



BSI Standards Publication

Cylindrical involute gears and gear pairs

Part 2: Calculation and measurement of tooth thickness and backlash

National foreword

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Cylindrical involute gears and gear pairs —

Part 2: Calculation and measurement of tooth thickness and backlash

Roues dentées et engrenages cylindriques en développante de cercle —

Partie 2: Calcul et mesurage de l'épaisseur de dent et du jeu de battement

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Foreword

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A list of all parts in the ISO 21771 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document replaces the sections of ISO/TR-10064-1 and ISO 21771-1 that used to provide information on calculation and measurement of tooth thickness. It is based on AGMA 2002, along with some material from ISO 21771-1:2007¹⁾ Annex A and ISO/TR 10064-2, and some new information on crossed axis gears and measurement using a three ball span device. Example calculations are provided in [Annex E](#).

1) Withdrawn.

Cylindrical involute gears and gear pairs —

Part 2: Calculation and measurement of tooth thickness and backlash

1 Scope

This document establishes the calculation procedures for determining specification limits for external and internal cylindrical involute gearing with constant gear ratio when the desired tooth thickness is known. It also shows the relationships between backlash and the tooth thickness, centre distance, and tooth deviations in a pinion and gear mesh.

This document applies to the following types of gears:

- external and internal parallel axis cylindrical involute spur and helical gears;
- involute worms and crossed axis gears;
- spur and helical racks;
- involute spur and helical sector gears.

The formulae in this document apply to gears of all sizes and materials, and to all manufacturing methods.

This document does not provide guidance on selecting the tooth thickness or the tolerance on the thickness.

2 Normative references

There are no normative references in this document.

3 Terms, definition and symbols

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1 Terms and definitions

3.1.1

backlash

amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the working pitch circles, i.e. the clearance between mating gear teeth

Note 1 to entry: See [Clause 13](#) for further information.

Note 2 to entry: Backlash can be defined in either a normal or a transverse section and either perpendicular to the flank or in a circumferential direction (tangential) at a specified diameter.

3.1.2

ball

calibrated gauge artifact which is spherical and has a known diameter and very tight tolerances on roundness and profile of surface

Note 1 to entry: See ISO 1101 for further information.

Note 2 to entry: For comparison, see the definition of *pin* (3.1.14).

3.1.3

addendum

h_a

radial distance from the reference diameter to the tip diameter of the gear

Note 1 to entry: See [Figure 1](#).

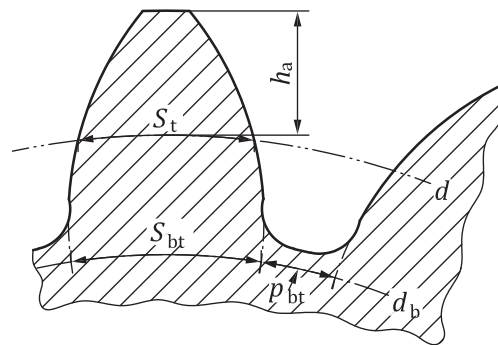


Figure 1 — External gear transverse tooth thickness and base transverse tooth thickness

3.1.4

chordal addendum

h_{cy}

radial distance from the mid point of the chordal measurement line to tip diameter of a gear

Note 1 to entry: See [Figure 2](#).

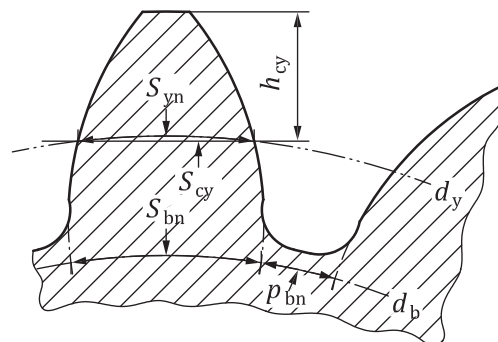
3.1.5

chordal radius

r_{cy}

radial distance from the datum axis to the midpoint of the chordal measurement line

Note 1 to entry: The chordal radius is normally found by subtracting the *chordal addendum* (3.1.4) from the tip radius.



NOTE 1 normal, chordal and base tooth thickness are shown

NOTE 2 The normal surface is shown projected onto a plane

Figure 2 — Tooth thickness on external helical gear normal surface

3.1.6**datum axis**

theoretical gear axis defined by the datum surfaces

Note 1 to entry: The datum axis cannot be directly defined; it only exists in relation to the datum surfaces. It is the axis to which the gear details, and in particular the pitch, profile, and helix tolerances are defined. It should correspond to the rotational axis. See ISO/TR 10064-3 for additional information.

3.1.7**tip involute limit diameter**
 d_{La}

diameter where the involute ends near the gear tip

Note 1 to entry: It is the smaller of the minimum tip relief diameter, d_{Ca} , the minimum tip form diameter, d_{Fa} , or the minimum tip diameter, $d_{a\min}$.

Note 2 to entry: For internal gears, the minimum tip (minor) diameter, $d_{a\min}$ has an absolute value greater than the absolute value of the maximum tip (major) diameter, $d_{a\max}$.

Note 3 to entry: In many cases, at the beginning of tip relief zone, the deviation from the involute is so small that it often can be ignored in tooth thickness measurements. In these cases, the value used for the limit diameter can be within the tip relief zone.

3.1.8**root involute limit diameter**
 d_{Lf}

diameter where the involute ends near the gear root

Note 1 to entry: It is the greater of the profile control diameter, d_{Cp} , maximum root relief diameter, $d_{C\alpha p}$ or the maximum root form diameter, d_{ff} .

Note 2 to entry: In many cases, at the beginning of root relief zone, the deviation from the involute is so small that it often can be ignored in tooth thickness measurements. In these cases, the value used for the limit diameter can be within the root relief zone.

3.1.9**working pitch diameter**

operating pitch diameter

diameter on a gear in mesh with its mate where there is a common pitch point between both gears at their actual centre distance position, and where the imaginary working pitch circles roll together without slipping

Note 1 to entry: The mean of the working pitch diameters of a pair of external parallel axis gears in mesh equals the working centre distance while the ratio of the working pitch diameters equals the gear ratio.

3.1.10**rack tool**

hypothetical rack type cutter that can directly generate the finished involute gear tooth flanks, including all machining allowances for the finished gear

3.1.11**rack reference plane**

A plane parallel to the datum surface of a rack where the tooth thickness equals the tooth space width, i.e., where both tooth thickness and space width are equal to one half pitch

3.1.12**rack reference line**

line in a plane parallel to the rack datum surface where the tooth thickness equals the space width

Note 1 to entry: The rack reference line was called the rack datum line in 21771:2007.

3.1.13**manufacturing profile shift coefficient****minimum**, $x_{E \min}$ **maximum**, $x_{E \max}$

amount by which the reference plane of the *rack tool* (3.1.10) is displaced from the reference cylinder of the gear away from the root of the gear, divided by the normal module

Note 1 to entry: The manufacturing profile shift coefficient of the gear directly relates to the finished tooth thickness of the gear, see 4.7.5. It is not to be confused with the zero-backlash profile shift coefficient, x , used in preliminary calculations.

Note 2 to entry: Positive values of manufacturing profile shift coefficient result in thicker teeth in both external and internal gears. Negative values result in thinner teeth.

3.1.14**pin**

calibrated gauge artifact in the form of a cylindrical rod with a known diameter and very tight tolerances on roundness and cylindricity

Note 1 to entry: Pins are sometimes referred to as cylinders, wires, or rolls. For comparison, see the definition of *ball* (3.1.2).

3.1.15**master gear test radius**

radial distance from the rotational axis of the master gear to the *rack reference line* (3.1.12) of a rack in tight mesh with that master

Note 1 to entry: This is also the radius of the V circle (see ISO 21771-1:2024, 4.6.2).

Note 2 to entry: the master gear test radius is dependent not only on its normal module, helix angle and pressure angle, but also on its calibrated tooth thickness and number of teeth.

3.1.16**product gear test radius**

difference between the *tight mesh* (3.1.17) centre distance of that product gear with a master gear and the master gear test radius

Note 1 to entry: The master gear test radius is determined by how it meshes with a rack and is therefore a constant. The product gear test radius is dependent on how it engages with a specific master gear during a double flank test. Therefore, the product gear test radius can be different depending upon the master employed and is not based solely on the product gear itself.

3.1.17**tight mesh**

condition where two gears are pushed together so that they operate with no *backlash* (3.1.1)

Note 1 to entry: For tooth thickness measurement this typically involves a master gear pressed lightly against the product gear being tested

3.1.18**normal tooth thickness** s_n, s_{yn}, s_{bn}

length, on a specified cylinder, of an arc on a helix that is normal to the two flanks of a gear tooth

Note 1 to entry: At the reference diameter d , the symbol s_n applies; at a specified diameter d_y , the symbol s_{yn} applies; at the base diameter d_b , the symbol s_{bn} applies.

Note 2 to entry: For spur gears, there is no difference between transverse and normal tooth thickness. See Figure 2.

3.1.19**normal rack tooth thickness** s_n, s_{yn}

length of a line segment, normal to the flank lines, between the two flanks of a rack tooth in a plane parallel to the reference plane

Note 1 to entry: The symbol s_n applies when measured at the reference plane, otherwise the symbol s_{yn} is used to indicate that the measurement applies at a specified plane.

3.1.20**transverse tooth thickness** s_t, s_{yt}, s_{bt}

length between the two sides of a spur gear, helical gear, or involute worm tooth, at a specified diameter along a circular arc in a transverse plane

Note 1 to entry: At the reference diameter d , the symbol s_t applies; at a specified diameter d_y , the symbol s_{yt} applies; at the base diameter d_b , the symbol s_{bt} applies.

Note 2 to entry: See [Figure 1](#).

3.1.21**transverse rack tooth thickness** s_t, s_{yt}

tooth thickness measured along the intersection of a specified plane that is parallel to the reference plane and a transverse plane between the plane flanks of the rack teeth

Note 1 to entry: The symbol s_t applies when measured at the reference plane, otherwise the symbol s_{yt} is used to indicate that the measurement applies at a specified plane.

3.1.22**chordal tooth thickness** s_{cy}

length of the chord (straight line) between the two points where a helix, that is normal to the helix at a specified diameter, intersects the tooth flanks

3.1.23**axis related tooth thickness**

tooth thickness which is measured in relation to the *datum axis* ([3.1.6](#))

Note 1 to entry: See [4.3](#) for further details.

3.1.24**non-datum related tooth thickness**

tooth thickness which is measured without referencing the *datum axis* ([3.1.6](#))

Note 1 to entry: See [4.3](#) for further details.

3.1.25**total radial composite deviation** F_{id}

difference between the maximum and minimum values of *tight mesh* ([3.1.17](#)) centre distance or *product gear test radius* ([3.1.16](#)) found after evaluating all of the teeth of a product gear in a double flank test

3.1.26**total radial composite tolerance** F_{idT}

maximum allowable *total radial composite deviation* ([3.1.25](#))

3.1.27

calculation diameter d_y

arbitrary diameter that defines a theoretical cylinder concentric to the gear axis

Note 1 to entry: often used as the desired contact diameter for a *ball* (3.1.2), *pin* (3.1.14), or measuring device.

3.2 Symbols and definitions

The symbols used in this document are listed alphabetically in [Table 1](#) along with a brief definition.

