

Implementation of
ISO 6336-3:1996

Calculation of load capacity of spur and helical gears —

Part 3: Calculation of tooth bending
strength

ICS 21.200

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- British Gear Association
- British Horological Federation
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- Engineering Equipment and Materials Users' Association
- Federation of Manufacturers of Construction Equipment and Cranes
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National foreword

This British Standard reproduces verbatim ISO 6336-3:1996 and implements it as the UK national standard.

This British Standard is published under the direction of the Engineering Sector Board whose Technical Committee MCE/5 has the responsibility to:

- aid enquirers to understand the text;
- present to the responsible international committee any enquiries on interpretation, or proposals for change, and keep UK interests informed;
- monitor related international and European developments and promulgate them in the UK.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, the ISO title page, page ii, pages 1 to 66, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

INTERNATIONAL STANDARD

ISO
6336-3

First edition
1996-06-15

Calculation of load capacity of spur and helical gears —

Part 3:

Calculation of tooth bending strength

*Calcul de la capacité de charge des engrenages cylindriques à dentures
droite et hélicoïdale —*

Partie 3: Calcul de la résistance à la flexion des dents



Reference number
ISO 6336-3:1996(E)

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liason with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the Technical Committees are circulated to the member bodies for voting. Publication as a International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard 6336-3 was prepared by Technical Committee ISO/TC60, Gears, Subcommittee SC2, Gear capacity calculation.

ISO 6336 consists of the following parts, under the general title *Calculation of load capacity of spur and helical gears*:

- *Part 1: Basic principles, introduction and general influence factors;*
- *Part 2: Calculation of surface durability (pitting);*
- *Part 3: Calculation of tooth bending strength;*
- *Part 5: Strength and quality of materials.*

Annex A of this part of ISO 6336 is for information only.

Introduction

The maximum tensile stress at the tooth-root (in the direction of the tooth height) which may not exceed the permissible bending stress for the material, is the basis for rating the bending strength of gear teeth. The stress occurs in the “tension fillets” of the working tooth flanks. If load-induced cracks are formed, the first of these often appears in the fillets where the compressive stress is generated; i.e. in the “compression fillets”, which are those of the non-working flanks. When the tooth loading is unidirectional and the teeth are of conventional shape, these cracks seldom propagate to failure. Crack propagation ending in failure is most likely to stem from cracks initiated in tension fillets.

The endurable tooth loading of teeth which are subjected to a reversal of loading during each revolution, such as “idler gears”, is less than the endurable unidirectional loading. The full range of stress, in such circumstances, is more than twice the tensile stress which occurs in the root fillets of the loaded flanks. This is taken into consideration when determining permissible stresses (see ISO 6336-5).

When gear rims are thin and tooth spaces adjacent to the root surface are narrow (conditions which can apply in particular to some internal gears), initial cracks commonly occur in the compression fillet. Since, in such circumstances, gear rims themselves can suffer fatigue breakage, special studies are necessary. See clause 1, 5.2.2 and 5.3.2.

Several methods for calculation of the critical tooth-root stress and for evaluating some of the relevant factors have been approved (see ISO 6336-1).

1 Scope

This part of ISO 6336 specifies the fundamental formulae for use in tooth bending stress calculations for involute internal and external spur and helical gears with a minimum rim thickness under the root of $s_R \leq 3,5 m_n$. All load influences on tooth stress are included insofar as they are the result of loads transmitted by the gearing and insofar as they can be evaluated quantitatively (see 4.1.1).

The given formulae are valid for spur and helical gears with tooth profiles in accordance with the basic rack standardized in ISO 53 (see introduction). They may also be used for teeth conjugate to other basic racks if the virtual contact ratio is less than $\epsilon_{\alpha n} = 2,5$.

NOTE 1 See 4.1.1 c) and 5.3 for restrictions in the case of method C.

The load capacity determined on the basis of permissible bending stress is termed “tooth bending strength”. The results are in good agreement with other methods for the range as indicated in ISO 6336-1.

The user of this part of ISO 6336 standard is cautioned that when the method specified is used for large helix angles and large pressure angles, the calculated results should be confirmed by experience as by method A.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO 6336. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO 6336 are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 53:1974, *Cylindrical gears for general and heavy engineering — Basic rack*.

ISO 6336-1:1996, *Calculation of load capacity of spur and helical cylindrical gears — Part 1: Basic principles, introduction and general influence factors*.

ISO 6336-5:1996, *Calculation of load capacity of cylindrical gears — Part 5: Strength and quality of materials*.

