

BS ISO 23509:2016



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Bevel and hypoid gear geometry

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National foreword

This British Standard is the UK implementation of ISO 23509:2016. It supersedes BS ISO 23509:2006 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee MCE/5/-/13, Bevel gears.

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Bevel and hypoid gear geometry

Géométrie des engrenages coniques et hypôïdes



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

The committee responsible for this document is ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

This second edition cancels and replaces the first edition (ISO 23509:2006), which has been technically revised with the following changes:

- minor corrections of several formulae;
- the figures have been reworked;
- explanations have been added in [4.4](#);
- the structure of [Formula \(129\)](#) has been changed to cover the case $\zeta_m = 0^\circ$;
- a formula for the calculation of c_{be2} has been added as [Formula \(F.160\)](#);
- the values for α_{nC} and α_{nD} in [Formulae \(F.318\)](#) and [\(F.319\)](#) have been extended to three decimal digits to prevent rounding errors.

Introduction

For many decades, information on bevel, and especially hypoid, gear geometry has been developed and published by the gear machine manufacturers. It is clear that the specific formulae for their respective geometries were developed for the mechanical generation methods of their particular machines and tools. In many cases, these formulae could not be used in general for all bevel gear types. This situation changed with the introduction of universal, multi-axis, CNC-machines, which in principle are able to produce nearly all types of gearing. The manufacturers were, therefore, asked to provide CNC programs for the geometries of different bevel gear generation methods on their machines.

This document integrates straight bevel gears and the three major design generation methods for spiral bevel gears into one complete set of formulae. In only a few places do specific formulae for each method have to be applied. The structure of the formulae is such that they can be programmed directly, allowing the user to compare the different designs.

The formulae of the three methods are developed for the general case of hypoid gears and to calculate the specific case of spiral bevel gears by entering zero for the hypoid offset. Additionally, the geometries correspond such that each gear set consists of a generated or non-generated wheel without offset and a pinion which is generated and provided with the total hypoid offset.

An additional objective of this document is that, on the basis of the combined bevel gear geometries, an ISO hypoid gear rating system can be established in the future.

Bevel and hypoid gear geometry

1 Scope

This document specifies the geometry of bevel gears.

The term bevel gears is used to mean straight, spiral, zerol bevel and hypoid gear designs. If the text pertains to one or more, but not all, of these, the specific forms are identified.

The manufacturing process of forming the desired tooth form is not intended to imply any specific process, but rather to be general in nature and applicable to all methods of manufacture.

The geometry for the calculation of factors used in bevel gear rating, such as ISO 10300 (all parts), is also included.

This document is intended for use by an experienced gear designer capable of selecting reasonable values for the factors based on his/her knowledge and background. It is not intended for use by the engineering public at large.

[Annex A](#) provides a structure for the calculation of the methods provided in this document.

2 Normative references

There are no normative references in this document.

3 Terms, definitions and symbols

For the purposes of this document, the terms and definitions given in ISO 1122-1 and the following apply.

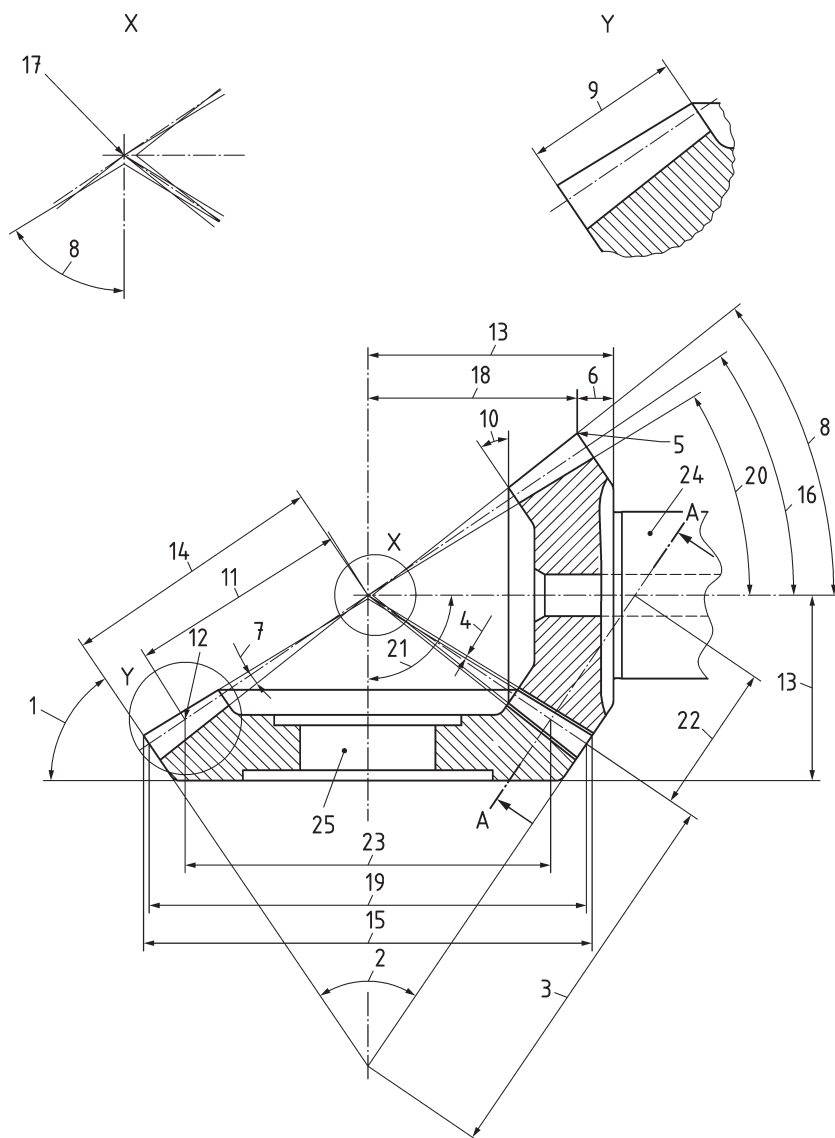
ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

NOTE 1 The symbols, terms and definitions used in this document are, wherever possible, consistent with other International Standards. It is known, because of certain limitations, that some symbols, their terms and definitions, as used in this document, are different from those used in similar literature pertaining to spur and helical gearing.

NOTE 2 Bevel gear nomenclature used throughout this document is illustrated in [Figure 1](#), the axial section of a bevel gear, and in [Figure 2](#), the mean transverse section. Hypoid nomenclature is illustrated in [Figure 3](#).

Subscript 1 refers to the pinion and subscript 2 to the wheel.

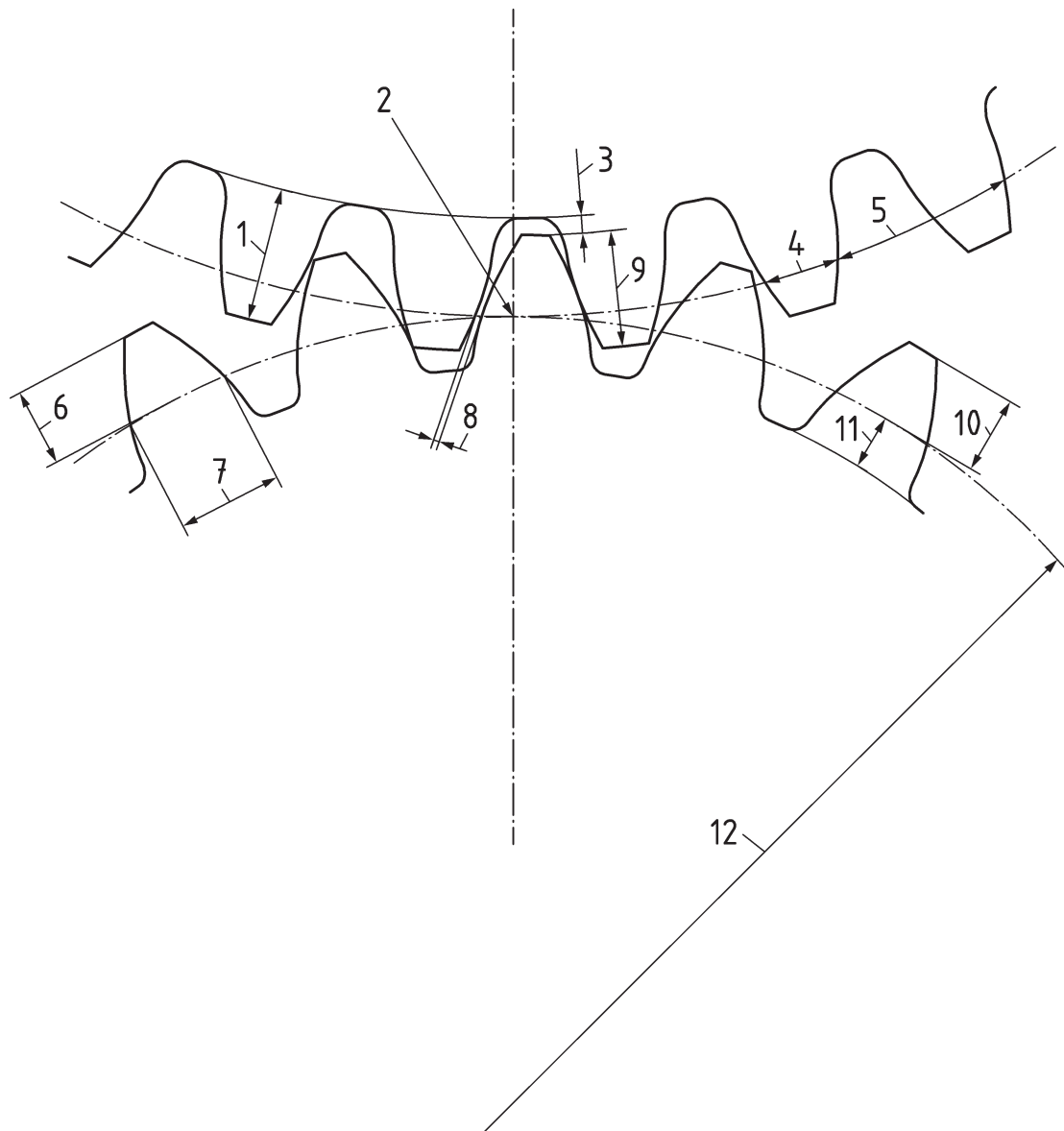


Key

1 back angle	10 front angle	19 outer pitch diameter, d_{e1} , d_{e2}
2 back cone angle	11 mean cone distance, R_m	20 root angle, δ_{f1} , δ_{f2}
3 back cone distance	12 mean point	21 shaft angle, Σ'
4 clearance, c	13 mounting distance	22 equivalent pitch radius
5 crown point	14 outer cone distance, R_e	23 mean pitch diameter, d_{m1} , d_{m2}
6 crown to back	15 outside diameter, d_{ae1} , d_{ae2}	24 pinion
7 dedendum angle, θ_{f1} , θ_{f2}	16 pitch angle, δ_1 , δ_2	25 wheel
8 face angle δ_{a1} , δ_{a2}	17 pitch cone apex	
9 face width, b	18 crown to crossing point, t_{x01} , t_{x02}	

NOTE See [Figure 2](#) for mean transverse section, A-A.

