



BSI Standards Publication

Cylindrical involute gears and gear pairs

Part 1: Concepts and geometry

National foreword

This British Standard is the UK implementation of ISO 21771-1:2024. It supersedes BS ISO 21771:2007, which is withdrawn.

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Standard**

ISO 21771-1

**Cylindrical involute gears and
gear pairs —**

**Part 1:
Concepts and geometry**

*Roues et engrenages cylindriques en développante de cercle —
Partie 1: Concepts et géométrie*

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771-1 cancels and replaces ISO 21771:2007, which has been technically revised.

The main changes are as follows:

- the sign convention for internal gears used in the ISO 6336 series^[8] has been adopted. The negative value for the number of teeth of an internal gear is applied to the diameters and centre distance, so these dimensions of internal gears have negative values;
- flank direction has been renamed as hand of helix and sign (+/-) of helix angle is used;
- a definition of normal surface has been added and this is used rather than normal plane;
- the annex on tooth thickness was removed because it is now addressed in ISO 21771-2.

Additional material has been added to cover:

- calculation of form diameters for tooth tip corner radius and tooth root fillet radius in the transverse plane for an involute cylindrical gear ([Clauses 10, 11](#) and [Annex B](#));
- calculation of the tooth tip corner radius for a specified form diameter and tip diameter of an involute cylindrical gear;
- calculation of a radius tangent to the involutes of adjacent teeth at root or tip diameter ([Annex A](#));
- generated tooth root fillet shape for individual involute cylindrical gears ([Annex B](#));
- concepts and parameters for involute cylindrical gear pairs with crossed axes ([Clause 6](#) and [Annex C](#));
- geometry of surfaces in contact ([Annex D](#));
- projection of a transverse plane profile of a tooth onto another plane ([Annex E](#));

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— interface to ISO 10828 for involute worm gear geometry ([Annex F](#)).

A list of all parts in the ISO 21771 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Cylindrical involute gears and gear pairs —

Part 1: Concepts and geometry

1 Scope

This document specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included. The formulae in this document apply to all pressure angles.

It also covers the concepts and parameters for involute cylindrical gear pairs with parallel or crossed axes, and a constant gear ratio. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

2 Normative references

There are no normative references in this document.

3 Terms, definitions, symbols, subscripts and units

3.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1.1

basic rack tooth profile for involute gear teeth

tooth profile of a normal section through the teeth of a basic rack which corresponds to an external spur gear with number of teeth $z = \infty$ and diameter $d = \infty$

Note 1 to entry: The tooth of the basic rack tooth profile is bounded by the tip line at the top and by the parallel root line at the bottom. The fillet between the straight part of the profile and the root line is a circular arc with a radius equal to ρ_{fp} .

3.1.2

counterpart rack tooth profile

rack tooth profile symmetrical to the basic rack tooth profile about the datum line P-P and displaced by half a pitch relative to it

3.1.3

nominal involute flank

pure involute flank prior to any modifications

Note 1 to entry: See [4.4](#) for more information on a gear tooth involute flank.

3.1.4**tip alteration coefficient** k

change to the addendum relating to the standard addendum of one normal module, it is made non-dimensional by dividing by the normal module

Note 1 to entry: Tip alteration is also known as addendum modification, tip shortening or truncation.

3.1.5**generating gear pair**

generating tool (rack, hob, pinion-type cutter, or grinding wheel) and the gear being machined during gear tooth machining process

3.2 Symbols

[Table 1](#) provides all the symbols used in this document.

Table 1 — Symbols

Symbol	Description	Subclause
A	pinion lower end point of meshing (near pinion root)	5.5.6.1
a_w	working centre distance of a cylindrical gear pair	5.3.3 , 5.3.5 , 6.3.12
a_{w0}	centre distance in the generating process with pinion-type cutter	11.2
B	pinion lower point of single tooth contact (LPSTC)	5.5.6.1
b	facewidth	4.3.9
b_{Ea}	length of relief near tip	8.5.2
b_{Ef}	length of relief near root	8.5.2
b_F	usable facewidth	4.3.9
b_w	active facewidth (the facewidth used)	5.5.9.2
C	working pitch point with subscript: amount of relief for modifications	5.3.5 8
C_{ay}	modification of the profile by function	8.6
C_{Ea}	amount of triangular end relief modification at tip	8.5.2
C_{Ef}	amount of triangular end relief modification at root	8.5.2
$C_{H\alpha}$	amount of transverse profile slope modification	8.3.3
$C_{H\beta}$	amount of flank line slope modification	8.4.2
$C_{i,j}$	amount of modification at point (i,j)	8.5.1
C_α	amount of profile crowning (barrelling)	8.3.4
$C_{\alpha a}$	amount of tip relief	8.3.2
$C_{\alpha f}$	amount of root relief	8.3.2
C_β	amount of flank line crowning	8.4.3
$C_{\beta I}, C_{\beta II}$	amount of end relief	8.4.1
$C_{\beta y}$	modification of the flank line by function	8.6
$C_{\Sigma y}$	modification of the flank surface by function	8.6
c	tip clearance	5.3.8 , 6.3.15
c_F	form over dimension	5.5.5
D	pinion highest point of single tooth contact (HPSTC)	5.5.6.1
d	reference diameter	4.3.4
d_a	tip diameter	4.6.5
d_{a0}	tip diameter of pinion-type cutter	11.1

Table 1 (continued)

Symbol	Description	Subclause
d_b	base diameter	4.4.2
d_{b0}	base diameter of the pinion-type cutter	11.1
d_{bK}	chamfering base diameter	8.2.2
$d_{C\alpha a}$	tip relief start diameter	8.3.2
$d_{C\alpha f}$	root relief start diameter	8.3.2
d_{Cf}	Profile control diameter	8.3.2
d_{Ea}	diameter for start of triangular end relief at tooth tip	8.5.2
d_{Ef}	diameter for start of triangular end relief at tooth root	8.5.2
d_{Fa}	tip form diameter	9.7
d_{Ff}	root form diameter	5.5.2 , 9.7
d_f	root diameter (nominal dimension)	4.6.7
d_{fE}	root diameter produced	9.6
d_{Na}	active tip diameter	5.5.2 , 6.4.3
d_{Nf}	start of active profile diameter (SAP diameter, active root diameter)	5.5.2 , 6.4.3
d_v	V-circle diameter	4.6.2
d_w	working pitch diameter	5.3.5
d_y	Y-circle diameter	4.6.3
d_0	reference diameter of pinion-type cutter	11.1.1
E	pinion upper end point of meshing (near pinion tip)	5.5.6.1
E_{sni}	lower tooth thickness limit deviation for normal tooth thickness at reference cylinder	9.4
E_{sns}	upper tooth thickness limit deviation for normal tooth thickness at reference cylinder	9.4
e_n	normal space width on the reference cylinder	4.8.7
e_p	space width of the basic rack tooth profile	4.3.3
e_t	transverse space width on the reference cylinder	4.8.4
e_{yn}	normal space width on the y-cylinder	4.8.7
e_{yt}	transverse space width on the y-cylinder	4.8.4
F	force	D.3
g_a	length of addendum path of contact	5.5.6.2 , 6.4.5.2
g_f	length of dedendum path of contact	5.5.6.2 , 6.4.5.2
$g_{yft}(\theta)$	angle from line of centres to point on the gear fillet generated with pinion-type cutter	B.4.2
g_α	length of path of contact	5.5.6.2
$g_{\alpha y}$	distance of a point Y from working pitch point C	5.7.5
g_β	overlap roll length (arc of contact)	5.5.9.4
h	tooth depth (between tip line and root line)	4.7.1
h_a	addendum from reference pitch circle	4.7.2
h_{aP}	addendum of the basic rack tooth profile	4.3.2
h_{aP0}	addendum of the counterpart of the basic rack tooth profile	9.2.2
h_{aw}	addendum from working pitch circle	4.7.2
h_{a0}	addendum of pinion-type cutter	11.1.1
h_{FaP0}	straight part of tip flank of tool-generating profile	9.2.2
h_{FfP}	portion of the dedendum to root form line, of the basic rack tooth profile	4.3.2
h_{FfP0}	portion of the dedendum to root form line of the counterpart rack tooth profile	9.2.1
h_{fP0}	dedendum of the counterpart rack tooth profile and rack tool	9.2.2
h_f	dedendum from reference circle	4.7.2

Table 1 (continued)

Symbol	Description	Subclause
h_{fp}	dedendum of the basic rack tooth profile	4.3.3
h_{fw}	dedendum from working pitch circle	4.7.2
h_K	height of the tip corner chamfering or tip corner rounding	8.2.2
h_p	tooth depth of basic rack tooth profile	4.3.2
h_w	working depth of teeth in a gear pair	5.3.7 , 6.3.14
i	transmission ratio of a gear pair	5.3.2
inv	involute function (not a variable)	4.4.5
j_{bn}	normal base backlash	5.6.4 , 6.5.2
j_{bt}	transverse backlash	5.6.2
j_r	radial backlash	5.6.5
j_t	circumferential backlash at the reference circle	5.6.3 , 6.5.3
j_{wn}	working normal backlash	6.5.2
j_{wt}	circumferential backlash at the working pitch circle	5.6.3 , 6.5.3
K_g	sliding factor	5.7.6
K_{ga}	sliding factor at tooth tip	5.7.6
K_{gf}	sliding factor at tooth root	5.7.6
k	tip alteration coefficient	3.1.3 , 4.6.4 , 5.3.9 , 6.3.16
L_{Ca}	tip relief roll length	8.3.2
L_{Cf}	root relief roll length	8.3.2
L_{Cl} , L_{CII}	length of end relief	8.4.1
L_{Ea}	tip roll length of triangular end relief modification	8.5.2
L_{Ef}	root roll length of triangular end relief modification	8.5.2
L_y	length of roll to y-cylinder	4.4.10
L_{yt}	length of involute profile to y-cylinder	4.4.11
l_{max}	maximum length of a contact line	5.5.10
M_y	a point on a tooth flank where radius of curvature is calculated	7.1
m	module	4.3.7
m_n	normal module	4.3.7
m_t	transverse module	4.3.7
m_x	axial module	4.3.7
N	number of tooth or pitch	4.2.6
n_a	rotational speed of driving gear (rpm)	5.3.2
n_b	rotational speed of driven gear (rpm)	5.3.2
O	centre of a circle	6.3.4
P_d	diametral pitch	4.3.8
P_{nd}	normal diametral pitch	4.3.8
p_{bn}	normal base pitch	4.5.5.1
p_{bt}	transverse base pitch	4.5.5.1
p_{en}	normal base pitch on the path of contact	4.5.5.3
p_{et}	transverse base pitch on the path of contact	4.5.5.2
p_n	normal pitch on the reference cylinder	4.5.2.2
p_t	transverse pitch on the reference cylinder	4.5.2.1
p_{wn}	normal pitch at the working diameter	6.3.6 , 6.5.2

Table 1 (continued)

Symbol	Description	Subclause
p_x	axial pitch	4.5.4
p_{yn}	normal pitch on the y-cylinder	4.5.3
p_{yt}	transverse pitch on the y-cylinder	4.5.3
p_z	lead	4.4.6
pr	protuberance of the tool (as seen in ISO 6336-3)	9.2.1
q	machining allowance on tooth flank	9.3
q_{Fs}	magnitude (amount) of undercut in transverse plane	8.2.1
R'	first principal radius of curvature of surface	7.4
R''	second principal radius of curvature of surface	7.4
R_c	radius of curvature of trochoid at point M	10.4
R_{fp}	radius of curvature of the basic rack profile at point Q	10.4
R_{tro-y}	radius of the fillet at point Q	10.4
r_{a0}	tip radius of the pinion-type cutter	11.1.1
r_{b0}	base radius of the pinion-type cutter	11.2
r_{ea}	x axis of ellipse	10.4
r_{eb}	y axis of ellipse	10.4
r_{Fa0}	tip form radius of the pinion-type cutter	11.1.1
r_{Ff}	root form radius	10.3
r_{inv}	radius to point on involute	10.1 , 11.4.4.1
r_{M0}	radius for the centre of the tool tooth tip rounding of the pinion-type cutter	11.1.1
r_{tro}	radius to point on trochoid	10.1
r_w	manufacturing pitch circle radius of the gear	11.4.3
r_{w0}	manufacturing pitch circle radius of the pinion-type cutter	11.4.3
$r_{ya0}(\theta)$	radial polar coordinate of a point on tip radius of pinion-type cutter	Figure B.1
$r_{yft}(\theta)$	radial polar coordinate of a point on the gear fillet generated with pinion-type cutter	B.4.2
S_α	twist of the transverse profile	8.5.3
S_β	twist of the flank line	8.5.3
s_{aK}	tip transverse tooth thickness when tip chamfering or tip rounding	8.2.2
s_n	normal tooth thickness at the reference diameter	4.8.6
s_{ni}	minimum normal tooth thickness at the reference diameter	9.4
s_{ns}	maximum normal tooth thickness at the reference diameter	9.4
s_p	tooth thickness of the basic rack tooth profile	4.3.3
s_{pr}	residual fillet undercut (on normal surface)	10.1
s_{prt}	residual fillet undercut (transverse plane)	11.4.1
s_t	transverse tooth thickness at the reference diameter	4.8.2
s_{wn}	normal tooth thickness at working diameter	6.3.6 , 6.5.2
s_{wt}	transverse tooth thickness at working diameter	6.3.8
s_{yn}	normal tooth thickness at the Y circle diameter	4.8.6
s_{yt}	transverse tooth thickness at the Y circle diameter	4.8.2
T	tangent point on base circle of line normal to involute	4.4.9
T_1	point of contact between the line of action and the base circle of pinion	5.5.6.1
T_2	point of contact between the line of action and the base circle of gear wheel	5.5.6.1
T_1	unit vector of reference helix	4.10
T_2	unit vector of generator	4.10

Table 1 (continued)