Table 1 — Symbols and definitions

Symbol	Term	Units	Formula ^c
$a_{dm \max}$	measured tight mesh centre distance, maximum ^a	mm	73
$a_{dm \min}$	measured tight mesh centre distance, minimum ^a	mm	74
$a_w \max$	working centre distance, maximum ^a	mm	215
$a_w \min$	working centre distance, minimum ^a	mm	216
$b_{\min}$	width of the unmodified portion of face ^a , minimum	mm	84 , 183
$b_{\min b}$	minimum required face width that will allow for span measurement	mm	85
C_{eo3}	factor for three ball measurement, even or odd number of teeth	-	177 , 178
c	distance between fixed ball centres, three ball span device ^a	mm	102
D_M	diameter of the ball or pin ^a	mm	111
D_{Ma}	ball or pin diameter for the ball or pin to be tangent to the tip diameter	mm	122 , 134
D_{Mdy}	ball or pin diameter for contact at the desired contact diameter, d_y	mm	130 , 136
D_{Mf}	ball or pin diameter for the ball or pin to be tangent to the root diameter	mm	124 , 133
D_{MLf}	ball or pin diameter for contact at the root involute limit diameter	mm	121 , 132
D_{MII}	ball or pin limit diameter for a valid numerical solution	mm	123
$D_{M \max}$	ball or pin diameter, maximum	mm	117 , 131
$D_{M \min}$	ball or pin diameter, minimum	mm	125 , 135
d	reference diameter	mm	1
$d_{a \text{ avg}}$	tip diameter, mean	mm	195
$d_{a \max}$	tip diameter, maximum ^a	mm	195
$d_{a \min}$	tip diameter, minimum ^a	mm	195
d_{am}	measured tip diameter of the product gear ^a	mm	204
d_{Bm}	measured dimension over blocks ^a	mm	192 , 193
d_b	base diameter	mm	7
d_{b1}	base diameter of the pinion	mm	217 , 218
d_{bL}	base diameter of the left flank	mm	144
d_{bR}	base diameter of the right flank	mm	143
$d_{c \max}$	contact diameter with maximum tooth thickness	mm	139
$d_{c \min}$	contact diameter with minimum tooth thickness	mm	140
d_{cL}	diameter where ball or pin contacts the left flank involute during measurement	mm	148
d_{cm}	diameter where ball or pin contacts the involute during measurement	mm	155 , 163 , 164 , 182 , 187
d_{cR}	diameter where ball or pin contacts the right flank involute during measurement	mm	147
$d_{f \max}$	root diameter, maximum ^a	mm	124
$d_{k \max}$	diameter through the centre of the ball or pin for maximum tooth thickness	mm	137

Table 1 (continued)

Symbol	Term	Units	Formula ^c
$d_{k\min}$	diameter through the centre of the ball or pin for minimum tooth thickness	mm	138
d_{k2m}	measured dimension over two balls or pins ^a	mm	161
d_{k3am}	measured dimension over three pins, axial plane method ^a	mm	186
d_{k3tm}	measured dimension over three balls or pins, transverse plane method ^a	mm	181
d_{La}	tip involute limit diameter ^a	mm	93 , 95
d_{Lf}	root involute limit diameter ^a	mm	94 , 96
d_M	measurement diameter	mm	99
$d_{wfi\max}$	functional working pitch diameter, pinion or gear ^b , maximum	mm	229
$d_{wfi\min}$	functional working pitch diameter, pinion or gear ^b , minimum	mm	230 , B.1 , B.2
$d_{wi\max}$	working pitch diameter, pinion or gear ^b , maximum	mm	215
$d_{wi\min}$	working pitch diameter, pinion or gear ^b , minimum	mm	216 , C.1 , C.2
d_y	calculation diameter ^a	mm	15
$d_{y\min}$	chordal contact diameter with minimum tooth thickness	mm	199
$d_{yt\min}$	contact diameter for the minimum tooth thickness at the specified chordal radius	mm	208
d_Σ	calculation diameter for imposed skew angle, product gear	mm	53
$d_{\Sigma 3}$	calculation diameter for imposed skew angle, master gear	mm	54
$d_{\Sigma w1}$	calculation diameter for working skew angle, pinion	mm	244
$d_{\Sigma w2}$	calculation diameter for working skew angle, gear wheel	mm	245
E	Error term	-	114 , 118 , 127
$E_{sn\min}$	lower normal tooth thickness limit deviation at reference cylinder ^a	mm	23
$E_{sn\max}$	upper normal tooth thickness limit deviation at reference cylinder ^a	mm	22
e_n	normal space width at reference diameter	mm	39
e_t	transverse space width at reference diameter	mm	34 , 141
e_{yn}	normal space width at diameter d_y	mm	38
e_{yt}	transverse space width at diameter d_y	mm	33
F_{idT}	total radial composite tolerance of the product gear ^a	µm	60
F_{idTi}	total radial composite tolerance for the pinion or gear ^{a,b}	µm	227 , 228
F_{pc}	multiplier to adjust conversion from total cumulative pitch to total radial composite tolerance	- -	228
F_{pTi}	total cumulative pitch (index) tolerance for the pinion or gear ^{a,b}	µm	228
F_{rc}	multiplier to adjust conversion from runout to total radial composite tolerance	- -	227
F_{rTi}	runout tolerance for the pinion or gear ^{a,b}	µm	227
$F_{\alpha Ti}$	total profile tolerance for the pinion or gear ^{a,b}	µm	227
$F_{\beta Ti}$	total helix tolerance for the pinion or gear ^{a,b}	µm	227
F_2	calibrated value of the tooth profile coefficient for the double tooth block ^a	- -	190
f_{pTi}	single pitch tolerance for the pinion or gear ^{a,b}	µm	227
$f_{\alpha s}$	modification coefficient for span profile limits	- -	82 , 83
$f_{\beta s}$	modification coefficient for span helix limit	- -	84
$H_{k\max}$	measurement limit over one ball or pin to the rack datum surface, maximum	mm	151
$H_{k\min}$	measurement limit over one ball or pin to the rack datum surface, minimum	mm	152
H_{km}	measured dimension over one ball or pin to the rack datum ^a	mm	154
h	distance from rack reference line to desired contact location	mm	136

Table 1 (continued)

Symbol	Term	Units	Formula ^c
h_a	addendum	mm	Figure 1
$h_{a \max}$	distance between the rack reference line and the rack tip line ^a , maximum	mm	134
h_{cy}	chordal addendum	mm	196
h_{cytw}	chordal addendum for measurement of transverse chordal tooth thickness	mm	212, 213, 214
h_{cyw}	chordal addendum to be used for the tooth thickness measurement	mm	203, 204
h_{ff}	distance between the rack reference line and the rack root form line ^a	mm	132
h_{fa}	distance between the rack reference line and the rack tip form line ^a	mm	131
$h_{f \min}$	distance between the rack reference line and the rack root line ^a , minimum	mm	133
$j_{bn \max}$	normal backlash, maximum	mm	255
$j_{bn \min}$	normal backlash, minimum	mm	256
$j_{bt \max}$	transverse backlash, maximum	mm	251, 253
$j_{bt \min}$	transverse backlash, minimum	mm	252, 254
$j_{r \max}$	radial backlash, maximum (cross axis gears)	mm	257
$j_{r \min}$	radial backlash, minimum (cross axis gears)	mm	258
$j_{wt \max}$	circumferential backlash, maximum	mm	237, 239
$j_{wt \min}$	circumferential backlash, minimum	mm	238, 240
$j_{wtz \max}$	circumferential backlash, maximum calculated from axis related tooth thickness	mm	223, 225
$j_{wtz \min}$	circumferential backlash, minimum calculated from axis related tooth thickness	mm	224, 226
$j_{\Sigma n \max}$	cross axis gear working normal backlash, maximum	mm	247, 249
$j_{\Sigma n \min}$	cross axis gear working normal backlash, minimum	mm	248, 250
k	number of teeth spanned on an external gear; or the number of tooth spaces spanned on an internal gear ^a	--	87, 92
$k_{\max a}$	maximum number of teeth spanned on an external gear based on the tip involute limit diameter	--	82
$k_{\max b}$	maximum number of teeth spanned based on face width of an external helical gear; or the maximum number of tooth spaces spanned based on the face width of an internal helical gear	--	84, 90
$k_{\max f}$	maximum number of tooth spaces spanned on an internal gear based on the root involute limit diameter	--	88
$k_{\min a}$	minimum number of tooth spaces spanned on an internal gear based on the tip involute limit diameter	--	89
$k_{\min f}$	minimum number of teeth spanned on an external gear based on the root involute limit diameter	--	83
L_p	critical pin length	mm	167
L_z	axial distance between the contact points of free floating pins	mm	165
M_2	calibrated value of the tooth profile coefficient for the single tooth block ^a	--	188, 189
$M_{B \max}$	measurement limit over blocks, maximum	mm	188, 190
$M_{B \min}$	measurement limit over blocks, minimum	mm	189, 191
$M_{cy \max}$	normal chordal tooth thickness limit at the desired contact diameter, d_y and chordal addendum h_{cy} , maximum	mm	201
$M_{cy \min}$	normal chordal tooth thickness limit at chordal addendum h_{cy} , minimum	mm	202
$M_{cyt \max}$	transverse chordal tooth thickness at the desired contact diameter, maximum	mm	210
$M_{cyt \min}$	transverse chordal tooth thickness at chordal radius r_{cy} , minimum	mm	211
$M_{d \max}$	tight mesh centre distance limit, maximum	mm	60, 64, 66, 68

Table 1 (continued)

Symbol	Term	Units	Formula ^c
$M_{d \min}$	tight mesh centre distance limit, minimum	mm	61 , 65 , 67 , 69
$M_{dk \max}$	measurement limit over two balls or pins for maximum tooth thickness	mm	157 , 159
$M_{dk \min}$	measurement limit over two balls or pins for minimum tooth thickness	mm	158 , 160
$M_{dkF \max}$	measurement limit over two pins, free pin method, maximum	mm	175
$M_{dkF \min}$	measurement limit over two pins, free pin method, minimum	mm	176
$M_{dk3a \max}$	measurement limit over three pins, axial plane method, maximum	mm	184
$M_{dk3a \min}$	measurement limit over three pins, axial plane method, minimum	mm	185
$M_{dk3t \max}$	measurement limit over three balls or pins, transverse plane method, for maximum tooth thickness	mm	179
$M_{dk3t \min}$	measurement limit over three balls or pins, transverse plane method, for minimum tooth thickness	mm	180
$M_{r \max}$	product gear test radius limit, maximum	mm	71
$M_{r \min}$	product gear test radius limit, minimum	mm	72
M_{rk}	measurement over one ball or pin on a gear	mm	153
$M_{rk \max}$	measurement limit over one ball or pin, maximum	mm	149
$M_{rk \min}$	measurement limit over one ball or pin, minimum	mm	150
m_n	normal module ^a	mm	2
m_{n3}	normal module of the master gear ^a	mm	40
P_{nd}	normal diametral pitch ^a	in ⁻¹	2
p_{bn}	normal base pitch	mm	11 , 40
p_n	normal pitch at the reference diameter	mm	9
$p_{rM \max}$	radial offset of the contact point, maximum	mm	171
$p_{rM \min}$	radial offset of the contact point, minimum	mm	172
$p_{twf \min}$	transverse pitch at the minimum functional working pitch diameter	mm	B.3 , B.4
$p_{wt \min}$	transverse pitch at the minimum working pitch diameter	mm	C.3 , C.4
p_x	axial pitch	mm	13
p_{yn}	normal pitch at diameter d_y		10
p_z	lead	mm	12
$p_{\Sigma wn}$	normal circular pitch for the working skew angle	mm	246
r_{am}	measured radius to the tip of the gear tooth being checked ^a	mm	203
r_{cy}	chordal radius to the midpoint of the chordal measurement line	mm	206
r_{k1m}	measured dimension over one ball or pin ^a	mm	155
$r_{m \max}$	radius to the centre point of the line between the pair of balls on a three ball span device with maximum tooth thickness	mm	100
$r_{m \min}$	radius to the centre point of the line between the pair of balls on a three ball span device with minimum tooth thickness	mm	101
r_{r3}	master gear test radius ^a	mm	70
s_{bn}	zero backlash normal base tooth thickness	mm	21
$s_{bn \max}$	non-datum related normal base tooth thickness, maximum	mm	19 , 22
$s_{bn \min}$	non-datum related normal base tooth thickness, minimum	mm	20 , 23
s_{bt}	base transverse tooth thickness	mm	17 , 18
s_n	normal tooth thickness at the reference diameter ^a	mm	27
$s_{n \max}$	non-datum related normal tooth thickness at the reference diameter, maximum	mm	5
$s_{n \min}$	non-datum related normal tooth thickness at the reference diameter, minimum	mm	6

Table 1 (continued)

Symbol	Term	Units	Formula ^c
$s_{nf1 \max}$	non-datum related normal tooth thickness at the reference diameter, pinion, maximum	mm	C.6
$s_{nf2 \max}$	non-datum related normal tooth thickness at the reference diameter, gear wheel, maximum	mm	C.7
$s_{n1 \max}$	non-datum related normal tooth thickness at the reference diameter, pinion, maximum	mm	B.6
$s_{n2 \max}$	non-datum related normal tooth thickness at the reference diameter, gear wheel, maximum	mm	B.7
s_{n3}	normal tooth thickness at the reference diameter, master gear ^a	mm	57
s_{nm}	measured non-datum related normal tooth thickness at the reference diameter	mm	98, 111, 161, 162, 181, 186, 192, 193
s_{ns}	mean normal circular tooth thickness of product gear	mm	41
$s_{nzi \max}$	axis related normal tooth thickness at the reference diameter for the pinion or gear ^b , maximum ^a	mm	219, 221
$s_{nzi \min}$	axis related normal tooth thickness at the reference diameter for the pinion or gear ^b , minimum ^a	mm	220, 222
s_{nzm}	measured axis related normal tooth thickness at the reference diameter	mm	154
$s_{nzm \max}$	measured axis related normal tooth thickness at the reference diameter, maximum	mm	73, 75
$s_{nzm \min}$	measured axis related normal tooth thickness at the reference diameter, minimum	mm	74, 76
s_t	transverse tooth thickness at the reference diameter ^a	mm	25
$s_{wtzi \max}$	axis related transverse tooth thickness at the working pitch diameter for pinion or gear ^b , for maximum tooth thickness	mm	219, 221
$s_{wtzi \min}$	axis related transverse tooth thickness at the working pitch diameter for pinion or gear ^b , for minimum tooth thickness	mm	220, 222
$s_{wti \max}$	non-datum related transverse tooth thickness at the functional working pitch diameter for pinion or gear ^b , for maximum tooth thickness	mm	233, 235
$s_{wti \min}$	non-datum related transverse tooth thickness at the functional working pitch diameter for pinion or gear ^b , for minimum tooth thickness	mm	234, 236
s_{yn}	normal tooth thickness at diameter d_y	mm	28, 29
$s_{yn \max}$	normal tooth thickness at diameter d_y , maximum	mm	194
$s_{yn \min}$	normal tooth thickness at diameter $d_{y \min}$, minimum	mm	200
s_{yt}	transverse tooth thickness at diameter d_y	mm	24, 25, 26, 27
$s_{yt \max}$	transverse tooth thickness at diameter d_y , maximum	mm	205
$s_{yt \min}$	transverse tooth thickness at diameter $d_{yt \min}$, minimum	mm	209
$s_{\Sigma n \max}$	product gear non-datum related normal circular tooth thicknesses at d_{Σ} , maximum	mm	58
$s_{\Sigma n \min}$	product gear non-datum related normal circular tooth thicknesses at d_{Σ} , minimum	mm	59
$s_{\Sigma n3}$	master gear normal circular tooth thicknesses at $d_{\Sigma3}$	mm	57
$s_{\Sigma nz \max}$	product gear axis related normal circular tooth thicknesses at d_{Σ} , maximum	mm	62
$s_{\Sigma nz \min}$	product gear axis related normal circular tooth thicknesses at d_{Σ} , minimum	mm	63
W_k	span measurement over k teeth on an external gear	mm	77
$W_{k \max}$	span measurement limit over k measured teeth or measured tooth spaces, maximum	mm	78, 80

