it is always recommended to use an orthogonal array plan as an inner array for design parameters, because many design parameters can be assigned in one orthogonal array.

The experimental data obtained for each combination of levels of control factors consist of multiple data with the corresponding number of noise levels. To find out the optimum values of design parameters for robustness, the sensitivity (mean value in case of nominal-the-best response), and the SN ratio should be calculated for each row of inner array, that is, for the combination of values of design parameters. Then

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the factorial effects of control factors on sensitivity and SN ratio should also be calculated, and they are summarized in factorial effect diagrams as shown in Figures 4 and 5. Specific calculation formulae are described in Clause 6. The optimum values of design parameters are selected using the diagrams on sensitivity and SN ratio. The sensitivity represents the mean value of the data set (in case of static characteristics), and the SN ratio represents robustness.

The factorial effect diagram shows how the system's function is affected by each design parameter incorporated into the experiment. If a factor has a large gradient, it has a large effect on the system's function. Two types of factorial effect diagram represent the degree of influence relating to SN ratio and sensitivity. An important point in two-step design is to pay more attention to SN ratio than to sensitivity. In the first step, the optimum level of control factor should be selected to maximize SN ratio in the factorial effect diagram on SN ratio (see Figure 4), and then, in the second step, typically just one design parameter should be used to adjust a mean value or linear slope, i.e. sensitivity (see Figure 5), to the target value. For this adjustment, it is desired to select one factor with maximal effect on sensitivity and minimal effect on SN ratio. The first step is to optimize the design for robustness by SN ratio, and the second step is to adjust the magnitude to the target value by sensitivity. This two-step design procedure is a very important concept for the design of robustness. This is the reason why parameter design for robustness is also called as a two-step optimization.

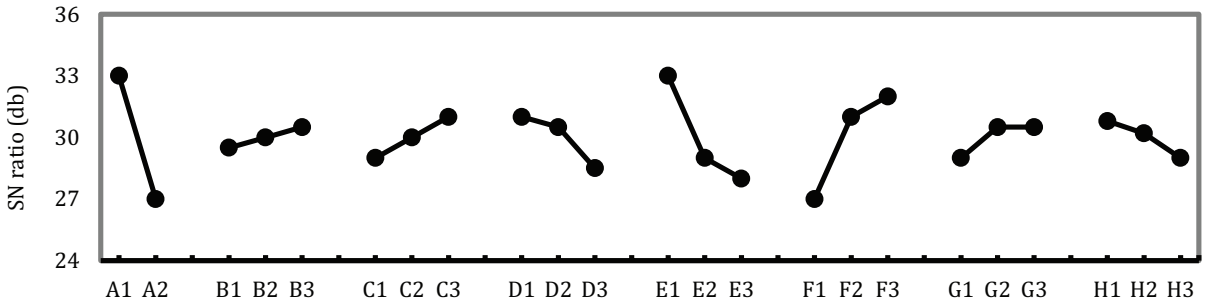

Figure 4 — Factorial effect diagram on SN ratio (robustness)

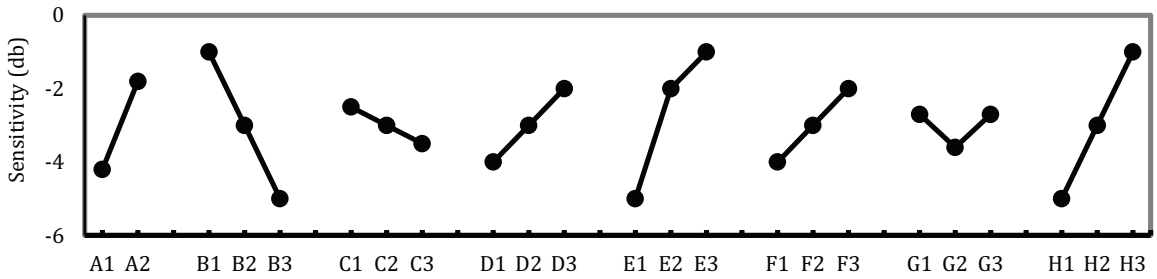

Figure 5 — Factorial effect diagram on sensitivity (mean value or linear slope)

Why is the two-step optimization important? Which is more difficult, the robustness optimization by SN ratio or the magnitude adjustment by sensitivity?