Symbol	Description	Subclause
T_{M1}	tangency point between the base cylinder of the pinion and a line normal to a contact line through point M_y	7.1
T_{M2}	tangency point on the base cylinder of the gear wheel and a line normal to a contact line through point M_y	7.1
U	involute point of origin	4.4.9
u	gear ratio	5.3.1
v	circumferential velocity at reference diameter	5.7.2
v_b	circumferential velocity at base diameter	5.7.2
v_g	sliding velocity	5.7.5
v_{ga}	sliding velocity at the active addendum	5.7.5
v_{gf}	sliding velocity at the active dedendum	5.7.5
v_{Mg}	sliding velocity at point M	6.6.4
v_n	normal velocity	5.7.3
v_{ry}	rolling velocity at diameter d_y	5.7.4
v_w	circumferential velocity of the working pitch circles	5.7.2 , 5.7.6
v_y	circumferential velocity at diameter d_y	5.7.2
x	profile shift coefficient	4.3.10
x_E	generating profile shift coefficient	9.2.1
x_{Ei}	lower limit generating profile shift coefficient	9.5
x_{Es}	upper limit generating profile shift coefficient	9.5
x_{EsV}	profile shift coefficient for rough machining, upper limit	9.5
x_{EiV}	profile shift coefficient for rough machining, lower limit	9.5
x_{Eu}	generating profile shift coefficient at undercut limit	9.8
x_0	profile shift coefficient of the pinion-type cutter	11.1.1
X_1	point on x axis of coordinate system of surface 1	D.1
X_2	point on x axis of coordinate system of surface 2	D.1
Y	any point on a tooth flank or involute	4.4.3
Y_1	point on y axis of coordinate system of surface 1	D.1
Y_2	point on y axis of coordinate system of surface 2	D.1
z	number of teeth	4.2.5
z_a	number of teeth of driving gear	5.3.2
z_b	number of teeth of driven gear	5.3.2
z_0	number of teeth of pinion-type cutter	11.1.1
Z_1	point on z axis of coordinate system of surface 1	D.1
Z_2	point on z axis of coordinate system of surface 2	D.1
α_a	pressure angle at tip circle	5.5.8.3
α_{Fa}	pressure angle at the tip form diameter d_{Fa}	A.2.1
α_{Fa0}	pressure angle on the radius r_{Fa0} of pinion-type cutter	11.1.1
α_{Ff}	pressure angle at root form circle	9.7
α_{KP0}	normal chamfering pressure angle of the counterpart rack tooth profile	8.2.2
α_{KP0t}	transverse chamfering pressure angle of the counterpart rack tooth profile	8.2.2
α_{Mt0}	transverse pressure angle for the radius at the point M of the pinion-type cutter	11.1.1
α_{NP}	pressure angle at start of active profile	5.5.2.2
α_n	normal pressure angle	4.4.4
α_p	pressure angle of the basic rack tooth profile	4.4.4

Table 1 (continued)

Symbol	Description	Subclause
α_{p0}	pressure angle of the counterpart rack tooth profile	9.2.1
α_{pr0}	pressure angle of the protuberance section of the counterpart rack tooth profile	9.2.1
α_t	transverse pressure angle	4.4.3
$\alpha_{w\min}$	working normal pressure angle at minimum (zero-backlash) centre distance	6.3.12
α_{wn}	working normal pressure angle of gear pair	6.3.13
α_{wt}	working transverse pressure angle of gear pair	5.3.4 , 6.3.7
α_{wt0}	working transverse pressure angle in the generating gear pair	11.2
α_{yn}	normal pressure angle at the y-cylinder	4.4.4
α_{yt}	transverse pressure angle at the y-cylinder	4.4.3
β	helix angle	4.4.7
β_b	base helix angle	4.4.7
β_R	Helix angle of right flanks	4.4.7
β_{M0}	helix angle on the circle of radius r_{M0}	11.1.1
β_L	Helix angle of left flanks	4.4.7
β_v	helix angle at diameter d_v	4.3.9
β_w	helix angle at working pitch diameter	5.3.6
β_y	helix angle at y-cylinder	4.4.7
β_0	helix angle of pinion-type cutter	11.1.1
γ	lead angle at reference cylinder	4.4.7
γ_y	lead angle at y-cylinder	4.4.7
Δ	sum of inverse radius of curvature of surfaces	7.4
δ	tilt angle of the contact line at the reference cylinder	4.10
δ_{pr0}	residual fillet undercut angle in transverse plane	11.4.1
δ_w	angle between the principal direction of curvature and working pitch plane	7.2
δ_y	tilt angle of the contact line at y-cylinder	4.10
ε_α	transverse contact ratio	5.5.9.1 , 6.4.7.1
$\varepsilon_{\alpha n}$	contact ratio in the common normal working plane of crossed axis gear pair	6.4.7.2
ε_β	overlap ratio (axial contact ratio)	5.5.9.3
ε_γ	total contact ratio	5.5.9.5
ζ	specific sliding	5.7.7
ζ_f	specific sliding at end points of path of contact	5.7.7
η	transverse space width half angle at reference circle	4.8.5
η_b	base transverse space width half angle	4.8.5 , 6.3.11
η_{inv}	angle between the start of involute profile on base circle and the point r_{inv} on involute	10.1
η_M	pressure angle at tip form radius minus the angle from tip form radius point to centre of tip rounding circle on pinion-type cutter.	11.1.1
η_{tro}	angle between the start of involute profile on base circle and the point r_{tro} on trochoid	10.1
η_{wt}	transverse space width half angle at working pitch diameter	6.3.8
η_y	transverse space width half angle at Y-circle	4.8.5
θ_{a1}	polar angle on the pinion tooth flank from pitch point to tip circle	5.5.8.3
θ_{a2}	polar angle on the gear tooth flank from pitch point to tip circle	5.5.8.3
θ_{aa1}	angle from the pitch point to the intersection of the tip circles on the pinion'	5.5.8.3
θ_{aa2}	angle from the pitch point to the intersection of the tip circles of an internal gear	5.5.8.3
θ_M	angle used to define point M on ellipse	11.4.5

Table 1 (continued)

Symbol	Description	Subclause
$\kappa_{\text{yft}}(\theta)$	angular polar coordinate of a point on the gear fillet generated with pinion-type cutter	B.4.2
λ	angle between the pitch line of the basic rack and the line joining the common normal at the generated point Q crossing the pitch point	10.1
λ_{M0}	polar angle on a pinion-type cutter from centre of tip rounding circle to point where the tip rounding circle transitions to the involute (at tip form diameter)	11.1.1
ξ_{Fa0}	roll angle at tip form circle of pinion-type cutter	9.7
ξ_{Ff}	roll angle at root form circle	9.7
ξ_{Na}	roll angle at active tip circle	5.5.2
ξ_{Nf}	roll angle at active root circle	5.5.2
ξ_{y}	roll angle of the involute at point Y	4.4.9
ρ_{A}	radius of curvature in transverse plane at point A, start of engagement	5.5.7
ρ_{a}	tooth tip corner radius of gear	Annex A
ρ_{aP0}	tooth tip corner radius of the hob	9.2.2
ρ_{a0n}	tooth tip corner radius of pinion-type cutter (on normal surface)	11.1.1
ρ_{B}	radius of curvature in transverse plane at inner point of single pair contact on the pinion	5.5.7 , 7.1
ρ_{C}	radius of curvature in transverse plane at point C, working pitch point	5.5.7 , 7.1
ρ_{D}	radius of curvature in transverse plane at outer point of single pair contact on the pinion	5.5.7 , 7.1
ρ_{E}	radius of curvature in transverse plane at end point of engagement	5.5.7 , 7.1
ρ_{f}	tooth root fillet radius of gear	Annex A
ρ_{fP}	tooth root fillet radius on the basic rack tooth profile	10.4
ρ_{fP0}	tooth root fillet radius of the counterpart rack tooth profile	8.2.2
ρ_{M}	radius of curvature in transverse plane at point M	7.1
ρ_{y}	radius of curvature of the involute at diameter d_{y}	4.4.10
ρ_{yn}	radius of curvature on normal surface at diameter d_{y}	4.9
ρ_{yt}	radius of curvature in transverse plane at diameter d_{y}	4.9
Σ	skew angle of crossed axis gear pair	6.3.5 , 6.3.12
Σ_{l}	sum of individual lines of contact	5.5.10
Σ_{R}	sum of inverse of main radii of curvature of equivalent ellipsoid	7.4
Σ^{x}	sum of profile shift coefficients	5.4
$\Sigma^{\text{x}}_{\text{E}}$	sum of profile shift coefficient, non-zero backlash	5.4
τ	angular pitch	4.5.1
ϕ_{yt}	angle of vector radius OM according Y axis	E.1
φ	trochoid/tip parameter	10.1
φ_{w}	angle between the principal radii of curvature in a crossed axis gear pair	7.3
φ_{j}	angular backlash	5.6.6
φ_{α}	transverse angle of transmission	5.5.9.1 , 6.4.7.1
φ_{β}	overlap angle	5.5.9.3
φ_{y}	total angle of transmission	5.5.9.5
$\chi_{\text{ya0t}}(\theta)$	angular polar coordinate of a point on tip radius of the pinion type cutter	Figure B.1
ψ	transverse tooth thickness half angle at reference circle	4.8.3
ψ_{b}	base transverse tooth thickness half angle	4.8.3
ψ_{y}	transverse tooth thickness half angle at Y-circle	4.8.3
ψ_{wt}	transverse tooth thickness half angle at working pitch diameter	6.3.8

Table 1 (continued)

Symbol	Description	Subclause
ω	angular velocity	5.5.8.3 , 5.7.1 , 6.6.2
ω_a	angular velocity of driving gear	5.3.2
ω_b	angular velocity of driven gear	5.3.2
ω_0	angle between the centre line of gear pair and centre point of tooth tip rounding of pinion-type cutter	11.4.2

The variables in [Table 1](#) can have an appended subscript 1 to indicate the variable is for a quantity associated with the pinion or a subscript 2 to indicate the variable is for a quantity associated with the gear wheel.

3.3 Subscripts

[Table 2](#) provides the subscripts used in this document.

Table 2 — Subscripts

Subscript	Description
—	no subscript designates quantities associated with the reference cylinder
a	for quantities associated with the tip of a tooth or for the driving gear
b	for quantities associated with the base cylinder or for the driven gear
E	relating to “generating” (e.g. quantities generated on the cylindrical gear) or to triangular end relief modification
e	for quantities associated with the plane of action
F	for quantities determining form circles and maximum usable flank area
f	for quantities associated with the root
g	for “sliding”
i	for the lower limit in the case of deviations or an index value
K	for quantities resulting from tip corner chamfering
k	for a number of teeth, tooth spaces, pitches or spans
L	for designating left flanks
l	for “left-hand”
M	for designating a measured value
M0	relating to centre of tip rounding circle or ellipse on tool tooth tip
m	for a mean value
max	for a maximum value
min	for a minimum value
N	for active circles
n	for quantities in a normal surface
P	for quantities of the basic rack tooth profile
P0	for quantities of the tool basic rack tooth profile and for counterpart rack tooth profile
R	for designating right flanks
r	for “right-hand”
s	relating to “tooth thickness” or for the upper limit in the case of deviations
t	for quantities in a transverse section
V	for working pitch element, for rough gear cutting
v	for quantities associated with the V-cylinder
W	for measuring base tangent length

Table 2 (continued)

Subscript	Description
w	for quantities associated with the working pitch cylinder and working values of a gear pair
x	for quantities in an axial section
y	for values at a point y (on the y-cylinder)
Z	for quantities associated with cylinder dimensions
α	for quantities associated with contact
β	for quantities associated with a tooth trace
γ	for total contact ratio
Σ	for “sum”
0	for quantities associated with the generating tool or the generating gear pair
1	for quantities associated with the pinion (smaller gear) of a gear pair
2	for quantities associated with the gear wheel (larger gear) or internal gear
I	for locating face
II	for the face opposite the locating face

3.4 Units and conventions

3.4.1 Units

The quantities dealt with in this document are to be stated in the following units:

- modules, lengths and linear dimensions in millimetres (mm);
- linear velocities (circumferential, normal, rolling, sliding) in metres per second (m/s);
- angles which are to be used in formulae in radians (rad);
- angles which can be used for entries or to display results in degrees (°);
- angular velocity in radians per second (rad/s).

3.4.2 Conventions

For internal gears, the number of teeth is given a negative value. So, in this document, as in the ISO 6336 series, the centre distance and diameters of internal gears have negative values. The notation $|z|$, denotes the absolute value, which is always positive, e.g. $|-50| = +50$. The expression $\frac{z}{|z|}$ is used to extract the sign of

the tooth number and is convenient for programming. In particular, it is often used to determine the appropriate sign for an element of an expression; the result is 1 for external gears and -1 for internal gears.

In this document, the driving gear is not used to define point A on the path of contact. Instead, point A is where the line of action intersects the active tip diameter, d_{Na2} , of the gear wheel. The last point, E, is where the line of action intersects the active tip diameter, d_{Na1} , of the pinion. See [5.5.6.1](#).

4 Individual cylindrical gears

4.1 General

In this clause, the geometry of gear teeth is described using a generation process based on zero-backlash engagement with a basic rack. The relationships are valid for any basic rack, but the standard basic rack tooth profile (see ISO 53) is used for illustration purposes. The standard basic rack tooth profile of the tooth system has straight flanks. Its datum line is the straight line on which the nominal dimensions of tooth

thickness and space width are defined as equal to half the pitch. The standard basic rack tooth profile has the same pressure angles for the left and right flanks and the dedendum equal to the addendum plus tip clearance for the mate. All the teeth have the same nominal helix angle. Formulae for calculation of start of involute diameter are given in [Clause 10](#) for hob and rack type cutters and in [Clause 11](#) for pinion-type cutters.

Information on projection of the transverse profile of a gear tooth to other planes is given in [Annex E](#).

Information helpful to interface this document to the geometry of worms defined with ISO-10828 is given in [Annex F](#).

4.2 Concepts for an individual gear

4.2.1 Gear, cylindrical gear, external gear and internal gear

A gear is a rotationally symmetrical object (gear blank) with a tooth system located at the rim. A cylindrical gear is a gear with a cylindrical reference surface. A distinction is made between external and internal gears according to the radial arrangement of the teeth in each case. The tips of the teeth point outwards in an external gear and inwards in an internal gear.