Figure 1 — Bevel gear nomenclature — Axial plane

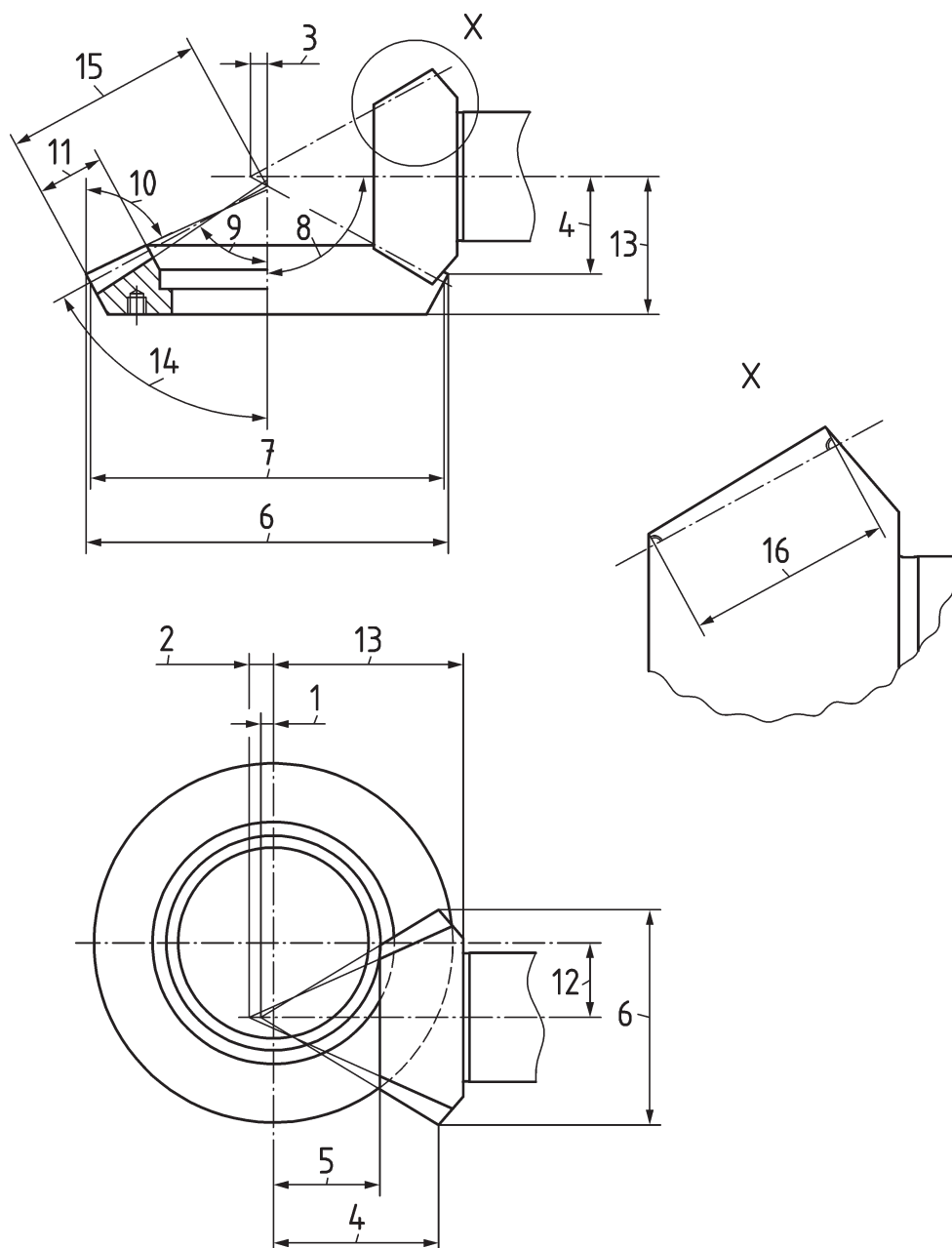


Key

1	whole depth, h_m	5	circular pitch	9	working depth, h_{mw}
2	pitch point	6	chordal addendum	10	addendum, h_{am}
3	clearance, c	7	chordal thickness	11	dedendum, h_{fm}
4	circular thickness	8	backlash	12	equivalent pitch radius

NOTE See A-A in [Figure 1](#).

Figure 2 — Bevel gear nomenclature — Mean transverse section



Key

1	face apex beyond crossing point, t_{zF1}	7	outer pitch diameter, d_{e1}, d_{e2}	13	mounting distance
2	root apex beyond crossing point, t_{zR1}	8	shaft angle, Σ	14	pitch angle, δ_2
3	pitch apex beyond crossing point, t_{z1}	9	root angle, δ_{f1}, δ_{f2}	15	outer cone distance, R_e
4	crown to crossing point, t_{x01}, t_{x02}	10	face angle of blank, δ_{a1}, δ_{a2}	16	pinion face width, b_1
5	front crown to crossing point, t_{xi1}	11	wheel face width, b_2		
6	outside diameter, d_{ae1}, d_{ae2}	12	hypoid offset, a		

NOTE Apex beyond crossing point values are positive when crossing point lies inside the respective cone.

Figure 3 — Hypoid nomenclature

3.1 Terms and definitions

3.1.1

mean chordal addendum

h_{amc1}, h_{amc2}

height from the top of the gear tooth to the chord subtending the circular thickness arc at the mean cone distance in a plane normal to the tooth face

3.1.2

mean addendum

h_{am1}, h_{am2}

height by which the gear tooth projects above the pitch cone at the mean cone distance

3.1.3

outer normal backlash allowance

j_{en}

amount by which the tooth thicknesses are reduced to provide the necessary backlash in assembly

Note 1 to entry: It is specified at the outer cone distance.

3.1.4

coast side

<by normal convention> convex pinion flank in mesh with the concave wheel flank

3.1.5

cutter radius

r_{c0}

nominal radius of the face type cutter or cup-shaped grinding wheel that is used to cut or grind the spiral bevel teeth

3.1.6

sum of dedendum angles

$\Sigma\theta_f$

sum of the pinion and wheel dedendum angles

3.1.7

sum of constant slot width dedendum angles

$\Sigma\theta_{fC}$

sum of dedendum angles for constant slot width

3.1.8

sum of modified slot width dedendum angles

$\Sigma\theta_{fM}$

sum of dedendum angles for modified slot width taper

3.1.9

sum of standard depth dedendum angles

$\Sigma\theta_{fS}$

sum of dedendum angles for standard depth taper

3.1.10

sum of uniform depth dedendum angles

$\Sigma\theta_{fU}$

sum of dedendum angles for uniform depth

3.1.11

mean dedendum

h_{fm1}, h_{fm2}

depth of the tooth space below the pitch cone at the mean cone distance

3.1.12

mean whole depth

h_m

tooth depth at mean cone distance

3.1.13

mean working depth

h_{mw}

depth of engagement of two gears at mean cone distance

3.1.14

direction of rotation

direction determined by an observer viewing the gear from the back looking towards the pitch apex

3.1.15

drive side

by normal convention, concave pinion flank in mesh with the convex wheel flank

3.1.16

face width

b

length of the teeth measured along a pitch cone element

3.1.17

mean addendum factor

c_{ham}

apportions the mean working depth between wheel and pinion mean addendums

Note 1 to entry: The gear mean addendum is equal to c_{ham} times the mean working depth.

3.1.18

mean radius of curvature

$\rho_{m\beta}$

radius of curvature of the tooth surface in the lengthwise direction at the mean cone distance

3.1.19

number of blade groups

z_0

number of blade groups contained in the circumference of the cutting tool

3.1.20

number of teeth

z_1, z_2

number of teeth contained in the whole circumference of the pitch cone

3.1.21

number of crown gear teeth

z_p

number of teeth in the whole circumference of the crown gear

Note 1 to entry: The number may not be an integer.

3.1.22

mean normal chordal tooth thickness

s_{mnc1}, s_{mnc2}

chordal thickness of the gear tooth at the mean cone distance in a plane normal to the tooth trace

3.1.23**mean normal circular tooth thickness** s_{mn1}, s_{mn2}

length of arc on the pitch cone between the two sides of the gear tooth at the mean cone distance in the plane normal to the tooth trace

3.1.24**tooth trace**

curve of the tooth on the pitch surface

3.1.25**mean point**

point where the calculation of basic geometry is executed

Note 1 to entry: Mean point does not necessarily coincide with middle point of face width.

Note 2 to entry: In all the methods listed in this document, the term “mean point” refers to “calculation point”. See [A.3](#) for calculation points.**3.2 Symbols****Table 1 — Symbols used in this document**

Symbol	Description	Unit
a	hypoid offset	mm
b_1, b_2	face width	mm
b_{e1}, b_{e2}	face width from calculation point to outside	mm
b_{i1}, b_{i2}	face width from calculation point to inside	mm
c	clearance	mm
c_{be2}	face width factor	—
c_{ham}	mean addendum factor of wheel	—
d_{ae1}, d_{ae2}	outside diameter	mm
d_{e1}, d_{e2}	outer pitch diameter	mm
d_{m1}, d_{m2}	mean pitch diameter	mm
F_{ax}	axial force	N
F_{mt1}, F_{mt2}	tangential force at mean diameter	N
F_{rad}	radial force	N
$f_{\alpha lim}$	influence factor of limit pressure angle	—
h_{ae1}, h_{ae2}	outer addendum	mm
h_{am1}, h_{am2}	mean addendum	mm
h_{amc1}, h_{amc2}	mean chordal addendum	mm
h_{e1}, h_{e2}	outer whole depth	mm
h_{fe1}, h_{fe2}	outer dedendum	mm
h_{fi1}, h_{fi2}	inner dedendum	mm
h_{fm1}, h_{fm2}	mean dedendum	mm
h_m	mean whole depth	mm
h_{mw}	mean working depth	mm
h_{t1}	pinion whole depth	mm
j_{en}	outer normal backlash	mm
j_{et}	outer transverse backlash	mm
j_{mn}	mean normal backlash	mm

Table 1 (continued)

Symbol	Description	Unit
j_{mt}	mean transverse backlash	mm
k_c	clearance factor	—
k_d	depth factor	—
k_{hap}	basic crown gear addendum factor (related to m_{mn})	—
k_{hfp}	basic crown gear dedendum factor (related to m_{mn})	—
k_t	circular thickness factor	—
m_{et}	outer transverse module	mm
m_{mn}	mean normal module	mm
n_1	pinion speed	min ⁻¹
P	power	kW
R_{e1}, R_{e2}	outer cone distance	mm
R_{i1}, R_{i2}	inner cone distance	mm
R_{m1}, R_{m2}	mean cone distance	mm
r_{c0}	cutter radius	mm
s_{mn1}, s_{mn2}	mean normal circular tooth thickness	mm
s_{mnc1}, s_{mnc2}	mean normal chordal tooth thickness	mm
T_1	pinion torque	Nm
t_{xi1}, t_{xi2}	front crown to crossing point	mm
t_{xo1}, t_{xo2}	pitch cone apex to crown (crown to crossing point, hypoid)	mm
t_{z1}, t_{z2}	pitch apex beyond crossing point	mm
t_{zF1}, t_{zF2}	face apex beyond crossing point	mm
t_{zi1}, t_{zi2}	crossing point to inside point along axis	mm
t_{zm1}, t_{zm2}	crossing point to mean point along axis	mm
t_{zR1}, t_{zR2}	root apex beyond crossing point	mm
u	gear ratio	—
u_a	equivalent ratio	—
W_{m2}	wheel mean slot width	mm
x_{hm1}	profile shift coefficient	—
x_{sm1}, x_{sm2}	thickness modification coefficient (backlash included)	—
x_{smn}	thickness modification coefficient (theoretical)	—
z_0	number of blade groups	—
z_1, z_2	number of teeth	—
z_p	number of crown gear teeth	—
α_{dC}	nominal design pressure angle on coast side	°
α_{dD}	nominal design pressure angle on drive side	°
α_{eC}	effective pressure angle on coast side	°
α_{eD}	effective pressure angle on drive side	°
α_{nD}	generated pressure angle on drive side	°
α_{nC}	generated pressure angle on coast side	°
α_{lim}	limit pressure angle	°
β_{e1}, β_{e2}	outer spiral angle	°
β_{i1}, β_{i2}	inner spiral angle	°
β_{m1}, β_{m2}	mean spiral angle	°
Δb_{x1}	pinion face width increment	mm

