



BSI Standards Publication

Sampling procedures for inspection by variables

Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

National foreword

This British Standard is the UK implementation of ISO 3951-1:2013.

The UK participation in its preparation was entrusted to Technical Committee SS/5, Acceptance sampling schemes.

A list of organizations represented on this committee can be obtained on request to its secretary.

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Sampling procedures for inspection by variables —

Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

Règles d'échantillonnage pour les contrôles par mesures —

*Partie 1: Spécification pour les plans d'échantillonnage simples
indexés d'après une limite de qualité acceptable (LQA) pour un
contrôle lot par lot pour une caractéristique-qualité unique et
une LQA unique*





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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 69, *Application of statistical methods*, SC 5, *Acceptance sampling*.

This second edition cancels and replaces the first edition (ISO 3951-1:2006), of which it constitutes a minor revision with the following changes:

- procedures have been introduced to accommodate measurement uncertainty;
- many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in [ISO 2859-1](#).

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

- *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*
- *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*
- *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*
- *Part 4: Procedures for assessment of declared quality levels*
- *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

This corrected version of ISO 3951-1:2013 incorporates the following corrections:

- [Clause 4](#), process mean (6th line): the symbol " m " has been replaced with " μ ", and "process mean" has been replaced with "unknown process mean";
- [16.2](#), before Example 1: " $Q_U \leq k$ " and " $Q_L \leq k$ " have been replaced with " $Q_U < k$ " and " $Q_L < k$ ";

- [16.4.2, Formula \(4\)](#): " $\left[\sqrt{(1 - Q\sqrt{3} / 2) / 2} \right]$ " has been replaced with " $\left[\sqrt{(1 - Q\sqrt{3} / 2) / 2} \right]$ ";
- [17.2](#), Example: "it is seen that for an AQL of 1,0 %" has been replaced with "it is seen that for an AQL of 0,65 %";
- [N.2, Formula \(N.2\)](#): " $\left[(1 - Q\sqrt{3} / 2) 2 \right]$ " has been replaced with " $\left[(1 - Q\sqrt{3} / 2) / 2 \right]$ ";
- [O.4, Formula \(O.6\)](#): in the fourth line, the question mark has been replaced with a "0", thus correcting a typographic error.

Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL) and is designed for users who have simple requirements. (A more comprehensive and technical treatment is given in [ISO 3951-2](#).) This part of ISO 3951 is complementary to [ISO 2859-1](#).

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

- a) an automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;
- b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is intended for application to a single quality characteristic that is measurable on a continuous scale. For two or more such quality characteristics, see [ISO 3951-2](#).

It is assumed in the body of this part of ISO 3951 that measurement error is negligible (see ISO 10576-1:2003). For information on allowing for measurement error, see Annex O, which was derived from Reference [20] in the Bibliography.

For double specification limits, this part of ISO 3951 treats combined control. For other types of control, refer to [ISO 3951-2](#).

CAUTION — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened for nonconforming items.

Inspection by variables for percent nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

- unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;
- a single specification limit, or combined control of double specification limits;
- normal inspection, tightened inspection, or reduced inspection.

[Table 1](#) is intended to facilitate the use of this part of ISO 3951 by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. The table only deals with [Clauses 15](#), [16](#), [20](#), [21](#), and [22](#); in every case, it is necessary, first of all, to have read the other clauses.

Table 1 — Summary table

Inspection	Single specification limit						Double specification limits with combined control					
	s-method			σ -method			s-method			σ -method		
	Claus- es or sub-claus- es	Tables/ Annexes	Charts	Claus- es or sub-claus- es	Tables/ Annexes	Charts	Claus- es or sub-claus- es	Tables/ Annexes	Charts	Claus- es or sub-claus- es	Tables/ Annexes	Charts
Normal inspection	16.1 , 16.2 , 16.3 , 21.1	A.1 , B.1 , B to R	B to R	17.1 , 17.2 , 21.1	A.1 , C.1 , B to R ^a	B to R ^a	16.1 , 16.4 , 21.1	A.1 , D.1 , E.1 (for $n = 3$), G.1 (for $n = 3$ or 4), B to R ^a	s-D to s-R, B to R ^a	17.1 , 17.3 and 21.1	A.1 , C.1 , E.1 , B to R ^a	B to R ^a
Switching between normal and tightened inspection	21.2 , 21.3	B.1 , B.2	B to R	21.2 , 21.3	C.1 , C.2	B to R ^a	21.2 , 21.3	D.1 , D.2	s-D to s-R, B to R ^a	21.2 , 21.3	C.1 , C.2 , E.1	B to R ^a
Switching between normal and reduced inspection	21.4 , 21.5	B.1 , B.3	B to R	21.4 , 21.5	C.1 , C.3 , I	B to R ^a	21.4 , 21.5	D.1 , D.3 , G.1 (for $n = 3$ or 4)	s-D to s-R, B to R ^a	21.4 , 21.5	C.1 , C.3 , E.1	B to R ^a
Switching between tightened and dis- continued inspection	22	B.2	B to R	22	C.2	B to R ^a	22	D.2	s-D to s-R, B to R ^a	22	E.1	B to R ^a
Switching between the s-method and σ -meth- od	23	Annex J		23	Annex J		23	Annex J		23	Annex E, Annex J	
^a But see 8.4 .												