The order of the optimizations is important to design a robust system efficiently. Consider an example of the case of voice recording. If audio data are recorded with high background noise, then adjusting the volume will hardly make the recorded sound any easier to listen to when it is played back. To extract

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information buried in the noise, techniques such as noise reduction to cancel the effects of noises or using a microphone that is less sensitive to the background noise shall be employed. Improving SN ratio requires advanced technical ability and counter-measures in recording. On the other hand, when the average recording level is too low, it can easily be improved by adjusting the volume during playback. The controlling of the average value of the playback sound by a volume can often be accomplished by a relatively simple method in this way. Adjusting the magnitude can be performed by sensitivity.

Another example is visual imaging function such as photographs and video pictures. The average tone level can easily be corrected, but a picture taken in dark conditions is often very noisy and results in a low-quality picture. There are also limits to how much the picture quality can be improved by image processing.

It is relatively easy to achieve adjusting the magnitude, because it needs just one adjusting parameter. It is easy to adjust the average energy level. On the other hand, it is not easy to improve robustness. It is desired to have as many control factors as possible. In designing a system, it is therefore wise to give greater priority to setting the optimum levels of design parameters to maximize SN ratio. This forms the foundation of the two-step optimization, where robustness optimization by the SN ratio should have the first priority.

4.6 Determination of the optimum design

Once parameter design experiment clarifies which design parameters affect SN ratio and which design parameters affect sensitivity as in factorial effect diagrams, one can select a set of optimum values of design parameters based on robustness. Then, the final optimum design can be determined by considering other constraints such as cost and delivery requirements.

Since a system's final optimum design should be determined by the overall balance of many constraints, it is preferable to select an experimental plan where each experimental factor covers a wider range in a design space. There is a possibility that the optimum value might exist in the range far from experiences; then it is recommended that the levels of control factors should cover a range as widely as possible.

In the parameter design, robustness optimization is performed through maximizing SN ratio. The SN ratio is a quantitative measure of the user's quality loss due to defects, failures, and quality problems caused by lack of robustness. The user's quality loss can include losses by malfunctions, by defects, by additional maintenance costs, etc.

According to Taguchi's quality loss function, the SN ratio can be transformed to user's quality loss in monetary unit. Total loss to the society caused by the product can be derived from the user's quality loss by adding other cost, such as product development cost, material cost, production cost, shipping cost, normal maintenance cost, disposal cost, etc. The total social loss should be a quality measure of the product. In product design stage, a product designer should consider the total social loss from the viewpoint of technology. However, it is difficult for the designer to forecast the total social loss in the design stage, but he should, at least, assess and optimize the product design from the viewpoint of robustness. Robust parameter design focuses on the user's quality loss from the viewpoint of robust engineering, that is, the variability in function of the product.

5 Assessment of robustness by SN ratio

5.1 Concepts of SN ratio

The variability in function of a system should be evaluated and optimized through parameter design for designing a robust product. When a subsystem is to be assessed for robustness, one should consider noise conditions at the whole system level in the users' hand. It is critical to ensure the robustness at whole system level.

A system's function can be defined as a functional form of input-output relationship at usage stage of the system. Users manipulate a signal to get an intended output response of the system. Signal is an input characteristic that is intentionally set to change the output of the system. A functional form

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that represents the ideal input-output relationship of the system's function is called as a system's ideal function. However, the ideal function might not be perfectly realized in a manufactured product, and also at actual usage stage; the function of the product can deviate from the ideal function due to noise conditions. Deviation from the ideal function should be evaluated at usage stage and expressed by one numerical measure of SN ratio in the first step of parameter design.

The user's conditions, under which the system is actually used, contain only signal and noises. As mentioned above, signal is an input to the system for intentional change of the output of the system. The signal should have a large effect on the system's output. On the other hand, the effect of noises has a negative impact on the system's output. The effect of the signal should be maximized and the effect of the noises should be minimized. In the experiment for assessing the robustness through SN ratio, the characteristic for input should be treated as a signal factor, and the noise sources should be treated as noise factors. Categorizing the variables in the user's conditions is important to clarify the purpose of the experiment.

The SN ratio is a measure that quantitatively expresses how close the actual input-output relationship is to the ideal function under various noise conditions. As SN ratio increases, the actual input-output relationship becomes closer to the ideal, and the loss to society will decrease. In the opposite case, it becomes farther to the ideal, and the loss to society will increase.

5.2 Types of SN ratio

There are three types of SN ratio in robustness assessment: SN ratio for dynamic characteristics, SN ratio for static or non-dynamic characteristics, and SN ratio for digital systems.