3 Tooth breakage and safety factors

Tooth breakage usually ends the service life of a transmission. Sometimes the destruction of all gears in a transmission can be a consequence of the breakage of one tooth. In some instances the transmission path between input and output shafts is broken. As a consequence, the chosen value of the safety factor S_F against tooth breakage should be larger than the safety factor against pitting.

General comments on the choice of the minimum safety factor can be found in ISO 6336-1, subclause 4.1.3. It is recommended that manufacturer and customer agree on the value of the minimum safety factor.

This part of ISO 6336 does not apply at stress levels above those permissible for 10^3 cycles, since stresses in this range may exceed the elastic limit of the gear tooth.

4 Basic formulae

NOTE 2 All symbols, terms and units are defined in ISO 6336-1.

The actual tooth-root stress σ_F and the permissible bending stress σ_{FP} shall be calculated separately for pinion and wheel; σ_F shall be less than σ_{FP} .

4.1 Tooth-root stress, σ_F

4.1.1 Methods for the determination of tooth-root stress, σ_F : Principles, assumptions, and application

According to this part of ISO 6336, the local tooth-root stress is determined as the product of nominal bending stress and a stress correction factor (methods B and C¹⁾).

a) Method A

In principle, the maximum tensile stress can be determined by any appropriate method (e.g. finite element analysis, integral equations, conformal mapping procedures, or experimentally by photo-elastic stress analysis, strain measurement, etc.). In order to determine the maximum tooth-root stress, the effects of load distribution over two or more engaging teeth and changes of stress with changes of meshing phase shall be taken into consideration.

It should be noted that the tooth-root tensile stress has relevance to the plane-strain condition. This is important when making comparisons with the results of photo-elastic stress evaluations (methods B and C) and the permissible stresses.

Method A is only used in special cases and, because of the great effort involved, is only justifiable in such cases.

b) Method B

This method involves the assumption that the determinant tooth-root stress occurs with application of load at the outer point of single pair tooth contact of spur gears or of the virtual spur gears of helical gears. However, in the latter case, the “transverse load” shall be replaced by the “normal load”, applied over the facewidth of the actual gear of interest.

For gears having virtual contact ratios in the range $2 \leq \epsilon_{\alpha n} < 3$, it is assumed that the determinant stress occurs with application of load at the inner point of double pair tooth contact. Formulae are provided for the calculation of the appropriate form factors Y_β for the nominal stress and Y_s for the stress correction factors. In the case of helical gears, the factor Y_β accounts for deviations from these assumptions.

Method B is suitable for more detailed calculations and is also appropriate for computer programming and for the analysis of pulsator tests (with a given point of application of loading).

c) Method C

This simplified method of calculation is derived from method B. The local stress for application of load at the tooth tip is calculated first (with factors Y_{Fa} and Y_{Sa}) and then converted to approximate the corresponding value, appropriate to contact at the outer point of single pair tooth contact, using the factor Y_ϵ .

The form factor Y_{Fa} for the nominal stress and the stress correction factor Y_{Sa} have been plotted on a series of graphs for a number of basic rack profiles.

Method C is only acceptable for gears when $\epsilon_{\alpha n} < 2$; it is also useful when no computer program is available. The method is sufficiently accurate for most cases and generally gives slightly higher values of stress than method B.

¹⁾ Stresses such as those caused by the shrink-fitting of gear rims, which are superimposed on stresses due to tooth loading, should be taken into consideration in the calculation of the tooth root stress σ_F or the permissible tooth root stress σ_{FP} .

4.1.2 Tooth-root stress, σ_F : Methods B and C

The total tangential load in the case of gear trains with multiple transmission paths (planetary gear trains, split-path gear trains) is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by inserting a distribution factor K_γ to follow K_A in equation (1) to adjust as necessary the average load per mesh.

$$\sigma_F = \sigma_{FO} K_A K_v K_{F\beta} K_{F\alpha} \leq \sigma_{FP} \quad \dots (1)$$

where

- σ_{FO} is the nominal tooth-root stress, which is the maximum local tensile stress produced at the tooth-root when an error-free gear pair is loaded by the static nominal torque.
- σ_{FP} is the permissible bending stress (see 4.2).
- K_A is the application factor (see ISO 6336-1). It takes into account load increments due to externally influenced variations of input or output torque.
- K_v is the dynamic factor (see ISO 6336-1). It takes into account load increments due to internal dynamic effects.
- $K_{F\beta}$ is the face load factor for tooth-root stress (see ISO 6336-1). It takes into account uneven distribution of load over the facewidth due to mesh-misalignment caused by inaccuracies in manufacture, elastic deformations, etc.
- $K_{F\alpha}$ is the transverse load factor for tooth-root stress (see ISO 6336-1). It takes into account uneven load distribution in the transverse direction, resulting for example, from pitch deviations.