Table 1 (continued)

Symbol	Description	Unit
Δg_{xi}	increment along pinion axis from calculation point to inside	mm
Δg_{xe}	increment along pinion axis from calculation point to outside	mm
$\Delta \Sigma$	shaft angle departure from 90°	°
δ_{a1}, δ_{a2}	face angle	°
δ_{f1}, δ_{f2}	root angle	°
δ_1, δ_2	pitch angle	°
ε_β	face contact ratio	-
η	wheel offset angle in axial plane	°
θ_{a1}, θ_{a2}	addendum angle	°
θ_{f1}, θ_{f2}	dedendum angle	°
ν	lead angle of cutter	°
ρ_b	epicycloid base circle radius	mm
ρ_{lim}	limit curvature radius	mm
ρ_{P0}	crown gear to cutter centre distance	mm
Σ	shaft angle	°
$\Sigma\theta_f$	sum of dedendum angles	°
$\Sigma\theta_{fC}$	sum of dedendum angles for constant slot width taper	°
$\Sigma\theta_{fS}$	sum of dedendum angles for standard taper	°
$\Sigma\theta_{fM}$	sum of dedendum angles for modified slot width taper	°
$\Sigma\theta_{fU}$	sum of dedendum angles for uniform depth taper	°
ζ_o	pinion offset angle in face plane	°
ζ_m	pinion offset angle in axial plane	°
ζ_{mp}	offset angle in pitch plane, pinion and wheel	°
ζ_R	pinion offset angle in root plane	°

4 Design considerations

4.1 General

Loading, speed, accuracy requirements, space limitations and special operating conditions influence the design. For details, see ISO 10300 (all parts), [Annex B](#) and handbooks of gear manufacturing companies.

“Precision finish”, as used in this document, refers to a machine finishing operation which includes grinding, skiving and hard cut finishing. However, the common form of finishing known as “lapping” is specifically excluded as a form of precision finishing.

Users should determine the cutting methods available from their gear manufacturer prior to proceeding. Cutting systems used by bevel gear manufacturers are heavily dependent upon the type of machine tool that will be used.

4.2 Types of bevel gears

4.2.1 General

Bevel gears are suitable for transmitting power between shafts at practically any angle or speed. However, the particular type of gear best suited for a specific application is dependent upon the mountings, available space and operating conditions.

4.2.2 Straight bevels

Straight bevel gears (see [Figure 4](#)) are the simplest form of bevel gears. Contact on the driven gear begins at the top of the tooth and progresses towards the root. They have teeth which are straight and tapered which, if extended inward, generally intersect in a common point at the axis.

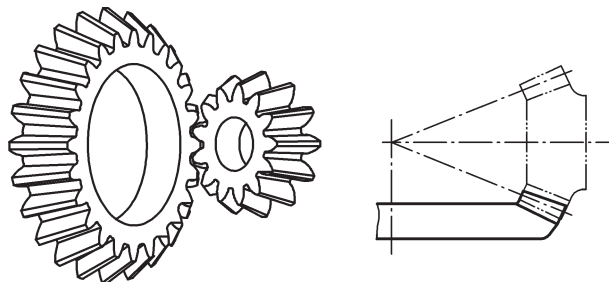


Figure 4 — Straight bevel

4.2.3 Spiral bevels

Spiral bevel gears (see [Figure 5](#)) have curved oblique teeth on which contact begins at one end of the tooth and progresses smoothly to the other end. They mesh with contact similar to straight bevels but as the result of additional overlapping tooth action, the motion will be transmitted more smoothly than by straight bevel or zerol bevel gears. This reduces noise and vibration especially noticeable at high speeds. Spiral bevel gears can also have their tooth surfaces precision-finished.

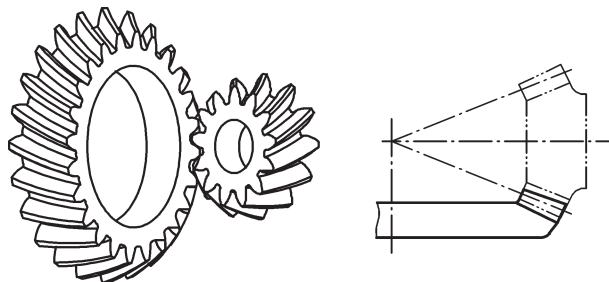


Figure 5 — Spiral bevel

4.2.4 Zerol bevels

Zerol bevel gears (see [Figure 6](#)), as well as other spiral bevel gears, with zero spiral angle have curved teeth which are in the same general direction as straight bevel teeth. They produce the same thrust loads on the bearings, can be used in the same mounting, have smooth operating characteristics and are manufactured on the same machines as spiral bevel gears. Zerol bevels can also have their tooth surfaces precision-finished. Gears with spiral angles less than 10° are sometimes referred to by the name “zerol”.

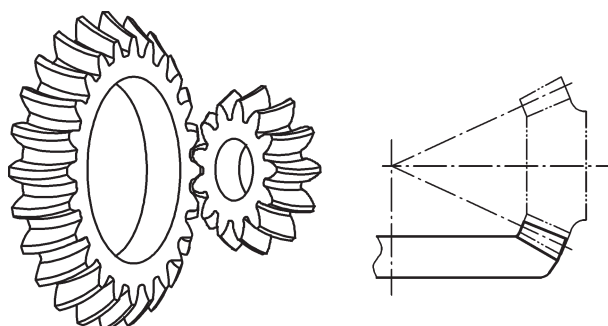


Figure 6 — Zerol bevel

4.2.5 Hypoids

Hypoid gears (see Figure 7) are similar to spiral bevel gears except that the pinion axis is offset above or below the wheel axis; see B.3. If there is sufficient offset, the shafts may pass one another and a compact straddle mounting can be used on the wheel and pinion. Hypoid gears can also have their tooth surfaces precision-finished.

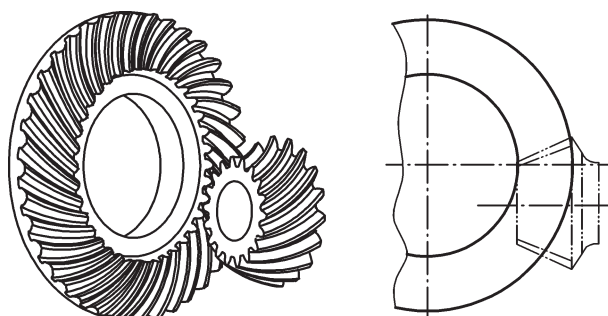


Figure 7 — Hypoid

4.3 Ratios

Bevel gears may be used for both speed-reducing and speed-increasing drives. The required ratio shall be determined by the designer from the given input speed and required output speed. For power drives, the ratio in bevel and hypoid gears may be as low as 1, but should not exceed approximately 10. High-ratio hypoids from 10 to approximately 20 have found considerable usage in machine tool design where precision gears are required. In speed-increasing applications, the ratio should not exceed 5.

4.4 Hand of spiral

The hand of spiral should be selected to give an axial thrust that tends to move both the wheel and pinion out of mesh when operating in the predominant working direction.

Often, the mounting conditions will dictate the hand of spiral to be selected. For spiral bevel and hypoid gears, both members should be held against axial movement in both directions.

A *right-hand spiral bevel gear* is one in which the outer half of a tooth is inclined in the clockwise direction from the axial plane through the midpoint of the tooth as viewed by an observer looking at the face of the gear. Figure 5 shows a right-hand wheel.

A *left-hand spiral bevel gear* is one in which the outer half of a tooth is inclined in the anticlockwise (counterclockwise) direction from the axial plane through the midpoint of the tooth as viewed by an observer looking at the face of the gear. Figure 5 shows a left-hand pinion.

To avoid the loss of backlash, the hand of spiral should be selected to give an axial thrust that tends to move the pinion out of mesh. See [Annex D](#).

For relation of the hand of spiral and the direction of hypoid offset, see [B.3](#).

4.5 Preliminary gear size

Once the preliminary gear size is determined (see [B.4.3](#)), the tooth proportions of the gears should be established and the resulting design should be checked for bending strength and pitting resistance. See ISO 10300 (all parts).

5 Tooth geometry and cutting considerations

5.1 Manufacturing considerations

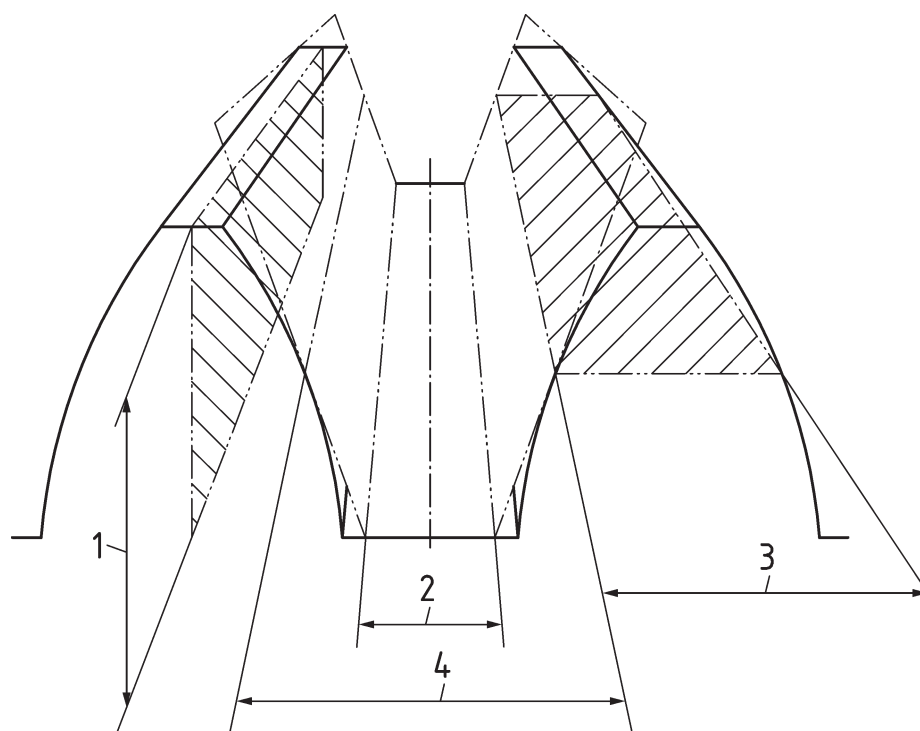
This clause presents tooth dimensions for bevel and hypoid gears in which the teeth are machined by a face mill cutter, face hob cutter, a planing tool or a cup-shaped grinding wheel. The gear geometry is a function of the cutting method used. For this reason, it is important that the user is familiar with the cutting methods used by the gear manufacturer. The following section is provided to familiarize the user with this interdependence.

5.2 Tooth taper

Bevel gear tooth design involves some consideration of tooth taper because the amount of taper affects the final tooth proportions and the size and shape of the blank.

It is advisable to define the following interrelated basic types of tapers (these are illustrated in [Figure 8](#), in which straight bevel teeth are shown for simplicity).

- Depth taper refers to the change in tooth depth along the face measured perpendicular to the pitch cone.
- Slot width taper refers to the change in the point width formed by a V-shaped cutting tool of nominal pressure angle, whose sides are tangent to the two sides of the tooth space and whose top is tangent to the root cone, along the face.
- Space width taper refers to the change in the space width along the face. It is generally measured in the pitch plane.
- Thickness taper refers to the change in tooth thickness along the face. It is generally measured in the pitch plane.