Table 1 (continued)

Symbol	Term	Units	Formula ^c
$W_{k \min}$	span measurement limit over k measured teeth or measured tooth spaces, minimum	mm	79, 81
W_{km}	measured span ^a	mm	98
$W_{3k \max}$	three ball span device measurement limit over k measured teeth, maximum	mm	107, 109
$W_{3k \min}$	three ball span device measurement limit over k measured teeth, minimum	mm	108, 110
x_E	manufacturing profile shift coefficient	--	26
$x_{E \max}$	manufacturing profile shift coefficient, maximum ^a	--	5
$x_{E \min}$	manufacturing profile shift coefficient, minimum ^a	--	6
Y_5	constant used in tight mesh calculation	--	43
Y_1	variable used in tight mesh calculation	--	44
Y_3	variable used in tight mesh calculation	--	45
y_{cm}	distance between the datum surface and the contact location on the rack from measurement over ball or pin	mm	156
y_D	distance from the rack reference line to the rack datum surface ^a	mm	151, 152
z	number of teeth ^a	--	1
z_i	number of teeth on pinion or gear ^{a,b}	--	215, 216
z_1	number of teeth, pinion ^a	--	215, 216
z_2	number of teeth, gear ^a	--	215, 216
z_3	number of teeth on the master gear ^a	--	42
$\alpha_{kt dy}$	transverse pressure angle to the centre of the ball or pin for the desired contact diameter, d_y	radian	128, 129
$\alpha_{kt Lf}$	transverse pressure angle to the centre of the ball or pin that contacts the form limit diameter	radian	119, 120
$\alpha_{kt \max}$	transverse pressure angle to the centre of the maximum diameter ball or pin	radian	115, 116
α_n	normal pressure angle at the reference diameter ^a	degree	3
α_{n3}	normal pressure angle of the master gear ^a	degree	40
α_{nL}	normal pressure angle at the reference diameter to left flank ^a	degree	141
α_{nR}	normal pressure angle at the reference diameter to right flank ^a	degree	141
α_t	transverse pressure angle at the reference diameter	degree	3
α_{wn3}	optimal tight mesh working normal pressure angle	radian	42
$\alpha_{wt \max}$	working transverse pressure angle, maximum	degree	217
$\alpha_{wt \min}$	working transverse pressure angle, minimum	degree	218, C.5
$\alpha_{wft \max}$	transverse pressure angle at functional working pitch diameter, maximum	degree	231
$\alpha_{wft \min}$	transverse pressure angle at functional working pitch diameter, minimum	degree	232
α_{yt}	transverse pressure angle at diameter d_y	degree	16
$\alpha_{yt \min}$	transverse pressure angle at diameter $d_{y \min}$	degree	197, 207
$\alpha_{\Sigma n}$	normal pressure angle for the imposed skew angle	radian	52
$\alpha_{\Sigma t}$	transverse pressure angle for the imposed skew angle, product gear	radian	55
$\alpha_{\Sigma t3}$	transverse pressure angle for the imposed skew angle, master gear	radian	56
$\alpha_{\Sigma wn}$	normal pressure angle for the working skew angle	radian	243
β	helix angle ^a	degree	4
β_b	base helix angle	degree	8
β_{bL}	base helix angle to left flank ^a	degree	148
β_{bR}	base helix angle to right flank ^a	degree	147
$\beta_{k \max}$	helix angle at the axis of pin for two pin transverse method	radian	166

Table 1 (continued)

Symbol	Term	Units	Formula ^c
β_y	helix angle at diameter d_y	degree	15
$\beta_{y \min}$	helix angle at measuring diameter, $d_{y \min}$ for chordal measurement	radian	198
β_w	tight mesh working helix angle of product gear at optimal skew angle	radian	47
β_{w3}	tight mesh working helix angle of master gear at optimal skew angle	radian	48
β_3	helix angle of the master gear ^a	degree	42
β_Σ	preliminary helix angle with imposed skew angle, product gear	radian	50
$\beta_{\Sigma 3}$	preliminary helix angle with imposed skew angle, master gear	radian	51
$\beta_{\Sigma w1}$	helix angle with working skew angle, pinion	radian	241
$\beta_{\Sigma w2}$	helix angle with working skew angle, gear wheel	radian	242
γ	lead angle	degree	4
$\Delta_{M \max}$	axial offset of the contact point, maximum	mm	168
$\Delta_{M \min}$	axial offset of the contact point, minimum	mm	169
δ_w	angle of rock	radian	97
δ_{wa}	angle of rock calculated from the tip involute limit diameter	radian	93, 95
δ_{wf}	angle of rock calculated from the root involute limit diameter	radian	94, 96
ζ_L	left flank angle, in a transverse plane, to the origin of an involute passing through the centre of the ball	radian	144
ζ_R	right flank angle, in a transverse plane, to the origin of an involute passing through the centre of the ball	radian	143
η	transverse space width half angle at reference diameter	radian	36, 142
η_b	transverse space width half angle at base diameter	radian	37
$\eta_{b \text{ avg}}$	mean transverse base space width half angle	radian	126
$\eta_{b \max}$	transverse base space width half angle, at maximum tooth thickness	radian	113
$\eta_{b \min}$	transverse base space width half angle, at minimum tooth thickness	radian	112
η_y	transverse space width half angle at diameter d_y	radian	35
$\theta_{M \max}$	tilt angle of the measurement axis, maximum	degree	173
$\theta_{M \min}$	tilt angle of the measurement axis, minimum	degree	174
Σ	optimal tight mesh skew angle for mean tooth thickness	radian	49
Σ_Σ	imposed shaft skew angle (angle used on double flank tester)	degree	50
$\phi_{1 \max}$	three ball span device measurement helix rotation from plane 0 to 1 at maximum tooth thickness	radian	102
$\phi_{1 \min}$	three ball span device measurement helix rotation from plane 0 to 1 at minimum tooth thickness	radian	103
$\phi_{3 \max}$	three ball span device measurement helix rotation from plane 0 to 3 at maximum tooth thickness	radian	104
$\phi_{3 \min}$	three ball span device measurement helix rotation from plane 0 to 3 at minimum tooth thickness	radian	105
ϕ_k	three ball span device measurement angle over k tooth spaces	radian	106
ϕ_L	left flank angle to intersection point of circle with an involute passing through the centre of the ball	radian	146
ϕ_R	right flank angle to intersection point of circle with an involute passing through the centre of the ball	radian	145
$\varphi_{ji \max}$	angular backlash of the pinion or gear ^b , maximum	degree	259
$\varphi_{ji \min}$	angular backlash of the pinion or gear ^b , minimum	degree	260
ψ	transverse tooth thickness half angle at reference diameter	radian	31
ψ_b	transverse tooth thickness half angle at base diameter	radian	32

Table 1 (continued)

Symbol	Term	Units	Formula ^c
ψ_v	transverse tooth thickness half angle at diameter d_v	radian	30
NOTE: ^a Input variable. ^b An “i” in a subscript denotes either the pinion (where the subscript i is replaced with a “1”) or the gear wheel (where the subscript i is replaced with a “2”). ^c Formula where the symbol calculated or first used.			

3.3 Units and conventions

3.3.1 Units

The quantities dealt with in this document are to be stated in the following units:

- modules, lengths and linear dimensions in millimetres (mm);
- composite and elemental tolerances in micrometres (μm);
- angles which are to be used in formulae in radians (rad);
- angles which can be used for entries or to display results in degrees (°);
- angular velocity in radians per second (rad/s).

3.3.2 Conventions

The conventions used in this document are:

- The number of teeth, z , for external gears has a positive value. For internal gears, the number of teeth has a negative value. Therefore, all internal gear diameters and radii will also have negative values. The centre distance of a pinion mated with an internal gear has a negative value. In visualizing internal gears, it is helpful to picture the tooth pointing up, so the tip diameter is that with the negative value closest to zero. For internal gears, the pitch and tooth thickness are always positive values but the lead is negative.
- In some formulae the term $\frac{z}{|z|}$ is used to make the formulae universal for external or internal gears. This term has a value of 1 for external gears and -1 for internal gears.
- The term “gear” refers to either a pinion or a gear (wheel) unless otherwise indicated.
- Gears with a left-hand helix have a negative helix angle, those with a right hand helix have a positive helix angle.
- Terms labelled as coefficients are dimensionless.
- For internal gears, the term tooth thickness refers to the tooth and not the space width.

4 Application

4.1 General

For cylindrical gears, unless chordal tooth thickness is explicitly mentioned, tooth thickness is defined on the surface of a specified cylinder. It is the length between the flanks of a tooth, measured either along a circular arc in a transverse plane or along a helix normal to the tooth flanks. The only exception to this is chordal tooth thickness, which is a straight-line measurement between points at a specified diameter on opposite flanks of a tooth.

Specification of gear tooth thickness is developed during the gear design process. The tooth thickness limits should be chosen based on the application and backlash requirements. Consideration should be given to the manufacturing cost associated with achieving the tolerances and the inherent limitations of the manufacturing process and inspection methods used.

Practical considerations and availability of test equipment can preclude the use of some tooth thickness measurement methods on some sizes of gears. Some methods are not appropriate for flexible materials (such as some plastics) if the use of the tooth thickness measuring tools and techniques result in deformation of the gear teeth.

Maximum and minimum backlash amounts depend on the gear's tooth thickness limits, elemental deviations or total radial composite deviation, and the centre distance range. The limit values of these characteristics should be specified to prevent interference (double flank contact) or excessive backlash. The minimum tooth thickness can be further controlled to meet tooth bending strength requirements.

The tolerance on axis related tooth thickness when expressed as a variation in tight mesh centre distance (or product gear test radius) should always include both the variation in tooth thickness and the effects related to total radial composite tolerance. So, the tolerance on tight mesh centre distance (or product gear test radius) should always exceed the total radial composite tolerance. See [4.3.3](#).

The specification of design tooth thickness for a given application is outside the scope of this document. This document assumes that either the design tooth thickness is known and that the measurement limits for a specified tooth thickness inspection method are to be established, or that the tooth thickness is to be determined based on measurement results.

This document describes each of the tooth thickness measurement methods, their associated underlying assumptions, and practical considerations that can limit measurement accuracy.

Once the specification limits have been established, if the gear is measured with the specified method and meets the specified limits, then the gear tooth thickness shall be considered acceptable.

This document also provides guidance on the prediction of backlash, including the effects of total radial composite or elemental manufacturing tolerances. See [Clause 13](#).

For additional information on geometry of involute gears, see ISO 21771-1^[8].

4.2 Datum surfaces and the datum axis

The datum axis is used by the manufacturer (and inspector) to define the geometry of the gear teeth on an individual component. It is the responsibility of the designer to ensure that the datum surfaces which define the datum axis are specified with sufficient clarity and precision to ensure that the requirements of the gear in relation to the functional rotational axis are met.

Very often this can be achieved by using the functional mounting surfaces as the datum surfaces.

In general, however, the datum surfaces shall be defined and then related to other surfaces with appropriate tolerances. This is particularly important for those surfaces used to mount the gear for either manufacturing or inspection. When the functional mounting surfaces for operation of the gear are not the datum surfaces, due consideration shall be given to the effect of the additional link in the tolerance chain. See ISO/TR 10064-3^[6] for additional information.

4.3 Gear tooth thickness specification systems

4.3.1 Types of tooth thickness specification systems

The tooth thickness of a cylindrical gear is commonly expressed as a circular or helical arc length on an implied cylindrical surface. This specification should clearly indicate if the tooth thickness value is in the transverse, normal, or axial plane. For racks, tooth thickness is expressed as a straight line length on a specified plane.