NOTE 3 See ISO 6336-1, subclause 4.1.8 for the sequence in which factors K_A , K_v , $K_{F\beta}$ and $K_{F\alpha}$ are calculated.

4.1.3 Nominal tooth-root stress, σ_{FO-B} : Method B

$$\sigma_{FO-B} = \frac{F_t}{b m_n} Y_F Y_S Y_\beta \quad \dots (2)$$

where

- F_t is the nominal tangential load, the transverse load tangential to the reference cylinder^a (see ISO 6336-1).
- b is the facewidth (for double helical gears $b = 2 b_P$). The value b , of mating gears, is the facewidth at the root circle, ignoring any intentional transverse chamfers or tooth-end rounding. If the facewidths of the pinion and wheel are not equal, it may be assumed that the load bearing width of the wider facewidth is equal to the smaller facewidth plus such extension of the wider that does not exceed one times the module at each end of the teeth.
- m_n is the normal module
- Y_F is the form factor (see clause 5). It takes into account the influence on nominal tooth-root stress, of the tooth form with load applied at the outer point of single pair tooth contact.
- Y_S is the stress correction factor (see clause 6). It takes into account the conversion of the nominal bending stress, determined for application of load at the outer point of single pair tooth contact, to the local tooth-root stress. Thus by means of Y_S , the following are taken into account:
 - a) the stress amplifying effect of change of section at the tooth-root; and
 - b) that evaluation of the true stress system, at the tooth-root critical section, is more complex than the simple system evaluation presented.
- Y_β is the helix factor (see clause 8). It compensates for the fact that the bending moment intensity at the tooth-root of helical gears is, as a consequence of the oblique lines of contact, less than the corresponding values for the virtual spur gears used as bases for calculation.

^a Subject to the condition that the gear rim under the tooth-root is sufficiently thick, i.e. that the rim thickness $s_R \geq 3.5 m_n$ (see scope). In all cases, even when $\epsilon_{\alpha n} > 2$, it is necessary to substitute the relevant total tangential load as F_t . Reasons for the choice of load application at the reference cylinder are given in subclause 5.5. See ISO 6336-1, subclause 4.2 for definition of F_t and comments on particular characteristics of double helical gearing.

4.1.4 Nominal tooth-root stress, σ_{FO-C} : Method C

$$\sigma_{FO-C} = \frac{F_t}{b m_n} Y_{Fa} Y_{Sa} Y_{\epsilon} Y_{\beta} = \frac{F_t}{b m_n} Y_{FS} Y_{\epsilon} Y_{\beta} \quad \dots (3)$$

where

- Y_{Fa} is the form factor (see clause 5). It takes into account the influence on nominal tooth-root stress of the tooth form, with load applied at the tooth tip;
- Y_{Sa} is the stress correction factor (see clause 6). It takes into account the conversion of the nominal bending stress determined for application of load at the tooth tip, to the local tooth-root stress. Thus, by means of Y_{Sa} , the following are taken into account:
- a) the stress amplifying effect of change of section at the tooth-root; and
 - b) that evaluation of the true stress system, at the tooth-root critical section, is more complex than the simple system evaluation presented, but the influence of the bending moment arm is neglected.
- Y_{ϵ} is the contact ratio factor (see clause 7). It takes into account the transformation of the local stress determined for application of load at the tooth tip, to approximate the value relevant to application of load at the outer point of single pair tooth contact. By means of this factor, account is taken of the influence on the stress correction factor of the load distribution over several points of contacts and that of the tooth bending moment.
- Y_{FS} is the tip factor, equal to $(Y_{Fa} Y_{Sa})$ (see clause 5). This factor accounts for all influences covered by Y_{Fa} and Y_{Sa} . Charts from which Y_{FS} may be read can be constructed for involute gears conjugate to any suitable basic rack.

Other relevant terms and symbols are discussed in 4.1.3.

4.2 Permissible bending stress, σ_{FP}

The limit value of tooth-root stresses (see clause 9) should preferably be derived from material tests using gears as test pieces, since in this way the effects of test piece geometry, as for example the effect of the fillet at the tooth-roots, are included in the results. The calculation methods provided constitute empirical means for comparing stresses in gears of different dimensions, with experimental results. The closer test gears and test conditions resemble the service gears and service conditions, the lesser will be the influence of inaccuracies in the formulation of the calculation expressions.

4.2.1 Methods for the determination of the permissible bending stress, σ_{FP} : Principles, assumptions and application

Several procedures for the determination of permissible bending stresses are acceptable. The method adopted shall be validated by carrying out careful comparative studies of well-documented service histories of a number of gears.

a) Method A

By this method, the values for σ_{FP} “permissible bending stress” or for σ_{FG} “tooth-root stress limit”, are obtained using equations (1) and (2) from the S-N curve or damage curve derived from results of testing facsimiles of the actual gear pair, under the appropriate service conditions.