4.2.2 Tooth system, external teeth and internal teeth

The tooth system refers to all the teeth and tooth spaces around the rim of a gear. As in [4.2.1](#), a distinction is made between internal and external gear teeth.

4.2.3 Tooth and tooth space

A tooth is a geometrical element on the gear that enables the transmission of force and motion. The form and dimensions of the teeth and the distance between consecutive teeth are defined by the tooth system parameters. The tooth space is the gap between two consecutive teeth, it is bounded by the tooth flanks, fillets, root, and tip cylinder.

4.2.4 Tooth system parameters

The nominal dimensions of involute cylindrical gear teeth are uniquely determined by the diameter of the reference cylinder, the associated basic rack and its position in relation to the reference cylinder. The nominal dimensions are defined by the following parameters, which are independent of each other:

- internal or external gear;
- number of teeth, z ;
- basic rack tooth profile;
- normal module, m_n ;
- helix angle, β , with sign or helix angle along with hand of helix;
- profile shift coefficient, x ;
- tip diameter, d_a ;
- facewidth, b ;
- root diameter, d_f .

4.2.5 Number of teeth and sign of number of teeth

The number of teeth around the rim of the gear is denoted by z .

The number of teeth, z , of an external cylindrical gear shall be taken as a positive value in the formulae in this document, while the number of teeth, z , in an internal cylindrical gear is to be taken as a negative value. In keeping with the ISO 6336 series, for internal gears, in addition to the number of teeth, the diameters and centre distances are taken as negative values.

NOTE 1 The negative sign for number of teeth z corresponds to the idea that in the transition from an external gear to an internal gear, the gear diameter d and number of teeth z is increased until $d=+\infty$ and $z=+\infty$ is reached. In the course of the transition, d and z change to $-\infty$ and then assume finite negative values.

With this rule and the definition of the sign of the profile shift coefficient it is possible to use the same formulae without any change for external and internal gears.

NOTE 2 For internal gears, although the diameters used in calculations are negative, on drawings the diameters are usually shown as positive numbers.

In the case of segments, the number of teeth, z , used in calculations is the number that there would be on the whole circumference.

4.2.6 Tooth number

When numbering teeth, the designations tooth 1, tooth 2, etc. are to be defined on a transverse surface (datum face) viewed in an agreed direction so that the teeth are numbered in ascending order (moving in a clockwise direction). If the letter N is used to denote a reference tooth, the next tooth in the direction of counting is denoted by $N + 1$ and the previous tooth going in the opposite direction by $N - 1$. Tooth z is followed by tooth 1 in the direction of counting, see [Figure 1](#).

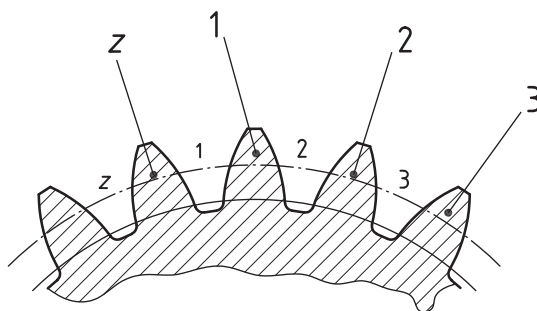


Figure 1 — Numbering of teeth and tooth spaces on datum face

4.2.7 Top land and bottom land

4.2.7.1 Top land

The top land of a tooth is the outermost (innermost in the case of internal gears) periphery of the tooth, it is concentric to the reference cylinder, see [Figure 2](#).

4.2.7.2 Bottom land

The bottom land is the innermost (outermost in the case of internal gears) periphery of the tooth space, it is concentric to the reference cylinder, see [Figure 2](#).

4.2.8 Tooth flanks and flank sections

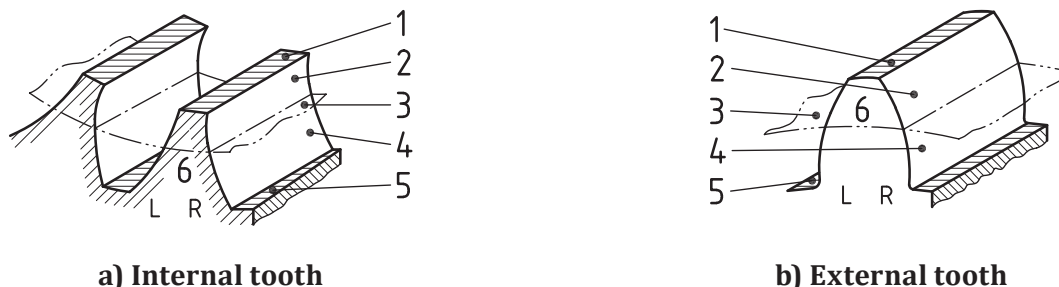
4.2.8.1 Tooth flank

Tooth flanks are those parts of the surface of a tooth that are located between the top land and the bottom land, see [Figure 2](#).

4.2.8.2 Right flank and left flank

The right flank (or left flank) is the tooth flank that an observer sees on the right-hand (or left-hand) side when viewing the datum face of a tooth when it is pointing upwards. This definition applies to both external and internal gears, see [Figure 2](#).

Right flank parameters are indicated by the subscript R and left flank parameters by the subscript L.



Key

1	top land	4	dedendum flank
2	addendum flank	5	bottom land
3	reference cylinder	6	datum face

Figure 2 — Top land, bottom land and tooth flank (from datum face)

4.2.8.3 Addendum flank

The addendum flank is that part of a tooth flank that is located between the reference cylinder and the top land, see [Figure 2](#).

4.2.8.4 Dedendum flank

The dedendum flank is that part of a tooth flank that is located between the reference cylinder and the bottom land, see [Figure 2](#).

4.2.8.5 Usable flank

The usable flank is that part of a tooth flank that can be used to engage with a mating flank. On a cylindrical gear, it is part of the involute helicoid including any flank modifications.

4.3 Reference surfaces, datum lines and reference quantities

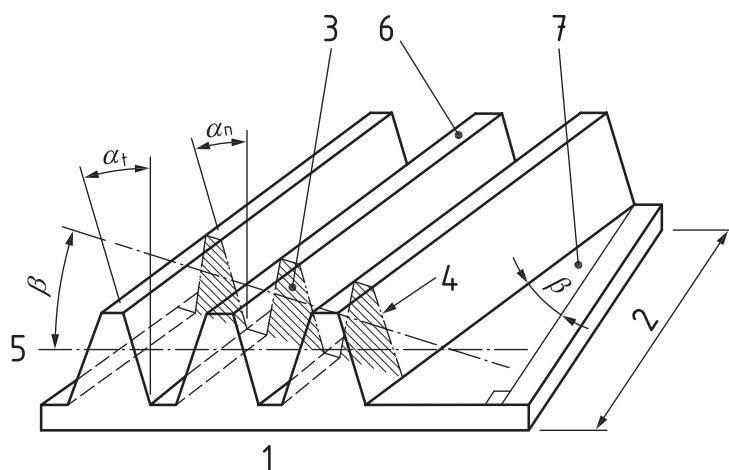
4.3.1 Reference surface and datum face

The reference surface of the teeth is an imaginary cylindrical surface to which the geometrical parameters relate. In the case of cylindrical gears, the reference surface is termed the reference cylinder, see [Figure 2](#).

The agreed front of the gear (usually used for text or suitably marked) is defined as the datum face, see [Figure 2](#). Parameters which relate to the datum face are denoted by the subscript I while parameters which relate to the opposite face are denoted by the subscript II.

4.3.2 Reference rack

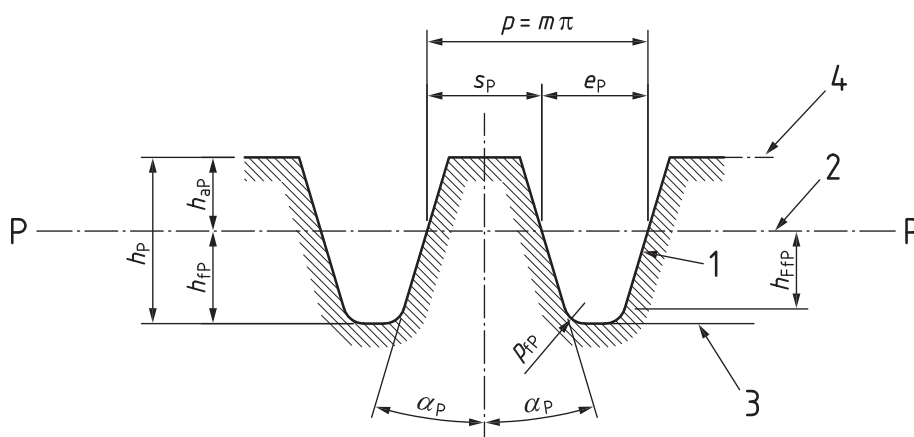
The reference rack is the rack that can be produced using the same gear-cutting tool, gear-cutting method and pitch point (pitch axis) as the actual cylindrical gear. It is characterized by its basic rack tooth profile, the direction of its teeth in relation to the pitch axis of the generating gear, tip plane, root plane and facewidth, see [Figure 3](#).



Key

- | | | | |
|---|--------------------------|---|------------|
| 1 | transverse section | 5 | pitch axis |
| 2 | facewidth | 6 | tip plane |
| 3 | normal section | 7 | root plane |
| 4 | basic rack tooth profile | | |

Figure 3 — Concepts and parameters relating to reference rack



Key

- | | | | |
|---|--------------------|---|-----------|
| 1 | basic rack profile | 3 | root line |
| 2 | datum line | 4 | tip line |

Figure 4 — Terms and parameters relating to basic rack tooth profile in normal section

4.3.3 Basic rack tooth profile for involute gear teeth

The basic rack tooth profile is defined in a normal section of the reference rack. The flank lines of the basic rack tooth profile of involute teeth are straight lines. Tooth thickness, s_p , and space width, e_p , on the datum line of the basic rack (P-P) in the normal section are equal, see [Figures 4](#) and [47](#).

Both flanks are symmetric with the same pressure angle and have a tooth root fillet radius ρ_{fP} with a maximum value corresponding to a full/complete radius. No peaks are allowed in the root.

The default basic rack tooth profile does not have any flank modifications, i.e. neither undercut near the root or tip chamfering are considered. Those aspects are considered in [Clauses 7](#) and [8](#).

This document is valid for any basic rack tooth profile. Some standardized rack tooth profiles are given in ISO 53, these should just be considered as examples of profiles that can be used.

4.3.4 Reference cylinder, reference circle and reference diameter

The reference cylinder is the reference surface for the cylindrical gear teeth. Its axis coincides with the axis of the gear (datum axis). The reference circle is the intersection of the reference cylinder with a transverse plane section. The reference diameter, d , is determined by [Formula \(1\)](#):

$$d = z m_t = \frac{z m_n}{\cos \beta} \quad (1)$$

NOTE 1 For internal gears, since the number of teeth is negative, the reference diameter is also negative.

NOTE 2 The reference diameter is the location where a gear's tooth thickness, pressure angle and helix angle are defined.

NOTE 3 At the reference diameter, the normal pressure angle is equal to the basic rack pressure angle ($\alpha_n = \alpha_p$).

4.3.5 Gear axis

The axis of a gear (datum axis) is the axis defined by the geometrical axis of the datum surfaces.

4.3.6 Sections through a cylindrical gear

4.3.6.1 Transverse section and transverse profile

The sectioning of cylindrical gear teeth by a plane perpendicular to the gear axis yields a transverse section. For helical gears, quantities in the transverse section are denoted by the subscript t . The intersection of a tooth with a transverse plane is termed the transverse profile.

NOTE The terms “transverse plane” and “transverse section” can be used interchangeably.

4.3.6.2 Normal surface, normal plane and normal profile

A normal plane is normal (perpendicular) to a flank line (see [4.3.6.4](#)) at a specified point. The sectioning of gear teeth by a normal plane yields a normal section. A normal section applies to racks and spur gears, it generally is not applicable to helical gears.

For a helical gear, for any helix (or flank line) on a given y -cylinder, a normal plane is normal to the helix at the considered point. This normal plane is not normal to other helices.

A complementary helix is normal to helices at the intersection of right and left flanks of a tooth with a y -cylinder. The complementary helix has a helix angle of $90^\circ - |\beta_y|$ (equivalent to lead angle γ_y).

A normal surface is defined by the complementary helices that intersect a radial axis of symmetry of a tooth. It is curved three dimensionally.

A normal profile is the intersection of a normal surface with the tooth flanks. For a helical gear, the normal profile does not lie in a plane.

Subscript “ n ” is used for parameters relating to the normal profile.

For spur gears and racks, as flank lines are all parallel straight lines, the normal surface as well as the normal profile are in a plane.

For spur gears the normal surface lies in a transverse plane, so the transverse profile and normal profile of a spur gear are identical.

NOTE In the past, the normal surface has been referred to as the normal plane; this is not correct for helical gears.

4.3.6.3 Axial section and axial profile

The sectioning of cylindrical gear teeth by a plane containing the gear axis yields an axial section.

Quantities in the axial section are denoted by the subscript x. The intersection of a tooth with an axial plane is termed the axial profile.

4.3.6.4 Cylindrical section and flank lines

The flank lines are the intersections of the right and left flanks with a cylinder that has an axis which coincides with the gear axis. Hence, right and left flank lines are to be distinguished.

The flank lines are helices in the case of helical gear teeth and straight lines in the case of spur gear teeth.

The reference flank line (tooth trace) is the intersection of the flank with the reference cylinder. The base flank line is the intersection of the unmodified involute flank - possibly imagined as extended - with the base cylinder. The origin of the involute helicoid is a base flank line. The tip flank line is the line of intersection of the unmodified involute flank - possibly imagined as extended - with the tip cylinder.

4.3.7 Module

The module of a basic rack is found as the pitch of the rack divided by the number π (see [Figure 4](#)). The normal module, m_n , of the cylindrical gear is found as the module of the basic rack tooth profile.

For a helical gear, the transverse module, m_t , is found as:

$$m_t = \frac{d}{z} = \frac{m_n}{\cos \beta} \quad (2)$$

and the axial module, m_x , as

$$m_x = \frac{m_n}{\sin |\beta|} = \frac{m_n}{\cos \gamma} = \frac{m_t}{\tan |\beta|} \quad (3)$$

For a spur gear, $m_t = m_n$.

