



BSI Standards Publication

Sampling procedures for inspection by variables

Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

National foreword

This British Standard is the UK implementation of ISO 3951-2:2013. It supersedes BS 6002-2:2007, which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee SS/5, Acceptance sampling schemes.

A list of organizations represented on this committee can be obtained on request to its secretary.

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Sampling procedures for inspection by variables —

Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

Règles d'échantillonnage pour les contrôles par mesures —

*Partie 2: Spécification générale pour les plans d'échantillonnage
simples indexés d'après une limite de qualité acceptable (LQA) pour le
contrôle lot par lot de caractéristiques-qualité indépendantes*





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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2. www.iso.org/directives

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received. www.iso.org/patents

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

The committee responsible for this document is ISO/TC 69, *Application of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: http://www.iso.org/iso/home/standards_development/resources-for-technical-work/foreword.htm

This second edition cancels and replaces the first edition (ISO 3951-2:2006), of which it constitutes a minor revision with the following changes:

- procedures have been introduced to accommodate measurement uncertainty;
- many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in ISO 2859-1.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

- *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*
- *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*
- *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*
- *Part 4: Procedures for assessment of declared quality levels*
- *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

This corrected version of ISO 3951-2:2013 incorporates the following corrections:

- [Clause 1](#), letter f), points 2) and 3): “ $x \geq U$ ” has been replaced with “ $x \leq U$ ”;
- [Clause 4](#), process mean (6th line): “process mean” has been replaced with “unknown process mean”;

- [Clause 11](#), Note 1: reference to “ISO 16269-4” has been replaced with a reference to “ISO 5479”;
- [Clause 11](#), Note 2: reference to “Clause 2 of ISO 5725-2” has been replaced with a reference to “ISO 2854”;
- [16.3.2.5](#), under “Information needed”, last line: “(from Table G.1 as it is normal inspection)” has been replaced with “(from Table D.1, with 2,5 % AQL)”, and the value “0,115 4” has been replaced with “0,064 66”;
- [16.3.2.5](#), last line before the Note: “which is less than the acceptability constant p^* . The lot is therefore accepted.” has been replaced with “which is greater than the acceptability constant p^* . The lot is therefore not accepted.”;
- [18.2](#), second paragraph: “ $\bar{x}_U [=U - ks]$ ” has been replaced with “ $\bar{x}_U [=U - k\sigma]$ ”;
- [18.2](#), Example, second paragraph, fourth line: “, it is seen that for an AQL of 1,0 %,” has been replaced with “, it is seen that for an AQL of 0,65 %,”;
- [18.3](#), Example: the sample size has been corrected to be “19” instead of “20” (several occurrences) and the value 488 Ω for the resistance has been deleted; under “Further information needed” and “Alternative further information needed”, the values have been corrected accordingly;
- [20.2](#), paragraph below Table 4: reference to “Table G.1” has been replaced with a reference to “Table E.1”;
- [0.2](#): the acceptability constant k has been corrected from “1,962” to “1,963” and the example has been recalculated;
- [P.4](#), last line before Figure P.1: “As $\bar{x} = 12,990 > 12,975$,” has been replaced with “As $\bar{x} = 12,980 > 12,975$,”;
- Bibliography: ISO 2854 and ISO 5479 have been added.

Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL) and is of a technical nature, aimed at users who are already familiar with sampling by variables or who have complicated requirements. (A more introductory treatment is given in ISO 3951-1.)

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

- a) automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;
- b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is either implicitly or explicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is complementary to ISO 2859-1. When specified by the responsible authority, both this part of ISO 3951 and ISO 2859-1 may be referenced in a product specification, contract, inspection instructions, or other documents, and the provisions set forth therein govern. The responsible authority shall be designated in one of the above documents.

Caution — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

Inspection by variables for percent nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

- unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;
- a single specification limit, or double specification limits with combined, separate, or complex control;
- univariate or multivariate cases;
- three inspection severities, namely normal inspection, tightened inspection, or reduced inspection.

[Table 1](#) is intended to facilitate the use of this part of ISO 3951 by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. [Table 1](#) only deals with [Clauses 15, 16, 17, 18, 19, 23, 24, and 25](#); in every case, it is necessary first of all to have read all the preceding clauses.