The cost required for this method is in general, only justifiable for the development of new products, failure of which would have serious consequences (e.g. for manned space flights).

Similarly, in line with this method, the allowable stress values may be derived from consideration of dimensions, service conditions and performance of carefully monitored reference gears.

b) Method B

Damage curves characterized by the nominal stress number (bending), $\sigma_F \lim$, and the factor Y_{NT} have been determined for a number of common gear materials and heat treatments from results of gear load or pulsator testing of standard reference test gears. Material values so determined are converted to suit the dimensions of the gears of interest, using the relative influence factors for notch sensitivity, $Y_{\delta \text{ rel } T}$, for surface roughness, $Y_{R \text{ rel } T}$ and for size, Y_X .

Method B is recommended for the calculation of reasonably accurate gear ratings whenever bending strength values are available from gear tests, from special tests or, if the material is similar, from ISO 6336-5.

c) Methods C and D

In these methods, which are derived from method B, the influence factors $Y_{\delta \text{ rel T}}$, $Y_{R \text{ rel T}}$ and Y_X are determined using simplified procedures. These methods are more easily and quickly applied than are those of method B. The results obtained tend to err on the side of safety. The experimental procedure for determination of strength values is as described for method B.

d) Methods B_k, C_k and D_k

The permissible bending stress is to be derived from the bending stress number, ($\sigma_{k \text{ lim}}$, and life factor Y_{Nk} results, usually presented as S-N or damage curves, of the pulsator fatigue testing of notched, flat test pieces. As in the case for method B, the test data shall be transformed to suit the gears of interest, using the influence factors appropriate to both the method and the test piece: $Y_{\delta \text{ rel k}}$ for notch sensitivity, $Y_{R \text{ rel k}}$ for surface roughness and the size factor Y_X , in accordance with method B.

The influence factors appropriate to methods C_k and D_k are determined by simpler equations than those of method B_k.

These methods can be applied when values obtained from test gears are not available. These methods are particularly suitable for evaluating, relative to one another, the tooth-root strength values for different materials.

e) Methods B_p, C_p and D_p

The permissible bending stresses are to be derived from the bending stress number, $\sigma_{p \text{ lim}}$, and life factor Y_{Np} results, usually presented as S-N or damage curves, of the pulsator fatigue testing of plain, polished test pieces. As in the case for method B, the test data are to be transformed to suit the gears of interest, using the (absolute) influence factors appropriate to the method and the test piece: Y_{δ} for notch sensitivity, Y_R for surface roughness, and the size factor Y_X in accordance with method B.

These methods can be applied when values obtained from either gears or notched test pieces are not available. These methods are particularly suitable for evaluating, relative to one another, the tooth-root strength values for different materials.

4.2.2 Permissible bending stress, σ_{FP} : Methods B, C and D

Subject to the reservations given in 4.2.2 a) and b), equation (4) is to be used for this purpose.

$$\begin{aligned}\sigma_{FP} &= \frac{\sigma_{F \text{ lim}} Y_{ST} Y_{NT}}{S_{F \text{ min}}} Y_{\delta \text{ rel T}} Y_{R \text{ rel T}} Y_X = \\ &= \frac{\sigma_{FE} Y_{NT}}{S_{F \text{ min}}} Y_{\delta \text{ rel T}} Y_{R \text{ rel T}} Y_X = \frac{\sigma_{FG}}{S_{F \text{ min}}} \quad \dots (4)\end{aligned}$$

where

$\sigma_{F \text{ lim}}$	is the nominal stress number (bending) from reference test gears (see ISO 6336-5). It is the bending stress limit value relevant to the influences of the material, the heat treatment, and the surface roughness of the test gear root fillets.
σ_{FE}	is the allowable stress number for bending. The basic bending strength of the unnotched test piece, under the assumption that the material condition (including heat treatment) is fully elastic. $\sigma_{FE} = (\sigma_{F \text{ lim}} Y_{ST})$.
Y_{ST}	is the stress correction factor, relevant to the dimensions of the reference test gears (see 6.5).
Y_{NT}	is the life factor for tooth-root stress, relevant to the dimensions of the reference test gear (see clause 10). It takes into account the higher load bearing capacity for a limited number of load cycles.
σ_{FG}	is the tooth-root stress limit $\sigma_{FG} = (\sigma_{FP} S_{F \text{ min}})$.
$S_{F \text{ min}}$	is the minimum required safety factor for tooth-root stress (see clause 3 and 4.3).