Fifteen annexes are provided. Annexes A to I provide the tables needed to support the procedures. Annex J indicates how the sample standard deviation, s , and the presumed known value of the process standard deviation, σ , should be determined. Annex K provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection as well as for the s -method and σ -method. Annex L provides similar information for the producer's risks. Annex M gives the general formula for the operating characteristic of the σ -method. Annex N provides the statistical theory underlying the estimation of the process fraction nonconforming under the s -method for sample sizes 3 and 4, which, for technical reasons, are treated differently from the other sample sizes in this part of ISO 3951. Annex O provides procedures for accommodating measurement uncertainty.

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Sampling procedures for inspection by variables —

Part 1:

Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

1 Scope

This part of ISO 3951 is primarily designed for use under the following conditions:

- a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process;
- b) where only a single quality characteristic, x , of these products is taken into consideration, which must be measurable on a continuous scale;
- c) where production is stable (under statistical control) and the quality characteristic, x , is distributed according to a normal distribution or a close approximation to the normal distribution;
- d) where a contract or standard defines a lower specification limit, L , an upper specification limit, U , or both; an item is qualified as conforming if and only if its measured quality characteristic, x , satisfies the appropriate one of the following inequalities:
 - 1) $x \geq L$ (i.e. the lower specification limit is not violated);
 - 2) $x \leq U$ (i.e. the upper specification limit is not violated);
 - 3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are called cases with a single specification limit and 3), a case with double specification limits.

Where double specification limits apply, it is assumed in this part of ISO 3951 that conformance to both specification limits is equally important to the integrity of the product. In such cases, it is appropriate to apply a single AQL to the combined percentage of a product outside the two specification limits. This is referred to as combined control.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

[ISO 2859-1](#), *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

[ISO 2859-2](#), *Sampling procedures for inspection by attributes — Part 2: Sampling plans indexed by limiting quality (LQ) for isolated lot inspection*

[ISO 3534-1](#), *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

[ISO 3534-2](#), *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

[ISO 3951-2](#), *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in [ISO 2859-1](#), [ISO 3534-1](#), and [ISO 3534-2](#) and the following apply.

3.1

inspection by variables

inspection by measuring the magnitude of a characteristic of an item

[SOURCE: ISO 3534-2]

3.2

sampling inspection

inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2]

3.3

acceptance sampling inspection

acceptance sampling

sampling inspection ([3.2](#)) to determine whether or not to accept a lot or other amount of product, material, or service

[SOURCE: ISO 3534-2]

3.4

acceptance sampling inspection by variables

acceptance sampling inspection ([3.3](#)) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

3.5

process fraction nonconforming

rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

3.6

acceptance quality limit

AQL

worst tolerable *process fraction nonconforming* ([3.5](#)) when a continuing series of lots is submitted for *acceptance sampling* ([3.3](#))

Note 1 to entry: See [Clause 5](#).

3.7

quality level

quality expressed as a rate of occurrence of nonconforming units

3.8

limiting quality

LQ

quality level ([3.7](#)), when a lot is considered in isolation, which, for the purposes of *acceptance sampling inspection* ([3.3](#)), is limited to a low probability of acceptance

[SOURCE: ISO 3534-2]

Note 1 to entry: See [14.1](#).

Note 2 to entry: In this part of ISO 3951: 10 %.

3.9

nonconformity

non-fulfilment of a requirement

3.10

nonconforming unit

unit with one or more nonconformities

[SOURCE: ISO 3534-2]

3.11

s-method acceptance sampling plan

acceptance sampling (3.3) plan by variables using the sample standard deviation

[SOURCE: ISO 3534-2]

Note 1 to entry: See [Clause 16](#).

3.12

σ -method acceptance sampling plan

acceptance sampling (3.3) plan by variables using the presumed value of the process standard deviation

[SOURCE: ISO 3534-2]

Note 1 to entry: See [Clause 17](#).

3.13

specification limit

conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2]

3.14

lower specification limit

L

specification limit (3.13) that defines the lower conformance boundary

[SOURCE: ISO 3534-2]

3.15

upper specification limit

U

specification limit (3.13) that defines the upper conformance boundary

[SOURCE: ISO 3534-2]

3.16

combined control

requirement when both upper and lower limits are specified for the quality characteristic and an *AQL* (3.6) that applies to the combined percent nonconforming beyond the two limits is given

Note 1 to entry: See [5.3](#).

Note 2 to entry: The use of combined control implies that nonconformity beyond either *specification limit* (3.13) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.17
acceptability constant
 k

constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2]

Note 1 to entry: See 16.2 and 17.2.

3.18
quality statistic
 Q

function of the *specification limit* (3.13), the sample mean, and the sample or process standard deviation used in assessing the acceptability of a lot

[SOURCE: ISO 3534-2]

Note 1 to entry: For the case of a single *specification limit* (3.13), the lot may be sentenced on the result of comparing Q with the *acceptability constant* (3.17) k .

Note 2 to entry: See 16.2 and 17.2.

3.19
lower quality statistic
 Q_L

function of the *lower specification limit* (3.14), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *lower specification limit* (3.14), the lot is sentenced on the result of comparing Q_L with the *acceptability constant* (3.17) k .

[SOURCE: ISO 3534-2]

Note 2 to entry: See Clause 4, 16.2, and 17.2.

3.20
upper quality statistic
 Q_U

function of the *upper specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *upper specification limit* (3.15), the lot is sentenced on the result of comparing Q_U with the *acceptability constant* (3.17) k .

[SOURCE: ISO 3534-2]

Note 2 to entry: See Clause 4, 16.2, and 17.2.