It is important to understand whether the specification of the implied cylindrical surface for tooth thickness measurement is in relation to the datum axis, and thus an axis related tooth thickness, or to the gear tooth profile itself and hence independent of the datum, and thus a non-datum related tooth thickness.

4.3.2 Non-datum related tooth thickness

Non-datum related gear tooth thickness is specified (or inherently understood) as being independent of the datum axis. This is the most common method used. The non-datum related system uses as its basis a tooth that has no runout and is perfect in shape with only a tolerance on thickness. When using the non-datum related system, the specification of either elemental tolerances or total radial composite tolerance is required to further control the functionality of the design and hence control the backlash in the system. In order to verify that a gear conforms to the specified non-datum related tooth thickness limits, measurement methods that are independent of the datum axis may be used (i.e. span, over two balls or pins). For a method to establish tooth thickness specifications in the non-datum related system, see [Annex B](#).

WARNING — When using the non-datum related system, failure to properly account for either elemental tolerances or total radial composite tolerance can result in insufficient backlash and possibly gear failure.

All tooth thickness measurements are influenced by deviations in base pitch and tooth profile. The non-datum related inspection methods assume that these deviations do not influence the calculation of the measurement limits and are therefore 'inclusive' within the tooth thickness tolerance range itself.

4.3.3 Axis related tooth thickness

A specification method known as axis related tooth thickness can be used instead of non-datum related tooth thickness. The axis related system uses as its basis a tooth with all allowances for radial composite or elemental tolerances already included in the maximum and minimum tooth thickness specification. In order to verify that a gear conforms to the specified axis related tooth thickness limits, measurement methods such as tight mesh centre distance on a double flank tester or measurement using a coordinate measuring machine (CMM) may be used. With these methods, the gear is either mounted on its datum surfaces, or with alternative surfaces that have a very tight tolerance to the datum surfaces. For a method to establish tooth thickness specifications in the axis related system, see [Annex C](#).

4.3.4 Relationship between the measurement methods and specification systems

Generally, gears specified in the non-datum related system are measured by non-datum related methods. They may be tested with the double flank method if the total composite deviation is specified. Gears specified in the non-datum related system and failing a non-datum related measurement method, e.g. span, can sometimes pass an axis related measurement method with acceptable backlash results when the total radial composite deviation relative to its tolerance is low. Such a condition is functionally acceptable even though its non-datum related measurement method shows rejection. In such cases, there should be agreement between the manufacturer and purchaser of the gear if the non-datum related measurement specification is needed.

Gears specified in the axis related system should only be measured using axis related methods. Although it can be said that the inclusion of a total radial composite tolerance allows for the conversion of the axis related tooth thickness into non-datum related tooth thickness limits, the use of non-datum related measurement methods can reject more product than intended by the original axis related specifications if the total radial composite deviation relative to its tolerance is low. See Reference [\[15\]](#).

4.3.5 Mean tooth thickness

The mean tooth thickness determined from multiple measurements around the gear will, within measurement deviation, be the same for all measurement methods, both non-datum related, and axis related, except for chordal measurements with gear tooth caliper. The mean thickness should be determined from measurements spaced equally around the gear; in some cases, as few as two measurements are sufficient. The mean tooth thickness is well suited for control of manufacturing process variables such as cutting tool infeed.

4.4 Gear tooth thickness measurement methods

[Table 2](#) lists tooth thickness measurement methods associated with the type of tooth thickness measurement system employed. This document describes how circular arc tooth thickness limits can be converted to measurement limits based on each of the described methods.

Table 2 — Tooth thickness specification system and corresponding measurement methods

Tooth thickness specification system	Measurement method
Non-datum related (not related to datum axis)	Span between two points or lines Three ball span device Over or between two or three balls or pins Measurement with blocks Chordal, when measured from tooth tip
Axis related (based on datum axis)	Pitch or index Double flank test Single flank test Over or between one ball or pin and the datum axis Chordal, when referenced to the datum axis

The types of tooth thickness measurement methods are categorized as either non-datum related or axis related. The non-datum related measurement methods do not reference the datum axis, whereas the axis related methods give the tooth thickness with reference to the datum axis. When only some of the teeth are measured, the teeth with the maximum or minimum thickness usually will not be measured.

In order to select the appropriate tooth thickness inspection method for an application, factors such as gear diameter, tooth size, measurement uncertainty, repeatability, reliability, and cost should be considered. The measurement method should not result in any tooth deflections which add error into the results. This is especially a problem for easily deformed materials such as polymers.

The tooth thickness measurement methods discussed in this document include methods that can survey either a few or all of the teeth on each gear. Single or double flank radial composite testing can quickly survey the flanks of all the teeth on a given gear. Measurement by pitch or index is generally done by point contact in a mid-plane on the tooth flanks of all the teeth. Over balls, over pins, span and tooth calliper (chordal) methods evaluate the tooth thickness of a single tooth (or pairs of teeth) between two points or two contact lines, so should be repeated at several positions to better characterize the gear. Depending on the needs of the application, it should be determined whether or not the entire gear can be characterized by just one or two measurements, or if a more comprehensive set of measurements should be taken.

The various measurement methods can lead to slightly different tooth thickness results on any given gear. Variations in profile, pitch, helix, runout, and measurement location will have different effects depending on the measuring method employed, see [Table 3](#). These variations cannot be separated from the measurement of tooth thickness. Therefore, a precise prediction of a measurement result from one method in relation to another method cannot be accurately made, although the different measuring methods will usually be reasonably consistent.

The gear deviations shown in [Table 3](#) can impact the tooth thickness measurement results as indicated. All of the gear deviations will affect the actual backlash since they will all affect the axis related tooth thickness even if they do not affect the measured non-datum related tooth thickness.

While the elemental deviations can affect the tooth thickness measurements, in general it is not possible to infer the magnitude of the deviation from the tooth thickness measurement. The two major exceptions to this are that runout can be estimated from any of the methods that measure axis related tooth thickness and measurement by pitch or index will give the single and cumulative pitch deviations.

The differences between results from the various measurement methods of the same type (non-datum related or axis related) are usually minor. If more than one method is specified, the gear shall meet the

requirements of at least one of the methods specified unless there is a note on the drawing specifically stating that the gear tooth thickness must meet all of the limits of the listed methods.

Maximum and minimum tooth thickness, along with an indication if these are non-datum related or axis related thickness, may be listed on a drawing, usually as a reference dimension.

Table 3 — Gear deviations that can affect tooth thickness measurement

Measurement method	Profile	Single pitch	Cumulative pitch	Helix	Runout	Tip diameter
Span	X	X	X	X		
Three ball span device	X	X	X	X		
Over two or three balls or pins	X	X	X		X	
Measurement with blocks	X	X	X		X	
Chordal, when measured from tooth tip	X					X
Pitch or index	X	X	X		X	
Double flank test	X	X	X	X	X	
Single flank test	X	X	X	X	X	
Over one ball or pin	X	X			X	
Chordal, when referenced to the datum axis	X				X	

4.5 Reporting of gear tooth thickness

Any report of measurements shall indicate the method that was used to establish the result. When the measurement cannot be performed using the specified method, agreement on an alternative method shall be reached with the purchaser. A non-datum related method shall not be used as a substitute for an axis related method. If a conversion from another method is presented, it shall be clearly labelled and the result from the method actually used shall also be presented. For example, if an over pins measurement is specified but the gear is measured on a CMM, the report shall state that a CMM was used and give the maximum, minimum and mean transverse axis related tooth thickness as measured by the CMM along with the calculated over pins measurements from the CMM.

4.6 Measurement diameter versus contact diameter

Conversion of measurement results to another method is not recommended. However, when a gear measuring machine (GMM) or a CMM is used to measure pitch, and then a calculation is made for the expected result from one of the non-datum related methods, the correlation is sometimes improved if the contact diameter for the non-datum related method is used as the measurement diameter. On a CMM it is also possible to use extra measurements at the axial contact positions of the non-datum related measurement instead of using the points used to evaluate pitch deviation.

4.7 Gear geometry calculations

4.7.1 Use of radians or degrees

There is extensive use of trigonometry throughout this document. If all angles are converted to radians, and the mode of the trigonometric functions is radians, then all formulae will work properly. Many of the formulae will also work properly if degrees are used for both the angles and the mode of the trigonometric functions. Formulae that will only work properly in radians are marked with “(use radians)” following the formula, however formulae that use the involute function but otherwise do not require radians are not so marked. It is assumed that the involute function will be properly executed using radians.

There can be issues with accuracy when using computing devices. It is recommended that enough significant digits (at least double precision or binary 64 as defined in IEEE 754) be used in the calculations to obtain an

accurate result. When using a spreadsheet, it is recommended that the PI() function be used for the value of π .

NOTE Examples are provided in [Annex E](#).

4.7.2 Reference diameter

The reference cylinder is the reference surface for the cylindrical gear teeth. Its axis coincides with the axis of the gear (gear axis). The reference circle is the intersection of the reference cylinder with a transverse plane section. The reference diameter, d , is determined as shown in Formula (1):

$$d = z \frac{m_n}{\cos \beta} \quad (1)$$

NOTE 1 The reference diameter is the location where a gear's tooth thickness, pressure angle and helix angle are defined.

NOTE 2 On a gear, the tooth thickness is often different than the tooth space width at the reference diameter.

NOTE 3 For internal gears, the number of teeth and all diameters are stated as negative values.

When normal diametral pitch is specified, the normal module can be calculated using:

$$m_n = \frac{25,4}{P_{nd}} \quad (2)$$

4.7.3 Transverse pressure angle

The transverse pressure angle is:

$$\alpha_t = \arctan \left(\frac{\tan \alpha_n}{\cos \beta} \right) \quad (3)$$

NOTE See ISO 21771-1:2024, 4.4.3

4.7.4 Lead angle

The lead angle, γ , is the complement of the helix angle. It is usually specified instead of helix angle for cylindrical involute worms. If the helix angle is not known, it can be calculated from the lead angle:

$$\beta = 90 - \gamma \quad (\text{use degrees}) \quad (4)$$

NOTE See ISO 21771-1:2024, 4.4.7

4.7.5 Normal tooth thickness from manufacturing profile shift coefficient

Profile shift coefficients can be very useful as part of a comprehensive system of gear design. However, during gear design, zero backlash profile shift coefficients are normally used. These use the symbol x and are also commonly called x factors or x coefficients. These do not relate directly to the tooth thickness. To calculate limits for finished tooth thickness measurement, some thinning for backlash should be applied to one or both members of a gear pair. Procedures to do this are given in DIN standards (e.g. DIN 3967) and in some proprietary company procedures, but they are beyond the scope of this document. It is not uncommon for gear drawings to list both the zero backlash profile shift coefficient (x factor) and measurement limits for a specified measurement method. When this is the case, the specified measurement method and its associated limits shall be used, and the x factor shall be ignored, at least for the thickness determination.

A profile shift coefficient listed on the drawing shall not be used to calculate measurement limits unless it has been positively verified to be a manufacturing profile shift coefficient for the finished gear, and both $x_{E \max}$ and $x_{E \min}$ are given.

For both internal and external gears, the maximum and minimum non-datum related normal tooth thicknesses are related to the finished gear maximum and minimum manufacturing profile shift coefficients using the following relationships:

$$s_{n \max} = m_n \left(2x_{E \max} \tan \alpha_n + \frac{\pi}{2} \right) \quad (5)$$

$$s_{n \min} = m_n \left(2x_{E \min} \tan \alpha_n + \frac{\pi}{2} \right) \quad (6)$$

NOTE 1 When using the axis related system, the profile shift coefficients are axis related so the results of [Formulae \(5\)](#) and [\(6\)](#) are $s_{nz \max}$ and $s_{nz \min}$.

NOTE 2 These are based on ISO 21771-1:2024, Formula (54).

4.7.6 Base diameter

The base diameter of a gear is :

$$d_b = d \cos \alpha_t \quad (7)$$

NOTE See ISO 21771-1:2024, [\[8\]](#) 4.4.2

4.7.7 Base helix angle

The base helix angle is:

$$\beta_b = \arcsin(\sin \beta \cos \alpha_n) \quad (8)$$

NOTE See ISO 21771-1:2024, 4.4.7

4.7.8 Normal pitch at the reference diameter

The normal pitch at the reference diameter is:

$$p_n = \pi m_n \quad (9)$$

NOTE See ISO 21771-1:2024, 4.5.2.2

4.7.9 Normal pitch at calculation diameter d_y

The normal pitch at calculation diameter d_y is:

$$p_{yn} = \frac{\pi d_y}{z} \cos \beta_y \quad (10)$$

4.7.10 Normal base pitch

The normal base pitch of a gear is :

$$p_{bn} = p_n \cos \alpha_n \quad (11)$$

4.7.11 Lead of a helical gear

The lead of a helical gear is the axial distance for one complete revolution of any helix.

$$p_z = \frac{\pi z m_n}{\sin|\beta|} \quad (12)$$

NOTE For internal gears, the lead is stated as a negative value.