Table 1 — Summary table

	Single specification limit				Double specification limits with combined control			
	s-method		σ -method		s-method		σ -method	
	Clauses or sub-clauses	Tables/Annexes	Clauses or sub-clauses	Tables/Annexes	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables/Annexes
Normal inspection	16.1, 16.2, 16.3, 17.1, 17.2, 20, 24.1	A.1, B.1	18.1, 18.2, 19, 20, 24.1	A.1, G.3	16.1, 16.3, 17.1, 17.2, 20, 24.1 Annex L	A.1, D.1, Annex F (for $n = 3$), G.1	18.1, 18.3, 19, 20, 24.1	A.1, C.1, E.1
Switching between normal and tightened inspection	24.2, 24.3	B.1, B.2	24.2, 24.3	C.1, C.2	24.2, 24.3	D.1, D.2, F.1, F.2	24.2, 24.3	E.1, G.1, G.2
Switching between normal and reduced inspection	24.4, 24.5	B.1, B.3, I.1	24.4, 24.5	C.1, I.1	24.4, 24.5	D.1, D.3, F.1, F.3, I.1	24.4, 24.5	E.1, G.1, G.3, I.1
Switching between tightened and discontinued inspection	22, 25	B.2	25	C.2	22, 25	D.2, E.2	25	E.1, G.2
Switching between the s-method and σ -method	26	I.1	26	K.2, I.1	26 L.2.1 L.3, L.4, L.5	I.1	26 L.2.2	K.2, I.1
Normal inspection	16.1, 17.1, 17.2, 20, 24.1, Annex L	A.1, D.1, Annex F (for $n = 3$), G.1	18.1, 18.2, 18.3, 19, 20, 24.1	Annex A, C.1, E.1	16.1, 16.3.4, 17.1, 17.2, 20, 24.1, Annex L	A.1, D.1, Annex F (for $n = 3$), G.1	18.1, 18.3, 19, 20, 24.1	A.1, C.1, E.1
Switching between normal and tightened inspection	24.2, 24.3	D.1, D.2, F.1, F.2	24.2, 24.3	E.1, E.2, G.2	24.2, 24.3	D.1, D.2, F.1, F.2	24.2, 24.3	E.1, E.2, G.3

	Single specification limit				Double specification limits with combined control			
	s-method		σ -method		s-method		σ -method	
	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables/ Annexes
Switching between normal and reduced inspection	24.4 , 24.5	D.1 , D.3 E.1 , E.3 I.1	24.4 , 24.5	E.1 , E.3 , G.2 , I.1	24.4 , 24.5	D.1 , D.3 E.1 , E.3 I.1	24.4 , 24.5	E.1 , E.3 , G.3 , I.1
Switching between tightened and dis-continued inspection	22 , 25	D.2 E.2	25	E.2 G.2	22 , 25	D.2 E.2	25	E.2 G.3
Switching between the s-method and σ -method	26 L.2.1 L.3, L.4, L.5	I.1	26 L.2.2	I.1 , K.2	26 L.2.1 L.3, L.4, L.5	I.1	26 L.2.2	I.1 , K.2

16 annexes are provided. [Annexes A](#) to [J](#) provide the tables needed to support the procedures. [Annex K](#) indicates how the sample standard deviation, s , and the presumed known value of the process standard deviation, σ , should be determined. [Annex L](#) provides formulae for the estimation of the process fraction nonconforming, together with a highly accurate approximation for use when the process standard deviation is unknown. [Annex M](#) provides formulae for the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection under the s -method and σ -method. [Annex N](#) provides similar information for the producer's risks. [Annex O](#) gives the general formula for the operating characteristic of the σ -method. [Annex P](#) provides procedures for accommodating measurement uncertainty.

Sampling procedures for inspection by variables —

Part 2:

General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

1 Scope

This part of ISO 3951 is primarily designed for use under the following conditions:

- a) where the inspection procedure is to be applied to a **continuing series of lots** of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 shall be applied to each one separately;
- b) where the **quality characteristics** of the items of product are **measurable on a continuous scale**;
- c) where the measurement error is negligible (i.e. with a standard deviation no more than 10 % of the corresponding process standard deviation). However, procedures are also provided in [Clause 9](#) and [Annex P](#) for accommodating measurement error when it has a non-negligible standard deviation;
- d) where production is stable (under statistical control) and the quality characteristics are distributed, at least to a close approximation, according to **normal distributions**;
- e) where, in the case of multiple quality characteristics, the characteristics are independent, or almost independent, of one another;
- f) where a contract or standard defines a **lower specification limit, L , an upper specification limit, U , or both** on each of the quality characteristics. If there is only one quality characteristic, an item is qualified as conforming if its measured quality characteristic x satisfies the appropriate one of the following inequalities:
 - 1) $x \geq L$ (i.e. the lower specification limit is not violated);
 - 2) $x \leq U$ (i.e. the upper specification limit is not violated);
 - 3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

If there are two or more, say m , quality characteristics, then, designating the lower and upper limits for the i^{th} quality characteristic by L_i and U_i respectively, an item of product is qualified as nonconforming if one or more of its m measured quality characteristics, x_i , fails to satisfy the appropriate one of the following inequalities:

- 4) $x_i \geq L_i$;
- 5) $x_i \leq U_i$;
- 6) $x_i \geq L_i$ and $x_i \leq U_i$.