3.21
maximum sample standard deviation
MSSD

$S_{\max}$
largest sample standard deviation for a given sample size code letter, inspection severity, and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criteria for the combined control of double *specification limits* (3.13) when the process variability is unknown

Note 1 to entry: See 16.4.

3.22 maximum process standard deviation MPSD

$\sigma_{\max}$
largest process standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double specification limits with a combined *AQL* (3.6) requirement under tightened inspection with known process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See 17.3.

3.23 switching rule

instruction within an *acceptance sampling* (3.3) scheme for changing from one *acceptance sampling* (3.3) plan to another of greater or lesser severity based on demonstrated quality history

[SOURCE: ISO 3534-2]

Note 1 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of 'severity'.

Note 2 to entry: See Clause 21.

3.24 measurement

set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2]

4 Symbols

The symbols used are as follows:

c_U	factor for determining the upper control limit for the sample standard deviation (See Annex H.)
f_s	factor that relates the maximum sample standard deviation to the difference between U and L (See Annex D)
f_σ	factor that relates the maximum process standard deviation under tightened inspection to the difference between U and L (See Annex E)
k	Form k acceptability constant for use with a single quality characteristic and a single specification limit (See Annex B for the s -method or Annex C for the σ -method)
L	lower specification limit (As a subscript to a variable, it denotes its value at L .)
μ	unknown process mean
N	lot size (number of items in a lot)
n	sample size (number of items in a sample)
$\hat{p}$	estimate of the process fraction nonconforming
$\hat{p}_L$	estimate of the process fraction nonconforming below the lower specification limit
$\hat{p}_U$	estimate of the process fraction nonconforming above the upper specification limit

p^* maximum acceptable value for the estimate of the process fraction nonconforming

P_a probability of acceptance

Q quality statistic

Q_L lower quality statistic

NOTE Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.

Q_U upper quality statistic

NOTE Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.

s sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}}$$

(See Annex J.)

$s_{\max}$ maximum sample standard deviation (MSSD)

σ standard deviation of a process that is under statistical control

NOTE σ^2 , the square of the process standard deviation, is known as the process variance.

$\sigma_{\max}$ maximum process standard deviation (MPSD)

U upper specification limit (As a suffix to a variable, it denotes its value at U .)

x_j measured value of the quality characteristic for the j^{th} item of the sample

$\bar{x}$ the arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$$

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the AQL may be accepted with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep the process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action

is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification or in the contract, or by the responsible authority. Where both upper and lower specification limits are given, this part of ISO 3951 addresses only the case of an overall AQL applying to the combined percent nonconforming beyond the two limits; this is known as “combined control.” (See [ISO 3951-2](#) for “separate” and “complex” control of double specification limits.)

5.4 Preferred AQLs

The 16 AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. They are only preferred in the sense that they are the AQL values used in the tabulations and charts. It follows that, if for any product or service, an AQL other than a preferred AQL is designated, then this part of ISO 3951 is not applicable. (See [14.2](#).)

5.5 Caution

From the definition of the AQL in [5.1](#), it follows that the desired protection can only be ensured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming product.

6 Switching rules for normal, tightened, and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral and, therefore, obligatory procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see [20.1](#)) that the variability is in statistical control, consideration should be given to switching to the σ -method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see [Clause 23](#)).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in [Clauses 21](#), [22](#), and [23](#).

7 Relation to ISO 2859-1

7.1 Similarities

The similarities are as follows.

- a) This part of ISO 3951 is complementary to [ISO 2859-1](#); the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.
- b) Both use the AQL to index the sampling plans, and the preferred values used in this part of ISO 3951 are identical with those given for percent nonconforming in [ISO 2859-1](#) (i.e. from 0,01 % to 10 %).
- c) In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. General tables give the sample size to be taken and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the s -method and σ -method, and for normal, tightened, and reduced inspection.
- d) The switching rules are essentially equivalent.

7.2 Differences

- a) **Determination of acceptability.** Acceptability for an [ISO 2859-1](#) attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated value of the process mean from the specification limit(s) in terms of the estimated or presumed value of the process standard deviation. In this part of ISO 3951, two methods are considered: the s -method, for use when the process standard deviation, s , is unknown, and the σ -method, for use when s is presumed to be known. In the case of a single specification limit, the acceptability may be calculated from a formula (see [16.2](#) and [17.2](#)), but for the s -method, it is also easily established by a graphical method (see [16.3](#)). In the case of combined control of double specification limits under the s -method, this part of ISO 3951 provides only for a graphical method of determining acceptability (see [16.4](#)); for combined control of double specification limits under the σ -method, a numerical method is given.
- b) **Normality.** In [ISO 2859-1](#), there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements be distributed according to a normal distribution or a close approximation to a normal distribution.
- c) **Operating characteristic curves (OC curves).** The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in [ISO 2859-1](#). The curves for unknown process standard deviation have been matched by minimizing the area between the curves representing the squares of the OC values, a method that gives greater emphasis to the match at the top of the OC curves. In most cases, the resulting match between the OC curves is so close that for most practical purposes, the attributes and variables OC curves may be considered to be identical. The plans for known process standard deviation were derived by minimizing the area between the squared OC functions subject to keeping the same form p^* acceptability constant as for the corresponding case for unknown process standard deviation, i.e. only the sample size was open to choice, so the match was, in general, less perfect.
- d) **Producer's risk.** For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in [ISO 2859-1](#).

NOTE The producer's risks of the plans are given in Annex L.