Key

- 1 depth
- 2 slot width
- 3 thickness
- 4 space width

Figure 8 — Bevel gear tooth tapers

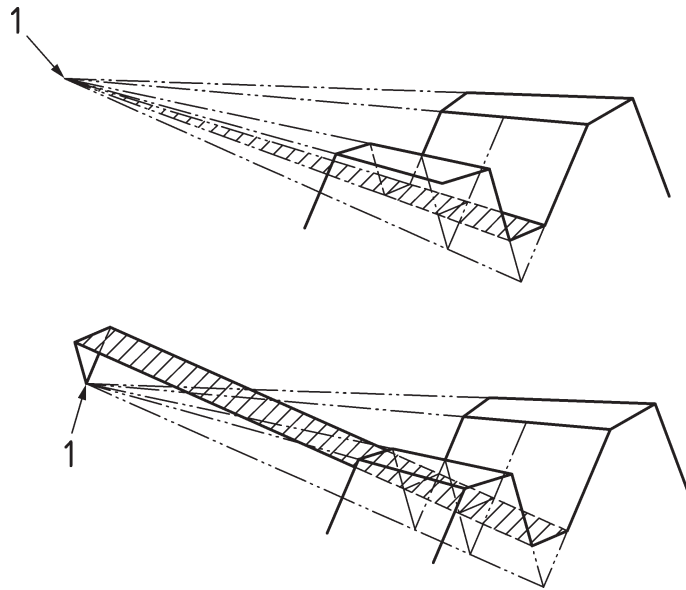
The taper of primary consideration for production is the slot width taper. The width of the slot at its narrowest point determines the point width of the cutting tool and limits the edge radius that can be placed on the cutter blade.

The taper which directly affects the blank is the depth taper through its effect on the dedendum angle, which is used in the calculation of the face angle of the mating member.

The slot width taper depends upon the lengthwise curvature and the dedendum angle. It can be changed by varying the depth taper, i.e. by tilting the root line as shown in [Figure 9](#), in which the concept is simplified by illustrating straight bevel teeth. In spiral bevel and hypoid gears, the amount by which the root line is tilted is further dependent upon a number of geometric characteristics including the cutter radius.

This relationship is discussed more thoroughly in [5.3](#).

The root line is generally rotated about the mid-section at the pitch line in order to maintain the desired working depth at the mean section of the tooth.



Key

1 pitch cone apex

Figure 9 — Root line tilt

5.3 Tooth depth configurations

5.3.1 Taper depth

5.3.1.1 Standard depth

Standard depth pertains to the configuration where the depth changes in proportion to the cone distance at any particular section of the tooth. If the root line of such a tooth is extended, it intersects the axis at the pitch cone apex, as illustrated in [Figure 10](#), but the face cone apex does not. The sum of the dedendum angles of pinion and wheel for standard depth taper, $\Sigma\theta_{fs}$, does not depend on cutter radius. Most straight bevel gears are designed with standard depth taper.

5.3.1.2 Constant slot width

This taper represents a tilt of the root line such that the slot width is constant while maintaining the proper space width taper. The slot width taper is zero on both members.

The formula for the sum of the dedendum angles is given in [C.5.1](#).

Formula (C.4), for the sum of the dedendum angles, indicates that the cutter radius, r_{c0} , has a significant effect on the amount by which the root line is tilted. For a given design, the following tendencies should be noted.

- A large cutter radius increases the sum of the dedendum angles. If the radius is too large, the resultant depthwise taper could adversely affect the depth of the teeth at either end, i.e. too shallow at inner end for proper tooth contact, and too deep at the outer end, which can cause undercut and narrow toplands. Therefore, the cutter radius should not be too large and an upper limit of r_{c0} approximately equal to R_{m2} is suggested.
- A small cutter radius decreases the sum of the dedendum angles. In fact, if r_{c0} equals $R_{m2} \sin \beta_{m2}$, the sum of the dedendum angles becomes zero, which results in uniform depth teeth. If r_{c0} is less than $R_{m2} \sin \beta_{m2}$, reverse depthwise taper would exist and the teeth would be deeper at the inner

end than at the outer. In order to avoid excessive depth (undercut and narrow toplands) at the inner end, a minimum value of r_{c0} , equal to $1,1 R_{m2} \sin \beta_{m2}$, is suggested.

NOTE For gears cut with a planing tool, the cutter centre is considered to be at infinity and root lines are not tilted. Standard taper is the norm for gears produced in this manner.

5.3.1.3 Modified slot width

This taper is an intermediate one in which the root line is tilted about the mean point. In this case, the slot width of the gear member is constant along the tooth length and any slot width taper is on the pinion member.

For the case where the root line is tilted to permit finishing the gear in one operation, the amount of tilt is somewhat arbitrary, but should fall within the following guidelines (see [Table C.4](#)):

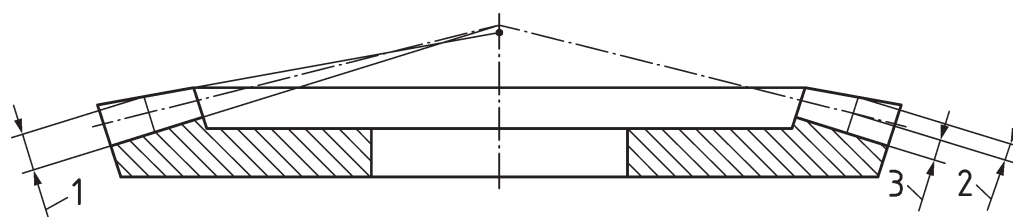
- the sum of the pinion and wheel dedendum angles for modified slot width taper, $\Sigma\theta_{fM}$, should not exceed 1,3 times the sum of the dedendum angles for standard depth taper, $\Sigma\theta_{fS}$, nor should it exceed the sum of the dedendum angles for constant slot width taper, $\Sigma\theta_{fC}$;
- in practice, the smaller of the values, $1,3 \Sigma\theta_{fS}$ or $\Sigma\theta_{fC}$, is used.

5.3.2 Uniform depth

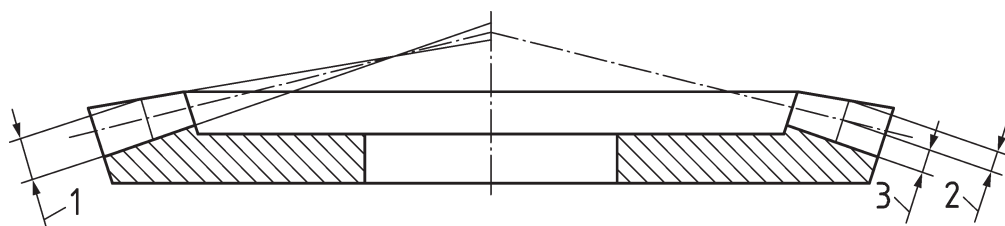
Uniform depth is the configuration where the tooth depth remains constant along the face width regardless of cutter radius. In this case, the root line is parallel to an element of the face cone, as illustrated in [Figure 10](#). The sum of the dedendum angles of pinion and wheel for uniform depth, $\Sigma\theta_{fU}$, equals zero.

For the uniform depth tooth, the cutter radius, r_{c0} , should be greater than $R_{m2} \sin \beta_{m2}$, but not more than 1,5 times this value. This approximation of lengthwise involute curvature, in conjunction with the uniform depth, holds the variation along the face width in normal circular thickness on the pinion and wheel to a minimum.

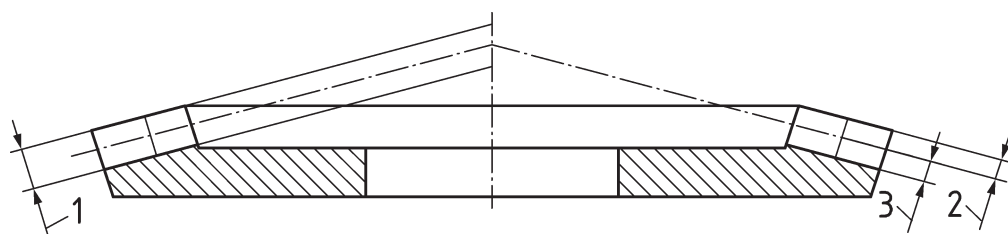
If narrow inner toplands occur on the pinion, a small tooth tip chamfer may be provided (see [Figure 11](#)).



a) Standard depth taper



b) Constant and modified slot width

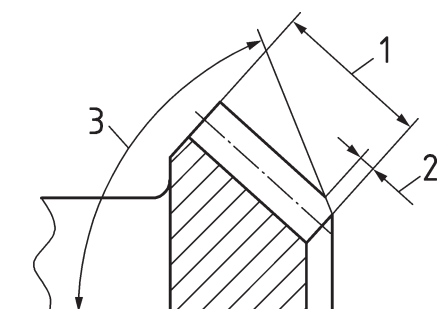


c) Uniform depth

Key

- 1 mean whole depth
- 2 mean addendum
- 3 mean dedendum

Figure 10 — Bevel gear depthwise tapers



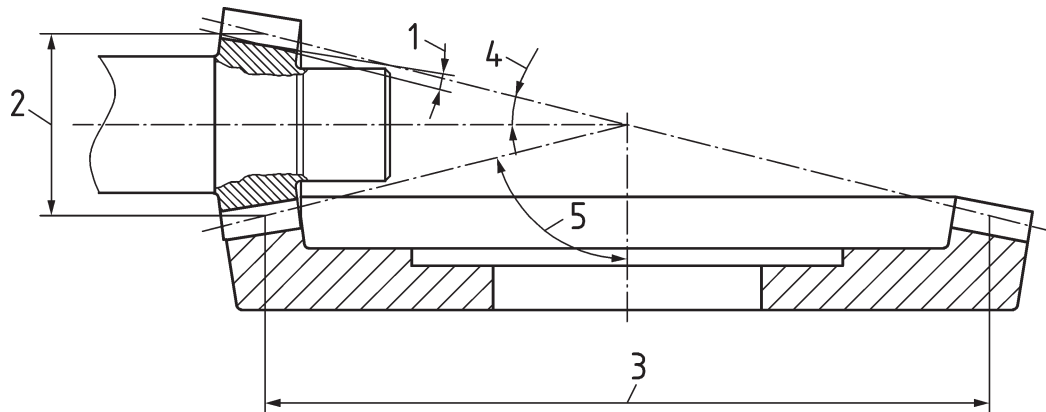
Key

- 1 face width, b_1
- 2 length of chamfer
- 3 angle of chamfer

Figure 11 — Tooth tip chamfering on the pinion

5.4 Dedendum angle modifications

To avoid cutter interference with a hub or shoulder, the wheel and pinion root line can be rotated about the mean point as shown in [Figure 12](#). A dedendum angle modification, when desired, normally ranges between -5° and $+5^\circ$.