4.3.8 Diametral pitch

The transverse diametral pitch is the ratio of the number of teeth to the reference diameter in inches. The diametral pitch has units of inverse inches.

$$P_d = \frac{z}{d} = \frac{\pi}{p_t} \quad (4)$$

NOTE P_d is a deviation from the symbol convention where “t” is used to denote the transverse plane.

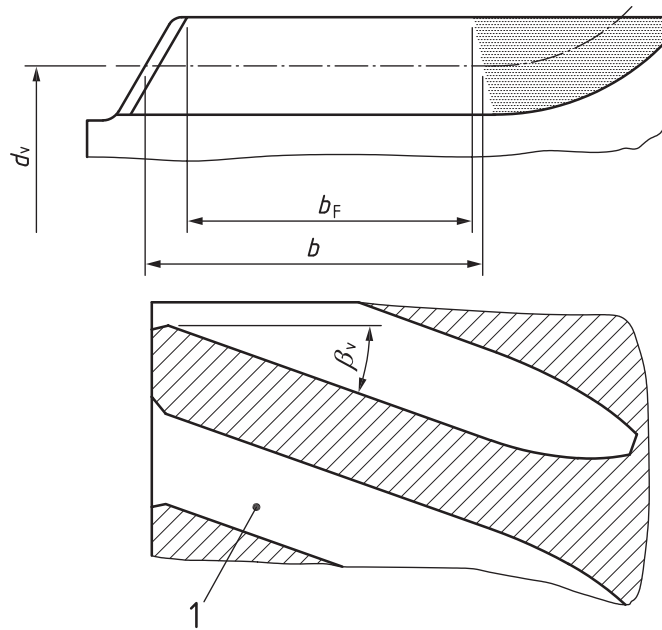
The normal diametral pitch is the value of diametral pitch on a normal surface of a helical gear or worm.

$$P_{nd} = \frac{25,4}{m_n} = \frac{P_d}{\cos \beta} \quad (5)$$

4.3.9 Facewidth

The facewidth, b , is the length of the toothed part of the cylindrical gear measured in the axial direction on the V-cylinder. See [4.6.2](#).

The usable facewidth, b_F , is the distance between two transverse sections that contain the fully developed height of the tooth flank. See [Figure 5](#).

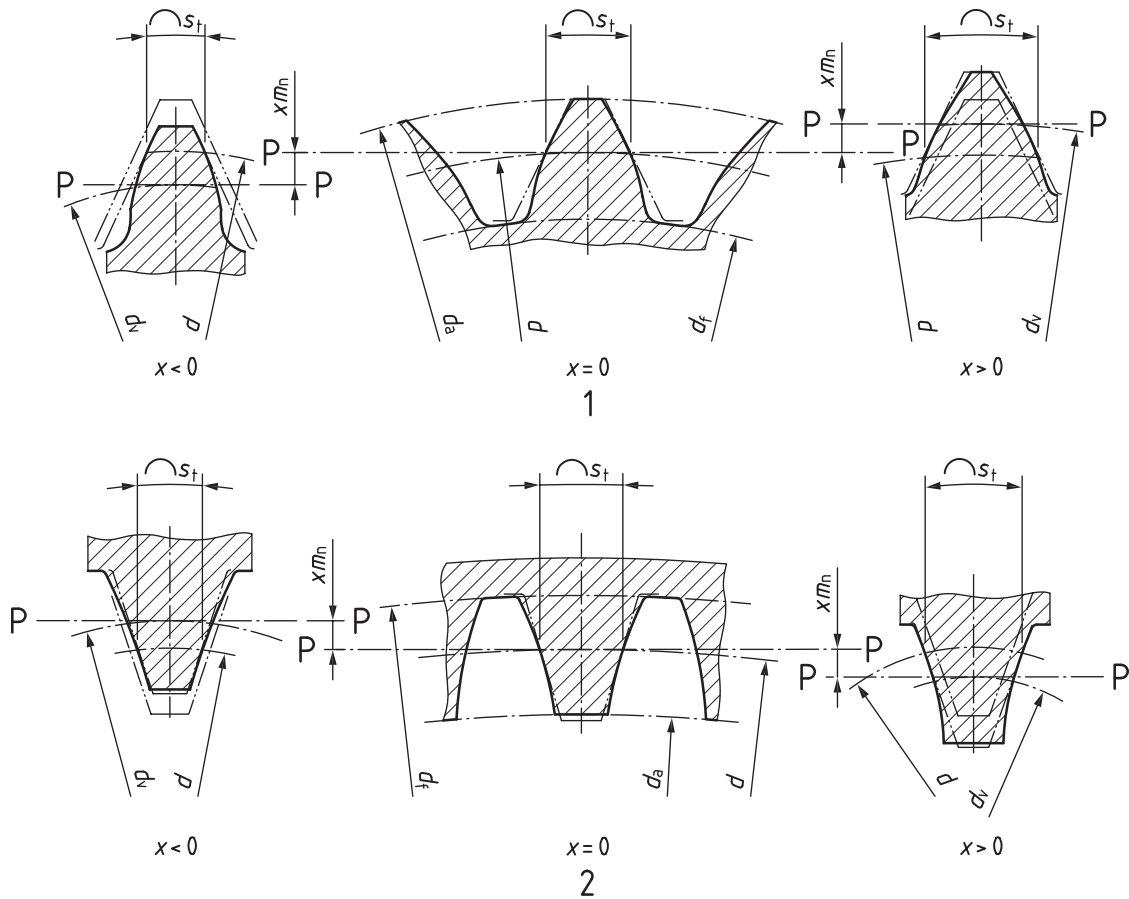
**Key**

1 developed view of V-cylinder

Figure 5 — Facewidth b , usable facewidth b_F

4.3.10 Profile shift, profile shift coefficient and sign of profile shift

The profile shift, xm_n , for involute gear teeth is the radial displacement of the basic rack datum line from the reference cylinder of the gear away from (or towards) the centre of the gear. The magnitude of the profile shift can be made non-dimensional by dividing by the normal module, and it is then expressed by the profile shift coefficient, x . Positive profile shift increases the tooth thickness on the reference cylinder on both external and internal gears. See [Figure 6](#).



Key

 measurement along an arc
P-P datum line of basic rack

1 external gear
2 internal gear

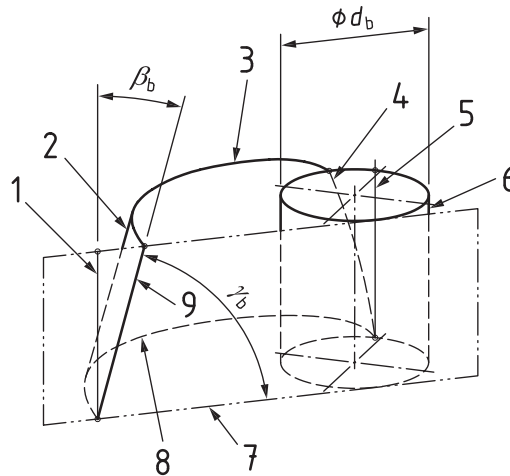
NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings, the diameters are usually shown as positive numbers.

Figure 6 — Profile shift for external and internal gear teeth

4.4 Involute helicoids

4.4.1 Generator of involute helicoids

Figure 7 shows that the intersection of the theoretical flank (key 3) with the base cylinder (key 6) is a helix (key 4). If the base cylinder is unwrapped to a plane [base cylinder tangential plane (key 7)] the flank line becomes a straight line (key 9) inclined by the base helix angle, β_b , to an axial line (key 1), and this is the generator of the involute helicoid.



Key

1	developed axial line	6	base cylinder
2	involute helicoid surface	7	developed base cylinder envelope
3	involute	8	involute
4	base helix	9	straight line generator
5	base cylinder axial line		

Figure 7 — Base cylinder with generator and involute helicoid

4.4.2 Base cylinder, base circle and base diameter

The base cylinder is that cylinder coaxial with the gear axis that is determinative for the generation of the involute helicoids, see [Figures 7](#) and [8](#). Quantities associated with the base cylinder are denoted by the subscript b.

The base circle is the intersection of the base cylinder with a transverse section. The involutes from the base circle form the transverse profiles of the gearing. The base diameter, d_b , is given by [Formula \(6\)](#):

$$d_b = d \cos \alpha_t = z m_t \cos \alpha_t = z m_n \frac{\cos \alpha_t}{\cos \beta} = z m_n \frac{\cos \alpha_n}{\cos \beta_b} = \frac{z m_n}{\sqrt{\tan^2 \alpha_n + \cos^2 \beta}} \quad (6)$$

NOTE For internal gears, since the number of teeth is negative, the base diameter is also negative.

4.4.3 Transverse pressure angle at a point and transverse pressure angle

In a transverse section, the tangent to the involute through an arbitrary point Y is inclined to the radius to the involute at that point by the transverse pressure angle, α_{yt} , (see [Figure 8](#)):

$$\cos \alpha_{yt} = \frac{d_b}{d_y} = \frac{d}{d_y} \cos \alpha_t \quad (7)$$

The transverse pressure angle, α_t , is the acute angle between the tangent to the involute at the point of intersection with the reference circle and the radius through this point of intersection. It is expressed by [Formula \(8\)](#):

$$\cos \alpha_t = \frac{d_b}{d} \text{ or } \tan \alpha_t = \frac{\tan \alpha_n}{\cos \beta} \quad (8)$$

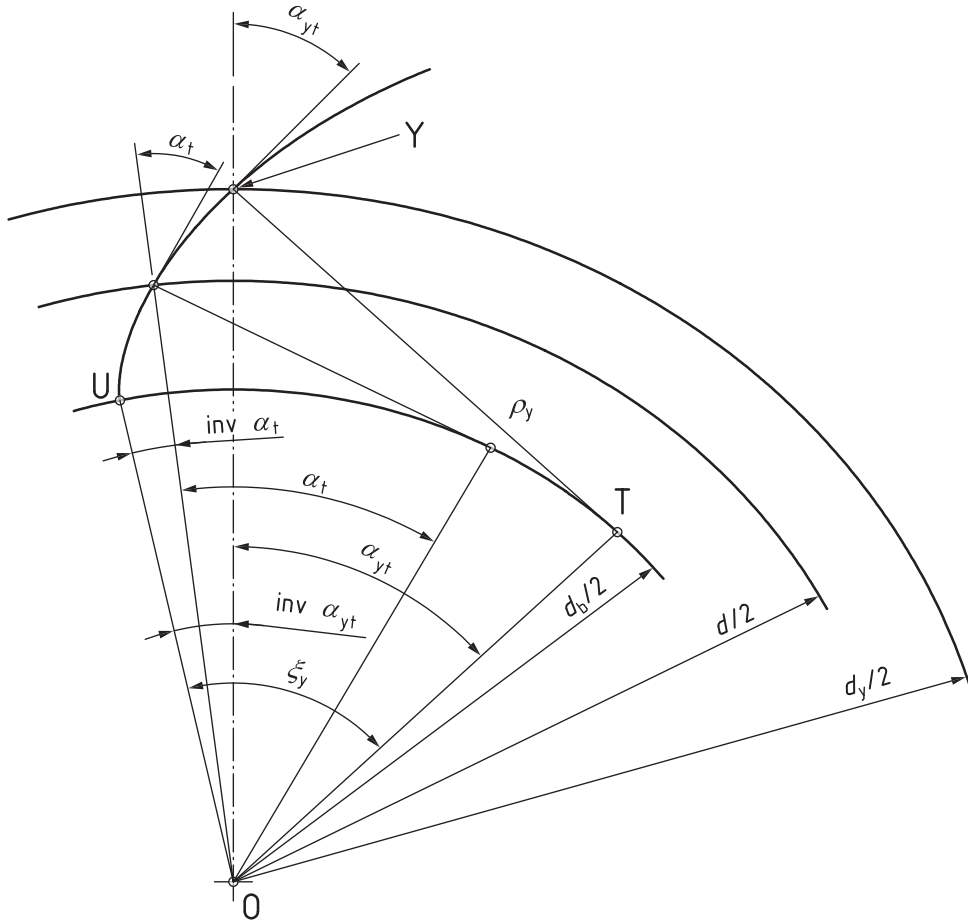


Figure 8 — Parameters relating to involute

4.4.4 Normal pressure angle at a point and normal pressure angle

At an arbitrary point Y on the flank, the tangent to the normal surface of the involute helicoid is inclined to the radius through Y by the normal pressure angle at that point, α_{yn} . The corresponding angle of inclination at the reference cylinder is the normal pressure angle, α_n , this is equal to the pressure angle, α_p , of the basic rack tooth profile.

$$\tan \alpha_n = \tan \alpha_t \cos \beta \quad (9)$$

$$\tan \alpha_{\text{yn}} = \tan \alpha_{\text{yt}} \cos \beta_{\text{y}} \quad (10)$$

For a spur gear, $\alpha_n = \alpha_t$ and $\alpha_{yn} = \alpha_{yt}$.

4.4.5 Involute function

The angular difference, $\xi_y - \alpha_{yt}$, is termed the involute function of angle α_{yt} and is denoted by $\text{inv } \alpha_{yt}$ (to be read as “involute α_{yt} ”):

$$\text{inv}\alpha_{\text{yt}} = \xi_{\text{y}} - \alpha_{\text{yt}} = \tan\alpha_{\text{yt}} - \alpha_{\text{yt}} \quad (11)$$

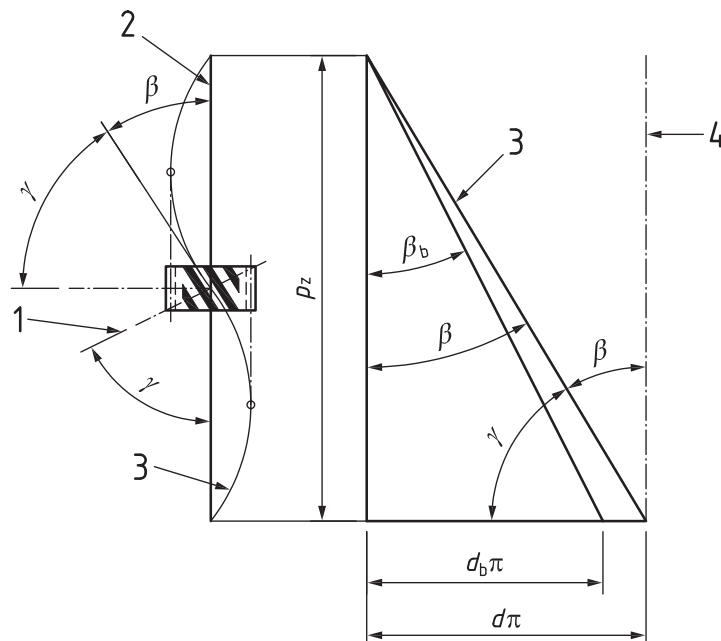
Radians shall be used in [Formula \(11\)](#).