Inequalities 1), 2), 4), and 5) are called cases with a **single specification limit** while 3) and 6) are called cases with **double specification limits**. For double specification limits, a further distinction is made between combined control, separate control, and complex control. If there is only one quality characteristic, then

— combined control is where a single AQL applies to nonconformity beyond both limits.

- separate control is where separate AQLs apply to nonconformity beyond each of the limits, and
- complex control is where one AQL applies to nonconformity beyond the limit that is of greater seriousness and a larger AQL applies to the total nonconformity beyond both limits.

If there are two or more quality characteristics, this generalizes as follows:

- combined control is where nonconformity beyond both limits on a variable belongs to the same class, to which a single AQL applies;
- separate control is where nonconformity beyond the two limits on a variable belongs to separate classes, to each of which a single AQL applies;
- complex control is where nonconformity beyond the limit that is of greater seriousness belongs to one class to which a single AQL applies, and the total nonconformity beyond both limits belongs to another class to which a larger AQL applies.

Note that, in the case of two or more quality characteristics, nonconformity on more than one quality characteristic may belong to the same class.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-1:2005, *Sampling procedures for inspection by variables — Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1, and ISO 3534-2 and the following apply.

3.1 inspection by variables

inspection by measuring the magnitude of a characteristic of an item

[SOURCE: ISO 3534-2]

3.2 sampling inspection

inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2]

3.3**acceptance sampling inspection****acceptance sampling**

sampling inspection (3.2) to determine whether or not to accept a lot or other amount of product, material, or service

[SOURCE: ISO 3534-2]

3.4**acceptance sampling inspection by variables**

acceptance sampling inspection (3.3) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

3.5**process fraction nonconforming**

rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

3.6**acceptance quality limit****AQL**

worst tolerable *process fraction nonconforming* (3.5) when a continuing series of lots is submitted for *acceptance sampling* (3.3)

Note 1 to entry: See [Clause 5](#).

3.7**quality level**

quality expressed as a rate of occurrence of nonconforming items

3.8**consumer's risk quality****CRQ**

quality level (3.7) of a process which, in the acceptance sampling plan, corresponds to a specified consumer's risk

Note 1 to entry: In this part of ISO 3951, the *quality level* (3.7) is the process fraction nonconforming.

Note 2 to entry: In this part of ISO 3951, the consumer's risk quality corresponds to a consumer's risk of 10 %.

3.9**producer's risk****PR**

probability of non-acceptance when the quality level has a value stated by the plan as acceptable

Note 1 to entry: Quality level relates to the *process fraction nonconforming* (3.5) and acceptable relates to the *acceptance quality limit* (3.6).

3.10**nonconformity**

non-fulfilment of a requirement

Note 1 to entry: Nonconformity will generally be classified by its degree of seriousness such as the following:

Class A. Nonconformity of a type considered to be of the highest concern for the product or service. Such types of nonconformity will typically be assigned very small AQL values;

Class B. Nonconformity of a type considered to have the next lower degree of concern; this is typically assigned a larger AQL value than that in class A and smaller than that in class C if a third class exists and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

3.11

nonconforming unit

unit with one or more nonconformities

[SOURCE: ISO 3534-2]

3.12

s-method acceptance sampling plan

acceptance sampling (3.3) plan by variables using the sample standard deviation(s)

[SOURCE: ISO 3534-2]

Note 1 to entry: See [Clause 15](#).

3.13

σ -method acceptance sampling plan

acceptance sampling (3.3) plan by variables using the presumed value(s) of the process standard deviation(s)

[SOURCE: ISO 3534-2]

Note 1 to entry: See [Clause 16](#).

3.14

specification limit

conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2]

3.15

lower specification limit

L

specification limit (3.14) that defines the lower conformance boundary

[SOURCE: ISO 3534-2]

3.16

upper specification limit

U

specification limit (3.14) that defines the upper conformance boundary

[SOURCE: ISO 3534-2]

3.17

combined control

requirement when nonconformance beyond both the *lower specification limit* (3.15) and the *upper specification limit* (3.16) of a quality characteristic belongs to the same class, to which a single *AQL* (3.6) applies

Note 1 to entry: See [5.3](#), [16.3.2](#), [18.3](#).

Note 2 to entry: The use of a combined *acceptance quality limit* (3.6) requirement implies that nonconformance beyond either *specification limit* (3.14) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.18

separate control

requirement when nonconformance beyond the *lower specification limit* (3.15) and the *upper specification limit* (3.16) of a quality characteristic belong to different classes, to which separate *acceptance quality limits* (3.6) are applied

Note 1 to entry: See [5.3](#), [16.3.3](#), [17.2](#).

3.19**complex control**

requirement when nonconformance beyond the *lower specification limit* (3.15) and the *upper specification limit* (3.16) of a quality characteristic belongs to one class and nonconformance beyond either the *upper specification limit* (3.16) or the *lower specification limit* (3.15) belongs to a different class, with separate *acceptance quality limits* (3.6) being applied to the two classes

Note 1 to entry: See 5.3, 16.3.4, 18.3.