The SN ratio for dynamic characteristic represents the stability of the relationship between the signal and the corresponding outputs. The SN ratios for dynamic characteristics can be subdivided into three types by functional forms of the system's ideal function; zero-point proportional ideal function, reference-point proportional ideal function, and linear formula ideal function. The choice of the functional form of the ideal function depends on the physics of the objective system. In many cases, the ideal function can be expressed by a zero-point proportional formula because of proportional nature of physics.

The SN ratio for static or non-dynamic characteristic represents the stability of the output of the system. The output target is fixed and the signal is constant. The SN ratios for static characteristics can be subdivided into three types by the values of system's fixed target; nominal-the-best, smaller-the-better, and larger-the-better characteristics. The choice of the fixed target depends on the system's intent. The value of fixed target is finite for nominal-the-best, zero for smaller-the-better, and infinity for larger-the-better.

SN ratio for a digital system can be applied to evaluate the performance of a digital system which has binary inputs and outputs that can only take a value of 0 or 1. In a digital system, when an input is 0 or 1, the output of the system should be 0 or 1 respectively. This input-output relationship is the ideal function of a digital system. SN ratio for a digital system represents the ability of a digital system after threshold calibration.

Procedures to formulate each type of SN ratio will be shown in the following subclauses.

5.3 Procedure of the quantification of robustness

The procedure to formulate SN ratio and sensitivity for robustness should be as follows.

— **(Step 1) Clarify the system's ideal function.**

Function is a work that a system performs in order to fulfil its objective. A function has an input signal to represent the operator's intention in a dynamic case. Output response of the system is varied by the input signal to fulfil the system's objective. Function can be expressed by a mathematical form of the relation between input signals and the output responses.

Define ideal function, i.e. intended relationship between input signals and output responses based on the objective of the system's function. Ideal function represents the system's work expected.

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In case of static/non-dynamic case, define the ideal output, the target value of the system's output, then go to step 3.

— **(Step 2) Select signal factor and its range.**

A signal for a dynamic characteristic case is an input to a system that is set actively or passively to vary the output response of the system as intended. In an experiment, the characteristic manipulating the input signal should be treated as a signal factor. A signal factor is an input variable with which the operator in the experiment sets to obtain the system's output response. The range of the signal factor should cover all the range used by users in the market.

— **(Step 3) Select measurement method of output response.**

Select an appropriate measuring method for the output characteristic. Output characteristic is also called as an output response. In dynamic function case, output is the quantity which users expect to get. There can be some difficulties in measuring time-depending responses. In such cases, appropriate measuring method should be developed for the purpose of robustness assessment.

— **(Step 4) Develop noise strategy, and select noise factors and their levels.**

Conditions that disrupt the ideal function under actual usage conditions during system's operation are called as noise conditions or error conditions. In an experiment to evaluate SN ratios, noise conditions should be generated by noise factors. Noise factors are variables that vary the output response in actual operating stages. It is recommended to discuss all possible noise conditions and to define efficient and effective noise conditions for assessing robustness. Noise factors come from "usage environment", "aging/wear", and "manufacturing variation". Noise factors should preferably include as many different types of factors as possible. Also, the levels of values of noise factors should be spread across a wide range by taking the usage conditions of ordinary users into consideration. It is possible to expand the system's lifetime, by taking noise factors due to wear through prolonged use.

— **(Step 5) Conduct experiment and collect data.**

Experimental layout for evaluating robustness by SN ratio is now defined. The layout shows which combination of levels of signal factor and noise factors will be applied in the experiment. Typical assignment is a two-way full factorial layout of a signal factor and a noise factor. In other words, the output response will be measured under various combinations of signal and noise levels. Then conduct experiment and collect data in the experimental layout. Then a dynamic SN ratio is ready to be calculated. In a static/non-dynamic characteristic case, there is no signal factor so obtain data under the noise conditions. The experimental layout for evaluating robustness corresponds to the outer array of a direct product plan of robust parameter design.

— **(Step 6) Calculate SN ratio, η, and sensitivity, S.**

SN ratio and sensitivity are calculated based on the data obtained in Step 5. Calculation formulae are shown in 5.4. The calculation formula is based on the system's ideal function defined in Step 1. SN ratio is to assess the functional variation due to noise conditions. SN ratio is a robustness measure of a system, and sensitivity, S, is an indicator representing the magnitude of the efficiency.