Key

- 1 dedendum angle modification
- 2 mean pitch diameter of pinion, d_{m1}
- 3 mean pitch diameter of wheel, d_{m2}
- 4 pitch angle of pinion, δ_1
- 5 pitch angle of wheel, δ_2

Figure 12 — Angle modification required because of extension in pinion shaft

5.5 Cutter radius

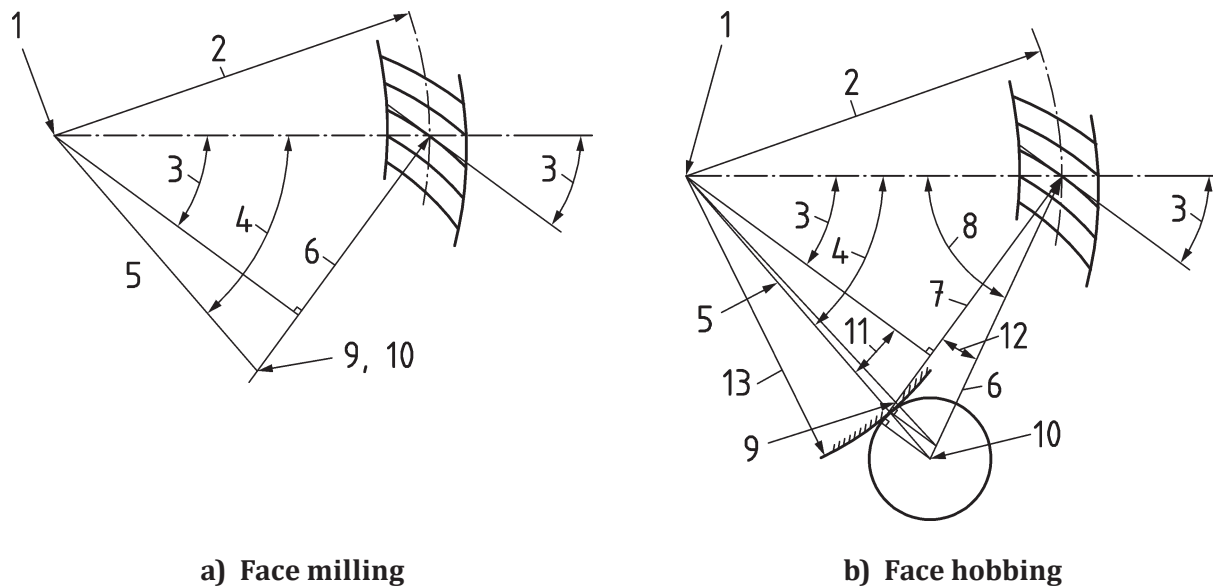
Most spiral bevel gears are manufactured with face cutters. The selection of the cutter radius depends on the cutting system used. A list of nominal cutter radii is contained in [Annex E](#).

5.6 Mean radius of curvature

Two types of cutting processes are used in the industry. In the process which will be referred to as the “face milling process”, the cradle axis and the work axis roll together in a timed relationship. In the process which will be referred to as the “face hobbing process”, the cradle axis, work axis and cutter axis roll together in a timed relationship.

With the face milling process, the mean radius of tooth curvature is equal to the cutter radius [see [Figure 13 a\)](#)].

With the face hobbing process, the curve in the lengthwise direction of the tooth is an extended epicycloid and is a function of the relative roll between the workpiece and the cutter. The radius of curvature is somewhat smaller than the cutter radius.



Key

- | | | | |
|---|--|----|---|
| 1 | crown gear centre | 8 | first auxiliary angle, λ |
| 2 | mean cone distance, R_{m2} | 9 | centre of curvature |
| 3 | spiral angle, β_{m2} | 10 | cutter centre |
| 4 | intermediate angle | 11 | second auxiliary angle, η_1 |
| 5 | crown gear to cutter centre, ρ_{P0} | 12 | lead angle of cutter, ν |
| 6 | cutter radius, r_{c0} | 13 | epicycloid base circle radius, ρ_b |
| 7 | lengthwise tooth mean radius of curvature, $\rho_{m\beta}$ | | |

Figure 13 — Geometry of face milling and face hobbing processes

5.7 Hypoid design

An infinite number of pitch surfaces exists for any hypoid pair. However, starting with the initial data given in Table 2, the result is only one pitch surface for each method. The design procedures used in the industry will be referred to as Method 1, Method 2 and Method 3, and, for bevel gears, as Method 0.

In Method 1 and Method 3, the pitch surfaces are selected such that the hypoid radius of curvature matches the cutter radius of curvature at the mean point for gears to be manufactured by the face milling process and such that it matches the mean epicycloidal curvature at the mean point for gears cut by the face hobbing process.

Method 2 is a method for designing gears to be cut by the face hobbing process. In this case, the wheel pitch apex, pinion pitch apex and cutter centre lie on a straight line.

5.8 Most general type of gearing

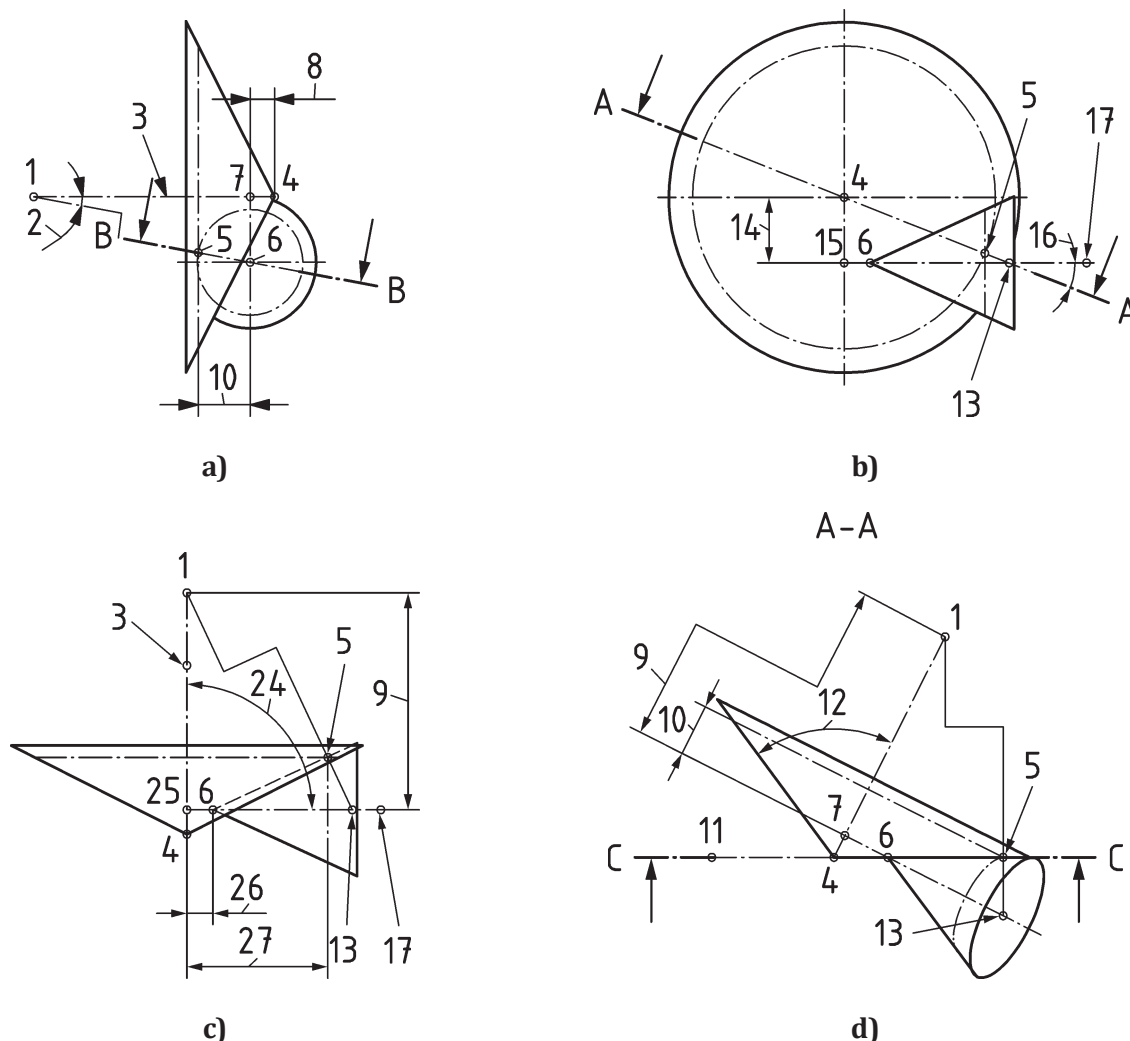
Hypoid gears are the most general type of gearing. The wheel and pinion axes are skew and non-intersecting. The teeth are curved in the lengthwise direction. All other types of gears can be considered subsets of the hypoid. Spiral bevel gears are hypoid gears with zero offset between the axes. Straight bevel gears are hypoid gears with zero offset and zero tooth curvature. Helical gears are hypoid gears with zero shaft angle and zero tooth curvature.

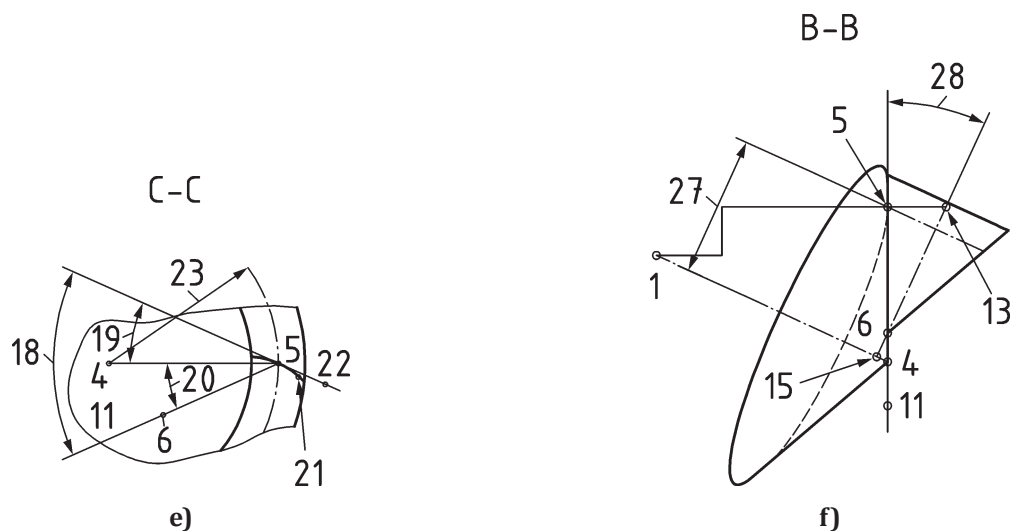
5.9 Hypoid geometry

5.9.1 Basics

Whenever a most general case is defined, the definition becomes complex. Hypoid gear geometry is no exception. [Figure 14](#) shows the major angles and quantities involved. [Figure 14 a\)](#) is a side view looking along the pinion axis. [Figure 14 b\)](#) is a front view looking along the wheel axis. [Figure 14 c\)](#) is a top view showing the shaft angle between the wheel and pinion axes. [Figure 14 d\)](#) is a view of the wheel section along the plane making the offset angle, key 16, in the pinion axial plane. [Figure 14 e\)](#) is a view of the pitch plane. [Figure 14 f\)](#) is a view of the pinion section along the plane making the offset angle, key 2, in the wheel axial plane.

The scope of this document does not permit an adequate explanation or derivation of the formulae involved.





Key

1 intersection of contact normal through mean point at wheels axis	10 crossing point to mean point along wheel axis, t_{zm2}	20 offset angle in pitch plane, pinion and wheel, ζ_{mp}
2 wheel offset angle in axial plane, η	11 pitch plane	21 sliding velocity vector
3 wheel axis	12 wheel pitch angle, δ_2	22 tangent to tooth trace at mean point
4 wheel pitch apex	13 intersection point of contact normal through mean point at pinion axis	23 wheel mean cone distance, R_{m2}
5 mean point, P	14 offset, a	24 shaft angle, Σ
6 pinion pitch apex	15 crossing point at pinion axis	25 common normal to pinion and wheel axis through crossing point
7 crossing point at wheel axis	16 pinion offset angle in axial plane, ζ_m	26 pinion pitch apex beyond crossing point, t_{z1}
8 wheel pitch apex beyond crossing point, t_{z2}	17 pinion axis	27 crossing point to mean point along pinion axis, t_{zm1}
9 distance along wheel axis between crossing point and contact normal intersection	18 pinion spiral angle, β_{m1}	28 pinion pitch angle, δ_1
	19 wheel spiral angle, β_{m2}	29 pinion mean cone distance, R_{m1}

Figure 14 — Hypoid geometry

5.9.2 Crossing point

Crossing point, O_c , is the point of intersection of bevel gear axes; it is also the apparent point of intersection of axes in hypoid gears, when projected to a plane parallel to both axes (see [Figure 15](#)).