3.20**acceptability constant**

k, p^*

constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2]

Note 1 to entry: See 16.2 and 16.3.

3.21**quality statistic**

Q

function of the *specification limit* (3.14), the sample mean, and the sample or process standard deviation used in assessing the acceptability of a lot

[SOURCE: ISO 3534-2]

Note 1 to entry: For the case of a single *specification limit* (3.14), the lot may be sentenced on the result of comparing Q with the *acceptability constant* (3.20) k .

Note 2 to entry: See 16.2 and 16.3.

3.22**lower quality statistic**

Q_L

function of the *lower specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *lower specification limit* (3.15), the lot is sentenced on the result of comparing Q_L with the *acceptability constant* (3.20) k .

Note 2 to entry: [SOURCE: ISO 3534-2].

Note 3 to entry: See Clause 4, 16.2 and 16.3.

3.23**upper quality statistic**

Q_U

function of the *upper specification limit* (3.16), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *upper specification limit* (3.16), the lot is sentenced on the result of comparing Q_U with the *acceptability constant* (3.20) k .

Note 2 to entry: [SOURCE: ISO 3534-2].

Note 3 to entry: See Clause 4, 16.2, and 18.3.

3.24
maximum sample standard deviation
MSSD

$s_{\max}$

largest sample standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double *specification limits* (3.14) with a combined *acceptance quality limit* (3.6) requirement and unknown process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See 16.3.2.1 and Annex F.

3.25
maximum process standard deviation
MPSD

$\sigma_{\max}$

largest process standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double *specification limits* (3.14) with a combined *acceptance quality limit* (3.6) requirement under tightened inspection with known process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See 17.2, 17.3.

3.26
switching rule

instruction within an *acceptance sampling* (3.3) scheme for changing from one *acceptance sampling* (3.3) plan to another of greater or lesser severity based on demonstrated quality history

[SOURCE: ISO 3534-2]

Note 1 to entry: See Clause 23.

Note 2 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of greater or lesser severity.

3.27
measurement
set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2]

4 Symbols

4.1 Univariate symbols

The symbols used when there is only one quality characteristic in the class are as follows:

c_U	factor for determining the upper control limit for the sample standard deviation (See Annex I.)
f_s	factor that relates the maximum sample standard deviation (MSSD) to the difference between U and L (See Annex F.)
f_σ	factor that relates the maximum process standard deviation (MPSD) to the difference between U and L (See Annex G.)
k	Form k acceptability constant, for use with a single specification limit and a single quality characteristic (See Annexes B and C.)
L	lower specification limit (As a subscript to a variable, denotes its value at L .)
μ	unknown process mean

N	lot size (number of items in a lot)
n	sample size (number of items in a sample)
$\hat{p}$	estimate of the process fraction nonconforming (See Annex L .)
p_L	process fraction nonconforming below the lower specification limit
$\hat{p}_L$	estimate of the process fraction nonconforming below the lower specification limit
p_U	process fraction nonconforming above the upper specification limit
$\hat{p}_U$	estimate of the process fraction nonconforming above the upper specification limit
p^*	Form p^* acceptability constant, the maximum acceptable value of the estimate of the process fraction nonconforming (See Annexes D and E .)
P_a	probability of acceptance
Q	quality statistic
Q_L	lower quality statistic NOTE Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.
Q_U	upper quality statistic NOTE Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.
s	sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e. $s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}}$ (See Annex K .)
$s_{\max}$	maximum sample standard deviation (MSSD)
σ	standard deviation of a process that is under statistical control NOTE σ^2 , the square of the process standard deviation, is known as the process variance.
$\sigma_{\max}$	maximum process standard deviation (MPSD)
U	upper specification limit (As a suffix to a variable, denotes its value at U .)
x_j	measured value of the quality characteristic for the j^{th} item of the sample
$\bar{x}$	the arithmetic mean of the measured values of the quality characteristic in the sample, i.e. $\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$
$\bar{x}_L$	lower acceptance value for $\bar{x}$
$\bar{x}_U$	upper acceptance value for $\bar{x}$

4.2 Multivariate symbols

Other symbols used when there are two or more quality characteristics in a class are as follows:

L_i	lower specification limit for the i^{th} quality characteristic
y	number of quality characteristics in the class
$\hat{p}_i$	estimate of the process fraction nonconforming for the i^{th} quality characteristic
$\hat{p}_{i,L}$	estimate of the process fraction nonconforming below the lower specification limit for the i^{th} quality characteristic
$\hat{p}_{i,U}$	estimate of the process fraction nonconforming above the upper specification limit for the i^{th} quality characteristic
s_i	sample standard deviation for the i^{th} quality characteristic, i.e. $s_i = \sqrt{\frac{\sum_{j=1}^n (x_{ij} - \bar{x}_i)^2}{n-1}}$
σ_i	process standard deviation for the i^{th} quality characteristic
U_i	upper specification limit for the i^{th} quality characteristic
x_{ij}	measured value of the i^{th} quality characteristic for the j^{th} item in the sample
$\bar{x}_i$	sample mean value of the i^{th} quality characteristic, i.e. $\bar{x}_i = \frac{\sum_{j=1}^n x_{ij}}{n}$

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the AQL may be accepted with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, in the contract, or by the responsible authority. In all cases, one AQL shall be specified for each class of nonconformity. (See [3.10](#).)