$Y_{\delta \text{ rel T}}$	is the relative notch sensitivity factor. It is the quotient of the notch sensitivity factor of the gear of interest divided by the standard test gear factor (see clause 11). Enables the influence of the notch sensitivity of the material to be taken into account.
$Y_{R \text{ rel T}}$	is the relative surface factor. It is the quotient of the surface roughness factor of tooth-root fillets of the gear of interest divided by the tooth-root fillet factor of the reference test gear (see clause 12); it enables the relevant surface roughness of tooth-root fillet influences to be taken into account.
Y_X	is the size factor relevant to tooth-root strength (see clause 13); it is used to take into account the influence of tooth dimensions on tooth bending strength.

a) Permissible bending stress (reference)

The permissible bending stress (reference), $\sigma_{\text{FP ref}}$, is derived from equation (4), with $Y_{\text{NT}} = 1$ and the influence factors $\sigma_{\text{F lim}}$, Y_{ST} , $Y_{\delta \text{ rel T}}$, $Y_{R \text{ rel T}}$, Y_X and $S_{\text{F min}}$ calculated in accordance with the specified method B, C or D.

b) Permissible bending stress (static)

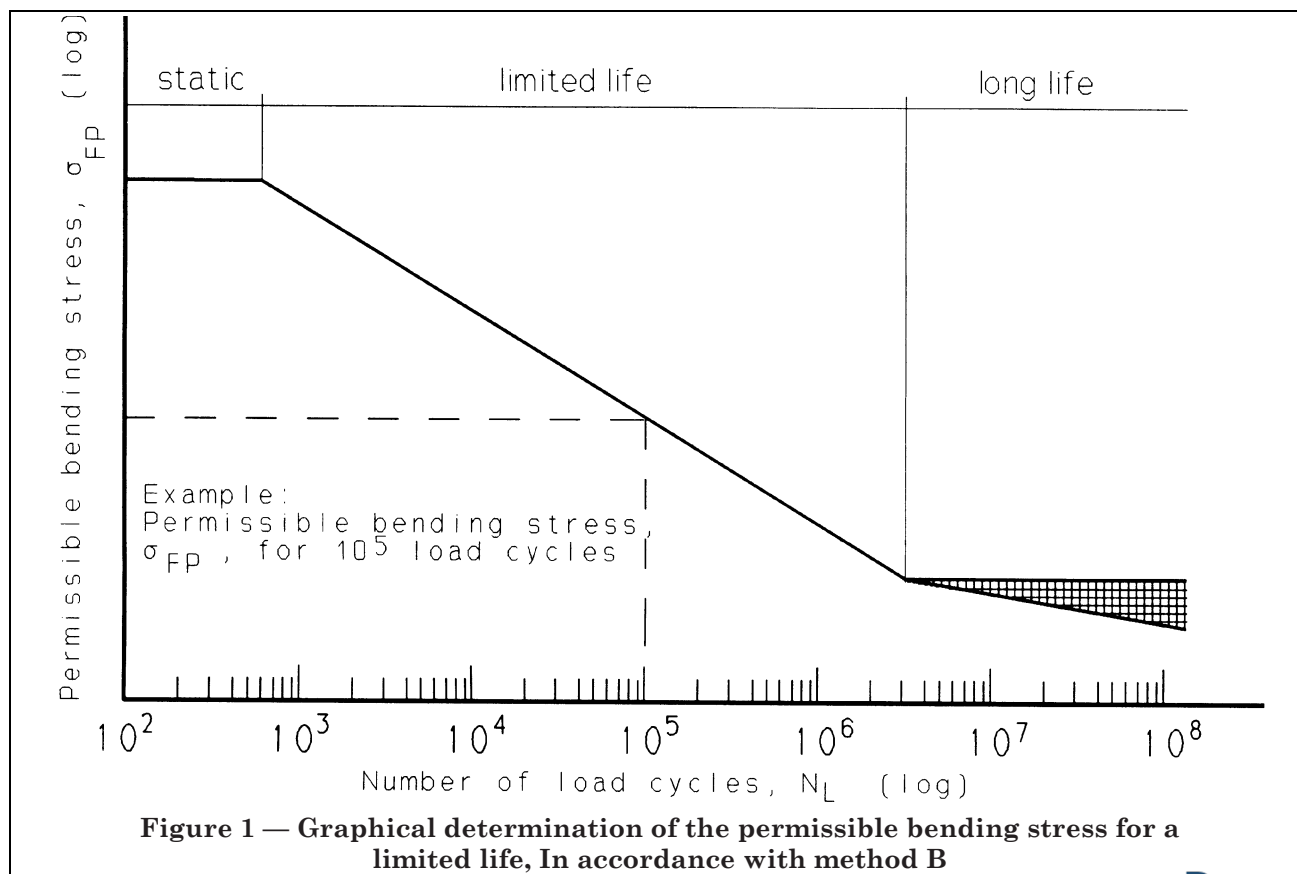
The permissible bending stress (static), $\sigma_{\text{FP stat}}$, is determined in accordance with equation (4) with the factors $\sigma_{\text{F lim}}$, Y_{NT} , Y_{ST} , $Y_{\delta \text{ rel T}}$, $Y_{R \text{ rel T}}$, Y_X and $S_{\text{F min}}$ calculated in accordance with the specified method B, C or D (for static stress).