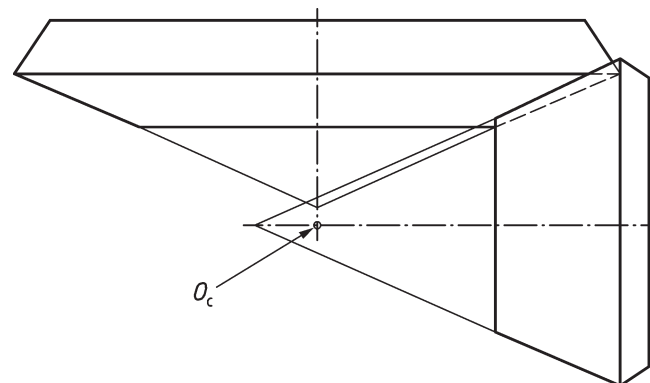


Figure 15 — Crossing point for hypoid gears

6 Pitch cone parameters

6.1 Initial data for pitch cone parameters

The recommended procedure in this document is to begin each method with the calculation of the pitch cone parameters; see [Annex A](#). For the calculation of the pitch cone parameters, a set of initial data is necessary, as given in [Table 2](#).

Table 2 — Initial data for pitch cone parameters

Symbol	Description	Method 0	Method 1	Method 2	Method 3
Σ	shaft angle	X	X	X	X
a	hypoid offset	0,0	X	X	X
$z_{1,2}$	number of teeth	X	X	X	X
d_{m2}	mean pitch diameter of wheel	—	—	X	—
d_{e2}	outer pitch diameter of wheel	X	X	—	X
b_2	wheel face width	X	X	X	X
β_{m1}	mean spiral angle of pinion	—	X	—	—
β_{m2}	mean spiral angle of wheel	X	—	X	X
r_{c0}	cutter radius	X	X	X	X
z_0	number of blade groups (only face hobbing)	X	X	X	X

[Annex A](#) provides a structure for calculating the methods provided in this document.

6.2 Determination of pitch cone parameters for bevel and hypoid gears

6.2.1 Method 0

Gear ratio, u

$$u = \frac{z_2}{z_1} \quad (1)$$

Pinion pitch angle, δ_1

$$\delta_1 = \arctan\left(\frac{\sin\Sigma}{\cos\Sigma + u}\right) \quad (2)$$

Wheel pitch angle, δ_2

$$\delta_2 = \Sigma - \delta_1 \quad (3)$$

Outer cone distance, R_e

$$R_{e1,2} = \frac{d_{e2}}{2\sin\delta_2} \quad (4)$$

Mean cone distance, R_m

$$R_{m1,2} = R_{e2} - \frac{b_2}{2} \quad (5)$$

Spiral angle, β_{m1} , pinion

$$\beta_{m1} = \beta_{m2} \quad (6)$$

Face width factor, c_{be2}

$$c_{be2} = 0,5 \quad (7)$$

6.2.2 Method 1

Gear ratio, u

$$u = \frac{z_2}{z_1} \quad (8)$$

Desired pinion spiral angle, $\beta_{\Delta 1}$, pinion

$$\beta_{\Delta 1} = \beta_{m1} \quad (9)$$

Shaft angle departure from 90° , $\Delta\Sigma$

$$\Delta\Sigma = \Sigma - 90^\circ \quad (10)$$

Approximate wheel pitch angle, $\delta_{\text{int}2}$

$$\delta_{\text{int}2} = \arctan \left(\frac{u \cos \Delta \Sigma}{1,2 (1 - u \sin \Delta \Sigma)} \right) \quad (11)$$

Wheel mean pitch radius, $r_{\text{mpt}2}$

$$r_{\text{mpt}2} = \frac{d_{e2} - b_2 \sin \delta_{\text{int}2}}{2} \quad (12)$$

Approximate pinion offset angle in pitch plane, ε'_i

$$\varepsilon'_i = \arcsin \left(\frac{a \sin \delta_{\text{int}2}}{r_{\text{mpt}2}} \right) \quad (13)$$

Approximate hypoid dimension factor, K_1

$$K_1 = \tan \beta_{\Delta 1} \sin \varepsilon'_i + \cos \varepsilon'_i \quad (14)$$

Approximate pinion mean radius, $r_{\text{mn}1}$

$$r_{\text{mn}1} = \frac{r_{\text{mpt}2} K_1}{u} \quad (15)$$

Start of iteration

Wheel offset angle in axial plane, η

$$\eta = \arctan \left[\frac{a}{r_{\text{mpt}2} (\tan \delta_{\text{int}2} \cos \Delta \Sigma - \sin \Delta \Sigma) + r_{\text{mn}1}} \right] \quad (16)$$

Intermediate pinion offset angle in axial plane, ε_2

$$\varepsilon_2 = \arcsin \left(\frac{a - r_{\text{mn}1} \sin \eta}{r_{\text{mpt}2}} \right) \quad (17)$$

Intermediate pinion pitch angle, $\delta_{\text{int}1}$

$$\delta_{\text{int}1} = \arctan \left(\frac{\sin \eta}{\tan \varepsilon_2 \cos \Delta \Sigma} + \tan \Delta \Sigma \cos \eta \right) \quad (18)$$

Intermediate pinion offset angle in pitch plane, ε'_2

$$\varepsilon'_2 = \arcsin \left(\frac{\sin \varepsilon_2 \cos \Delta \Sigma}{\cos \delta_{\text{int}1}} \right) \quad (19)$$

Intermediate pinion mean spiral angle, $\beta_{m,int1}$

$$\beta_{m,int1} = \arctan\left(\frac{K_1 - \cos\varepsilon'_2}{\sin\varepsilon'_2}\right) \quad (20)$$

Increment in hypoid dimension factor, ΔK

$$\Delta K = \sin\varepsilon'_2 \left(\tan\beta_{\Delta 1} - \tan\beta_{m,int1} \right) \quad (21)$$

Pinion mean radius increment, Δr_{mpt1}

$$\Delta r_{mpt1} = r_{mpt2} \frac{\Delta K}{u} \quad (22)$$

Pinion offset angle in axial plane, ε_1

$$\varepsilon_1 = \arcsin\left(\sin\varepsilon_2 - \frac{\Delta r_{mpt1}}{r_{mpt2}} \sin\eta\right) \quad (23)$$

Pinion pitch angle, δ_1

$$\delta_1 = \arctan\left(\frac{\sin\eta}{\tan\varepsilon_1 \cos\Delta\Sigma} + \tan\Delta\Sigma \cos\eta\right) \quad (24)$$

Pinion offset angle in pitch plane, ε'_1

$$\varepsilon'_1 = \arcsin\left(\frac{\sin\varepsilon_1 \cos\Delta\Sigma}{\cos\delta_1}\right) \quad (25)$$

Spiral angle, β_{m1} , pinion

$$\beta_{m1} = \arctan\left(\frac{K_1 + \Delta K - \cos\varepsilon'_1}{\sin\varepsilon'_1}\right) \quad (26)$$

Spiral angle, β_{m2} , wheel

$$\beta_{m2} = \beta_{m1} - \varepsilon'_1 \quad (27)$$

Wheel pitch angle, δ_2

$$\delta_2 = \arctan\left(\frac{\sin\varepsilon_1}{\tan\eta \cos\Delta\Sigma} + \cos\varepsilon_1 \tan\Delta\Sigma\right) \quad (28)$$

Mean cone distance, R_{m1} , pinion

$$R_{m1} = \frac{r_{mn1} + \Delta r_{mpt1}}{\sin\delta_1} \quad (29)$$

Mean cone distance, R_{m2} , wheel

$$R_{m2} = \frac{r_{mpt2}}{\sin\delta_2} \quad (30)$$

Mean pinion radius, r_{mpt1}

$$r_{\text{mpt1}} = R_{\text{m1}} \sin \delta_1 \quad (31)$$

Limit pressure angle, α_{lim}

$$\alpha_{\text{lim}} = \arctan \left[- \frac{\tan \delta_1 \tan \delta_2}{\cos \varepsilon'_1} \left(\frac{R_{\text{m1}} \sin \beta_{\text{m1}} - R_{\text{m2}} \sin \beta_{\text{m2}}}{R_{\text{m1}} \tan \delta_1 + R_{\text{m2}} \tan \delta_2} \right) \right] \quad (32)$$

Limit radius of curvature, ρ_{lim}

$$\rho_{\text{lim}} = \frac{\sec \alpha_{\text{lim}} (\tan \beta_{\text{m1}} - \tan \beta_{\text{m2}})}{- \tan \alpha_{\text{lim}} \left(\frac{\tan \beta_{\text{m1}}}{R_{\text{m1}} \tan \delta_1} + \frac{\tan \beta_{\text{m2}}}{R_{\text{m2}} \tan \delta_2} \right) + \frac{1}{R_{\text{m1}} \cos \beta_{\text{m1}}} - \frac{1}{R_{\text{m2}} \cos \beta_{\text{m2}}}} \quad (33)$$

For face hobbled gears:

Number of crown gear teeth, z_p

$$z_p = \frac{z_2}{\sin \delta_2} \quad (34)$$

Lead angle of cutter, ν

$$\nu = \arcsin \left(\frac{R_{\text{m2}} z_0}{r_{\text{c0}} z_p} \cos \beta_{\text{m2}} \right) \quad (35)$$

First auxiliary angle, λ

$$\lambda = 90^\circ - \beta_{\text{m2}} + \nu \quad (36)$$

Crown gear to cutter centre distance, ρ_{P0}

$$\rho_{\text{P0}} = \sqrt{R_{\text{m2}}^2 + r_{\text{c0}}^2 - 2 R_{\text{m2}} r_{\text{c0}} \cos \lambda} \quad (37)$$

Second auxiliary angle, η_1

$$\eta_1 = \arccos \left[\frac{R_{\text{m2}} \cos \beta_{\text{m2}}}{\rho_{\text{P0}} z_p} (z_p + z_0) \right] \quad (38)$$

Lengthwise tooth mean radius of curvature, $\rho_{\text{m}\beta}$

$$\rho_{\text{m}\beta} = R_{\text{m2}} \cos \beta_{\text{m2}} \left[\tan \beta_{\text{m2}} + \frac{\tan \eta_1}{1 + \tan \nu (\tan \beta_{\text{m2}} + \tan \eta_1)} \right] \quad (39)$$

For face milled gears:

Lengthwise tooth mean radius of curvature, $\rho_{m\beta}$

$$\rho_{m\beta} = r_{c0} \quad (40)$$

Change η and recalculate from [Formula \(17\)](#) to [Formula \(33\)](#) until $\frac{\rho_{m\beta}}{\rho_{\text{lim}}} - 1 \leq 0,01$

End of iteration

Face width factor, c_{be2}

$$c_{be2} = \frac{\frac{d_{e2}}{2\sin\delta_2} - R_{m2}}{b_2} \quad (41)$$

6.2.3 Method 2

Lead angle of cutter, ν (see [Figure 13](#))

$$\nu = \arcsin\left(\frac{z_0 d_{m2} \cos\beta_{m2}}{2z_2 r_{c0}}\right) \quad (42)$$