- e) **Sample sizes.** The variables sample sizes for given combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes. This is particularly true

for the σ -method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.

- f) **Double sampling plans.** Double sampling plans by variables are presented separately, in ISO 3951-3.
- g) **Multiple sampling plans.** No multiple sampling plans by variables are given in this part of ISO 3951.
- h) **Average outgoing quality limit (AOQL).** The AOQL concept is mainly of value when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

8 Consumer protection

8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal, and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Some users may select specific individual plans from this part of ISO 3951 and use them without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as “inspection in compliance with ISO 3951-1.” When used in such a way, ISO 3951-1 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

8.2 Consumer’s risk quality (CRQ) tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give a consumer’s risk quality (CRQ) not worse than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer’s risk quality and a consumer’s risk to be associated with it. Annex K gives values of consumer’s risk quality for the s -method and σ -method corresponding to a consumer’s risk of 10 %.

However, application of this part of ISO 3951 to isolated lots is deprecated, as the theory of sampling by variables applies to a process. For isolated lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from [ISO 2859-2](#). (See also Reference [5] in the Bibliography.)

8.3 Producer’s risk tables

Annex L gives the probability of non-acceptance under the s -method and σ -method for lots produced when the process fraction nonconforming equals the AQL. This probability is called the producer’s risk.

8.4 Operating characteristic (OC) curves

The tables for consumer’s risk quality and producer’s risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any process quality level may, however, be judged from its operating characteristic curve. OC curves for the normal inspection s -method sampling plans of this part of ISO 3951 are given in Charts B to R, which should be consulted when choosing a sampling plan. Also given are tables of process qualities at nine standard probabilities of acceptance for all of the s -method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the s -method. Most of them also provide a good approximation to the σ -method and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the σ -method, refer to Annex M.

9 Allowing for measurement uncertainty

The master tables of this part of ISO 3951 are based on the assumption that the quality characteristic, X , of the items in the lots is normally distributed with unknown process mean, μ , and either known or unknown process standard deviation σ . The assumption is also made that X can be measured without measurement error, i.e. that measurement of an item with the true value, x_i , results in the value x_i . However, the master tables can also be used, with appropriate adjustments, in the presence of measurement error.

If the measurement standard deviation is no greater than 10 % of the process standard deviation, it can be ignored. For measurement standard deviation greater than 10 % of the process standard deviation, the sample size will need to be increased, although the acceptability constant remains the same. Moreover, if neither the measurement standard deviation nor the process standard deviation is known, more than one measurement will need to be made on each sampled item and the total variability of the measurements will need to be separated into the components due to the measurements and to the process.

Details are provided in Annex O.

10 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge both of statistics and the product to be inspected. [Clauses 11 to 13](#) of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

11 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

- a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time consuming and costly per item.
- b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely how good the product is. Earlier warning may, therefore, be given if the quality is slipping.
- c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in [16.4.2](#) and [16.4.4](#).)
- d) From a comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes, such as from [ISO 2859-1](#), and the standard plans in this part of ISO 3951, it will be seen that the smallest samples are generally required by the σ -method (used when the process standard deviation is presumed to be known). The sample sizes for the s -method (used when the process standard deviation is unknown) are also, in general, substantially smaller than for sampling by attributes.

- e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.
- f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.
- g) A variables scheme becomes relatively more complicated to operate as the number of measurements to be taken on each item increases. (For two or more quality characteristics, this part of ISO 3951 does not apply. See [ISO 3951-2](#) for details.)
- h) The use of this part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of the quality characteristic is normal. In case of doubt, the responsible authority should be consulted.

NOTE 1 Departure from normality may be caused by the presence of outliers, the assessment and accommodation of which is considered in [ISO 16269-4](#).

NOTE 2 ISO 5479 gives detailed procedures for tests for departure from normality.

12 Choice between the s -method and σ -method

If it is desired to apply inspection by variables, the next question is whether to use the s -method or the σ -method. The σ -method is the most economical in terms of sample size, but before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the s -method, but subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, whether it will be economical to change to the σ -method. The size of the sample will generally be smaller and the acceptability criterion simpler under the σ -method. (See [17.2](#)) On the other hand, it will still be necessary to calculate the sample standard deviation, s , for record purposes and to keep the control charts up to date. (See [Clause 20](#).) The calculation of s can appear daunting at first sight, but the difficulty is more apparent than real; this is especially true if a calculator or computer is available. Methods of determining s and σ are given in Annex J.

13 Choice of inspection level and AQL

For a standard sampling plan, the inspection level, in conjunction with the size of the lots and the AQL, determines the size of the sample to be taken and governs the severity of the inspection. The appropriate OC curve from Charts B to R or the appropriate table from Tables B to R shows the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

Three inspection levels, I, II, and III, are given in [Table 1](#) for general use. The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate. Level I may be used when less discrimination is needed or level III when greater discrimination is required. Four additional special levels, S-1, S-2, S-3, and S-4, are also given in [Table 1](#) and may be used when relatively small sample sizes are necessary and larger sampling risks can be tolerated.

14 Choice of sampling scheme

14.1 Standard plans

The standard procedure can be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the *s*-method, has been found in practice to produce workable sampling plans; but it rests on the assumption that the order of priority is first the AQL, second the sample size, and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see [Clauses 21](#), [22](#), and [23](#)), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE It should also be remembered that the limiting quality is the quality which, if offered for inspection, would have a 10 % probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

If, in certain circumstances, the limiting quality has a higher priority than the sample size, a suitable plan in this part of ISO 3951 may be selected by using Chart A. Construct a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample size code letter of a standard normal inspection plan that meets the specified requirements. This should be verified by inspecting the OC curve from among Charts B to R relating to this code letter and AQL.