4.2.3 Permissible bending stress, σ_{FP} , for limited and long life: Methods B, C and D.

σ_{FP} for a given number of load cycles N_L is determined by means of graphical or calculated interpolation along the S-N curve, between the value obtained for reference stress in accordance with 4.2.2 a) and the value obtained for static stress in accordance with 4.2.2 b). Also see clause 10.

4.2.3.1 Graphical values

Calculate $\sigma_{\text{FP ref}}$ for the reference stress and $\sigma_{\text{FP stat}}$ for the static stress in accordance with 4.2.2 and plot the S-N curve corresponding to the life factor, Y_{NT} . See Figure 1 for the principle. σ_{FP} for the relevant number of load cycles N_L can be read from this graph.



4.2.3.2 Determination by calculation

Calculate $\sigma_{FP\text{ ref}}$ for the reference stress and $\sigma_{FP\text{ stat}}$ for the static stress in accordance with 4.2.2 and, using these results, determine σ_{FP} for the relevant number of load cycles N_L in the limited life range, as follows:

$$\sigma_{FP} = \sigma_{FP\text{ ref}} Y_N = \sigma_{FP\text{ ref}} \left(\frac{3 \times 10^6}{N_L} \right)^{\text{exp}} \quad \dots (5)$$

a) For structural and through hardened steel, perlitic or bainitic nodular cast iron, perlitic malleable cast iron (limited life range as shown in Figure 36: $10^4 < N_L \leq 3 \times 10^6$):

$$\text{exp} = 0,4037 \log \frac{\sigma_{FP\text{ stat}}}{\sigma_{FP\text{ ref}}} \quad \dots (6)$$

b) For case hardened or surface hardened steel; through hardened steel or nitriding steel, gas nitrided; through hardened steel and case hardening steel, nitrocarburized; ferritic nodular cast iron, or grey cast iron (limited life range as shown in Figure 36: $10^3 < N_L \leq 3 \times 10^6$):

$$\text{exp} = 0,2876 \log \frac{\sigma_{FP\text{ stat}}}{\sigma_{FP\text{ ref}}} \quad \dots (7)$$

Corresponding calculations may be determined for the range of long life.

4.2.4 Permissible bending stress, σ_{FP} : Methods B_k , C_k and D_k

4.2.4.1 σ_{FP} for static stress and reference stress

Following these methods the permissible bending stress is calculated on the basis of the strength of a notched test piece from the following equation:

$$\sigma_{FP} = \frac{\sigma_{k\text{ lim}} Y_{Sk} Y_{Nk}}{S_{F\text{ min}}} Y_{\delta\text{ rel } k} Y_{R\text{ rel } k} Y_X = \frac{\sigma_{FG}}{S_{F\text{ min}}} \quad \dots (8)$$

where

$\sigma_{k\text{ lim}}$	is the nominal notched-bar stress number (bending). It is the bending stress limit value of the notched-bar test piece relevant to its material, heat treatment and surface condition in relation to its dimensions. Differences due to conditions of manufacture, between the properties of the heat treated materials, application of stresses and sections of test piece and gear of interest, should be taken into consideration.
Y_{SK}	is the stress correction factor, relevant to the notched test piece.
Y_{Nk}	is the life factor for tooth-root stress, relevant to the notched test piece. It is used in order to take into account the higher load bearing capacity for a limited number of load cycles.
$Y_{\delta\text{ rel } k}$	is the relative notch sensitivity factor. It is the quotient of the notch sensitivity factor of the gear of interest divided by the notched test piece factor (see clause 11). It enables the influence of the notch sensitivity of the material to be taken into account.
$Y_{R\text{ rel } k}$	is the relative roughness factor. It is the quotient of the tooth-root fillet roughness factor of the gear of interest divided by the notched test piece factor (see clause 12). It enables relevant surface roughness of tooth-root fillet influences to be taken into account.

Other relevant terms and symbols are defined in 4.2.2.