Where both upper and lower specification limits are given for a quality characteristic, the following three cases may be identified:

- a) *combined* control of double specification limits, where nonconformity beyond both limits belongs to the same class, to which a single AQL applies;
- b) *separate* control, where nonconformity beyond both limits belongs to different classes, to which separate AQLs apply;
- c) *complex* control, where nonconformity beyond the limit that is of greater seriousness belongs to one class to which one AQL applies, and nonconformity beyond both limits combined belongs to another class to which a larger AQL applies.

In other words, for a single quality characteristic that has a lower specification limit, L , an upper specification limit, U , an unknown process fraction nonconforming below L of p_L , and an unknown process fraction nonconforming above U of p_U , combined control seeks simply to control the sum $p_L + p_U$ within one class of nonconformity, to which a single AQL applies. Separate control seeks to control p_L within one class to which one AQL applies and to separately control p_U within another class to which a second AQL applies. Complex control seeks to control $p_L + p_U$ within one class, to which one AQL applies, and to separately control either p_L or p_U , whichever is relevant, within another class to which a lower AQL applies.

Including the control of single specification limits, there are therefore four types of control. A class may contain nonconformities under any number of these types of control.

An acceptance test shall be carried out according to the provisions of this part of ISO 3951 for each class of nonconformity. The lot shall only be accepted if all classes of nonconformity satisfy their respective acceptance tests.

5.4 Preferred AQLs

The 16 AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL other than a preferred AQL is designated, then this part of ISO 3951 is not applicable. (See [14.2](#).)

5.5 Caution

From the definition of the AQL in 5.1, it follows that the desired protection can only be ensured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming items of product.

6 Switching rules for normal, tightened, and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving the production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see [23.1](#)) that the variability is in statistical control, consideration should be given to switching to the σ -method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see [Clause 26](#)).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in [Clauses 24, 25, and 26](#).

7 Relation to ISO 2859-1 and ISO 3951-1

7.1 Relation to ISO 2859-1

7.1.1 Similarities to ISO 2859-1

The similarities are as follows.

- a) This part of ISO 3951 is complementary to ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.
- b) Both use the AQL to index the sampling plans and the preferred values used in this part of ISO 3951 are identical with those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).
- c) In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. General tables give the sample size to be taken and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the s -method and σ -method and for normal, tightened, and reduced inspection.
- d) The switching rules are essentially equivalent.
- e) The classification of nonconformities by degree of seriousness into class A, class B, etc., remains unchanged.

7.1.2 Differences from ISO 2859-1

The differences are as follows.

- a) **Determination of acceptability.** Acceptability for an ISO 2859-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated process mean from the specification limit(s) in terms of the estimated process standard deviation. In this part of ISO 3951, two methods are considered: the s -method, for use when the process standard deviation, σ , is unknown, and the σ -method, for use when σ is presumed to be known. In the case of a class containing a single quality characteristic with a single specification limit, acceptability is determined most easily by comparing a quality statistic with a "Form k " acceptability constant (see [16.2](#) and [17.2](#)). For more complicated classes with multiple quality characteristics and/or combined or complex control of double specification limits, acceptability is determined by comparing an estimate of the process fraction nonconforming for that class with a "Form p^* " acceptability constant.
- b) **Normality.** In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements on each quality characteristic should be distributed according to a normal distribution or at least a close approximation to a normal distribution.

- c) **Independence.** In ISO 2859-1, there is no requirement relating to the independence of multiple quality characteristics. However, in this part of ISO 3951, for the efficient operation of a plan, it is necessary that the measurements for all quality characteristics in a class shall be independent or at least approximately so.
- d) **Operating characteristic curves (OC curves).** The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves for unknown process standard deviation have been matched by minimizing the area between the curves representing the *squares* of the OC values, a method that gives greater emphasis to the match at the top of the OC curves. In most cases, the resulting match between the OC curves is so close that for most practical purposes, the attributes and variables OC curves may be considered to be identical. The plans for known process standard deviation were derived by minimizing the area between the squared OC functions subject to keeping the same Form p^* acceptability constant as for the corresponding case for unknown process standard deviation, i.e. only the sample size was open to choice, so the match was, in general, less perfect.
- e) **Producer's risk.** For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1. (The producer's risks of the plans are given in [Annex N](#).)
- f) **Sample sizes.** The variables sample sizes for combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes for the same letters. This is particularly true for the σ -method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.
- g) **Double sampling plans.** Double sampling plans by variables are presented separately, in ISO 3951-3.
- h) **Multiple sampling plans.** No multiple sampling plans by variables are given in this part of ISO 3951.
- i) **Average Outgoing Quality Limit (AOQL).** The AOQL concept is mainly of value when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

7.2 Relation to ISO 3951-1

7.2.1 Similarities to ISO 3951-1

The similarities are as follows.