4.7.12 Axial pitch

The axial pitch of a helical gear or involute worm is:

$$p_x = \frac{p_n}{\sin|\beta|} = \frac{\pi m_n}{\sin|\beta|} = \pi m_x = \frac{p_z}{z} = \frac{\pi m_t}{\tan|\beta|} = \frac{p_{yt}}{\tan|\beta_y|} = \frac{p_{yn}}{\sin|\beta_y|} \quad (13)$$

NOTE See ISO 21771-1:2024, 4.5.4

4.7.13 The involute function

The involute function of any angle θ in radians is:

$$\text{inv}\theta = \tan\theta - \theta \quad (\text{use radians}) \quad (14)$$

Angle θ shall be expressed in radians for this calculation.

NOTE See [Annex D](#) for a method to calculate the inverse involute (arcinv) function.

4.7.14 Helix angle at calculation diameter d_y

The helix angle at any diameter, d_y , is:

$$\beta_y = \arctan\left(\frac{d_y}{d} \tan\beta\right) \quad (15)$$

NOTE For internal gears, all diameters are stated as negative values.

4.7.15 Transverse pressure angle at calculation diameter d_y

The transverse pressure angle at any diameter, d_y , is:

$$\alpha_{yt} = \arccos\left(\frac{d_b}{d_y}\right) \quad (16)$$

4.7.16 Tooth thickness calculations

4.7.16.1 Transverse base tooth thickness

The transverse base tooth thickness can be calculated from the normal tooth thickness at the reference diameter, see [Formulae \(5\)](#) and [\(6\)](#):

$$s_{bt} = d_b \left(\frac{s_n}{d \cos\beta} + \text{inv}\alpha_t \right) \quad (17)$$

or with the transverse tooth thickness at the reference diameter:

$$s_{bt} = d_b \left(\frac{s_t}{d} + \text{inv}\alpha_t \right) \quad (18)$$

4.7.16.2 Normal base tooth thickness

A preliminary step for some measurement limit calculations is to find the non-datum related normal base tooth thickness and the normal base pitch. [Formula \(11\)](#) may be used to calculate the normal base pitch.

When starting with the manufacturing profile shift coefficient, x_E , [Formulae \(5\)](#) and [\(6\)](#) may be used to find the non-datum related normal tooth thickness at the reference diameter.

Given the non-datum related normal tooth thickness at the reference diameter, the maximum and minimum non-datum related normal base tooth thicknesses are:

$$s_{bn \max} = \frac{\cos \alpha_t \cos \beta_b}{\cos \beta} (s_{n \max} + m_n z \operatorname{inv} \alpha_t) \quad (19)$$

$$s_{bn \min} = \frac{\cos \alpha_t \cos \beta_b}{\cos \beta} (s_{n \min} + m_n z \operatorname{inv} \alpha_t) \quad (20)$$

NOTE For internal gears, span is calculated over a number of tooth spaces. However, in the calculations, the values of tooth thickness as opposed to space width are used.

4.7.16.3 Normal base tooth thickness from zero backlash tooth thickness

When starting with a zero-backlash x factor, the zero backlash base tooth thickness is:

$$s_{bn} = s_{bt} \cos(\beta_b) = m_n \cos \alpha_n \left(\frac{\pi}{2} + 2x \tan \alpha_n + z \operatorname{inv} \alpha_t \right) \quad (21)$$

Then the maximum and minimum manufactured normal base tooth thickness should be:

$$s_{bn \max} = s_{bn} + E_{sn \max} \cos \alpha_n \quad (22)$$

$$s_{bn \min} = s_{bn} + E_{sn \min} \cos \alpha_n \quad (23)$$

4.7.16.4 Transverse tooth thickness at diameter d_y

The transverse tooth thickness at any diameter d_y can be calculated in several ways depending on the tooth thickness definition available.

Given the transverse base tooth thickness:

$$s_{yt} = d_y \left(\frac{s_{bt}}{d_b} - \operatorname{inv} \alpha_{yt} \right) \quad (24)$$

Given the transverse tooth thickness at the reference diameter:

$$s_{yt} = d_y \left(\frac{s_t}{d} + (\operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt}) \right) \quad (25)$$

Given the half angle or profile shift coefficient:

$$s_{yt} = |d_y| \psi_y = |d_y| \left[\psi + \frac{z}{|z|} (\operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt}) \right] = d_y \left[\frac{\pi + 4x_E \tan \alpha_n}{2z} + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt} \right] \quad (26)$$

Given the normal tooth thickness at the reference diameter:

$$s_{yt} = d_y \left(\frac{s_n}{d \cos \beta} + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt} \right) \quad (27)$$

4.7.16.5 Normal tooth thickness at diameter d_y

In general, multiplying a transverse circular arc dimension at any diameter by the cosine of the helix angle at that diameter will give the corresponding normal dimension. This normal dimension is the length between flanks on a helix normal to the tooth flanks on the specified cylinder.

The normal tooth thickness at any diameter d_y , calculated based on the transverse tooth thickness at the same diameter d_y , and the helix angle at that same diameter is:

$$s_{yn} = s_{yt} \cos \beta_y \quad (28)$$

If diameter d_y is also the reference diameter, then the helix angle at the reference diameter is used.

The normal tooth thickness at diameter d_y can also be calculated as:

$$s_{yn} = d_y \left(\frac{s_n}{d \cos \beta} + (\text{inv} \alpha_t - \text{inv} \alpha_{yt}) \right) \cos \beta_y \quad (29)$$

4.7.16.6 Tooth thickness half angle

Tooth thickness angles are angles at the centre in a transverse section which are enclosed by the radii bounding a transverse tooth thickness, see [Figure 3](#). The corresponding tooth thickness half angles, ψ_y , for any transverse tooth thickness, s_{yt} , are expressed by

$$\psi_y = \frac{s_{yt}}{|d_y|} = \psi + \frac{z}{|z|} (\text{inv} \alpha_t - \text{inv} \alpha_{yt}) \quad (30)$$

The following tooth thickness half angle applies to the reference circle:

$$\psi = \frac{\pi + 4x_E \tan \alpha_n}{2|z|} \quad (31)$$

In the case of the base circle, the base tooth thickness half angle is produced by

$$\psi_b = \psi + \frac{z}{|z|} \text{inv} \alpha_t \quad (32)$$

4.7.17 Space width calculations

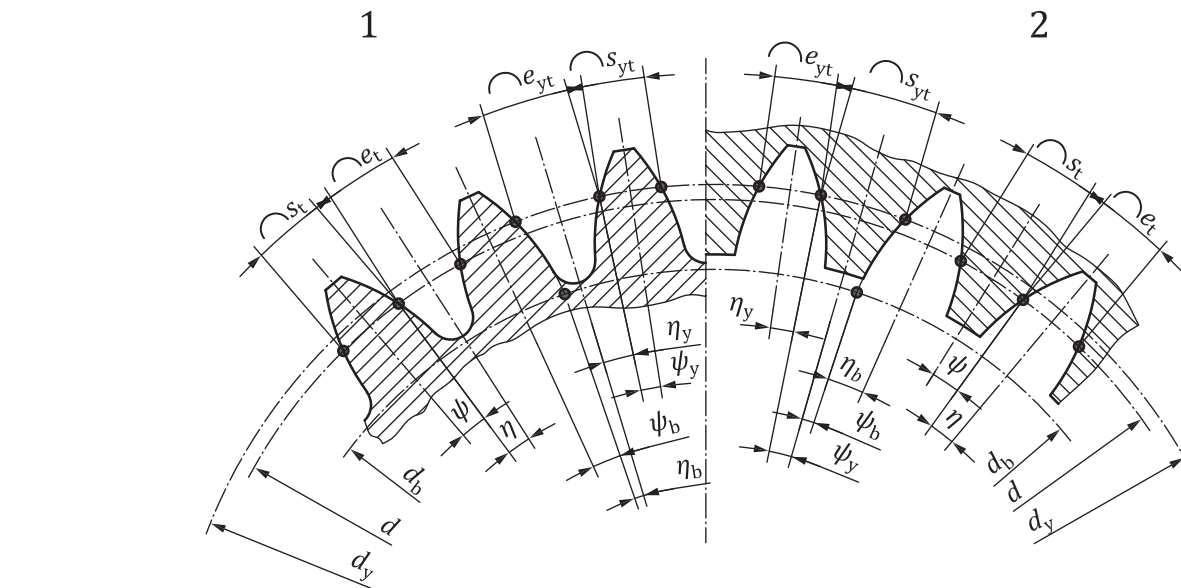
4.7.17.1 Space width

The space width, e_{yt} , in the transverse section is the length of a circular arc of diameter, d_y , between the two involute helicoids of a space width. See [Figure 3](#). It can be calculated as:

$$\begin{aligned} e_{yt} &= |d_y| \eta_y = |d_y| \left[\eta - \frac{z}{|z|} (\text{inv} \alpha_t - \text{inv} \alpha_{yt}) \right] \\ &= d_y \left[\frac{\pi - 4x_E \tan \alpha_n}{2z} - (\text{inv} \alpha_t - \text{inv} \alpha_{yt}) \right] = p_{yt} - s_{yt} \end{aligned} \quad (33)$$

The space width, e_t , on the reference circle is produced by:

$$e_t = |d| \eta = d \left(\frac{\pi - 4x_E \tan \alpha_n}{2z} \right) = \frac{m_n}{\cos \beta} \left(\frac{\pi}{2} - 2x_E \tan \alpha_n \right) = p_t - s_t \quad (34)$$



Key

measurement along an arc

- 1 external gear
- 2 internal gear

Figure 3 — Tooth thickness and space width (external and internal gear teeth)

4.7.17.2 Space width half angle

Space width angles are angles at the centre in a transverse section which are enclosed by the radii bounding a space width, see [Figure 3](#). At a specified diameter d_y , the space width half angle, η_y , corresponding to the space width, e_{yt} , is produced by:

$$\eta_y = \frac{e_{yt}}{d_y} = \eta - \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{yt}) \quad (35)$$

The following space width half angle applies to the reference circle:

$$\eta = \frac{\pi - 4x_E \tan \alpha_n}{2|z|} = \frac{\pi - \frac{s_n}{m_n}}{|z|} \quad (36)$$

At the base circle, the base space width half angle is produced by

$$\eta_b = \eta - \frac{z}{|z|} \text{inv } \alpha_t \quad (37)$$

4.7.17.3 Normal space width

The normal space width is the space width in a normal section of the gear teeth. It is the length of the helical arc on the respective cylinder between and normal to the two involute helicoids of a space width. The normal space width, e_{yn} , for any cylinder is calculated using

$$e_{yn} = e_{yt} \cos \beta_y \quad (38)$$

The following normal space width applies to the reference cylinder:

$$e_n = e_t \cos \beta = m_n \left(\frac{\pi}{2} - 2x_E \tan \alpha_n \right) \quad (39)$$

5 Measurement by pitch (index)

5.1 Principle of measurement

Pitch (or index) measurements, which are commonly used to determine single and cumulative pitch deviations, can be used to determine the axis related tooth thickness provided that measurements are taken relative to the datum axis and that measurements from both flanks are based on the same coordinate system. The circular arc distance between a point on one flank and a point on the other flank at the same diameter and in the same transverse plane is the transverse tooth thickness at that diameter. The circular arc distance can be found directly from the radius from the datum axis to these points and the angle between these two points.

[Formulae \(17\)](#) through [\(29\)](#) may be used to convert the tooth thickness at the given diameter to the tooth thickness at the reference diameter, as well as to convert between normal and transverse tooth thickness.

5.2 Underlying assumptions and limitations

The pitch or index measurements are taken relative to the gear's datum axis, therefore the results correspond to the axis related tooth thickness.

NOTE See ISO/TR 10064-1 for more information on measurement practice.

The inspection machine should be regularly calibrated and operated by trained personnel according to the inspection machine manufacturer's recommendations. See [Annex A](#) for additional information on use of GMM or CMM for tooth thickness measurement.

The effect of mounting fixtures on the measurements should be carefully considered. Stylus tip size can be problematic with fine pitch gears. The stylus tip shall be able to fit in the tooth space and shall reach the depth required. Machines for very large gears are unlikely to be available.

A single point per tooth flank is normally used, but accuracy can be increased if multiple points are measured on the flank.

5.3 Advantages and disadvantages of measurement by pitch

The data gathered during pitch (or index) measurement with a GMM or CMM can be used to find the axis related tooth thickness of each tooth. Therefore, if an elemental measurement is being performed, the tooth thickness can be found without any additional measuring time.

NOTE Pitch measurement can also be used while troubleshooting the manufacturing process, since it can often reveal the source of problems.