However, use of this method is deprecated for isolated lots or short series of lots. (See [8.2](#).)

EXAMPLE Suppose that an acceptable value for the limiting quality is 6,0 % nonconforming and that the desired quality with a 95 % probability of acceptance is 2,0 % nonconforming. A vertical line on Chart A at 6,0 % nonconforming and a horizontal line at 2,0 % nonconforming intersect just below the sloping line indexed by the letter L. Examining Chart L, it is seen that a plan with sample size code letter L and AQL 1,5 % meets the requirements.

If the horizontal and vertical lines intersect at a point above the line marked R in Chart A, this implies that the specification cannot be met by any of the plans in this part of ISO 3951.

14.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

15 Preliminary operations

Before starting inspection by variables:

- a) check that production is considered to be continuing and that the distribution of the quality characteristic can be considered to be normal;

NOTE 1 For tests for departure from normality, see, for example, ISO 16269-3.

NOTE 2 If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

- b) check whether the *s*-method is to be used initially or whether the standard deviation is stable and known, in which case the σ -method should be used;
- c) check that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

- d) check, for a quality characteristic with double specification limits, that nonconformities beyond each limit are of equal importance. If this is not the case, refer to [ISO 3951-2](#);
- e) check that an AQL has been designated, and that it is one of the preferred AQLs for use with this part of ISO 3951. If it is not, then the tables are not applicable.

16 Standard procedures for the s-method

16.1 Obtaining a plan, sampling, and preliminary calculations

The procedure for obtaining and implementing a plan is as follows:

- a) With the inspection level given (normally this will be level II) and with the lot size, obtain the sample size code letter using [Table A.1](#).
- b) For a single specification limit, enter [Table B.1](#), [B.2](#), or [B.3](#) as appropriate with this code letter and the AQL and obtain the sample size, n , and the acceptability constant, k . For combined control of double specification limits when the sample size is 5 or more, find the appropriate acceptance curve from among Charts s-D to s-R.
- c) Take a random sample of size n , measure the characteristic x in each item and then calculate $\bar{x}$, the sample mean, and s , the sample standard deviation (see Annex J). If $\bar{x}$ lies outside the specification limit(s), the lot can be judged unacceptable without even calculating s . It is, however, necessary to calculate s for record purposes.

16.2 Acceptability criteria for single specification limits

If single specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s} \quad (1)$$

or

$$Q_L = \frac{\bar{x} - L}{s} \quad (2)$$

as appropriate, then compare the quality statistic (Q_U or Q_L) with the acceptability constant, k , obtained from [Table B.1](#), [B.2](#), or [B.3](#) for normal, tightened, or reduced inspection, respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if it is less, the lot is not acceptable.

Thus, if only the upper specification limit, U , is given, the lot is

acceptable if $Q_U \geq k$, and

not acceptable if $Q_U < k$,

or, if only the lower specification limit, L , is given, the lot is

acceptable if $Q_L \geq k$, and

not acceptable if $Q_L < k$.

EXAMPLE 1 Single upper specification limit.

The maximum temperature of operation for a certain device is specified as 60 °C and the operating temperature is known from previous experience to be normally distributed. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL of 2,5 % is to be used. From [Table A.1](#), the sample size code letter is found to be F; from [Table B.1](#), it is seen that a sample size of 13 is required and that the acceptability constant, k , is 1,426. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Compliance with the acceptability criterion is to be determined.

Information needed

Values obtained

Sample size: n

13

Sample mean: $\bar{x} = \sum x / n$

54,62 °C

Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$

3,330 °C

(See K.1.2)

Specification limit (upper): U

60 °C

Upper quality statistic: $Q_U = (U - \bar{x}) / s$

1,617

Form k acceptability constant: k (from [Table B.1](#))

1,426

Acceptability criterion: Is $Q_U \geq k$?

Yes (1,617 > 1,426)

The lot meets the acceptability criterion and is therefore acceptable.

EXAMPLE 2 Single, lower specification limit, requiring the use of an arrow in the master table.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. The process standard deviation is unknown. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. From [Table A.1](#), it is seen that the sample size code letter is J. However, on entering [Table B.1](#) with sample size code letter J and AQL of 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, i.e. sample size 28 and acceptability constant $k = 2,580$. A random sample of size 28 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95	6,04	6,68	6,63	6,65	6,52	6,59	6,40	6,44	6,34	6,04	6,15	6,29	6,63
6,44	7,15	6,70	6,59	6,51	6,80	5,94	6,35	7,17	6,83	6,25	6,96	7,00	6,38

Compliance with the acceptability criterion is to be determined.

Information needed

Values obtained

Sample size: n

28

Sample mean: $\bar{x} = \sum x / n$

6,551 s

Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$

0,325 1 s

(See K.1.2)

Lower specification limit: L

4,0 s

Lower quality statistic: $Q_L = (\bar{x} - L) / s$ 7,847

Form k acceptability constant: k (from [Table B.1](#)) 2,580

Acceptability criterion: Is $Q_L \geq k$? Yes ($7,847 > 2,580$)

The lot meets the acceptability criterion, so it is acceptable.