4.4.6 Lead

The lead, p_z , is the distance between successive intersections of an axial line with the involute helicoid, see [Figure 9](#). The lead is independent of the diameter of the cylinder.

$$p_z = \frac{z m_n \pi}{\sin|\beta|} = \frac{z m_t \pi}{\tan|\beta|} = z p_x \quad (12)$$

NOTE For internal gears, since the number of teeth is negative, the lead is also negative.



Key

- | | | | |
|---|---|---|-----------------------------|
| 1 | normal surface | 3 | trace of reference helix |
| 2 | reference cylinder generator, gear axis | 4 | projection of the gear axis |

Figure 9 — Lead triangle, lead, helix angle and lead angle

4.4.7 Helix angle and lead angle

The helix angle, β , is the angle between a tangent to a reference helix and an axial line on the reference cylinder through the tangent contact point. In special cases, the helix angle, β_R , of right flanks can differ from the helix angle, β_L , of left flanks; however, all formulae in this document are based on the left and right flanks having equal helix angles.

In some calculations, such as for crossed axis gears, the sign of the helix angles is required.

For an external gear with right-hand lead (right-hand helix) the helix angle is positive, for a left-hand lead (left-hand helix) the helix angle is negative.

For an internal gear the signs are opposite those of an external gear, i.e. an internal gear with a right-hand lead has a negative helix angle and an internal left-hand lead has a positive helix angle.

Therefore, the sum of the helix angles of a parallel axis gear pair is zero.

NOTE On gear drawings, the absolute values of the helix angles are usually listed together with the direction of the helix (r for right-hand, l for left-hand).

Relationships between β and the base helix angle, β_b , (helix angle on the base cylinder):

$$\tan \beta_b = \tan \beta \cos \alpha_t \quad (13)$$

$$\sin \beta_b = \sin \beta \cos \alpha_n \quad (14)$$

$$\cos \beta_b = \cos \beta \frac{\cos \alpha_n}{\cos \alpha_t} = \frac{\sin \alpha_n}{\sin \alpha_t} = \frac{\sin \alpha_{yn}}{\sin \alpha_{yt}} = \cos \alpha_n \sqrt{\tan^2 \alpha_n + \cos^2 \beta} \quad (15)$$

On a cylinder with arbitrary diameter, d_y , the helix angle, β_y , can be found:

$$\tan \beta_y = \frac{d_y}{d} \tan \beta = \tan \beta \frac{\cos \alpha_t}{\cos \alpha_{yt}} = \frac{d_y}{d_b} \tan \beta_b = \frac{\tan \beta_b}{\cos \alpha_{yt}} \quad (16)$$

$$\sin \beta_y = \sin \beta \frac{\cos \alpha_n}{\cos \alpha_{yn}} = \frac{\sin \beta_b}{\cos \alpha_{yn}} \quad (17)$$

$$\cos \beta_y = \frac{\tan \alpha_{yn}}{\tan \alpha_{yt}} = \frac{\cos \alpha_{yt} \cos \beta_b}{\cos \alpha_{yn}} \quad (18)$$

The lead angle, γ , is the angle between a tangent to a reference helix and a transverse plane through the tangent contact point, see [Figure 9](#). Lead angle does not apply to internal gears. For external gears:

$$|\beta + \gamma| = 90^\circ \quad (19)$$

$$\gamma_y = (90^\circ - |\beta_y|) \frac{\beta_y}{|\beta_y|} \quad (20)$$

For spur gears, $\beta = 0^\circ$ and $\gamma = 90^\circ$.

4.4.8 Hand of helix

The flank lines can follow either a right-hand helix or a left-hand helix. See [Figure 10](#).

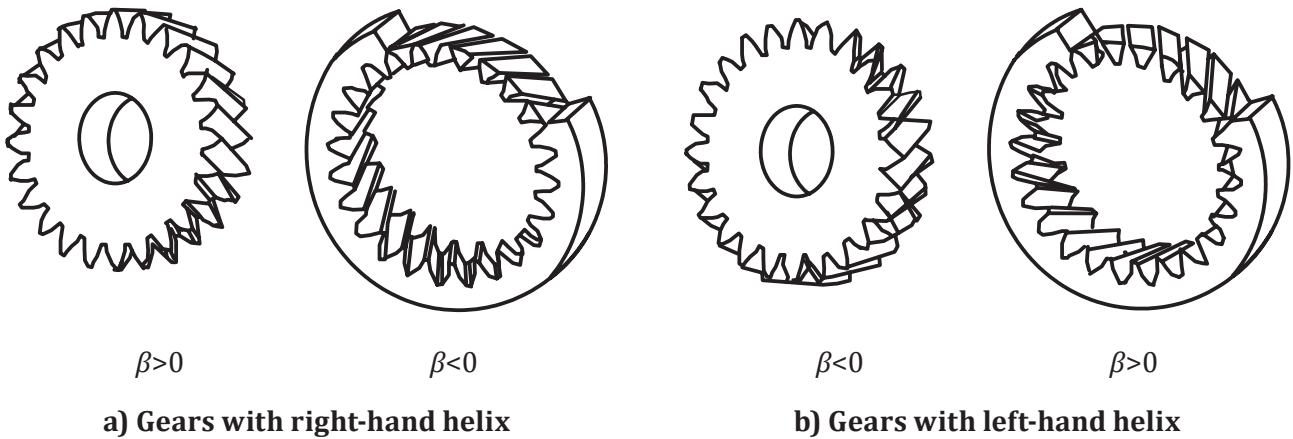


Figure 10 — Direction of helix

4.4.9 Roll angle of the involute

In the transverse plane, the angle at the centre over the base circle arc from the origin, U, of the involute to the contact point, T, of the tangent from point Y to the base circle is the roll angle, ξ_y , of the involute, see [Figure 8](#). The length of the base circle arc, UT, is equal to the length of the tangent portion, YT, hence

$$\xi_y = \tan \alpha_{yt} \quad (21)$$

NOTE ξ_y is in radians.

4.4.10 Radius of curvature of the involute and length of roll

The tangent portion, YT, is the radius of curvature, ρ_y , of the involute at point Y and at the same time the length of roll, L_y , belonging to point Y, i.e. the length of the developed base circle arc from the involute origin, U. (See [Figure 8](#).) In the triangle OTY it is the side opposite the transverse pressure angle, α_{yt} , at the centre of the circle O:

$$L_y = \rho_y = \frac{d_b}{2} \xi_y = \frac{d_b}{2} \tan \alpha_{yt} = \frac{z}{|z|} \frac{\sqrt{d_y^2 - d_b^2}}{2} \quad (22)$$

NOTE 1 For internal gears, since the number of teeth is negative, the length of roll is also negative.

NOTE 2 The length of roll is measured in a transverse plane.

4.4.11 Length of involute profile, L_{yt}

The length of an involute profile from its origin on the base circle to point Y with pressure angle α_{yt} (length along profile from U to Y on [Figure 8](#)) is:

$$L_{yt} = r_b \frac{\tan^2 \alpha_{yt}}{2} \quad (23)$$

4.5 Angular pitch and pitches

4.5.1 Angular pitch

The angular pitch, τ , is that angle laying in a transverse section that results from the dividing of the complete periphery of a circle into z equal parts.

$$\tau = \frac{2\pi}{|z|} = \frac{2p_{yt}}{|d_y|} \text{ in radians} \quad (24)$$

$$\tau = \frac{360}{|z|} = \frac{360p_{yt}}{\pi|d_y|} \text{ in degrees} \quad (25)$$

4.5.2 Pitches on the reference cylinder

4.5.2.1 Transverse pitch

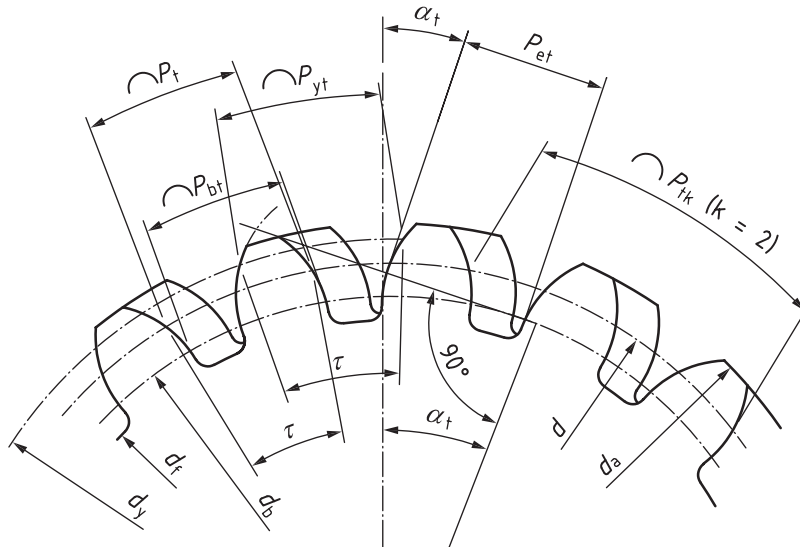
The transverse pitch, p_t , is the length of the reference circle arc between two successive right or left tooth flanks (see [Figure 11](#)):

$$p_t = \frac{\pi m_n}{\cos \beta} = \frac{|d|}{2} \tau = \frac{\pi d}{z} = \pi m_t \quad (26)$$

4.5.2.2 Normal pitch

The normal pitch, p_n , is the length of the helical arc on the reference cylinder that is between and normal to the flank lines of two successive right or left tooth flanks (see [Figure 12](#)):

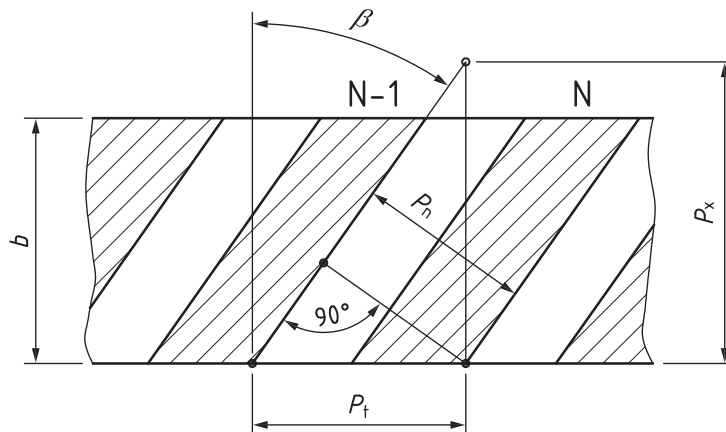
$$p_n = \pi m_n = p_t \cos \beta \quad (27)$$



Key

 measurement along an arc

Figure 11 — Diameter, angular pitch, and transverse pitches on helical cylindrical gear



NOTE This is a developed view of a reference cylinder.

Figure 12 — Geometrical relations between transverse, normal and axial pitch

4.5.3 Pitches on any cylinder

It is necessary to distinguish between the transverse pitch, p_{yt} , and the normal pitch, p_{yn} , on a cylinder of any diameter, d_y , (y-cylinder):

$$p_{yt} = \frac{|d_y|}{2} \tau = \frac{\pi d_y}{z} = \frac{d_y}{d} p_t \quad (28)$$

$$p_{yn} = p_{yt} \cos \beta_y \quad (29)$$

4.5.4 Axial pitch

The axial pitch, p_x , of a helical gear is the portion of a generation line of a cylinder concentric with the gear axis between two successive right or left tooth flanks, see [Figure 12](#). The axial pitch is independent of the diameter of the cylinder. Axial pitch does not apply to spur gears. It is expressed by:

$$p_x = \frac{\pi m_n}{\sin |\beta|} = \pi m_x = \frac{p_z}{z} = \frac{\pi m_t}{\tan |\beta|} = \frac{p_{yt}}{\tan |\beta_y|} = \frac{p_{yn}}{\sin |\beta_y|} \quad (30)$$

4.5.5 Base pitch

4.5.5.1 General

The distance between successive right or left tooth flanks on the developed base cylinder tangential plane is the base pitch.

— Transverse base pitch:

$$p_{bt} = \frac{|d_b|}{2} \tau = p_t \cos \alpha_t = p_{yt} \cos \alpha_{yt} = \frac{\pi d_b}{z} = \frac{d_b}{d} p_t \quad (31)$$

— Normal base pitch:

$$p_{bn} = p_n \cos \alpha_n = p_{bt} \cos \beta_b \quad (32)$$

4.5.5.2 Transverse base pitch along a tangent to the base circle

The transverse base pitch, p_{et} , measured along a line tangent to the base circle in the transverse plane, is the distance between two successive right or left tooth flanks (see [Figure 11](#)):

$$p_{et} = p_{bt} \quad (33)$$

NOTE For a gear pair, the path of contact is on a line that is tangent to both base circles.

4.5.5.3 Normal base pitch on the plane of contact

In a gear pair, the plane of contact is a plane tangent to the two base cylinders. The normal base pitch, p_{en} , measured in the plane of contact is the distance between two parallel tangential planes which contact two successive right or left tooth flanks:

$$p_{en} = p_{bn} \quad (34)$$

4.6 Diameters of gear teeth

4.6.1 General

The position of the basic rack tooth profile relative to the reference cylinder gives rise to the following cylindrical surfaces and diameters with respect to the gear teeth.