5.4 Selection of measurement diameter

Measurements of the pitch are generally taken at mid flank height. Other diameters may be used when necessary, such as when the available stylus tip is too large for the space between teeth. If an attempt is made to compare the tooth thickness found from pitch or index measurement to another measurement technique, it can be helpful to use a measurement diameter that corresponds to the contact diameter of that technique (see 4.6). A better comparison is sometimes achieved when extra measurements are taken at the axial contact positions of the non-datum related measurement instead of using the points used to evaluate pitch deviation.

6 Measurement with a master gear on a double flank tester

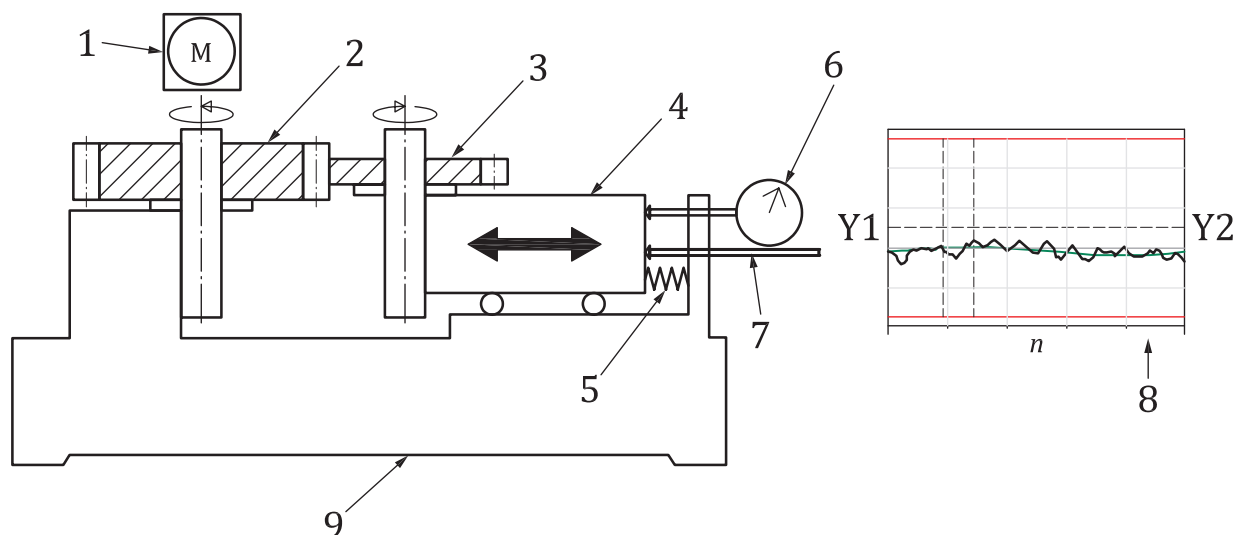
6.1 Principle of measurement

6.1.1 Main principles

Tooth thickness can be controlled with the use of a double flank tester. When a product gear is held in tight mesh with a master gear, the centre distance is related to the product gear's tooth thickness and the cumulative effects of runout, profile, pitch and helix deviations. This technique provides a complete analysis of the maximum effective material (peak surface) condition of the active tooth surfaces provided the master gear is properly designed to engage the product gear over its functional surface. Depending on the design of this master, local deviations can be averaged due to contact across the face width and overlapping of tooth pairs during testing.

NOTE 1 The intent of double flank testing is to measure the test gear against its product specifications and not to test the functionality of a gear mesh. The master gear is likely considerably different than the gear the product gear will mate with, while at the same time operating at zero backlash as opposed to normal operation with backlash. The working pitch cylinders are therefore likely different than in the application itself.

The test uses an apparatus of general layout as shown in [Figure 4](#) with two spindles (also known as mandrels or broaches) on which a gear pair is mounted: a driving spindle and a second spindle mounted on a carrier that can float in a direction parallel to the centre distance. A typical configuration is shown in [Figure 5](#). A calibrated master gear is mounted on a fixed base with only rotational freedom. A product gear is mounted on a floating slide mechanism which allows rotation of the product gear and movement along an axis between the line of centres of the master and product gear. A spring (with an adjustable pre-set force) pushes the floating slide resulting in tight mesh (zero backlash) double flank contact (i.e. on both left and right flanks) between the product gear and master gear.



Key

Y1	tight mesh centre distance	Y2	product gear test radius
1	drive motor (optional)	6	dial indicator
2	master gear	7	linear sensor (optional)
3	product gear	8	recording device (optional)
4	floating slide	9	base
5	spring	<i>n</i>	tooth number or degrees rotation

Figure 4 — General arrangement of a double flank tester

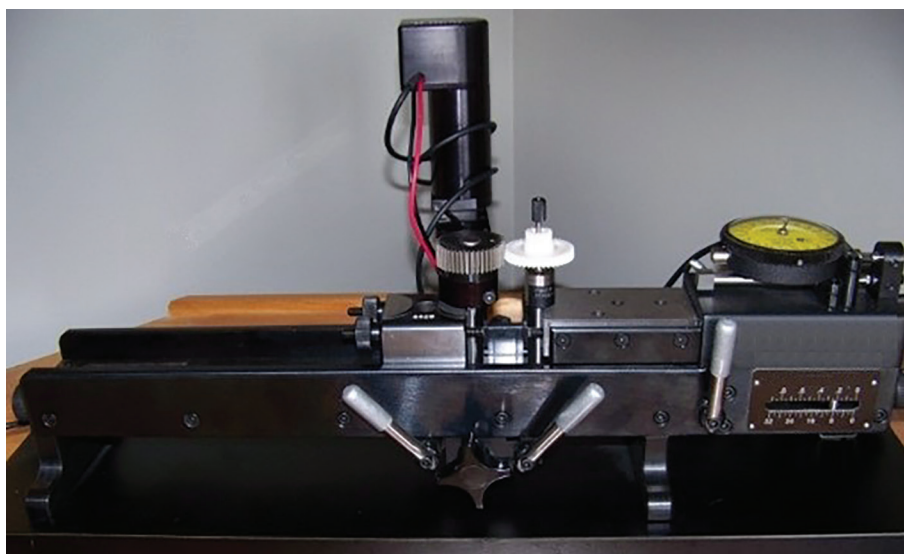


Figure 5 — Double flank test of a parallel axis cylindrical gear

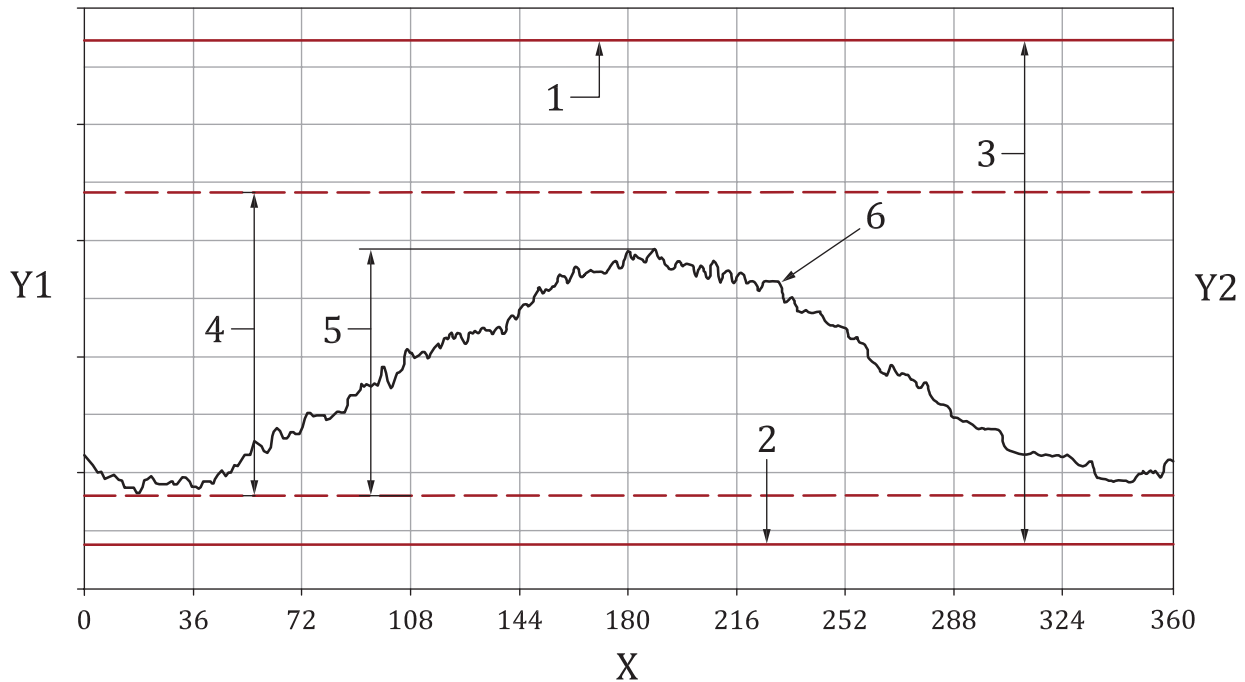
When the master gear is rotated (by hand or by motor), the product gear follows. Involute gear theory dictates that perfectly formed teeth will result in no movement of the floating slide along the line between centres. Since no gear is perfect, there will always be some movement of the floating slide as the gears are rotated. The magnitude of this movement relative to the rotational position of the product gear is measured with either a mechanical indicator or electronic sensor that contacts the slide mechanism.

The apparatus can measure:

- tight mesh centre distance or product gear test radius;

— total radial composite deviation.

A sample result from a double flank test is shown in [Figure 6](#). Tooth-to-tooth radial composite deviation can also be measured, although this measurement is not used in this document.



Key

X tooth number or degrees rotation

Y1 tight mesh centre distance

Y2 product gear test radius

1 maximum tight mesh centre distance/product gear test radius limit based on maximum axis related tooth thickness

2 minimum tight mesh centre distance/product gear test radius limit based on minimum axis related tooth thickness

3 tight mesh centre distance tolerance range based on axis related tooth thickness tolerance range

4 total radial composite tolerance

5 total radial composite deviation

6 radial composite deviation trace

NOTE The tight mesh centre distance tolerance is directly related to the axis related tooth thickness tolerance.

Figure 6 — Sample double flank test report

This document provides double flank test tight mesh centre distance limit calculations for involute cylindrical gearing. There are calculations both for parallel axis gears (spur or helical, see [Figure 5](#)) and crossed axis gears (see [Figures 7](#) and [8](#)).

Crossed axis gears in tight mesh are theoretically conjugate at a unique relation between tight mesh centre distance and tight mesh skew angle. Since a change in tooth thickness results in a change in tight mesh centre distance, ideally the double flank tester would vary the skew angle as the centre distance changes. However, since this adjustment is impractical, the test is often set up based on the mean value of the tooth thickness, see [6.5.3](#). The residual skew angle deviation is included in the measurement results and is usually neglected.

Crossed axis helical gears may be measured at skew angles other than the optimal angle. This is accounted for in the centre distance limit calculations ([Clause 6.6.2](#)). A skew angle other than the optimal skew angle will result in increased tight mesh centre distance.

NOTE 2 While all gears exhibit some degree of non-conjugacy due to imperfections in the tooth profiles, with crossed axis gears there will be additional non-conjugacy if the skew angle is not at the optimum value. This additional non-conjugacy can result in measuring different values of tooth-to-tooth deviations.

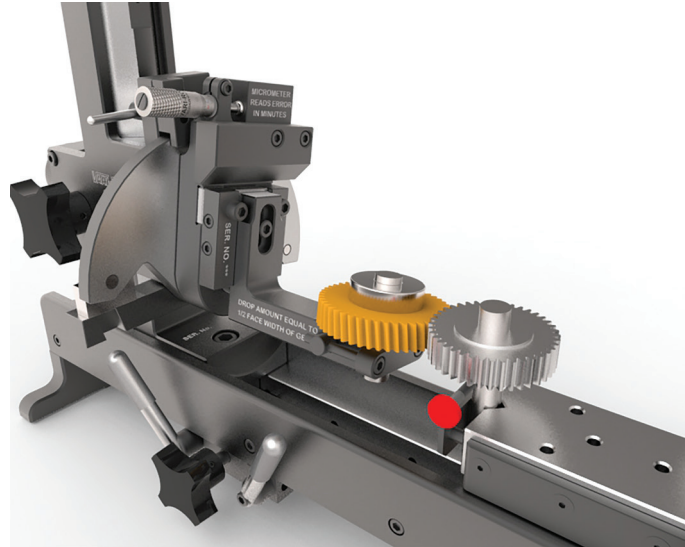


Figure 7 — Double flank test of crossed axis gears

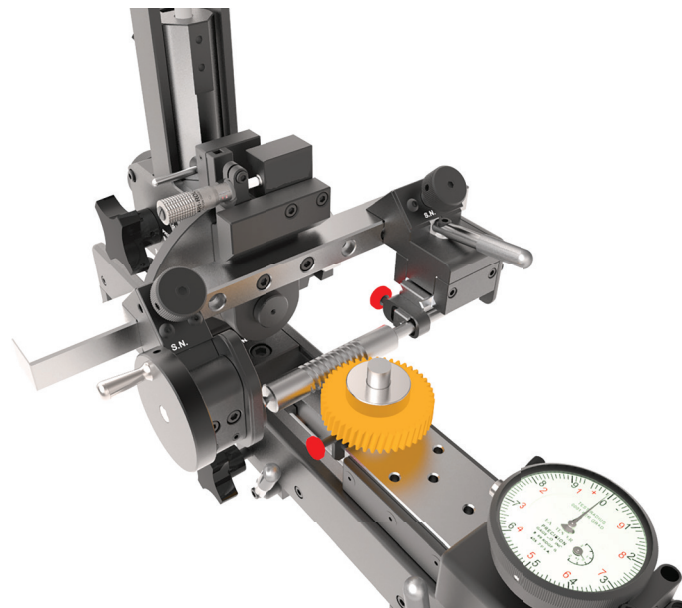


Figure 8 — Double flank test of a worm and helical gear

6.1.2 Tight mesh centre distance

Tight mesh centre distance (and product gear test radius) can be measured directly on a double flank tester if a calibration has been performed to relate the position of the floating slide to the actual distance between the centres of the master and product gears.