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5.4 Formulation of SN ratio: Calculation using decomposition of total sum of squares

5.4.1 Zero-point proportional formula (dynamic characteristic)

When the signal is zero, then the output response is zero, and the output response proportionally increases as the signal increases. If this is the ideal state, as in many cases, the ideal function should be expressed by a zero-point proportional formula such as Formula (1):

$$y = \beta \mathrm{M} \tag{1}$$

where the output response and the input signal are denoted by "y" and "M", respectively.

This function is called a zero-point proportional ideal function. The coefficient β denotes the sensitivity coefficient.

a) Data set for computing SN ratio for zero-point proportional ideal function

The data set for zero-point proportional formula are shown in Table 1, where the signal factor has k levels and the noise factor has n levels.

Table 1 — Data set for zero-point proportional ideal function

Signal		M1	M2		Mk	Linear form
Noise level	N1	y_{11}	y_{12}		y_{1k}	L_1
	N2	y_{21}	y_{22}		y_{2k}	L_2
	...	...	...		...	...
	Nn	y_{n1}	y_{n2}		y_{nk}	L_n

b) Decomposition of total sum of squares for zero-point proportional formula

Total sum of squares

$$S_{\mathrm{T}} = y_{11}{}^2 + y_{12}{}^2 + \cdots + y_{nk}{}^2 \quad (f_{\mathrm{T}} = n \times k) \tag{2}$$

Sum of squares of input signal levels/effective divisor

$$r = M_1^2 + M_2^2 + \cdots + M_k^2 \tag{3}$$

Linear forms for noise levels:

$$\begin{aligned} L_1 &= M_1 y_{11} + M_2 y_{12} + \cdots + M_k y_{1k} \\ L_2 &= M_1 y_{21} + M_2 y_{22} + \cdots + M_k y_{2k} \\ &\ldots \\ L_n &= M_1 y_{n1} + M_2 y_{n2} + \cdots + M_k y_{nk} \end{aligned} \tag{4}$$

Sum of squares due to linear slope β

$$S_\beta = \frac{(L_1 + L_2 + \cdots + L_n)^2}{n \times r} \quad (f_\beta = 1) \tag{5}$$

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Sum of squares due to the variation of linear slope β between noise levels

$$S_{N\times\beta}=\frac{L_1^2+L_2^2+\cdots+L_n^2}{r}-S_\beta \qquad (f_{N\times\beta}=n-1) \tag{6}$$

Sum of squares due to error

$$S_{\mathrm{e}}=S_{\mathrm{T}}-S_\beta-S_{\mathrm{N}\times\beta} \quad (f_{\mathrm{e}}=f_{\mathrm{T}}-f_\beta-f_{\mathrm{N}\times\beta}=n\times k-n) \tag{7}$$

Variance due to error/error variance

$$V_{\mathrm{e}}=\frac{S_{\mathrm{e}}}{f_{\mathrm{e}}}=\frac{S_{\mathrm{e}}}{n\times k-n} \tag{8}$$

Variance due to pooled error/variance due to error and noise

$$V_{\mathrm{N}}=\frac{S_{\mathrm{N}\times\beta}+S_{\mathrm{e}}}{f_{\mathrm{N}\times\beta}+f_{\mathrm{e}}} \tag{9}$$

SN ratio, η, and sensitivity, S

$$\eta=10\log\frac{\frac{1}{n\times r}(S_\beta-V_{\mathrm{e}})}{V_{\mathrm{N}}} \quad \text{(db)} \tag{10}$$

$$S=10\log\frac{1}{n\times r}(S_\beta-V_{\mathrm{e}}) \quad \text{(db)} \tag{11}$$

5.4.2 Linear formula (dynamic characteristic)

The zero-point proportional formula has very wide applications. When input energy is zero, the output energy will be zero. However, there are some other cases where the origin point is not defined and/or only linearity between input and output is desired. In such cases, one can assume the ideal function as the linear formula such as Formula (12):

$$y=\alpha+\beta\mathrm{M} \tag{12}$$

This function is called a linear formula ideal function. The coefficient β denotes sensitivity coefficient and parameter α denotes zero-section.

a) Data set for computing SN ratio for linear formula ideal function

The data set for a linear formula are shown Table 2, where the signal factor has k levels and the noise factor has n levels.