4.6.2 V-cylinder and V-circle diameter

The V-cylinder is the cylinder concentric to the gear axis which is tangent to the datum line of the basic rack in its generating position (see [Figure 6](#)). Its nominal diameter, d_v , (V-circle diameter), is calculated using [Formula \(35\)](#).

$$d_v = d + 2xm_n \quad (35)$$

NOTE The V-cylinder is also the pitch surface at generation with a rack tool.

4.6.3 y-cylinder and y-circle diameter

The y-cylinder is a cylinder of arbitrary diameter, d_y , concentric to the gear axis that is used for calculation purposes.

4.6.4 Tip alteration coefficient

Tip alteration is a change to the addendum relating to the standard addendum of one normal module. The tip alteration is made non-dimensional by dividing by the normal module, and it is then expressed as the tip alteration coefficient, k .

The value to be used for k is signed.

A negative value yields a shorter addendum for either external or internal gears.

NOTE The value for k is specified by the designer.

4.6.5 Tip cylinder, tip circle and tip diameter

The tip cylinder is the cylinder that defines the tips of the gear tooth system. A transverse section yields the tip circle. The nominal dimension of the tip diameter, d_a , is calculated [Formula \(36\)](#).

$$d_a = d + 2(xm_n + h_{ap} + km_n) \quad (36)$$

NOTE 1 For a standard addendum of one normal module, $h_{ap} = m_n$.

NOTE 2 The value for the tip alteration coefficient, k , is determined by the gear designer. The value of k can be positive or negative. See [5.3.9](#).

4.6.6 Profile control diameter, d_{cf}

The profile control diameter, also known as start of profile evaluation diameter, is a specified diameter beyond which the tooth profile is required to conform to the specified design. If not specified, the profile form diameter, d_{ff} , is used as the profile control diameter. Note that $d_{ff} \leq d_{cf}$.

NOTE If the mating gear is known, the designer sometimes selects the minimum start of active profile diameter, d_{Nf} , as the profile control diameter.

4.6.7 Root cylinder, root circle and root diameter

The root cylinder is the cylindrical envelope surface that forms the bottom of the tooth space. A transverse section yields the root circle. The nominal dimension of the root diameter, d_f , is [Formula \(37\)](#).

$$d_f = d - 2(h_{fp} - xm_n) \quad (37)$$

For additional information and formulae related to detailed root geometry, see form factor in ISO 6336-3.

4.7 Gear tooth depth

4.7.1 Tooth depth

The tooth depth, h , of cylindrical gear (or rack) teeth is the difference between the tip and root radius:

$$h = \frac{d_a - d_f}{2} = h_{aP} + km_n + h_{fP} \quad (38)$$

NOTE The tooth depth is sometimes referred to as the tooth height.

4.7.2 Addendum, working addendum, dedendum and working dedendum

The addendum, h_a , and the dedendum, h_f , of a cylindrical gear are stated on the basis of the reference circle. Their values are obtained using [Formulae \(39\)](#) and [\(40\)](#):

$$h_a = \frac{d_a - d}{2} = h_{aP} + xm_n + km_n \quad (39)$$

$$h_f = \frac{d - d_f}{2} = h_{fP} - xm_n \quad (40)$$

The working addendum, h_{aw} , and working dedendum, h_{fw} , are stated on the basis of the working pitch circle. Their values are obtained [Formulae \(40\)](#) and [\(41\)](#):

$$h_{aw} = \frac{d_a - d_w}{2} \quad (41)$$

$$h_{fw} = \frac{d_w - d_f}{2} \quad (42)$$

The direction of sliding on the working addendum is opposite of that on the working dedendum.

4.8 Tooth thickness and space width

4.8.1 General

The formulae in [4.8](#) yield the tooth thickness and space width and their half angles for any value of profile shift coefficient x . If x is the nominal profile shift coefficient, then nominal sizes for the tooth thickness and space width and their half angles result. If the generating profile shift coefficient, x_E , is used, then generated sizes for the tooth thickness and space width and their half angles result. See ISO 21771-2 for additional information on tooth thickness.

For cylindrical gears, unless chordal tooth thickness is explicitly mentioned, tooth thickness is defined on the surface of a specified cylinder. It is the length between the flanks of a tooth, measured either along a circular arc in a transverse plane or along a helix normal to the tooth flanks. The only exception to this is chordal tooth thickness, which is a straight-line measurement between points at a specified diameter on opposite flanks of a tooth.

4.8.2 Transverse tooth thickness

For spur and helical gears and involute worms, the transverse tooth thickness, s_{yt} , is the length between the two flanks of a gear tooth, measured at a specified diameter along a circular arc in the transverse plane. See [Figure 13](#).

$$s_{yt} = |d_y| \psi_y = |d_y| \left[\psi + \frac{z}{|z|} (\text{inv} \alpha_t - \text{inv} \alpha_{yt}) \right] = d_y \left[\frac{\pi + 4x \tan \alpha_n}{2z} + \text{inv} \alpha_t - \text{inv} \alpha_{yt} \right] \quad (43)$$

NOTE The diameters of internal gears have negative values, so the absolute value function is used in some formulae.

The transverse tooth thickness, s_t , on the reference circle is produced from:

$$s_t = |d|\psi = d \left(\frac{\pi + 4x \tan \alpha_n}{2z} \right) = \frac{m_n}{\cos \beta} \left(\frac{\pi}{2} + 2x \tan \alpha_n \right) \quad (44)$$

4.8.3 Transverse tooth thickness half angle

Transverse tooth thickness angles are angles at the centre in a transverse section which are enclosed by the radii bounding a transverse tooth thickness, see [Figure 13](#). The corresponding transverse tooth thickness half angles, ψ_y , for any transverse tooth thickness, s_{yt} , are expressed by:

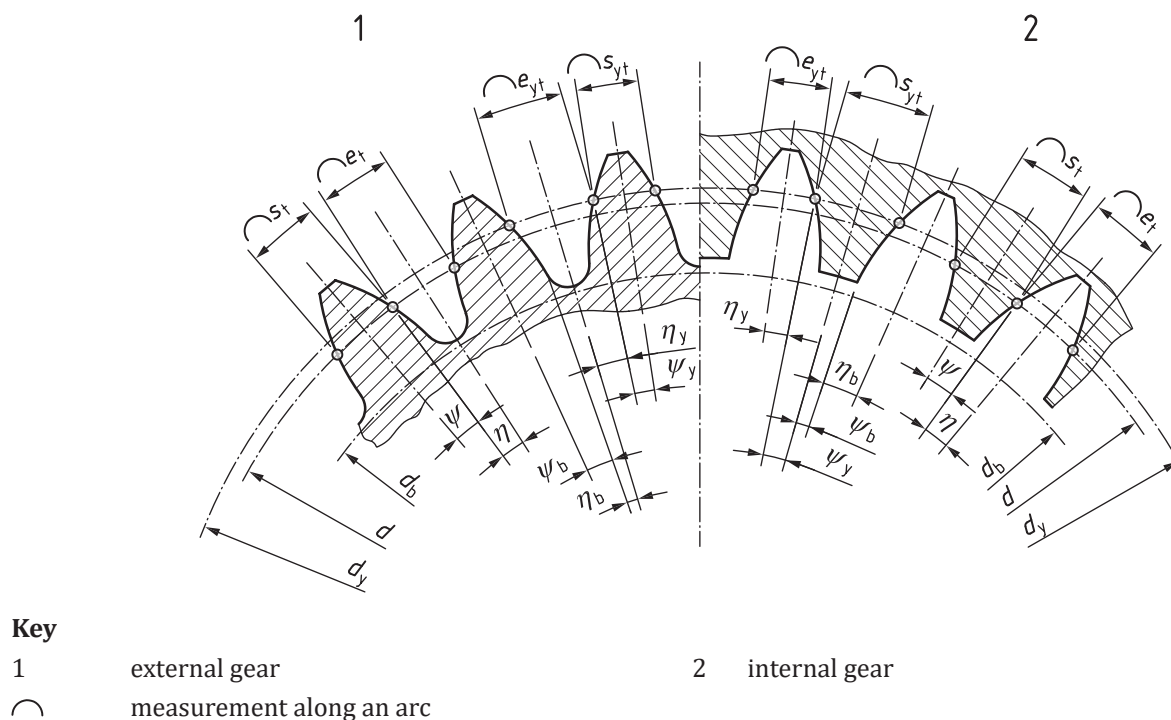
$$\psi_y = \frac{s_{yt}}{|d_y|} = \psi + \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{yt}) \quad (45)$$

The following transverse tooth thickness half angle applies to the reference circle:

$$\psi = \frac{\pi + 4x \tan \alpha_n}{2 |z|} \quad (46)$$

In the case of the base circle, the base transverse tooth thickness half angle is produced by:

$$\psi_b = \psi + \frac{z}{|z|} \text{inv } \alpha_t \quad (47)$$



NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings, the diameters are usually shown as positive numbers.

Figure 13 — Transverse tooth thickness and space width (external and internal gear teeth)

4.8.4 Transverse space width

The transverse space width, e_{yt} , is the length in a transverse section of a circular arc of diameter, d_y , between the two involute helicoids of a tooth space (see [Figure 13](#)):

$$e_{yt} = |d_y| \eta_y = |d_y| \left[\eta - \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{yt}) \right] = d_y \left[\frac{\pi - 4x \tan \alpha_n}{2z} - (\text{inv } \alpha_t - \text{inv } \alpha_{yt}) \right] = p_{yt} - s_{yt} \quad (48)$$

The transverse space width, e_t , on the reference circle is produced by:

$$e_t = |d| \eta = d \left(\frac{\pi - 4x \tan \alpha_n}{2z} \right) = \frac{m_n}{\cos \beta} \left(\frac{\pi}{2} - 2x \tan \alpha_n \right) = p_t - s_t \quad (49)$$

4.8.5 Transverse space width half angle

Space width angles are angles at the centre in a transverse section which are enclosed by the radii bounding a tooth space, see [Figure 13](#). The corresponding transverse space width half angles, η_y , for any transverse space width, e_{yt} , are produced by

$$\eta_y = \frac{e_{yt}}{|d_y|} = \eta - \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{yt}) \quad (50)$$

The following transverse space width half angle applies to the reference circle:

$$\eta = \frac{\pi - 4x \tan \alpha_n}{2|z|} \quad (51)$$

At the base circle, the base transverse space width half angle is produced by:

$$\eta_b = \eta - \frac{z}{|z|} \text{inv } \alpha_t \quad (52)$$

4.8.6 Normal tooth thickness

For helical gears and involute worms, the normal tooth thickness is the length between the two sides of a gear tooth that is measured at a specified diameter along a helical arc that is normal to the helix along the tooth flank. The normal tooth thickness, s_{yn} , on a cylinder with diameter d_y is calculated using [Formula \(53\)](#):

$$s_{yn} = s_{yt} \cos \beta_y \quad (53)$$

The following normal tooth thickness applies to the reference cylinder:

$$s_n = s_t \cos \beta = m_n \left(\frac{\pi}{2} + 2x \tan \alpha_n \right) \quad (54)$$

4.8.7 Normal space width

The normal space width is the length of a helical arc that is between and is normal to the two involute helicoids bounding a tooth space. The normal space width, e_{yn} , on a cylinder with diameter d_y is calculated using [Formula \(55\)](#):

$$e_{yn} = e_{yt} \cos \beta_y = p_{yn} - s_{yn} \quad (55)$$

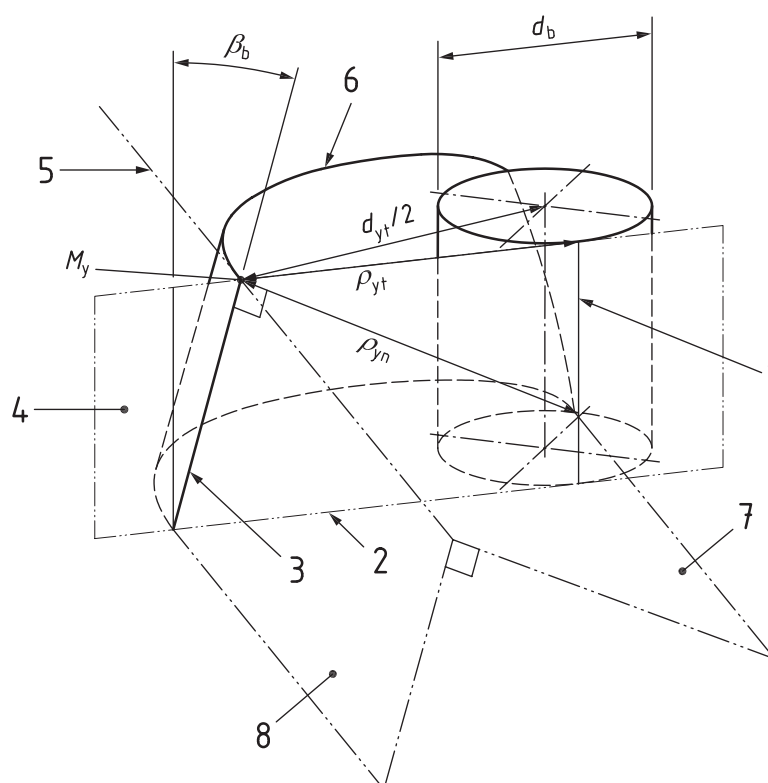
The following normal space width applies to the reference cylinder:

$$e_n = e_t \cos \beta = m_n \left(\frac{\pi}{2} - 2x \tan \alpha_n \right) = p_n - s_n \quad (56)$$

4.9 Radii of curvature of the tooth flanks in different planes

In general, at a specified point on the tooth flank of an involute cylindrical gear, the radius of curvature in any plane perpendicular to the surface can be determined. There are three important planes related to the radius of curvature at a specified point on an involute tooth flank.

- The tangent plane to involute helicoid: a plane tangent to the tooth flank surface at the specified point. This plane (key 8 in [Figure 14](#)) is established by the tangent to the involute in the transverse plane (key 5 in [Figure 14](#)) and the straight-line generator (key 3 in [Figure 14](#)).
- The base plane: a plane that is tangent to the base cylinder and that contains both the specified point and the straight-line generator (key 3 in [Figure 14](#)).
- The normal plane: a plane containing the specified point that is perpendicular to both the tangent plane to involute helicoid and the base plane.