If the product gear is specified using non-datum related tooth thickness, then the tight mesh centre distance measurement limits shall include total radial composite tolerances.

The tight mesh centre distance can be measured over the entire product gear's active rolling profile revealing a range of values for tight mesh centre distance over all the measured teeth of a product gear. The maximum and minimum tight mesh centre distance results can be related back to the maximum and minimum axis related normal tooth thickness. If the measured maximum and minimum tight mesh centre distance are within the specified centre distance range, then the gear's axis related tooth thickness is within tolerance. The maximum and minimum axis related tooth thickness values are also useful in predicting backlash as outlined in [Clause 13](#). The mean axis related and non-datum related tooth thicknesses are identical on any given product gear.

If the tight mesh centre distance (or product gear test radius) results are within the allowable specifications, it will not ensure that the gear's non-datum related tooth thickness is within tolerance, since a gear that has very low total radial composite variation could have a non-datum related tooth thickness that is thicker or thinner than specified, yet still be within the axis related tooth thickness working limits. See [4.4](#).

6.1.3 Product gear test radius

The product gear test radius is sometimes specified instead of tight mesh centre distance. The product gear test radius is the difference between the tight mesh centre distance and the calibrated value of the master gear test radius. It is used in the same way and for the same purposes as tight mesh centre distance. It is considered by some as convenient to use a specification which is not sensitive to the tooth thickness or number of teeth of the master gear, so that different masters can be used to measure the same product gear, thereby eliminating the need for multiple tight mesh centre distance specifications that are master gear specific. The assumption is that regardless of the master gear test radius, the product gear test radius limits are constant. This assumption is only true if the sum of the mean tooth thickness for the master and product gear is equal to the normal pitch of the mesh. Since this is rarely the case, the use and application of this simplification can lead to errors which can only be avoided if the normal tooth thickness and number of teeth for the master gear are specified along with the product gear test radius specification limits. This being the case, there is no advantage to specifying the product gear test radius and therefore tight mesh centre distance should be used to avoid confusion. For more information on the potential magnitude of errors associated with the use of test radius, see ISO/TR 10064-2 and Reference [\[17\]](#).

6.1.4 Total radial composite deviation

Total radial composite deviation, as shown in [Figure 6](#), is the difference between the maximum and minimum positional readings of the floating slide taken over all of the measured teeth of the product gear.

6.2 Underlying assumptions and limitations

The total radial composite deviation can provide an analysis of the entire active tooth flanks, provided that the master gear is designed to engage the product gear over its functional surface. The design of the master can mask local tooth surface deviations since contact is based on the maximum material condition and overlap of tooth pairs can miss hollow surfaces. Careful design of the master gear can minimize the chance of this condition occurring.

An underlying assumption is that the master gear has a minimum contact ratio greater than 1,0 with the product gear. For accurate measurement, double flank contact shall be maintained at all times. Therefore, in addition to using a master gear that has a contact ratio greater than one with the product gear, the selection of the pre-set spring force and the speed of rotation of the gears should be given careful consideration to limit measurement errors. The pre-set spring force should be selected specifically for the product gear's design considering the material's and spindle's ability to resist deformation under the load. The correct pre-set spring force is the minimum force needed to maintain reliable continuous double flank contact. If this force results in distortion of the product gear or deflection of the spindles, then changes are needed to allow use of a lower force. A large pre-set force can distort the gear into conformity or can mask the effect of burrs or other asperities that otherwise can lead to rejection of the product gear. A large pre-set spring force can also deflect the supporting spindles on the master or product gear resulting in an error in the measurement.

If, during rotation, there is excessive resistance torque coming from the mounting of either the product gear or the master gear on their mounts, then this resistance will result in a separating force. A large separating force can overcome the pre-set spring force resulting in a loss of double flank contact. As the master and

product gear rotate and separate, the gears eventually roll to a position where the spring force abruptly pushes them back into double flank contact (i.e. the floating slide moves abruptly). If double flank contact is lost, the test results are invalid and the resistance in the mandrels should be corrected.

The speed of the rotation of the gears should be selected considering the natural response of the mechanical and electrical (if so equipped) elements of the tester.

At least 30 data points per tooth shall be available in the data set collected to ensure sufficient sensitivity of the results.

The use of double flank radial composite measurements can be limited by the ability to produce a suitable master gear or by the size of the product gear to be tested relative to an available tester. Consult equipment suppliers for capacities and limitations of these types of testing machines. Furthermore, the datum structure of an individual product gear can limit the ability to mount it on the apparatus. Care should be taken to avoid misalignment or eccentricity introduced by any locating fixtures.

For additional information on double flank radial composite testing, see ISO/TR 10064-2 and Reference [17](#).

6.3 Advantages and disadvantages of double flank radial composite measurement

Double flank radial composite measurement can provide a real time in-process check that can alert when a process has deviated from the ideal manufacturing condition. Both non-datum related and axis related tooth thickness along with the cumulative effects of the runout, profile, pitch and helix deviations can be controlled with the use of a double flank tester. However, if a product gear is rejected by this method, the analysis does not in itself provide a key as to the specific characteristic(s) which have led to non-conformance. Further analysis is needed to identify the exact characteristics that need correction. When this occurs, other common techniques of analysis can include (but are not limited to) span, over pins, or elemental measurement.

Double flank testing of parallel axis gears will detect raised surfaces caused by nicks, handling damage, burrs, or in the case of net shape gears, parting line flash. This makes double flank testing useful in identifying noise concerns related to these types of defects.

For crossed axis gears, only one line on the flank will be measured, unlike parallel axis gears where the entire flank of the product gear can be assessed. Detection of raised surfaces, nicks, handling damage, burrs, or in the case of net shape gears, parting line flash depends on the relative axial position of the gear mesh during testing.

A disadvantage is the double flank tester must be properly calibrated for measurement of centre distance and an appropriate master gear must be available.

6.4 Master gear design considerations

6.4.1 Criteria for proper meshing

Master gears used in double flank radial composite measurement shall meet the following criteria to mesh properly with a product gear:

- a) The master gear shall be verified for suitability by evaluating its meshing condition over the allowable manufacturing tolerance range of the product gear. The master shall have the potential to function with proper conjugate action with a product gear at minimum tooth thickness and maximum root and tip diameters and at maximum tooth thickness with minimum root and tip diameters. In both conditions, the issues to be evaluated are contact ratio, and start of active profile on both the master and the product gears. The use of "standard" commercially available master gears does not preclude the need for this check.
- b) The tip of the master gear shall not contact the product gear below its form diameter. This applies to initial contact and to any type of secondary contact in the fillet or root zone due to inadequate clearance.

- c) The tip of the product gear shall not contact the master gear below its form diameter. This applies to initial contact and to any type of secondary contact in the fillet or root zone due to inadequate clearance.
- d) The minimum total contact ratio of the double flank test shall not be less than 1,0 when considering the maximum tooth thickness, minimum tip diameter, maximum root diameter, and maximum tip radius of the product gear. Should the contact ratio drop below 1,0, the meshing action of the gears on test will generate an immediate jump in the double flank result for every tooth meshing cycle. This happens as the spring of the slide on the double flank tester compensates for the loss of mesh force by abruptly pushing the gears together.
- e) The face width of the master gear shall be greater than the product gear if the full face width of the product gear is to be checked in a single test.
- f) When operating with parallel axes, the master gear and the product gear shall have the same reference base helix angle (and for external gears, opposite hand).
- g) The master gear and the product gear shall have the same normal base pitch. In most cases this is accomplished by the master gear having the same normal module and normal pressure angle as the product gear. In theory these variables can be different, which can be useful in some special circumstances, provided that the normal base pitches are equal according to the following relation:

$$p_{bn} = \pi m_{n3} \cos \alpha_{n3} = \pi m_n \cos \alpha_n \quad (40)$$

- h) For crossed axis gears the tight mesh centre distance specifications shall be based on the specified skew angle setting of the double flank tester. See [6.5.1](#).

6.4.2 Additional recommendations

The following additional recommendations for good practice can also be of use. These recommendations are particularly helpful for the determination of other parameters, such as total radial composite and tooth-to-tooth radial composite deviations:

- a) For measurement of tooth-to-tooth radial composite deviations, the maximum total contact ratio of the double flank test should be less than 2,0 considering minimum tooth thickness, maximum tip diameter, minimum root diameter, and minimum tip radius of the product gear. High contact ratios on the double flank tester promote more overlapping of the mesh and can hide some defects in the product gear.

Helical gears can have a total contact ratio greater than 2,0 when run against a master gear covering its full face width. In such cases a decision should be made to either accept the possible smoothing out of deviations that would result with this high contact ratio, or to possibly reduce the face width of the master gear and measure the helical product gear in different contact zones along the product gear's axis while maintaining an overall contact ratio of less than 2,0.

When a master gear has a face width that is less than the active face width of the product gears, then the tooth thickness will not be fully evaluated.

Unlike a double flank test, the actual application of the product gear usually includes backlash. Therefore, the double flank mesh will not replicate the intended application in every way possible. The design of the master gear shall be agreed upon between the supplier and purchaser.

- b) In the case of crossed axis gears where the driver is an involute worm, an involute worm may be considered for the master gear. This can provide an advantage for the product gear in that only the functional zone is measured and other tooth deviations that will not be seen in the actual product mesh will be ignored. In some cases, a helical product gear designed to mate with a worm can be tested with a narrow face width helical gear master to provide a similar result in a parallel axis arrangement.

In applications where a worm master is used, it is sometimes necessary to add to the double flank mesh the proper viscosity lubricant to assist sliding action in the mesh without causing reading errors induced by inappropriate friction conditions.

- c) The extent of the master gear's reach (i.e. the master gear's tip diameter) into the product gear should be carefully chosen. Although as stated, the mesh under test shall have a minimum contact ratio of 1,0 and should have a maximum contact ratio of less than 2,0, there shall also be no contact of the master below the root form diameter of the product gear. There can be a wide range of choices in between those requirements when establishing the tip diameter of the master gear.

The decision on what master design to use can be based on the cost and availability of existing or commercially available master gears or can be based on measuring a product gear to at least its start of active profile location in the actual application.

- d) In order to machine and produce high quality master gears by grinding, the bore of the master gear must be sufficiently large for a mandrel to hold the master gear stable during machining and measurement. Ground master gears with bores less than 6 mm should be carefully considered for the effect on master gear precision from a small diameter machining mandrel.

6.5 Skew angle for double flank testing

6.5.1 General

The first working parameter that must be established in planning the double flank testing of any involute cylindrical gear is the skew angle. For parallel axis spur and helical gearing, the skew angle is set to zero degrees to maintain parallelism between the product gear and master gear axis. For crossed axis helical gears, including where an involute worm is driving a helical or spur gear, there is more complexity in establishing the skew angle. See [6.5.3](#).

6.5.2 Skew angle for parallel axis gear pairs

In the case of parallel axis gear pairs, the skew angle shall be set to zero degrees.

6.5.3 Skew angle for crossed axis gear pairs

6.5.3.1 General

For any specified crossed axis pair of gears (in this case a master and a product gear) there is a unique skew angle that results in both optimal conjugacy and minimum centre distance during double flank testing. This optimal skew angle will also minimize the recorded tooth-to-tooth deviation. Selection of any skew angle other than this optimal skew angle, will result in a loss of conjugacy and can increase measured tooth-to-tooth deviations which may not actually be due to poor product quality. The magnitude of these deviations depends on several factors and is not necessarily detrimental to the measurement result.

There can be practical reasons why the optimal skew angle is not chosen, such as limitations in the test apparatus, application issues, test environment, or even convenience in performing the measurements. Usually, the skew angle used for the test is within about two degrees of the optimal skew angle for the test.

6.5.3.2 Calculation of the optimal skew angle

The optimal skew angle for tight mesh can be found since the two gears will have the same working normal pressure angle. The tight mesh centre distance is directly related to the product gear's tooth thickness. Since tooth thickness has tolerance associated with it, the skew angle would need to be able to be adjusted during a test to maintain theoretical conjugacy. This adjustment is not practical due to apparatus limitations. Hence, the skew angle for test is based on the centre of the product gear's tooth thickness tolerance. The residual errors resulting from tooth thickness deviations from the mean value are usually neglected.