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Table 2 — Data set for linear formula ideal function

Signal		M1	M2		Mk	sum
Noise level	N1	y_{11}	y_{12}		y_{1k}	N_1
	N2	y_{21}	y_{22}		y_{2k}	N_2
	...	...	...		...	...
	Nn	y_{n1}	y_{n2}		Y_{nk}	N_n
	sum	Y_1	Y_2		Y_k	

b) Decomposition of sum of squares for linear formula

Total sum of squares

$$S_{\mathrm{T}} = y_{11}{}^2 + y_{12}{}^2 + \cdots y_{nk}{}^2 \quad (f_{\mathrm{T}} = n \times k) \tag{13}$$

Sum of squares due to mean

$$S_{\mathrm{m}} = \frac{(y_{11} + y_{12} + \cdots + y_{nk})^2}{n \times k} \quad (f_{\mathrm{m}} = 1) \tag{14}$$

The mean of signal levels

$$\overline{M} = \frac{(M_1 + M_2 + \cdots + M_k)}{k} \tag{15}$$

Sum of squares due to input signal levels from the mean

$$r = (M_1 - \overline{M})^2 + (M_2 - \overline{M})^2 + \cdots + (M_k - \overline{M})^2 \tag{16}$$

Sum of squares due to linear slope β

$$S_\beta = \frac{\left[(M_1 - \overline{M})Y_1 + (M_2 - \overline{M})Y_2 + \cdots + (M_k - \overline{M})Y_k\right]^2}{n \times r} \quad (f_\beta = 1) \tag{17}$$

Sum of squares due to the main effect of noise

$$S_{\mathrm{N}} = \frac{N_1^2 + N_2^2 + \cdots + N_n^2}{k} - S_{\mathrm{m}} \quad (f_{\mathrm{N}} = n - 1) \tag{18}$$

Sum of squares due to error

$$S_{\mathrm{e}} = S_{\mathrm{T}} - S_{\mathrm{m}} - S_\beta - S_{\mathrm{N}} \quad (f_{\mathrm{e}} = n \times k - 1 - n) \tag{19}$$

Variance due to error/error variance

$$V_{\mathrm{e}} = \frac{S_{\mathrm{e}}}{f_{\mathrm{e}}} = \frac{S_{\mathrm{e}}}{n \times k - 1 - n} \tag{20}$$

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Variance due to pooled error

$$V_{\mathrm{N}} = \frac{S_{\mathrm{N}} + S_{\mathrm{e}}}{f_{\mathrm{N}} + f_{\mathrm{e}}} \tag{21}$$

SN ratio, η, and sensitivity, S

$$\eta = 10 \log \frac{\frac{1}{n \times r}(S_{\beta} - V_{\mathrm{e}})}{V_{\mathrm{N}}} \quad \text{(db)} \tag{22}$$

$$S = 10 \log \frac{1}{n \times r}(S_{\beta} - V_{\mathrm{e}}) \quad \text{(db)} \tag{23}$$

NOTE 1 In the computation of SN-ratio, pure replication trial is not applied, but multiple trials are performed under the situation where the combinations of levels of noise factor(s) are changed and other conditions are the same. Noise factors are intentionally introduced in the replication rather than random noise conditions. Evaluation of variability under intentionally arranged noise conditions is a feature of the assessment of SN ratio.

NOTE 2 It should be noted that the formulae for the decomposition of sum of squares vary with the cases of formulae's forms. For example, sum of squares due to mean (S_{m}) is calculated for a linear formula case, but not for a zero-point proportional formula case. There are some other differences in the formulae depending on the functional forms.

5.4.3 Reference-point proportional formula (dynamic characteristic)

One fixed point, which is called the reference point, is treated like an origin point in the space. SN ratio for reference-point proportional ideal function can be calculated from the data (M', y') transformed as follows:

$$(M'\, y') = (\mathrm{M}, y) - (M_0, y_0) \tag{24}$$

Subtracting the reference point (M_0, y_0) from each data (M, y), then the ideal function can be described by a reference-point proportional formula such as,

$$(y - y_0) = \beta\,(M - M_0) \tag{25}$$

This ideal function is called a reference-point proportional ideal function. The coefficient β denotes linear slope or sensitivity coefficient.

Calculation formulae of SN ratio are the same as those for a zero-point proportional ideal function when the transformed data (M', y') are treated as the original data (M, y).

5.4.4 Nominal-the-best response (static/non-dynamic characteristic)

When a system has one finite fixed target, the system's output response is called nominal-the-best characteristic or nominal-the-best response. SN ratio for nominal-the-best response can be applied in this case in robustness assessment.

a) Data set for computing SN ratio for nominal-the-best response

Data set for static/non-dynamic response is shown in Table 3, where noise factor has n levels, and there is no signal factor. Data is collected under one design specification, such as design A. This data configuration is common for all the types of static/non-dynamic characteristic.