- a) This part of ISO 3951 is complementary to ISO 3951-1 and the two documents both present single sampling procedures for inspection by variables.
- b) The procedures of ISO 3951-1 are included in this part of ISO 3951 and referred to as "Form k ".

7.2.2 Differences from ISO 3951-1

The differences are as follows.

- a) This part of ISO 3951 is more general than ISO 3951-1 as it includes multivariate procedures for independent quality characteristics and also includes procedures for separate or complex control of double specification limits.
- b) Because Form k procedures may only be used for a single quality characteristic with a single AQL, this part of ISO 3951 also includes the more general Form p^* procedures.

NOTE For users who are familiar with MIL-STD-414,^[19] Form *k* corresponds to form 1 of the Military Standard, and Form *p** corresponds to form 2. The new terminology is considered to be more helpful.

8 Consumer protection

8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal, and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Sometimes, specific individual plans are selected from this part of ISO 3951 and used without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as “inspection in compliance with ISO 3951-2”. When used in such a way, ISO 3951-2 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

8.2 Consumer’s risk quality (CRQ) tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give consumer’s risk quality not more than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer’s risk quality and a consumer’s risk to be associated with it. [Annex M](#) gives values of consumer’s risk quality levels for the *s*-method and σ -method corresponding to a consumer’s risk of 10 %.

However, application of this part of ISO 3951 to isolated or short series of lots is deprecated, as the theory of sampling by variables applies to a *process*. For isolated or short series of lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from ISO 2859-2. (See also Reference [\[14\]](#) in the Bibliography.)

8.3 Producer’s risk tables

[Annex N](#) gives the probability of non-acceptance under the *s*-method and σ -method for lots produced when the process fraction nonconforming equals the AQL. This probability is called the producer’s risk.

8.4 Operating characteristic (OC) curves

The tables for consumer’s risk quality and producer’s risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any process quality level may, however, be judged from its operating characteristic curve. OC curves for the normal inspection *s*-method sampling plans of this part of ISO 3951 are given in Charts B to R of ISO 3951-1, which should be consulted when choosing a sampling plan. Also given in ISO 3951-1 are Tables B to R of process quality levels at nine standard probabilities of acceptance for all the *s*-method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the *s*-method. Most of them also provide a good approximation to the σ -method and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the σ -method, refer to [Annex O](#).

9 Accommodating measurement variability

The master tables of this part of ISO 3951 are based on the assumption that the quality characteristic *X* of the items in the lots is normally distributed with unknown process mean, μ , and either known or unknown process standard deviation, σ . The assumption is also made that the measurement *X* is

corrected for bias (if any) and contains no measurement variability, i.e. that the measurement of an item with the true value x_i results in the value x_i . However, the master tables can also be used, with appropriate adjustments, in the presence of measurement error.

If the measurement standard deviation is no greater than 10 % of the process standard deviation, it can be ignored. For measurement standard deviation greater than 10 % of the process standard deviation, the sample size will need to be increased, although the acceptability constant remains the same. Moreover, if neither the measurement standard deviation nor the process standard deviation is known, more than one measurement will need to be made on each sampled item and the total variability of the measurements will need to be separated into the components due to the measurements and to the process.

Details are provided in [Annex P](#).

10 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge both of statistics and the product to be inspected. [Clauses 11](#) to [13](#) of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

11 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

- a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time consuming and costly per item.
- b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely how good the product is. Earlier warning will, therefore, be given if the quality is slipping.
- c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in [16.3.2.2](#) and [16.3.2.4](#).)
- d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859-1) and the standard plans in this part of ISO 3951 reveals that the smallest samples tend to be required by the σ -method (used when the process standard deviation is presumed to be known). The sample sizes for the s -method (used when the process standard deviation is unknown) are also, in general, substantially smaller than for sampling by attributes.
- e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.
- f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.
- g) A variable scheme becomes relatively more complicated to operate as the number of quality characteristics and the number of measurements to be taken on each item increases.
- h) This part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of each quality characteristic is normal and that the quality characteristics are independent. In case of doubt, the responsible authority should be consulted.

NOTE 1 ISO 5479 gives detailed procedures for tests for departure from normality.