16.3 Graphical method for a single specification limit

When a graphical criterion is desired, draw the line

$$\bar{x} = U - ks \text{ (for an upper limit)}$$

or

$$\bar{x} = L + ks \text{ (for a lower limit)}$$

as appropriate on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. When the inspection concerns an upper specification limit, the accept zone is the zone below the line. When a lower specification limit is considered, the accept zone is the zone above the line. Plot the point $(s, \bar{x})$ on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

EXAMPLE Graphical method for upper specification limit.

Using the data given in EXAMPLE 1 of 15.2, mark the point $U = 60$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $-k$ [As $k = 1,426$, this means that the line passes through the points $(s = 1, \bar{x} = 58,574)$, $(s = 2, \bar{x} = 57,148)$, $(s = 3, \bar{x} = 55,722)$, etc.]. Select a suitable point and draw a straight line through both it and $(s = 0, \bar{x} = 60)$, i.e. $(0, U)$. The accept zone is then the area under this line.

The calculated values of s and $\bar{x}$ are 3,330 and 54,62. The point $(s, \bar{x})$ is seen from [Figure 1](#) to lie within the accept zone; the lot is therefore acceptable.

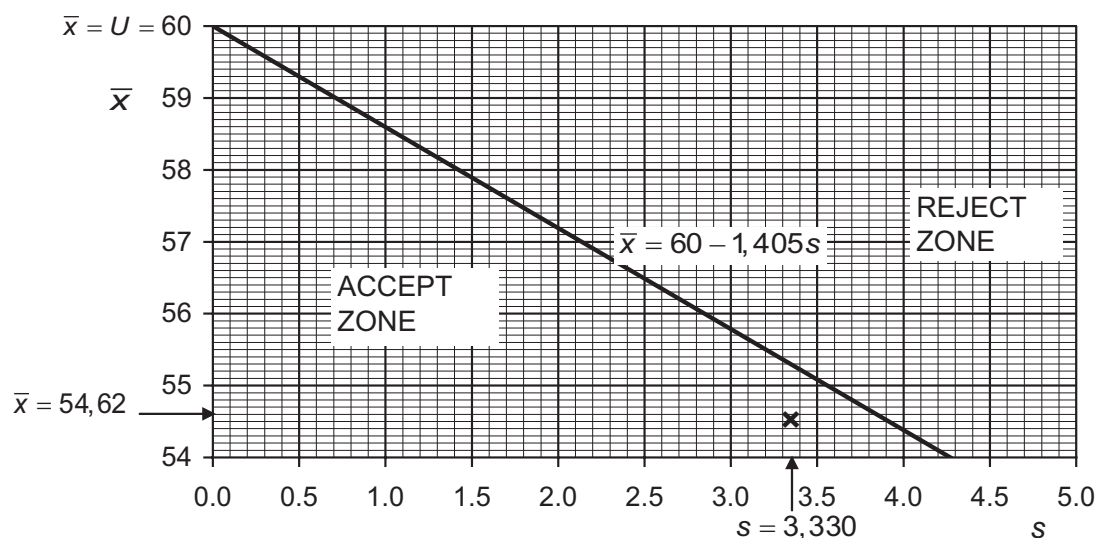


Figure 1 — Example of the use of an acceptance chart for a single specification limit, s —method

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot, the point $(s, \bar{x})$ may be plotted to determine whether or not the lot is acceptable.

16.4 Acceptability criterion for combined control of double specification limits

16.4.1 General

For the s -method with combined control of the upper and lower specification limits, i.e. with an overall AQL for the percentage of items from the process outside the specification limits, this part of ISO 3951 provides a graphical method for determining lot acceptability for all sample sizes except for sample sizes 3 and 4. (ISO 3951-2 provides numerical methods only.) The larger the sample variability, the less likely it is that the requirement is satisfied. If the value of s exceeds the value of the maximum sample standard deviation (MSSD) obtained from Table D.1, D.2, or D.3, no further calculation or reference to graphs is required, for the lot shall be immediately judged unacceptable.

Numerical methods are provided for the combined control of double specification limits for sample sizes 3 and 4.

16.4.2 Procedure for sample size 3

It may be seen from Annex B that there are three cases where the required sample size is 3 for the s -method, i.e. for sample size code letter B under normal inspection for an AQL of 4 %, for sample size code letter B under tightened inspection for an AQL of 6,5 %, and for sample size code letter range B to D under reduced inspection for an AQL of 1,5 %.

For sample size 3, proceed as follows. After calculating the sample mean, $\bar{x}$, and the sample standard deviation, s , find the applicable value of the coefficient f_s from Table D.1, D.2, or D.3. Determine the MSSD (i.e. the maximum allowable) from Formula (3).

$$\text{MSSD} = s_{\max} = (U - L)f_s \quad (3)$$

Then compare s with $s_{\max}$. If s is greater than $s_{\max}$, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x}) / s$ and $Q_L = (\bar{x} - L) / s$. Multiply Q_U and Q_L by $\sqrt{3}/2$ (i.e. approximately 0,866) and use Table F.1 to determine the estimates $\hat{p}_U$ and $\hat{p}_L$ of the fraction of items in the process that are nonconforming beyond the upper and lower limits, respectively.

NOTE 1 Negative values of Q correspond to estimates of the process fraction nonconforming in excess of 0,5 at that specification limit and will consequently always result in lot non-acceptance under the provisions of this part of ISO 3951. However, in order to obtain a numerical value for record-keeping purposes, the estimate of the process fraction nonconforming may be obtained by entering Table F.1 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1,0. E.g. if $Q_U = -0,156$, then $\sqrt{3}Q_U/2 = -0,135$; entering Table F.1 with 0,135 gives an estimate of 0,456 9; subtracting this from 1,0 gives $\hat{p}_U = 0,543 1$.