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Table 3 — Data set for static/non-dynamic characteristic

Signal		Design specification A
Noise level	N1	y_1
	N2	y_2
	...	...
	Nn	y_n

b) Decomposition of sum of squares for computing SN ratio for nominal-the-best response

Total sum of squares

$$S_T = y_1^2 + y_2^2 + \cdots + y_n^2 \quad (f_T = n) \tag{26}$$

Sum of squares due to mean

$$S_m = \frac{(y_1 + y_2 + \cdots + y_n)^2}{n} \quad (f_m = 1) \tag{27}$$

Sum of squares due to error

$$S_e = S_T - S_m \quad (f_e = n - 1) \tag{28}$$

Variance due to noise and error

$$V_e = \frac{S_e}{f_e} = \frac{S_e}{n-1} \tag{29}$$

SN ratio, η, and sensitivity, S

$$\eta = 10 \log \frac{\frac{1}{n}(S_m - V_e)}{V_e} \quad \text{(db)} \tag{30}$$

$$S = 10 \log \frac{1}{n}(S_m - V_e) \quad \text{(db)} \tag{31}$$

5.4.5 Smaller-the-better response (static/non-dynamic characteristic)

When a system has a non-negative output response and its ideal value is zero, the system's output response is called smaller-the-better characteristic or smaller-the-better response. SN-ratio for smaller-the-better response should be applied in this case in robustness assessment.

a) Data set for computing SN ratio for smaller-the-better response

The data set for smaller-the-better response has the common configuration of the data set for static/non-dynamic characteristic, as shown in Table 3, where the noise factor has n levels.

b) Computing SN ratio for smaller-the-better response

The mean squared deviation from the ideal value of zero is given by

$$\hat{\sigma}^2 = \frac{1}{n}(y_1^2 + y_2^2 + \cdots + y_n^2) \tag{32}$$

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In this International Standard, the notation $\hat{\sigma}^2$ is used for an estimate of a mean squared deviation (MSD).

SN ratio, η is calculated as

$$\eta = -10 \log \hat{\sigma}^2 = 10 \log \frac{1}{\hat{\sigma}^2} \quad \text{(db)} \tag{33}$$

5.4.6 Larger-the-better response (static/non-dynamic characteristic)

When a system's output response is desired to be as large as possible, this means the ideal value is infinity. The system's output response is called larger-the-better characteristic or larger-the-better response. SN ratio for larger-the-better response can be applied in the robustness assessment in this case.

In cases where the limit value y_0 is theoretically fixed, after data transformation to the difference from the limit value, $y' = y_0 - y$, the new variable y' can be considered as a smaller-the-better characteristic or nominal-the-best characteristic. It is recommended to transform the data and apply the static/non-dynamic formulation. Percentage data is an example.

a) Data set for computing SN ratio for larger-the-better response

The data set for larger-the-better response has the common configuration of the data set for static/non-dynamic characteristic, as shown in Table 3, where the noise factor has n levels.

b) Computing SN ratio for larger-the-better response

For a larger-the-better response, the inverse of y, $1/y$, is treated as a smaller-the-better characteristic. Therefore, the estimate of mean squared deviation $\hat{\sigma}^2$ is given by Formula (33):

$$\hat{\sigma}^2 = \frac{1}{n}\left(\frac{1}{y_1^2} + \frac{1}{y_2^2} + \cdots + \frac{1}{y_n^2}\right) \tag{34}$$

SN ratio, η, is calculated as

$$\eta = -10 \log \hat{\sigma}^2 = 10 \log \frac{1}{\hat{\sigma}^2} \quad \text{(db)} \tag{35}$$

5.4.7 SN ratio for digital characteristics

In computers, control systems, digital communication systems, and the like, the input and the resulting output data consist of only two digital sates, namely ones and zeroes. In such digital input and digital output systems, robustness should be evaluated by standardized SN ratio. Standardized SN ratio means "SN ratio after optimized by adjusting the threshold level". This adjustment is also called as calibration or levelling.

a) Data set for computing standardized SN ratio for digital characteristic

Considering a communication function, receiving a 0 when a 1 is transmitted constitutes an error. The rate of such errors is denoted by p. Receiving a 1 when a 0 is transmitted is also an error, and this error rate is denoted by q. This can be summarized as shown in Table 4.

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