NOTE 2 Departure from normality is also dealt with in ISO 2854, which provides examples of graphical methods that can be used to verify that the distribution of the data are sufficiently normal to justify the use of sampling by variables.

12 Choice between the *s*-method and σ -method

If it is desired to apply inspection by variables, the next question is whether to use the *s*-method or the σ -method. The σ -method is usually the most economical in terms of sample size, but, before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the *s*-method but, subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the σ -method? The size of the sample will generally be smaller and the acceptability criterion simpler under the σ -method. On the other hand, it will still be necessary to calculate the sample standard deviation, *s*, for record purposes and to keep the control charts up to date. (See [Clause 22](#).) The calculation of *s* can appear daunting, but the difficulty is more apparent than real; this is especially true if an electronic calculator is available. Methods of calculating *s* are given in [Annex K](#).

13 Choice of inspection level and AQL

For a standard sampling plan, the inspection level, in conjunction with the size of the lots and the AQL, determines the size of the sample to be taken and governs the severity of the inspection. The appropriate OC curve from Charts B to R of ISO 3951-1 or the appropriate table from Tables B to R of ISO 3951-1 shows the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

14 Choice of sampling scheme

14.1 Standard plans

The standard procedure can be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the *s*-method, has been found in practice to produce workable sampling schemes; but it rests on the assumption that the order of priority is the AQL first, the sample size second, and the limiting quality last.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see [Clauses 23](#), [24](#), and [25](#)), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE It should also be remembered that the limiting quality is the quality which, if offered for inspection, would have a 10 % probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

If, in certain circumstances, the limiting quality has a higher priority than the sample size, a suitable plan from this part of ISO 3951 may be selected by using Chart A. Construct a vertical line through the

acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample size code letter of a standard normal inspection plan that meets the specified requirements. (This may be verified by inspecting the OC curve from among Charts B to R of ISO 3951-1 relating to this code letter and AQL.)

However, the use of this method is deprecated for isolated lots or short series of lots. (See 8.2.)

EXAMPLE Suppose that an acceptable value for the limiting quality is 6,0 % nonconforming and that the desired quality with a 95 % probability of acceptance is 2,0 % nonconforming. A vertical line on Chart A at 6,0 % nonconforming and a horizontal line at 2,0 % nonconforming intersect just below the sloping line indexed by the letter L. Examining Chart L, it is seen that a plan with a sample size code letter L and an AQL of 1,5 % meets the requirements.

If the horizontal and vertical lines intersect at a point above the line marked R in Chart A, this implies that the specification cannot be met by any of the plans in this part of ISO 3951.

14.2 Special plans

If none of the standard plans are acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special plan is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

15 Preliminary operations

Perform the following checks before starting inspection by variables.

- a) check that production is considered to be continuing and that the distribution of the quality characteristics can be considered to be normal and independent;

NOTE 1 For tests for departure from normality, see, for example, ISO 16269-4.

NOTE 2 If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

- b) check separately for each quality characteristic whether the s -method is to be used initially or whether the process standard deviation is stable and known, in which case the σ -method should be used;
- c) check that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;
- d) check, for every quality characteristic with double specification limits, whether the limits are under combined, separate, or complex control and to which class of nonconformity each limit has been assigned. For combined control, check that nonconformity beyond each limit is of equal importance;
- e) check that an AQL has been designated for each class of nonconformity and that it is one of the preferred AQLs for use with this part of ISO 3951. If it is not, then the tables are not applicable.

16 Standard procedures for the univariate s-method

16.1 Obtaining a plan, sampling, and preliminary calculations

The procedure for obtaining and implementing a plan is as follows.

- a) With the inspection level given (normally, this will be level II) and with the lot size, obtain the sample size code letter using [Table A.1](#).
- b) For a single specification limit, enter [Table B.1](#), [B.2](#), or [B.3](#) as appropriate with this code letter and the AQL and obtain the sample size, n , and the Form k acceptability constant, k . For separate control of double specification limits, do this for both limits. For combined control of double specification limits, enter [Table D.1](#), [D.2](#), or [D.3](#) as appropriate and obtain the sample size n and the Form p^* acceptability constant. For complex control of double specification limits, enter [Table D.1](#), [D.2](#), or [D.3](#) as appropriate twice, once with the combined control part of the specification and once with the smaller AQL applying to the specification limit of greater concern.
- c) Take a random sample of size n , measure the characteristic x in each item, and then calculate the sample mean, $\bar{x}$, and the estimate s of the process standard deviation (see Annex K). If $\bar{x}$ lies outside the specification limit(s), the lot can be judged unacceptable without even calculating s . It is, however, necessary to calculate s for record purposes. (See [Clause 22](#).)