First auxiliary angle, λ

$$\lambda = 90^\circ - \beta_{m2} + \nu \quad (43)$$

Gear ratio, u

$$u = \frac{z_2}{z_1} \quad (44)$$

First approximate pinion pitch angle, δ_{1app}

$$\delta_{1app} = \arctan\left(\frac{\sin\Sigma}{u + \cos\Sigma}\right) \quad (45)$$

First approximate wheel pitch angle, δ_{2app}

$$\delta_{2app} = \Sigma - \delta_{1app} \quad (46)$$

First approximate pinion offset angle in axial plane, ζ_{mapp}

$$\zeta_{mapp} = \arcsin\left(\frac{\frac{2a}{d_{m2}}}{1 + \frac{\cos\delta_{2app}}{u \cos\delta_{1app}}}\right) \quad (47)$$

Approximate hypoid dimension factor, F_{app}

$$F_{\text{app}} = \frac{\cos\beta_{\text{m2}}}{\cos(\beta_{\text{m2}} + \zeta_{\text{mapp}})} \quad (48)$$

Approximate pinion mean pitch diameter, d_{m1app}

$$d_{\text{m1app}} = \frac{F_{\text{app}} d_{\text{m2}}}{u} \quad (49)$$

Intermediate angle, φ_2

$$\varphi_2 = \arctan \left[\frac{u \cos\zeta_{\text{mapp}}}{\frac{u}{\tan\delta_{2\text{app}}} + (F_{\text{app}} - 1) \sin\Sigma} \right] \quad (50)$$

Approximate mean radius of crown gear, R_{mapp}

$$R_{\text{mapp}} = \frac{d_{\text{m2}}}{2\sin\varphi_2} \quad (51)$$

Second auxiliary angle, η_1 (see [Figure 13](#))

$$\eta_1 = \arctan \left(\frac{r_{\text{c0}} \cos\nu - R_{\text{mapp}} \sin\beta_{\text{m2}}}{r_{\text{c0}} \sin\nu + R_{\text{mapp}} \cos\beta_{\text{m2}}} \right) \quad (52)$$

Intermediate angle, φ_3

$$\varphi_3 = \arctan \left[\frac{\tan(\beta_{\text{m2}} + \eta_1)}{\sin\varphi_2} \right] \quad (53)$$

Second approximate pinion pitch angle, δ_1''

$$\delta_1'' = \arctan \left[\frac{d_{\text{m1app}} \sin\Sigma}{d_{\text{m2}} \cos\zeta_{\text{mapp}} + d_{\text{m1app}} \cos\Sigma - \frac{2a}{\tan(\varphi_3 + \zeta_{\text{mapp}})}} \right] \quad (54)$$

Approximate wheel pitch angle, δ_2'' , projected into pinion axial plane along the common pitch plane (see [Figure 14](#))

$$\delta_2'' = \Sigma - \delta_1'' \quad (55)$$

Start of iteration

Improved wheel pitch angle, $\delta_{2\text{imp}}$

$$\delta_{2\text{imp}} = \arctan \left(\tan \delta_2'' \cos \zeta_{\text{mapp}} \right) \quad (56)$$

Auxiliary angle, η_p

$$\eta_p = \arctan \left[\frac{\sin \zeta_{\text{mapp}} \cos \delta_{2\text{imp}}}{\cos (\Sigma - \delta_{2\text{imp}})} \right] \quad (57)$$

Approximate wheel offset angle, η_{app}

$$\eta_{\text{app}} = \arctan \left[\frac{2a}{d_{m2} \tan \delta_{2\text{imp}} + d_{m1\text{app}} \frac{\cos \eta_p \sin (\beta_{m2} + \eta_1)}{\cos (\Sigma - \delta_{2\text{imp}})}} \right] \quad (58)$$

Improved pinion offset angle in axial plane, $\zeta_{m\text{imp}}$

$$\zeta_{m\text{imp}} = \arcsin \left[\frac{2a}{d_{m2}} - \frac{F_{\text{app}} \tan \eta_{\text{app}} \sin \delta_{2\text{imp}} \cos \eta_p}{u \cos (\Sigma - \delta_{2\text{imp}})} \right] \quad (59)$$

Improved pinion offset angle in pitch plane, $\zeta_{\text{mp imp}}$

$$\zeta_{\text{mp imp}} = \arctan \left[\frac{\tan \zeta_{m\text{imp}} \sin \Sigma}{\cos (\Sigma - \delta_{2\text{imp}})} \right] \quad (60)$$

Hypoid dimension factor, F

$$F = \frac{\cos \beta_{m2}}{\cos (\beta_{m2} + \zeta_{\text{mp imp}})} \quad (61)$$

Pinion mean pitch diameter, d_{m1}

$$d_{m1} = \frac{F d_{m2}}{u} \quad (62)$$

Intermediate angle, φ_4

$$\varphi_4 = \arctan \left(\frac{\sin \lambda \sin \Sigma}{\frac{d_{m2}}{2r_{c0}} - \cos \lambda \sin \delta_{2\text{imp}}} \right) \quad (63)$$

Improved pinion pitch angle, $\delta_1''_{\text{imp}}$

$$\delta_1''_{\text{imp}} = \arctan \left[\frac{d_{m1} \sin \Sigma}{d_{m2} \cos \zeta_{m \text{ imp}} + d_{m1} \cos \Sigma \cos \eta_p - \frac{2a}{\tan(\varphi_4 + \zeta_{m \text{ imp}})}} \right] \quad (64)$$

Improved wheel pitch angle, projected into pinion axial plane along the common pitch plane, $\delta_2''_{\text{imp}}$

$$\delta_2''_{\text{imp}} = \Sigma - \delta_1''_{\text{imp}} \quad (65)$$

Wheel pitch angle, δ_2

$$\delta_2 = \arctan \left(\tan \delta_2''_{\text{imp}} \cos \zeta_{mp \text{ imp}} \right) \quad (66)$$

Intermediate angle, φ_5

$$\varphi_5 = \arctan \left(\frac{\tan \delta_2}{\cos \zeta_{m \text{ imp}}} \right) \quad (67)$$

Improved auxiliary angle, η_{pimp}

$$\eta_{\text{pimp}} = \arctan \left[\frac{\tan \eta_{\text{app}} \sin \varphi_5}{\cos(\Sigma - \varphi_5)} \right] \quad (68)$$

Wheel offset angle in axial plane, η

$$\eta = \arctan \left[\frac{2a}{d_{m2} \tan \delta_2 + d_{m1} \frac{\cos \eta_{\text{pimp}} \sin \varphi_5}{\cos(\Sigma - \varphi_5)}} \right] \quad (69)$$

Pinion offset angle in axial plane, ζ_m

$$\zeta_m = \arcsin(\tan \delta_2 \tan \eta) \quad (70)$$

Pinion offset angle in pitch plane, ζ_{mp}

$$\zeta_{mp} = \arctan \left[\frac{\tan \zeta_m \sin \Sigma}{\cos(\Sigma - \delta_2)} \right] \quad (71)$$

Pinion spiral angle, β_{m1}

$$\beta_{m1} = \beta_{m2} + \zeta_{mp} \quad (72)$$

Pinion mean pitch diameter, d_{m1}

$$d_{m1} = \frac{d_{m2} \cos \beta_{m2}}{u \cos \beta_{m1}} \quad (73)$$

Auxiliary angle, ξ

If $\Sigma \neq 90^\circ$:

$$\xi = \arctan\left(\tan\Sigma \cos\zeta_m\right) - \delta_2 \quad (74)$$

If $\Sigma = 90^\circ$:

$$\xi = 90^\circ - \delta_2 \quad (75)$$

Pinion pitch angle, δ_1

$$\delta_1 = \arctan\left(\tan\xi \cos\zeta_{mp}\right) \quad (76)$$

Mean cone distance, R_{m1} , pinion

$$R_{m1} = \frac{d_{m1}}{2\sin\delta_1} \quad (77)$$

Mean cone distance, R_{m2} , wheel

$$R_{m2} = \frac{d_{m2}}{2\sin\delta_2} \quad (78)$$

Crown gear to cutter centre distance, ρ_{p0}

$$\rho_{p0} = \sqrt{r_{c0}^2 + R_{m2}^2 - 2r_{c0} R_{m2} \cos\lambda} \quad (79)$$

Intermediate angle, φ_6

$$\varphi_6 = \arcsin\left(\frac{r_{c0}\sin\lambda}{\rho_{p0}}\right) \quad (80)$$

Complementary angle, φ_{comp}

$$\varphi_{comp} = 180^\circ - \zeta_{mp} - \varphi_6 \quad (81)$$

Checking variable, R_{mcheck}

$$R_{mcheck} = \frac{R_{m2}\sin\varphi_6}{\sin\varphi_{comp}} \quad (82)$$

Change δ_{2imp} [[Formula \(56\)](#)] and recalculate to [Formula \(82\)](#) until $\left|\frac{R_{m1}}{R_{mcheck}} - 1\right| \leq 0,01$, increase δ_{2imp} , if $R_{m1} < R_{mcheck}$ and vice versa.

End of iteration

Face width factor, c_{be2}

$$c_{be2} = 0,5 \quad (83)$$

6.2.4 Method 3

Gear ratio, u

$$u = \frac{z_2}{z_1} \quad (84)$$

For the iteration that follows, start with hypoid dimension factor, F

$$F = 1 \quad (85)$$

Wheel pitch angle, δ_2

$$\delta_2 = \arctan \left(\frac{\sin \Sigma}{\frac{F}{u} + \cos \Sigma} \right) \quad (86)$$

Pinion pitch angle, δ_1

$$\delta_1 = \Sigma - \delta_2 \quad (87)$$

Start of iteration

Wheel mean pitch diameter, d_{m2}

$$d_{m2} = d_{e2} - b_2 \sin \delta_2 \quad (88)$$

Pinion offset angle in axial plane, ζ_m

$$\zeta_m = \arcsin \left[\frac{2a}{d_{m2} \left(1 + \frac{F \cos \delta_2}{u \cos \delta_1} \right)} \right] \quad (89)$$

Pinion pitch angle, δ_1

$$\delta_1 = \arcsin \left(\cos \zeta_m \sin \Sigma \cos \delta_2 - \cos \Sigma \sin \delta_2 \right) \quad (90)$$

Pinion offset angle in pitch plane, ζ_{mp}

$$\zeta_{mp} = \arcsin \left(\frac{\sin \zeta_m \sin \Sigma}{\cos \delta_1} \right) \quad (91)$$

Mean normal module, m_{mn}

$$m_{mn} = \frac{\cos\beta_{m2} d_{m2}}{z_2} \quad (92)$$

Spiral angle, β_{m1} , pinion

$$\beta_{m1} = \beta_{m2} + \zeta_{mp} \quad (93)$$

Hypoid dimension factor, F

$$F = \frac{\cos\beta_{m2}}{\cos\beta_{m1}} \quad (94)$$

Pinion mean pitch diameter, d_{m1}

$$d_{m1} = \frac{d_{m2}}{u} F \quad (95)$$

Mean cone distance, R_{m1} , pinion

$$R_{m1} = \frac{d_{m1}}{2\sin\delta_1} \quad (96)$$

Mean cone distance, R_{m2} , wheel

$$R_{m2} = \frac{d_{m2}}{2\sin\delta_2} \quad (97)$$

Lead angle of cutter, ν

$$\nu = \arcsin\left(\frac{z_0 m_{mn}}{2r_{c0}}\right) \quad (98)$$

Auxiliary angle, ϑ_m

$$\vartheta_m = \arctan(\sin\delta_2 \tan\zeta_m) \quad (99)$$

Intermediate variable, A_3

$$A_3 = r_{c0} \cos^2(\beta_{m2} - \nu) \quad (100)$$

Intermediate variable, A_4

$$A_4 = R_{m2} \cos(\beta_{m2} + \vartheta_m) \cos\beta_{m2} \quad (101)$$

Intermediate variable, A_5

$$A_5 = \sin\zeta_{mp} \cos\vartheta_m \cos\nu \quad (102)$$

Intermediate variable, A_6

$$A_6 = R_{m2} \cos\beta_{m2} + r_{c0} \sin\nu \quad (103)$$

Intermediate variable, A_7

$$A_7 = \cos\beta_{m1} \cos(\beta_{m2} + \vartheta_m) - \frac{\sin(\beta_{m2} + \vartheta_m - \nu) \sin\zeta_{mp}}{\cos(\beta_{m2} - \nu)} \quad (104)$$