The values of the factors related to the notched test piece ($\sigma_{k\text{ lim}}$, Y_{SK} and Y_{Nk}) shall be determined by tests or to be taken from literature (see 9.2). Evaluations of $\sigma_{k\text{ lim}}$, and all corresponding influence factors, shall be based on values of static stress and reference stress appropriate to the notched test piece.

The influence factors shall be determined in accordance with 4.2.2. and 4.2.3, using the more detailed method B_k or one of the more simplified methods, C_k or D_k .

4.2.4.2 σ_{FP} for limited life

The value of σ_{FP} shall be determined in accordance with the procedure described in 4.2.3.

4.2.5 Permissible bending stress, σ_{FP} : Methods B_p , C_p and D_p

4.2.5.1 σ_{FP} for static stress and reference stress

For these methods the permissible bending stress is calculated on the basis of the strength of a plain, polished test piece from the following equation:

$$\sigma_{FP} = \frac{\sigma_{p \text{ lim}} Y_{NP}}{S_{F \text{ min}}} Y_{\delta} Y_R Y_X = \frac{\sigma_{FG}}{S_{F \text{ min}}} \quad \dots (9)$$

where

- $\sigma_{p \text{ lim}}$ is the nominal plain-bar stress number (bending). It is the bending stress limit value of the plain bar test piece relevant to its material and heat treatment in relation to its dimensions. Differences between the properties of the heat treated materials of the test piece and gear of interest, due to conditions of manufacture, should be taken into consideration, as discussed in the case of $\sigma_{k \text{ lim}}$ in 4.2.4.
- Y_{NP} is the life factor for tooth-root stress, relevant to the plain, polished test piece. It is used in order to take into account the higher load bearing capacity for a limited number of cycles.
- Y_{δ} is the notch sensitivity factor of the gear of interest, as related to a plain, polished test piece (see clause 11). It enables the influence of the notch sensitivity of the material to be taken into account.
- Y_R is the surface factor of the gear of interest, as related to the plain, polished test piece. It enables relevant surface roughness influences to be taken into account.

Other relevant terms and symbols are defined in 4.2.2.

Evaluations of $\sigma_{p \text{ lim}}$ and Y_{NP} , for plain test pieces, shall be based on tests or obtained from the literature (see 9.2). Evaluations of $\sigma_{p \text{ lim}}$ and all corresponding influence factors shall be based on values of static stress and reference stress.

The influence factors shall be determined in accordance with 4.2.2 and 4.2.3, using the more detailed method B_p or one of the more simplified methods, C_p or D_p .

4.2.5.2 σ_{FP} for limited life

The value of σ_{FP} shall be determined in accordance with the procedure described in 4.2.3 and 4.2.4.

4.3 Safety factor for bending strength (safety against tooth breakage), S_F

Calculate S_F separately for pinion and wheel:

$$S_F = \frac{\sigma_{FG}}{\sigma_F} \geq S_{F \text{ min}} \quad \dots (10)$$

a) Method B

The values of σ_{FG} for reference stress and static stress are calculated in accordance with 4.2.2 a) and b), using equation (4). For limited life, σ_{FG} is determined in accordance with 4.2.3. σ_F is derived from equations (1) and (2).

b) Methods C and D

The values of σ_{FG} for reference stress and static stress are calculated in accordance with 4.2.2 a) and b), using equation (4). For limited life σ_{FG} is to be determined in accordance with 4.2.3. σ_F is derived from equations (1) and (3).

c) Methods B_k , C_k and D_k

These procedures follow the methods described in 4.3 a) or b), with σ_{FG} calculated in accordance with 4.2.4.

d) Methods B_p , C_p and D_p

These procedures follow the methods described in 4.3 a) or b), with σ_{FG} calculated in accordance with 4.2.5.

The values of tooth-root stress limit σ_{FG} , of permissible stress σ_{FP} and tooth-root stress σ_F , may each be determined by different methods. The method used for each value shall be stated in the calculation report.

NOTE 4 Safety factors in accordance with 4.3 are relevant to transmissible torque. See ISO 6336-1, subclause 4.1.3 for comments on numerical values for the minimum safety factor and risk of damage.