Key

- | | | | |
|---|----------------------------------|---|---|
| 1 | axial line | 5 | tangent to involute in transverse plane |
| 2 | developed base cylinder envelope | 6 | involute |
| 3 | straight line generator | 7 | normal plane |
| 4 | base plane | 8 | tangent plane to involute helicoid |

Figure 14 — Involute helicoid with generator and tangent to involute

The minimum and maximum radii of curvature are defined as the principal radii of curvature. These are always in two perpendicular planes that are also perpendicular to the tangent plane to involute helicoid. For a cylindrical involute gear, the first principal radius of curvature, ρ_{yn} , is in the normal plane and is the minimum radius of curvature. The second principal radius of curvature is in the base plane and is infinite since the generator of the flank surface is a straight line. See [Clause 7](#).

NOTE 1 For an involute gear pair with parallel axis, the straight-line generator is equivalent to contact lines between tooth flanks during meshing.

Once the radius of curvature in the normal plane is known, the radius of curvature in any inclined plane can be determined according to the Meusnier theorem. The curvature radius in the transverse plane at an involute point is the length of roll at this point. The rotation of the transverse plane around the tangent of the involute by the base helix angle delivers the normal plane and the normal radius of curvature (first principal radius of curvature).

Radius of curvature in a transverse plane at diameter d_y :

$$\rho_{yt} = \frac{z}{|z|} 0,5 \sqrt{d_y^2 - d_b^2} \quad (57)$$

Normal radius of curvature at diameter d_y :

$$\rho_{yn} = \frac{z}{|z|} \frac{\sqrt{d_y^2 - d_b^2}}{2 \cos \beta_b} = \frac{\rho_{yt}}{\cos \beta_b} \quad (58)$$

NOTE 2 The normal radius of curvature given in [Formula \(58\)](#) is also known as the first principal radius of curvature. The second principal radius of curvature is in a direction perpendicular to the first principal radius of curvature and is infinite except when there is face line crowning.

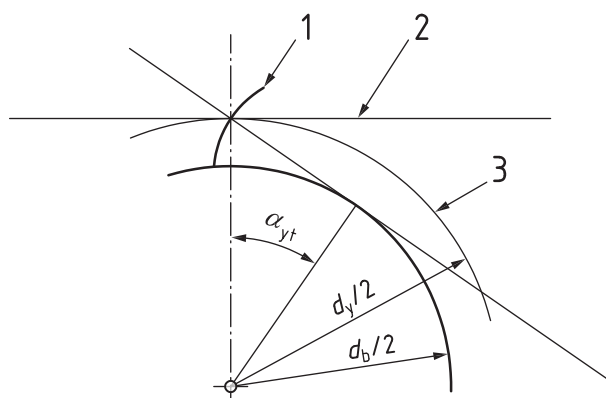
4.10 Tilt angle of contact line

For some considerations, the tilt angle between the tangent at the reference cylinder and the generator of the involute helicoid is of importance. It also describes the tilt angle between the instantaneous contact line between the basic rack and the helical gear tooth flank. See [Figure 15](#).



Figure 15 — Tilt angle of generatrices, δ

At a specified diameter, d_y , the tangent to the helix and the generator through that point are both members of the tangent plane at the point. See [Figure 16](#). The dot product of the two vectors delivers the angle between the vectors.



Key

- | | |
|---------------------------------------|------------------------------|
| 1 involute | 3 circle with diameter d_y |
| 2 tangent to circle of diameter d_y | |

Figure 16 — Tangent to the helix at diameter d_y and the generator

The unit vectors of helix, T_1 , and generator, T_2 , are:

$$T_1 = (\sin(\beta_y), 0, \cos(\beta_y)) \quad (59)$$

$$T_2 = (\cos(-\alpha_{yt}) \sin(\beta_b), \sin(-\alpha_{yt}) \sin(\beta_b), \cos(\beta_b)) \quad (60)$$

The scalar vector product delivers the tilt angle, δ_y , between the two vectors.

$$\cos \delta_y = \frac{\cos \beta_y}{\cos \beta_b} \text{ or } \tan \delta_y = \sin \alpha_{yn} \tan \beta_y \quad (61)$$

The tilt angle of the contact line varies with the diameter at the flank. For the counterpart rack, the tilt angle is constant and defined at reference diameter d .

$$\cos \delta = \frac{\cos \beta}{\cos \beta_b} \text{ or } \tan \delta = \sin \alpha_n \tan \beta \quad (62)$$

5 Parallel axis cylindrical gear pairs

5.1 General

The basic prerequisites for meshing of a parallel axis cylindrical gear pair (or rack and pinion) according to this document are:

- identical basic rack tooth profiles for gear and mating gear (rack), and
- the same base helix angle (or rack helix angle) with appropriate hands of the helices.

NOTE The concepts and formulae in [Clause 5](#) apply to rack and pinion sets when both the rack and pinion have the same normal base pitch, even if the rack and pinion have different helix angles. With this combination, the axis of the pinion will not be perpendicular to the transverse plane of the rack. When the helix angles do not match, there will be an additional sliding component across the faces of the rack and pinion, this sliding is not covered in this document.

5.2 Concepts for a gear pair

5.2.1 Mating gear and mating flank

The mating gear in a gear pair is the gear which meshes with the other gear in question. The mating flanks for the tooth system in question are the contacting tooth flanks of the mating gears.

5.2.2 Working flank and non-working flank

The tooth flank which transmits torque during meshing is called the working flank. The other flank of this tooth is the non-working flank.

5.2.3 External gear pair

The mating of two external cylindrical gears gives an external gear pair. The mating of an external gear with a rack gives a rack and pinion.

In the case of an external gear pair, the subscript 1 is used in formulae for the smaller gear (pinion) and the subscript 2 for the larger gear (gear wheel). When the gears are of the same size, the subscripts can be allocated as desired. In the case of a parallel axis external gear pair with helical gear teeth, one gear has a left-handed and the other gear (mating gear) has a right-handed helix.

5.2.4 Internal gear pair

The mating of an external cylindrical gear with an internal cylindrical gear gives an internal gear pair.

In the case of an internal gear pair, the subscript 1 is used in formulae for the external gear and the subscript 2 for the internal gear. In the case of an internal gear pair with helical gear teeth, both gears have the same hand of helix. Both are either right-handed or left-handed.

5.3 Mating quantities

5.3.1 Gear ratio

The gear ratio, u , of a gear pair is the ratio of the number of teeth of the gear wheel (or internal gear), z_2 , to the number of teeth of the pinion, z_1 :

$$u = \frac{z_2}{z_1} = \frac{d_{b2}}{d_{b1}} = \frac{d_2}{d_1} = \frac{d_{w2}}{d_{w1}}, \quad |u| \geq 1 \quad (63)$$

NOTE See 5.3.5 for d_{w1} and d_{w2} .

5.3.2 Driving gear, driven gear and transmission ratio

The driving gear introduces rotation to the gear pair and effects the rotation of the driven gear.

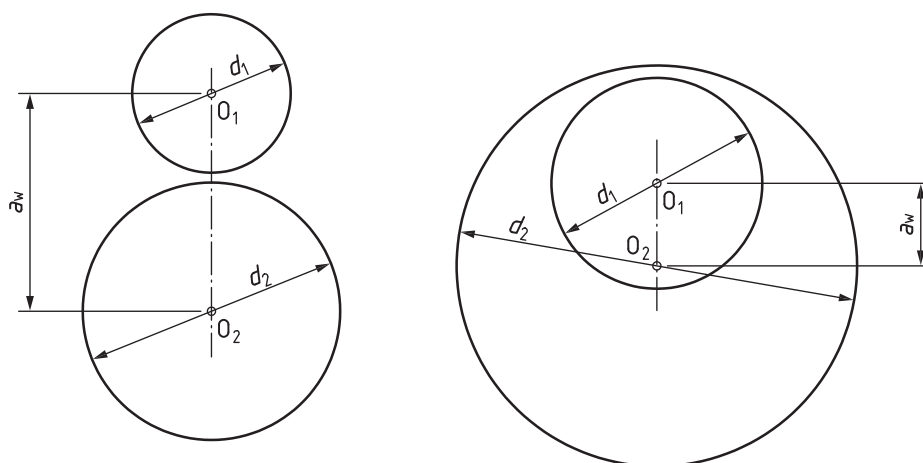
The transmission ratio, i , of a gear pair is the ratio of the angular velocity (rotational speed) of the driving gear (subscript a) to that of the driven gear (subscript b):

$$i = \frac{\omega_a}{\omega_b} = \frac{n_a}{n_b} = -\frac{z_b}{z_a} \quad (64)$$

In the case of an external gear pair, the two cylindrical gears rotate in opposite directions, i.e. their angular velocities or rotational speeds have opposite signs; the transmission ratio is negative. In the case of an internal gear pair, the two cylindrical gears have the same direction of rotation, i.e. their angular velocities or rotational speeds have the same sign; the transmission ratio is positive. If it is necessary to make a distinction, ratios such that $|i| > 1$ are said to be “speed reducing ratios” while ratios such that $|i| < 1$ are said to be “speed increasing ratios”.

5.3.3 Line of centres and centre distance

In a transverse section of two mating gears, the line which connects the two axes is called the line of centres. The centre distance, a_w , is the working distance between the gear axes of the two gears on the line of centres, see Figure 17.



NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative and the centre distance is negative. On drawings, the diameters are usually shown as positive numbers.

Figure 17 — Line of centres, centre distance

5.3.4 Working transverse pressure angle

The working transverse pressure angle, α_{wt} , is that pressure angle whose vertex lies on the working pitch circle. When the centre distance, a_w , is known, α_{wt} is calculated from:

$$\cos \alpha_{wt} = (z_1 + z_2) \left(\frac{m_n \cos \alpha_t}{2a_w \cos \beta} \right) \quad (65)$$

Alternatively, with nominal profile shift coefficients, α_{wt} results from:

$$\operatorname{inv} \alpha_{wt} = \operatorname{inv} \alpha_t + \frac{2 \tan \alpha_n}{z_1 + z_2} (x_1 + x_2) \quad (66)$$

NOTE The inverse involute function can be used to solve for the working pressure angle, α_{wt} . One method for calculating the inverse involute is given in ISO 21771-2:—¹⁾, Annex D.

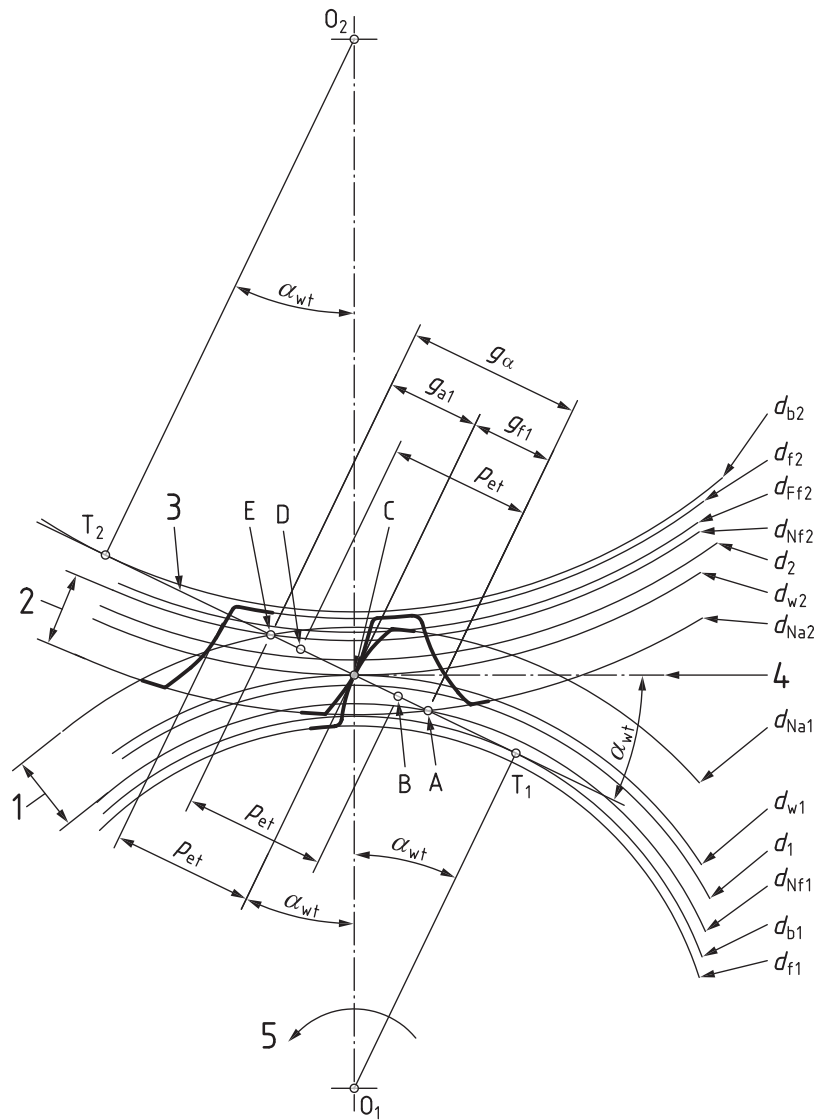
5.3.5 Working pitch point, pitch cylinders, pitch circles, pitch diameter and pitch axis

The working pitch point C divides the centre distance in the ratio of the tooth numbers. The working pitch cylinders (working pitch circles) are those cylinders (circles) which pass through the working pitch point. The working pitch diameter is the diameter of the working pitch circle. The working pitch axis is the axis through the working pitch point, parallel to the axis of the reference cylinders.

NOTE During operation, the circumferential velocities on the working pitch cylinders are the same.

The pitch circles established during the operation of a cylindrical gear pair (gear pair in a gear unit) are termed working pitch circles (d_w). (See [Figures 18](#) and [20](#).) The pitch circles established by a generating cutter during the generating of a tooth system in a generating gear pair are termed generating pitch circles.

1) Under preparation. Stage at the time of publication: ISO/DIS 21771-2:2024.