Intermediate variable, $R_{m \text{ int}}$

$$R_{m \text{ int}} = \frac{A_3 A_4}{A_5 A_6 + A_3 A_7} \quad (105)$$

Check:

$$\left| R_{m \text{ int}} - R_{m1} \right| < 0,000 \, 1 R_{m1} \quad (106)$$

Pinion pitch angle, δ_1

$$\delta_1 = \arcsin\left(\frac{d_{m1}}{2R_{m \text{ int}}}\right) \quad (107)$$

Wheel pitch angle, δ_2

$$\delta_2 = \arccos\left(\frac{\sin\delta_1 \cos\zeta_m \sin\Sigma + \cos\delta_1 \cos\zeta_{mp} \cos\Sigma}{1 - \sin^2\Sigma \sin^2\zeta_m}\right) \quad (108)$$

Repeat calculation from [Formula \(88\)](#) to [Formula \(108\)](#) until [Formula \(106\)](#) is true.

End of iteration

Face width factor, c_{be2}

$$c_{be2} = 0,5 \quad (109)$$

7 Gear dimensions

7.1 Initial data for tooth profile parameters

For the calculation of the gear dimensions, initial data for tooth profile parameters shown in [Table 3](#) are required in addition to the initial data for pitch cone parameters shown in [Table 2](#). The data in [Table 3](#) are defined at the calculation point of the wheel. Bevel and hypoid gear data may be supplied in either of the two commonly used forms: data type I or data type II; see [Table 3](#) and [Annex A](#).

Table 3 — Initial data for tooth profile parameters

Data type I		Data type II	
Symbol	Description	Symbol	Description
α_{dD}	nominal design pressure angle – drive side ^a		
α_{dC}	nominal design pressure angle – coast side ^a		
f_{alim}	influence factor of limit pressure angle ^a		
x_{hm1}	profile shift coefficient	c_{ham}	mean addendum factor of wheel
k_{hap}	basic crown gear addendum factor	k_d	depth factor
k_{hfp}	basic crown gear dedendum factor	k_c	clearance factor
x_{smn}	thickness modification coefficient	k_t	thickness factor or
		W_{m2}	wheel mean slot width
$j_{mn}, j_{mt2},$ j_{en}, j_{et2}	backlash (choice of four)		
θ_{a2}	addendum angle of wheel		
θ_{f2}	dedendum angle of wheel		

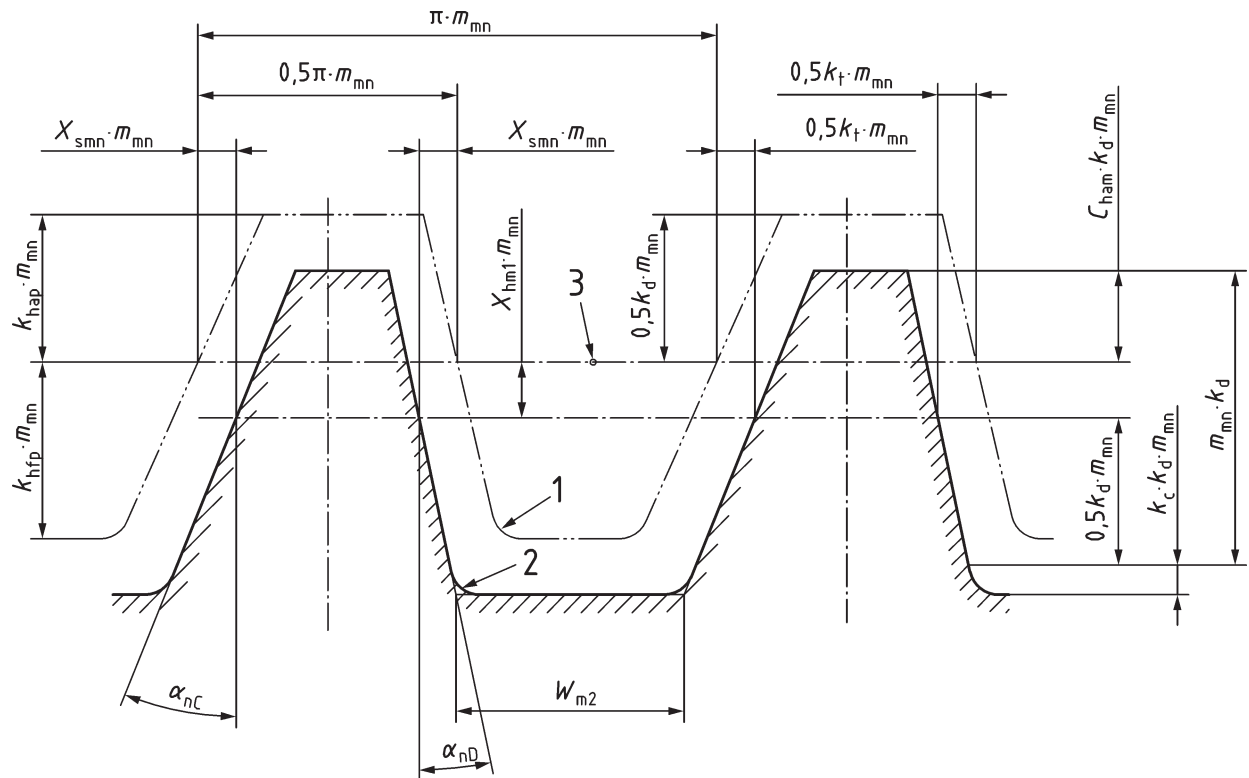
^a Generally, drive and coast side pressure angles are balanced in initial design. However, some applications may be optimized with unbalanced pressure angles; see [Annex C](#) for guidance.

[Annex C](#) gives suggestions for the data in [Table 3](#).

Data type II can be directly transformed into data type I and vice versa. [Table 4](#) shows the appropriate formulae.

Table 4 — Conversions between data type I and data type II

Conversion of data type II to data type I	Conversion of data type I to data type II
$x_{hm1} = k_d \left(\frac{1}{2} - c_{ham} \right)$	$c_{ham} = \frac{1}{2} \left(1 - \frac{x_{hm1}}{k_{hap}} \right)$
$k_{hap} = \frac{k_d}{2}$	$k_d = 2 k_{hap}$
$k_{hfp} = k_d \left(k_c + \frac{1}{2} \right)$	$k_c = \frac{1}{2} \left(\frac{k_{hfp}}{k_{hap}} - 1 \right)$
$x_{smn} = \frac{k_t}{2} = \frac{1}{2} \left[\frac{W_{m2}}{m_{mn}} + k_d \left(k_c + \frac{1}{2} \right) \left(\tan \alpha_{nD} + \tan \alpha_{nC} \right) - \frac{\pi}{2} \right]$	$k_t = 2 x_{smn}$



Key

- 1 basic rack tooth profile
- 2 tooth profile with profile shift and thickness modification
- 3 reference line

Figure 16 — Basic rack tooth profile of wheel at calculation point

7.2 Determination of basic data

Pinion mean pitch diameter, d_{m1}

$$d_{m1} = 2R_{m1} \sin \delta_1 \quad (110)$$

Wheel mean pitch diameter, d_{m2}

$$d_{m2} = 2R_{m2} \sin \delta_2 \quad (111)$$

Shaft angle departure from 90° , $\Delta\Sigma$

$$\Delta\Sigma = \Sigma - 90^\circ \quad (112)$$

Offset angle in the pinion axial plane, ζ_m

$$\zeta_m = \arcsin \left(\frac{2a}{d_{m2} + d_{m1} \frac{\cos \delta_2}{\cos \delta_1}} \right) \quad (113)$$

Offset angle in the pitch plane, ζ_{mp}

$$\zeta_{mp} = \arcsin \left(\frac{\sin \zeta_m \sin \Sigma}{\cos \delta_1} \right) \quad (114)$$

Offset in pitch plane, a_p

$$a_p = R_{m2} \sin \zeta_{mp} \quad (115)$$

Mean normal module, m_{mn}

$$m_{mn} = \frac{2R_{m2} \sin \delta_2 \cos \beta_{m2}}{z_2} \quad (116)$$

Limit pressure angle, α_{lim}

$$\alpha_{lim} = -\arctan \left[\frac{\tan \delta_1 \tan \delta_2}{\cos \zeta_{mp}} \left(\frac{R_{m1} \sin \beta_{m1} - R_{m2} \sin \beta_{m2}}{R_{m1} \tan \delta_1 + R_{m2} \tan \delta_2} \right) \right] \quad (117)$$

Generated normal pressure angle, α_{nD} , drive side

$$\alpha_{nD} = \alpha_{dD} + f_{\alpha_{lim}} \alpha_{lim} \quad (118)$$

Generated normal pressure angle, α_{nC} , coast side

$$\alpha_{nC} = \alpha_{dC} - f_{\alpha_{lim}} \alpha_{lim} \quad (119)$$

Effective pressure angle, α_{eD} , drive side

$$\alpha_{eD} = \alpha_{nD} - \alpha_{lim} \quad (120)$$

Effective pressure angle, α_{eC} , coast side

$$\alpha_{eC} = \alpha_{nC} + \alpha_{\text{lim}} \quad (121)$$

Outer pitch cone distance, R_{e2} , wheel

$$R_{e2} = R_{m2} + c_{be2} b_2 \quad (122)$$

Inner pitch cone distance, R_{i2} , wheel

$$R_{i2} = R_{e2} - b_2 \quad (123)$$

Outer pitch diameter, d_{e2} , wheel

$$d_{e2} = 2R_{e2}\sin\delta_2 \quad (124)$$

$$d_{i2} = 2R_{i2}\sin\delta_2 \quad (125)$$

Outer transverse module, m_{et2}

$$m_{et2} = \frac{d_{e2}}{z_2} \quad (126)$$

Wheel face width from calculation point to outside, b_{e2}

$$b_{e2} = R_{e2} - R_{m2} \quad (127)$$

Wheel face width from calculation point to inside, b_{i2}

$$b_{i2} = R_{m2} - R_{i2} \quad (128)$$

Crossing point to calculation point along wheel axis, t_{zm2}

$$t_{zm2} = \frac{d_{m1}\sin\delta_2}{2\cos\delta_1} - 0,5\cos\zeta_m \tan\Delta\Sigma \left(d_{m2} + \frac{d_{m1}\cos\delta_2}{\cos\delta_1} \right) \quad (129)$$

Crossing point to calculation point along pinion axis, t_{zm1}

$$t_{zm1} = \frac{d_{m2}}{2} \cos\zeta_m \cos\Delta\Sigma - t_{zm2}\sin\Delta\Sigma \quad (130)$$

Pitch apex beyond crossing point along axis, $t_{z1,2}$

$$t_{z1,2} = R_{m1,2}\cos\delta_{1,2} - t_{zm1,2} \quad (131)$$