NOTE 2 The basis of Table F.1 is given in Annex K. Instead of using Table F.1, the estimate of the process fraction nonconforming beyond each specification limit when $n = 3$ may be calculated directly as

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2 / \sqrt{3} \\ \frac{2}{\pi} \arcsin \left[\sqrt{(1 - Q\sqrt{3}/2) / 2} \right] & \text{if } -2 / \sqrt{3} \leq Q \leq 2 / \sqrt{3} \\ 1 & \text{if } Q < -2 / \sqrt{3} \end{cases} \quad (4)$$

These two estimates must be added to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^* given in Table G.1, the lot is considered to be acceptable; otherwise, the lot is considered unacceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 3.

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so combined control of the double specification limits is appropriate. The specification limits are set at 10 m either side of the point of aim at a distance of 1 km, with an AQL of 4 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From [Table A.1](#), the sample size code letter is found to be B. From [Table B.1](#), it is seen that a sample of size 3 is required. Three torpedoes are tested, yielding errors -5,0 m, 6,7 m, and 8,8 m. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed**Value obtained**Sample size: n

3

Sample mean: $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$

3,5 m

Sample standard deviation: $s = \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2 / (n-1)}$

7,436 m

(See [I.1.2.](#))Value of f_s for MSSD ([Table D.1](#))

0,475

MSSD, $s_{\max} = (U - L)f_s = [10 - (-10)] \times 0,475$

9,50

Since $s = 7,436 < s_{\max} = 9,50$, the lot may be acceptable, so continue with the calculations. $Q_U = (U - \bar{x}) / s = (10 - 3,5) / 7,436$

0,874 1

 $Q_L = (\bar{x} - L) / s = (3,5 + 10) / 7,436$

1,815

 $\sqrt{3}Q_U / 2$

0,757

 $\sqrt{3}Q_L / 2$

1,572

 $\hat{p}_U$

0,226 7

 $\hat{p}_L$ (from [Table F.1](#))

0,000 0

 $\hat{p} = \hat{p}_U + \hat{p}_L$

0,2267

 p^* (from [Table G.1](#), normal inspection)

0,1924

Since $\hat{p} > p^*$ the lot is not acceptable.

NOTE This lot is not acceptable even though all inspected items in the sample are within the specification limits.

16.4.3 Procedure for sample size 4

For sample size 4 under the s -method, proceed as follows. After calculating the sample mean, $\bar{x}$, and the sample standard deviation, s , find the applicable value of the coefficient f_s from [Table D.1](#), [D.2](#), or [D.3](#). Determine the MSSD (i.e. the maximum allowable) from [Formula \(5\)](#)

$$\text{MSSD} = s_{\max} = (U - L)f_s \quad (5)$$

Then, compare s with the MSSD. If s exceeds the MSSD, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x}) / s$ and $Q_L = (\bar{x} - L) / s$. Calculate

$$\hat{p}_U = \begin{cases} 1 & \text{if } Q_U \leq -1,5 \\ 0,5 - Q_U / 3 & \text{if } -1,5 < Q_U < 1,5 \\ 0 & \text{if } Q_U \geq 1,5 \end{cases} \quad (6)$$

and

$$\hat{p}_L = \begin{cases} 1 & \text{if } Q_L \leq -1,5 \\ 0,5 - Q_L / 3 & \text{if } -1,5 < Q_L < 1,5 \\ 0 & \text{if } Q_L \geq 1,5 \end{cases} \quad (7)$$

Add these two estimates to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^* , given in [Table G.1](#), the lot is considered to be acceptable; otherwise, the lot is considered unacceptable.

NOTE The basis of [Formulae \(6\)](#) and [\(7\)](#) is given in Annex N.

EXAMPLE Items are being manufactured in lots of size 25. The lower and upper specification limits on their diameters are 82 mm to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming using an AQL of 2,5 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From [Table A.1](#), the sample size code letter is found to be C. From [Table B.1](#), it is seen that a sample of size 4 is required. The diameters of four items from the first lot are measured, yielding diameters of 82,4 mm, 82,2 mm, 83,1 mm, and 82,3 mm. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed

Value obtained

Sample size: n

4

Sample mean: $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$

82,50 mm

Sample standard deviation: $s = \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2 / (n - 1)}$

0,408 2 mm

(See Annex J, [J.1.2](#).)

Upper specification limit: U

84,0 mm

Lower specification limit: L

82,0 mm

Value of f_s for MSSD ([Table D.1](#))

0,365

$s_{\max} = (U - L)f_s = (84 - 82) \times 0,365$

0,730 mm

Since $s = 0,408\ 2 < s_{\max} = 0,730$, the lot may be acceptable, so continue with the calculations.

Further information needed	Value obtained
$Q_U = (U - \bar{x}) / s = (84 - 82,5) / 0,408\ 2$	3,674 7
$Q_L = (\bar{x} - L) / s = (82,5 - 82) / 0,408\ 2$	1,224 9
$\hat{p}_U$ [from Formula (6) above]	0,000 0
$\hat{p}_L$ [from Formula (7) above]	0,091 7
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,091 7
p^* (from Table G.1 , normal inspection)	0,086 0

Since $\hat{p} > p^*$, the lot is not acceptable.