Key

- | | |
|--------------------------------------|---|
| 1 radial part of active flank gear 1 | 4 tangent to working pitch circles |
| 2 radial part of active flank gear 2 | 5 direction of rotation of driving pinion |
| 3 line of action | |

NOTE 1 See 5.5.6.1 for description of lettered points.

NOTE 2 Figure 18 is for a transverse section of an external gear pair.

Figure 18 — Meshing conditions and active ranges on working flanks, external gear pair

The diameters of the working pitch circles are:

$$d_{w1} = \frac{2z_1 a_w}{z_1 + z_2} = d_1 \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{d_{b1}}{\cos \alpha_{wt}} \quad (67)$$

$$d_{w2} = \frac{2z_2 a_w}{z_1 + z_2} = d_2 \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{d_{b2}}{\cos \alpha_{wt}} \quad (68)$$

NOTE For an internal gear, d_{w2} is negative and for an internal gear pair, a_w is negative.

This gives a working centre distance:

$$a_w = \frac{1}{2} (d_{w2} + d_{w1}) \quad (69)$$

5.3.6 Working helix angles

For parallel axis gears, the helix angle at the working pitch diameter of the pinion has the same magnitude but opposite hand from the helix angle at the working pitch diameter of the gear wheel:

$$\beta_{w1} = -\beta_{w2} \quad (70)$$

The working helix angle can be determined from the working pitch diameter, base diameter, and base helix angle:

$$\tan \beta_{w1} = \frac{d_{w1}}{d_{b1}} \cdot \tan \beta_{b1} \quad (71)$$

$$\tan \beta_{w2} = \frac{d_{w2}}{d_{b2}} \cdot \tan \beta_{b2} \quad (72)$$

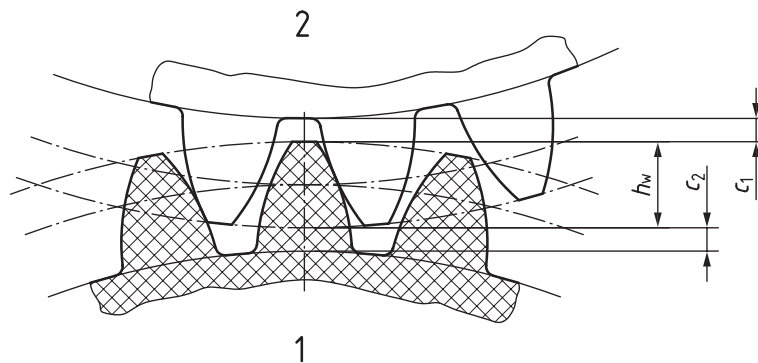
5.3.7 Working depth

The working depth, h_w , of a gear pair is the overlap of the tip circles of the two cylindrical gears on the line of centres, see [Figure 19](#):

$$h_w = \frac{d_{a1} + d_{a2}}{2} - a_w \quad (73)$$

5.3.8 Tip clearance

The tip clearance, c , is the distance by which the tip circle of a gear is separated from the root circle of the mating gear, see [Figure 19](#).



Key

1 pinion

2 gear wheel

Figure 19 — Working depth, h_w , and tip clearances c_1 and c_2 of a gear pair

The actual clearance follows from the centre distance, a_w , the manufactured tip diameter, d_a , and the generated root diameter, d_{fE} . For a pinion it is

$$c_1 = a_w - \frac{d_{a1}}{2} - \frac{d_{fE2}}{2} \quad (74)$$

and for a gear wheel

$$c_2 = a_w - \frac{d_{a2}}{2} - \frac{d_{fE1}}{2} \quad (75)$$

5.3.9 Calculation of tip alteration coefficient k (for parallel axis gear)

It is sometimes necessary for the addendum to be altered to suit the mating conditions and the specified minimum tip clearance of the mate. This is known as tip alteration, and is also sometimes called addendum modification, truncation, or tip shortening, and is usually given as a coefficient (i.e. normalized by the normal module so it is dimensionless). Tip alteration is usually used to shorten the teeth, but it can be used to enlarge the addendum.

If the tip clearance c_1 (or c_2) corresponding to the basic rack tooth profile is to be retained, then for many gear sets, the necessary tip alteration coefficient k (for both pinion and gear) can be estimated as:

$$\begin{aligned} k &= \frac{1}{m_n} \left(a_w - \frac{d_1 + d_2}{2} \right) - (x_1 + x_2) \\ &= \frac{a_w}{m_n} - \frac{z_1 + z_2}{2 \cdot \cos \beta} - (x_1 + x_2) \\ &= \frac{z_1 + z_2}{2 \cdot \cos \beta} \left(\frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right) - (x_1 + x_2) \end{aligned} \quad (76)$$

The tip alteration coefficient, k , calculated in this way is obtained with correct sign, that is to say:

- negative values in the case of external gear pairs, so that the tip diameters become smaller;
- positive values in the case of internal gear pairs, so that the absolute value of the tip diameter of the internal gear becomes smaller while the diameter of the pinion tip circle becomes larger.

The calculated values are often so small that they are cancelled out by the deeper infeed of the cutting tool which is necessary for producing the backlash and by the negative root diameter deviations so that the remaining effective tip clearance of the mate is altered to only a slight extent (or within permissible limits). See [5.3.8](#).

In the case of internal gear pairs, the tip alteration coefficient, k , which is always positive in this case can usually not be realized because the special engagement and manufacturing conditions of internal gear pairs can limit the usable addendum of the internal gear and pinion if tip-to-tip interference is to be avoided. See [5.5.8.3](#).

5.4 Calculation of the sum of the profile shift coefficients

The sum of the profile shift coefficients which corresponds to the zero-backlash condition is related to the basic tooth parameters and centre distance by [Formula \(77\)](#), with α_{wt} from [Formula \(65\)](#):

$$\sum x = x_1 + x_2 = \frac{(z_1 + z_2)(\text{inv } \alpha_{wt} - \text{inv } \alpha_t)}{2 \tan \alpha_n} \quad (77)$$

In the case of non-zero backlash, the normal backlash, j_{bn} (see [5.6.4](#)), is included in the calculation:

$$\sum x_E = x_{E1} + x_{E2} = \frac{(z_1 + z_2)(\text{inv } \alpha_{wt} - \text{inv } \alpha_t)}{2 \tan \alpha_n} - \frac{j_{bn}}{2 m_n \sin(\alpha_n)} \quad (78)$$

The way in which $\sum x = x_1 + x_2$ is distributed between the two gears can be decided on the basis of aspects such as permissible stress, sliding velocities or other specified dimensions of the gear teeth such as root diameter.

5.5 Tooth engagement

5.5.1 General

Tooth engagement refers to the meshing of a gear (or a rack) with its mating gear. The tooth engagement is influenced by the geometry of the gear pair (or rack and pinion), the mutual contact of the tooth flanks and the sliding conditions, see [Figures 18](#) and [20](#).

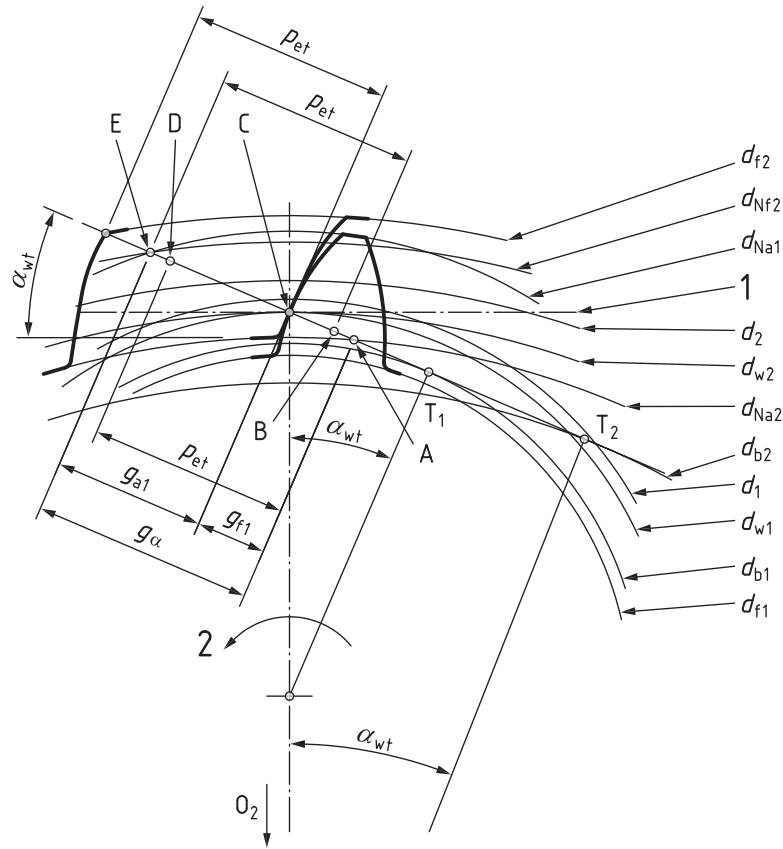
5.5.2 Active area of the tooth flanks, start of active profile and active tip diameters

5.5.2.1 General

The start of involute diameter (root form diameter), d_{ff} , is the start of the involute portion of the profile. For an external gear it is the greater of the base diameter, d_b , or the diameter of the intersection of the flank with the tooth root fillet or trochoid (taking into account undercut, if necessary, see [Clause 10](#)).

In a defined gear pair, the active tip diameter, d_{Na} , of a gear can be governed either by its tip form diameter, d_{Fa} , or by the start of involute of its mate, d_{ff} . The start of active profile (active root) diameter, d_{Nf} , can be governed either by the diameter of start of involute d_{ff} , or by the tip form diameter of its mate, d_{Fa} . During rotation, the active tip diameter of a gear will contact the active root diameter of the mating gear. The active area of the flank extends from the active tip diameter, d_{Na} , to the active root diameter, d_{Nf} , and so is dependent on the characteristics of both gears and the centre distance.

Usually, contact in the dedendum on both gears is limited by the tip form diameter of the mating gear.



Key

- 1 tangent to working pitch circles 2 direction of rotation of driving pinion

NOTE 1 See 5.5.6.1 for a description of points A, B, C, D and E.

NOTE 2 For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings the diameters are usually shown as positive numbers.

NOTE 3 Figure 20 is for a transverse section of an internal gear pair.

Figure 20 — Meshing conditions and active ranges on working flanks, internal gear pair

5.5.2.2 Pinion start of active profile and active tip diameters

When contact in the pinion dedendum is limited by the tip form diameter of the gear wheel:

$$d_{Nf1} = \sqrt{\left(2|a_w| \sin \alpha_{wt} - \sqrt{d_{Fa2}^2 - d_{b2}^2}\right)^2 + d_{b1}^2} \quad (79)$$

$$d_{Na2} = d_{Fa2} \quad (80)$$

However, if d_{Ff1} is greater than the quantity calculated for d_{Nf1} , then:

$$d_{Nf1} = d_{Ff1} \quad (81)$$

$$d_{Na2} = \frac{z_2}{|z_2|} \sqrt{\left(2a_w \sin \alpha_{wt} - \sqrt{d_{Ff1}^2 - d_{b1}^2}\right)^2 + d_{b2}^2} \quad (82)$$

See 9.7, 10.3 or 11.3 for d_{Ff} .

It is important to verify if the gear teeth need to be truncated to avoid gear mesh interference with the tooth root fillet of the pinion.

With α_{Nf1} from:

$$\cos \alpha_{\text{Nf1}} = \frac{d_{\text{b1}}}{d_{\text{Nf1}}} \quad (83)$$

The corresponding roll angles (see 4.4.9) are:

$$\xi_{\text{Nf1}} = \frac{z_2}{z_1} (\xi_{\text{wt}} - \xi_{\text{Na2}}) + \xi_{\text{wt}} \quad (84)$$

$$\xi_{\text{Na2}} = \tan \left(\arccos \frac{d_{\text{b2}}}{d_{\text{Na2}}} \right) \quad (85)$$

5.5.2.3 Gear wheel start of active profile and active tip diameters

When contact in the gear wheel dedendum is limited by the tip form diameter of the pinion:

$$d_{\text{Nf2}} = \frac{z_2}{|z_2|} \sqrt{\left(2a_w \sin \alpha_{\text{wt}} - \sqrt{d_{\text{Fa1}}^2 - d_{\text{b1}}^2} \right)^2 + d_{\text{b2}}^2} \quad (86)$$

$$d_{\text{Na1}} = d_{\text{Fa1}} \quad (87)$$

However, if d_{Ff2} is greater than the quantity calculated for d_{Nf2} , then:

$$d_{\text{Nf2}} = d_{\text{Ff2}} \quad (88)$$

$$d_{\text{Na1}} = \sqrt{\left(2|a_w| \sin \alpha_{\text{wt}} - \sqrt{d_{\text{Ff2}}^2 - d_{\text{b2}}^2} \right)^2 + d_{\text{b1}}^2} \quad (89)$$

It is important to verify if the pinion teeth need to be truncated to avoid gear mesh interference with the tooth root fillet of the gear wheel.

With α_{Nf2} from:

$$\cos \alpha_{\text{Nf2}} = \frac{d_{\text{b2}}}{d_{\text{Nf2}}} \quad (90)$$

The corresponding roll angles (see 4.4.9) are:

$$\xi_{\text{Nf2}} = \frac{z_1}{z_2} (\xi_{\text{wt}} - \xi_{\text{Na1}}) + \xi_{\text{wt}} \quad (91)$$

$$\xi_{\text{Na1}} = \tan \left(\arccos \frac{d_{\text{b1}}}{d_{\text{Na1}}} \right) \quad (92)$$

5.5.3 Plane of action, zone of action and contact line

The plane of action of a cylindrical gear pair is tangent to the base cylinders of the gear and mating gear. In the case of an external gear pair, the plane of action passes between the base cylinders. The intersection of the two planes of action (one for each tooth flank) is parallel to the gear axis, and is the working pitch axis (see 5.3.5). The zones of action are the parts of the planes of action which are limited by the active tip diameters d_{Na1} , d_{Na2} of the gear and mating gear and by the facewidth. A zone of action is linked to the flank that is normal to it. Hence, one of the planes of action is linked to the right flanks and the other to the left flanks.