The mean normal circular tooth thickness of the product gear can be calculated either from the non-datum related or the axis related tooth thickness limits as follows:

$$s_{ns} = \frac{s_{n \min} + s_{n \max}}{2} = \frac{s_{nz \min} + s_{nz \max}}{2} \quad (41)$$

NOTE 1 The non-datum related and axis related mean normal circular tooth thicknesses are equal.

NOTE 2 For the master gear a subscript “3” is used while no subscript is used for the product gear

The optimal tight mesh working normal pressure angle can be determined with [Formula \(42\)](#):

$$z \left\{ \operatorname{inv} \left[\arctan \frac{\tan \alpha_{wn}}{\sqrt{1 - \frac{(\sin \beta \cdot \cos \alpha_n)^2}{\cos^2 \alpha_{wn}}}} \right] - \operatorname{inv} \alpha_t \right\} + z_3 \left\{ \operatorname{inv} \left[\arctan \frac{\tan \alpha_{wn}}{\sqrt{1 - \frac{(\sin \beta_3 \cdot \cos \alpha_n)^2}{\cos^2 \alpha_{wn}}}} \right] - \operatorname{inv} \alpha_{t3} \right\} + \pi - \frac{(s_{ns} + s_{n3})}{m_n} = 0 \quad (42)$$

There is no direct solution to calculate the tight mesh working normal pressure angle, α_{wn} , but there are many formulae solving techniques that can be used (root finding algorithms such as bisection or other more advanced methods). A suggested method of solution uses a constant for an invariant part of the formula:

$$Y_5 = \pi - \frac{(s_{ns} + s_{n3})}{m_n} \quad (43)$$

The normal working pressure angle can be found by iterating through [Formulae \(44\)](#), [\(45\)](#), and [\(46\)](#), starting with the master gear normal pressure angle as the initial guess for α_{wn} :

$$Y_1 = \sqrt{1 - \frac{(\sin \beta \cdot \cos \alpha_n)^2}{\cos^2 \alpha_{wn}}} \quad (44)$$

$$Y_3 = \sqrt{1 - \frac{(\sin \beta_3 \cdot \cos \alpha_n)^2}{\cos^2 \alpha_{wn}}} \quad (45)$$

$$\alpha_{wn} = \arctan \frac{z \cdot \left(\arctan \frac{\tan \alpha_{wn}}{Y_1} + \operatorname{inv} \alpha_t \right) + z_3 \cdot \left(\arctan \frac{\tan \alpha_{wn}}{Y_3} + \operatorname{inv} \alpha_{t3} \right) - Y_5}{\frac{z}{Y_1} + \frac{z_3}{Y_3}} \quad (46)$$

The result of each iteration for α_{wn} should be used as the guess for the next iteration. The iteration should continue until the change in α_{wn} is less than 10^{-6} .

With this working normal pressure angle established, the tight mesh working helix angles are:

$$\beta_w = \arcsin \left(\frac{\sin \beta \cdot \cos \alpha_n}{\cos \alpha_{wn}} \right) \quad (47)$$

$$\beta_{w3} = \arcsin \left(\frac{\sin \beta_3 \cdot \cos \alpha_n}{\cos \alpha_{wn}} \right) \quad (48)$$

The optimal tight mesh skew angle for mean tooth thickness is:

$$\Sigma = \beta_w + \beta_{w3} \quad (49)$$

6.5.3.3 Skew angle used for test

For crossed axis helical gears, either the optimum skew angle, Σ , or some another convenient skew angle (See [6.5.3.1](#)) can be imposed on the tight mesh centre distance calculation. The imposed skew angle is designated as Σ_{Σ} .

6.6 Calculation of tight mesh centre distance limits for crossed axis gears

6.6.1 Preliminary calculations

The preliminary calculations are carried out with the imposed skew angle and data from the product gear and master gear. See Reference [\[13\]](#).

The helix angles are:

$$\beta_{\Sigma} = \arctan\left(\frac{\sin \beta_b \cdot \sin \Sigma_{\Sigma}}{\sin \beta_b \cdot \cos \Sigma_{\Sigma} + \sin \beta_{b3}}\right) \quad (50)$$

$$\beta_{\Sigma3} = \arctan\left(\frac{\sin \beta_{b3} \cdot \sin \Sigma_{\Sigma}}{\sin \beta_{b3} \cdot \cos \Sigma_{\Sigma} + \sin \beta_b}\right) \quad (51)$$

The normal pressure angle for the imposed skew angle is:

$$\alpha_{\Sigma n} = \arccos\left(\frac{\sin \beta_b}{\sin \beta_{\Sigma}}\right) = \arccos\left(\frac{\sin \beta_{b3}}{\sin \beta_{\Sigma3}}\right) \quad (52)$$

For [Formulae 50](#) to [52](#), if $\beta_b = 0$ ignore the formula using β_b , and if $\beta_{b3} = 0$ ignore the formula using β_{b3} .

With the imposed skew angle the calculation diameters are:

$$d_{\Sigma} = \frac{d_b \cdot \cos \beta_b}{\cos \alpha_{\Sigma n} \cdot \cos \beta_{\Sigma}} \quad (53)$$

$$d_{\Sigma3} = \frac{d_{b3} \cdot \cos \beta_{b3}}{\cos \alpha_{\Sigma n} \cdot \cos \beta_{\Sigma3}} \quad (54)$$

The transverse pressure angles for the imposed skew angles are:

$$\alpha_{\Sigma t} = \arccos\left(\frac{d_b}{d_{\Sigma}}\right) \quad (55)$$

$$\alpha_{\Sigma t3} = \arccos\left(\frac{d_{b3}}{d_{\Sigma3}}\right) \quad (56)$$

The master gear normal circular tooth thickness at $d_{\Sigma3}$ is:

$$s_{\Sigma n3} = d_{\Sigma3} \left[\frac{s_{n3}}{|z_3| m_n} + \frac{z_3}{|z_3|} (\text{inv } \alpha_{t3} - \text{inv } \alpha_{\Sigma t3}) \right] \cos \beta_{\Sigma3} \quad (57)$$

6.6.2 Tight mesh centre distance limits for crossed axis gears

6.6.2.1 General

The calculation of the final tight mesh centre distance limits is slightly different depending on whether the non-datum related or the axis related system is being used. When using non-datum related normal circular tooth thickness values, the calculation for tight mesh centre distance limits shall include an adjustment for total composite tolerance.

6.6.2.2 Non-datum related system

The non-datum related normal circular tooth thicknesses at d_Σ are:

$$s_{\Sigma n \max} = |d_\Sigma| \left[\frac{s_{n \max}}{|z|m_n} + \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{\Sigma t}) \right] \cos \beta_\Sigma \quad (58)$$

$$s_{\Sigma n \min} = |d_\Sigma| \left[\frac{s_{n \min}}{|z|m_n} + \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{\Sigma t}) \right] \cos \beta_\Sigma \quad (59)$$

The tight mesh centre distance limits for the imposed skew angles are:

$$M_{d \max} = \frac{d_\Sigma + d_{\Sigma 3}}{2} + \left(\frac{s_{\Sigma n \max} + s_{\Sigma n 3} - d_\Sigma \frac{\pi}{|z|} \cos \beta_\Sigma}{2 \tan \alpha_{\Sigma n}} \right) + \frac{F_{idT}}{2000} \quad (60)$$

$$M_{d \min} = \frac{d_\Sigma + d_{\Sigma 3}}{2} + \left(\frac{s_{\Sigma n \min} + s_{\Sigma n 3} - d_\Sigma \frac{\pi}{|z|} \cos \beta_\Sigma}{2 \tan \alpha_{\Sigma n}} \right) - \frac{F_{idT}}{2000} \quad (61)$$

The value (2 000) in the denominator of [Formulae \(60\)](#) and [\(61\)](#) adjust for both half the total composite tolerance applied to the maximum or minimum limits, while also converting the units of total composite tolerance from μm to mm .

6.6.2.3 Axis related system

The axis related normal circular tooth thicknesses at d_Σ are:

$$s_{\Sigma n z \max} = |d_\Sigma| \left[\frac{s_{nz \max}}{|z|m_n} + \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{\Sigma t}) \right] \cos \beta_\Sigma \quad (62)$$

$$s_{\Sigma n z \min} = |d_\Sigma| \left[\frac{s_{nz \min}}{|z|m_n} + \frac{z}{|z|} (\text{inv } \alpha_t - \text{inv } \alpha_{\Sigma t}) \right] \cos \beta_\Sigma \quad (63)$$

Adjustment for total composite tolerance is not needed when using the axis related normal tooth thickness input values because the maximum and minimum axis related tooth thickness limits already include total radial composite tolerance effects.

The tight mesh centre distance limits for the imposed skew angles in the axis related system are:

$$M_{d \max} = \frac{d_\Sigma + d_{\Sigma 3}}{2} + \left(\frac{s_{\Sigma n z \max} + s_{\Sigma n 3} - d_\Sigma \frac{\pi}{|z|} \cos \beta_\Sigma}{2 \tan \alpha_{\Sigma n}} \right) \quad (64)$$

$$M_{d \min} = \frac{d_\Sigma + d_{\Sigma 3}}{2} + \left(\frac{s_{\Sigma n z \min} + s_{\Sigma n 3} - d_\Sigma \frac{\pi}{|z|} \cos \beta_\Sigma}{2 \tan \alpha_{\Sigma n}} \right) \quad (65)$$

6.7 Tight mesh centre distance limits for parallel axis gears:

6.7.1 General

For parallel axis gears, the skew angle is zero and $\beta = -\beta_3$. See ISO 21771-1:2024 for additional details.

NOTE 1 A method to calculate arcinv (the inverse involute) is given in [Annex D](#).

NOTE 2 For internal gears, the reference diameter of the product gear has a negative value for the calculations, and the resulting tight mesh centre distance is also a negative value.

NOTE 3 [Formulae \(66\)](#) to [\(69\)](#) allow for a special case where the normal modules and helix angles between the product and master gears are different but their normal base pitches are still equal. See [Formula \(40\)](#).

6.7.2 For the non-datum related system:

When using non-datum related normal circular tooth thickness values, the calculation for tight mesh centre distance limits must include an adjustment for total composite tolerance. The value (2 000) in the denominator of [Formulae \(66\)](#) and [\(67\)](#) adjust for both half the total composite tolerance applied to the maximum or minimum limits, while also converting the units of total composite tolerance from μm to mm .

The tight mesh centre distance limits for the non-datum related system are:

$$M_{d \max} = \frac{(d + d_3) \cos \alpha_t}{2 \cos \left\{ \arcsin \left[\left(\frac{s_{n \max} + s_{n3} - \pi m_n}{(d + d_3) \cos \beta} \right) + \text{inv} \alpha_t \right] \right\}} + \frac{F_{idT}}{2000} \quad (66)$$

$$M_{d \min} = \frac{(d + d_3) \cos \alpha_t}{2 \cos \left\{ \arcsin \left[\left(\frac{s_{n \min} + s_{n3} - \pi m_n}{(d + d_3) \cos \beta} \right) + \text{inv} \alpha_t \right] \right\}} - \frac{F_{idT}}{2000} \quad (67)$$

6.7.3 For the axis related system:

Adjustment for total composite tolerance is not needed when using the axis related normal tooth thickness input values because the maximum and minimum axis related tooth thickness limits already include total radial composite tolerance effects.

The tight mesh centre distance limits for the axis related system are:

$$M_{d \max} = \frac{(d + d_3) \cos \alpha_t}{2 \cos \left\{ \arcsin \left[\left(\frac{s_{nz \max} + s_{n3} - \pi m_n}{(d + d_3) \cos \beta} \right) + \text{inv} \alpha_t \right] \right\}} \quad (68)$$

$$M_{d \min} = \frac{(d + d_3) \cos \alpha_t}{2 \cos \left\{ \arcsin \left[\left(\frac{s_{nz \min} + s_{n3} - \pi m_n}{(d + d_3) \cos \beta} \right) + \text{inv} \alpha_t \right] \right\}} \quad (69)$$

6.7.4 Differences between axis related and non-datum related systems

The difference between [Formulae \(66\)](#) and [\(67\)](#) compared to (68) and (69) is the inclusion of the total composite tolerance as a separate input value when non-datum related tooth thickness is used. This is not needed when the axis related tooth thickness input value is used because the maximum and minimum axis related tooth thickness limits already include total radial composite tolerance effects.

These differences are shown graphically in [Figure 9](#) for a gear with non-datum related gear tooth thickness specified. This shows the tolerance bands from various parts of formulae for external gears and hence the relationship between non-datum related and axis related normal tooth thicknesses.

The meshing conditions shall be checked with the tight mesh centre distance limits to avoid meshing interferences and evaluate contact ratio. See [6.4](#). The associated calculations for these verifications are out of the scope of this document.