16.4.4 Procedure for sample sizes greater than 4

After calculating the sample mean, $\bar{x}$, and the sample standard deviation, s , find the applicable value of the coefficient f_s from [Table D.1](#), [D.2](#), or [D.3](#). Determine the MSSD (i.e. the maximum allowable) from the formula

$$\text{MSSD} = s_{\max} = (U - L)f_s$$

Compare s with $s_{\max}$. If s is greater than $s_{\max}$, then the lot may be rejected without further calculation.

Otherwise, from among Charts s-D to s-R, consult the chart labelled with the appropriate sample size code letter and select the acceptance curve with the AQL specified for the two limits.

Calculate the values of $s / (U - L)$ and $(\bar{x} - L) / (U - L)$ and plot a point representing these values on a copy of the graph. If the point lies inside the curve, the lot is acceptable; if it lies outside, the lot is not acceptable.

It is recommended that before the inspection operations begin, the required acceptance curves for normal, tightened, and reduced inspection be copied. The scales should be adjusted so that s and $\bar{x}$ can be plotted directly (i.e. the upper specification limit is given instead of 1,0 and the lower specification limit instead of 0,0 on the vertical scale).

Plot a point on the chart representing the values of s and $\bar{x}$ found from the sample. If the point lies inside the curve, the lot is acceptable; if it lies outside, the lot is not acceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 5 or more.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 80 items. Inspection level II, normal inspection, with AQL of 1,5 %, is to be used. From [Table A.1](#), the sample size code letter is found to be E; from [Table B.1](#), it is seen that a sample of 13 is required, and from [Table D.1](#), that the value of f_s for the MSSD under normal inspection is 0,274. Suppose the measurements obtained are as follows: 63,5 °C; 61,9 °C; 65,2 °C; 61,7 °C; 68,4 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Sample size: n	13
Sample mean: $\bar{x} = \sum x / n$	64,223 °C

Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n - 1)}$ 2,789 9 °C

(See K.1.2)

Upper specification limit: U 70,0 °C

Lower specification limit: L 60,0 °C

Value of f_s for MSSD (Table D.1 for normal inspection) 0,274

MSSD = $s_{\max} = (U - L)f_s = (70 - 60) \times 0,274$ 2,74 °C

Since the value of s exceeds $s_{\max}$, the lot may immediately be adjudged unacceptable.

NOTE This lot is not acceptable even though all inspected items in the sample are within the specification limits.

Suppose that the AQL had been 2,5 % instead of 1,5 %. In this case, the value of f_s would be 0,285, so $s_{\max}$ is equal to $(70 - 60) \times 0,285 = 2,85$ °C. Since s is now less than $s_{\max}$, it is not possible to determine immediately whether or not the lot is acceptable.

The appropriate acceptance curve is taken from Chart s-E. If the scales have been adjusted to the real measurements, plot the point ($s = 2,79$, $\bar{x} = 64,22$), as on Figure 2. The point lies just outside the acceptance curve for an AQL of 2,5 % so the lot is not acceptable.

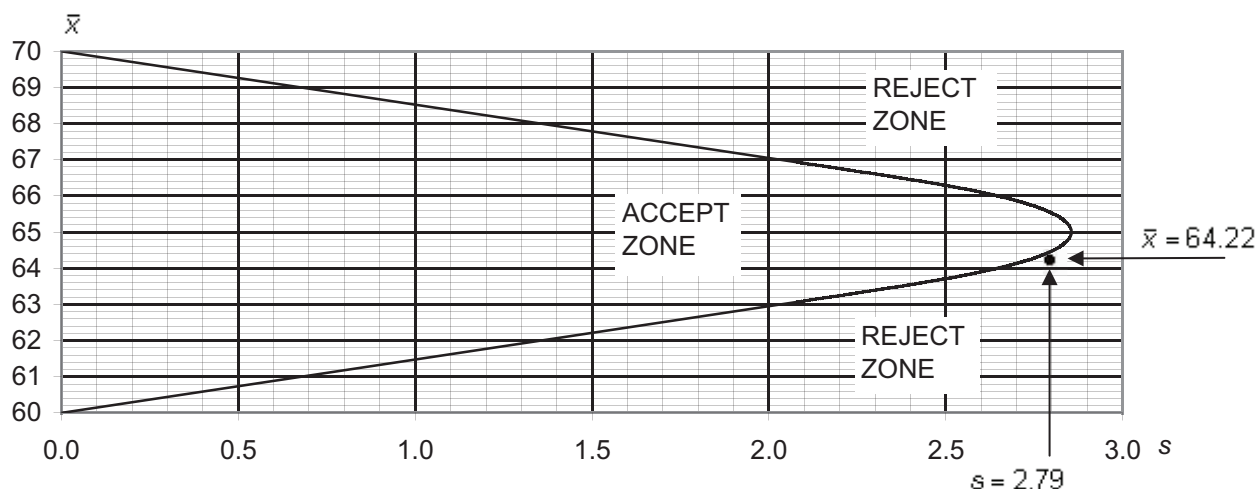


Figure 2 — Example of the use of an acceptance chart for combined control of double specification limits: s-method with actual scales

If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required:

- standardized sample mean: $(\bar{x} - L) / (U - L) = (64,22 - 60) / (70 - 60) = 0,422$;
- standardized sample standard deviation: $s / (U - L) = 2,79 / (70 - 60) = 0,279$.

The point (0,279, 0,422) is plotted on Figure 3. Since it lies outside the acceptance curve for AQL of 2,5 %, the lot is not acceptable.