16.2 Form k acceptability criterion for the s-method

If single specification limits are given, or separate control of double specification limits is required, the most straightforward procedure is as follows. Calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s} \quad (1)$$

and/or

$$Q_L = \frac{\bar{x} - L}{s} \quad (2)$$

as appropriate, then compare the quality statistic (Q_U or Q_L) with the Form k acceptability constant obtained from [Table B.1](#), [B.2](#), or [B.3](#) for normal, tightened, or reduced inspection, respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if it is less, the lot is not acceptable.

Thus, if only the upper specification limit, U , is given, the lot is

acceptable if $Q_U \geq k$, and

not acceptable if $Q_U < k$,

or, if only the lower specification limit, L , is given, the lot is

acceptable if $Q_L \geq k$, and

not acceptable if $Q_L < k$.

Under separate control of double specification limits, the Form k acceptability constants at L and U may be different. Denote them by k_L and k_U respectively. In this case, the lot is

acceptable if $Q_U \geq k_U$ and $Q_L \geq k_L$, and

not acceptable if $Q_U < k_U$ and/or $Q_L < k_L$.

EXAMPLE 1 Single upper specification limit.

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From [Table A.1](#), the sample size code letter is found to be F; from [Table B.1](#), it is seen that a sample size of 13 is required and that the acceptability constant k is 1,426. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size: n	13
Sample mean: $\bar{x} = \sum x / n$	54,615 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See K.1.2, Annex K .)	3,330 °C
Specification limit (upper): U	60 °C
Upper quality statistic: $Q_U = (U - \bar{x}) / s$	1,617
Form k acceptability constant: k (from Table B.1)	1,426
Acceptability criterion: Is $Q_U \geq k$?	Yes (1,617 > 1,426)

The lot meets the acceptability criterion and is therefore acceptable.

EXAMPLE 2 Single lower specification limit, requiring the following of an arrow in the master table.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. The process standard deviation is unknown. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. From [Table A.1](#), it is seen that the sample size code letter is J. However, on entering [Table B.1](#) with sample size code letter J and AQL 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, i.e. sample size 28 and acceptability constant $k = 2,580$. A random sample of size 28 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95	6,04	6,68	6,63	6,65	6,52	6,59	6,40	6,44	6,34	6,04	6,15	6,29	6,63
6,44	7,15	6,70	6,59	6,51	6,80	5,94	6,35	7,17	6,83	6,25	6,96	7,00	6,38

Compliance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size: n	28
Sample mean: $\bar{x} = \sum x / n$	6,551 s
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See Annex K , K.1.2.)	0,3251 s
Lower specification limit: L	4,0 s

Lower quality statistic: $Q_L = (\bar{x} - L) / s$	7,847
Form k acceptability constant: k (from Table B.1)	2,580
Acceptability criterion: Is $Q_L \geq k$?	Yes ($7,847 > 2,580$)

The lot meets the acceptability criterion, so it is acceptable.

16.3 Form p^* acceptability criterion for the s -method

16.3.1 Introduction

This part of ISO 3951 also provides a Form p^* method for determining lot acceptability. Whereas Form k applies only to a single quality characteristic with either a single specification limit or with double specification limits that are to be controlled separately, Form p^* applies much more generally to a class consisting of single or multiple quality characteristics with any combination of single or double specification limits with combined, separate, or complex control.

16.3.2 Combined control for the s -method

16.3.2.1 General

If, for the univariate s -method, combined or complex control of both the upper and lower specification limits is required, i.e. there is an overall AQL for the percentage of the process outside the two specification limits, the first step is to check that the sample standard deviation, s , is not so large that lot acceptability is impossible. If the value of s exceeds the value of the maximum sample standard deviation (MSSD) determined from [Table F.1](#), [F.2](#), or [F.3](#), no further calculation or reference to graphs is required and the lot shall be immediately judged unacceptable.

If the value of s does not exceed the value of the MSSD, the estimate $\hat{p}$ of the process fraction nonconforming shall be calculated and compared with the Form p^* acceptability constant. The lot is determined to be

acceptable if $\hat{p} \leq p^*$, and

not acceptable if $\hat{p} > p^*$,

where

$$\hat{p} = \hat{p}_L + \hat{p}_U \quad (3)$$

with

$$\hat{p}_L = G_{(n-2)/2} \left[\frac{1}{2} \left(1 - \frac{\bar{x} - L}{s} \frac{\sqrt{n}}{n-1} \right) \right] \quad (4)$$

$$\hat{p}_U = G_{(n-2)/2} \left[\frac{1}{2} \left(1 - \frac{U - \bar{x}}{s} \frac{\sqrt{n}}{n-1} \right) \right] \quad (5)$$

in which $G_m(\cdot)$ represents the distribution function of the symmetric beta distribution with both parameters equal to m . (See [Annex L](#) for details.)

Form p^* may also be applied to a single specification limit, although in that case, Form k is equivalent and easier to apply. However, an estimate of the process fraction nonconforming will not be obtained when using Form k .