5 Form factors, Y_F and Y_{Fa} ; Tip factor, Y_{FS}

5.1 General

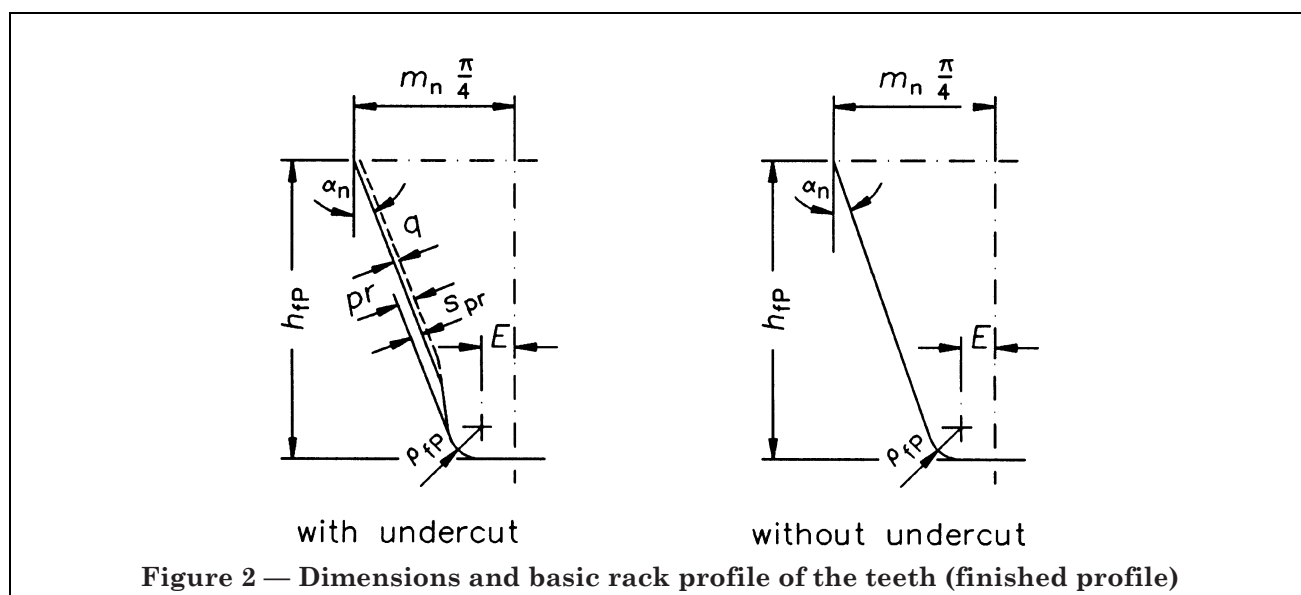
Y_F and Y_{Fa} are the factors by means of which the influence of tooth form on nominal bending stress is taken into account. See 4.1.1 for principles, assumptions and details of use. Y_F is relevant to application of load at the outer point of single pair tooth contact (method B), and Y_{Sa} to application of load at the tooth tip (method C).

The chord between the points at which the 30° tangents contact the root fillets defines the section to be used as the basis for calculation (see Figure 3 to Figure 6).

Determination of the values of Y_F , Y_S , Y_{FS} , Y_{Fa} and Y_{Sa} is based on the nominal tooth form with the rack shift coefficient x . Values can also be obtained from Figure 9 to Figure 32. In general, the effect of reduction of tooth thickness on the tooth bending strength of finished-cut cylindrical gears may be ignored. Since the tooth-roots of ground or shaved gear teeth are usually generated by cutting tools such as hobs, their shapes and dimensions are usually determined by the cutting depth settings.

Because of material allowances for finishing processes such as profile grinding, it is usually the case that the depth setting of the roughing tool, relative to the gear axis, includes the amount of nominal rack shift $x m_n$ plus a tolerance designed to ensure that the finishing allowance will be greater instead of less than the requisite minimum. Because of this, calculated values of tooth-root stresses usually err on the side of safety.

If the tooth thickness deviation near the root results in a thickness reduction of more than $0,05 m_n$, this shall be taken into account in the stress calculation, by taking the generated profile relative to the rack shift amount $m_n x_E$ instead of the nominal profile.



The equations in this part of ISO 6336 apply to all basic rack profiles (see Figure 2) with and without undercut, but with the following restrictions:

- a) the contact point of the 30° tangent shall lie on the tooth-root fillet generated by the root fillet of the basic rack;

- b) the basic rack profile of the gearing shall have a root fillet of with a radius $\rho_{FP} > 0$;
- c) the teeth shall be generated using tools such as hobs or rack type cutters;
- d) since calculated ratings refer to finished tooth forms, profile grinding and similar allowances, including tooth thickness allowances, can be neglected. In practice it can be assumed that the dimensions of the basic rack of the tool are the same as those of the counterpart basic rack of the gear.

The above comments apply to straight spur and helical gears. Values Y_F , Y_{Fa} , and Y_{FS} are determined for the virtual spur gears of helical gears; the virtual number of teeth z_n can be determined using equation (19) or (20), or an approximate value of z_n can be read from Figure 8. Y_F , Y_{Fa} and Y_{FS} are determined separately for the wheel and pinion.

5.2 Calculation of the form factor, Y_F : Method B

The determination of the normal chordal dimension S_{Fn} of the tooth-root critical section and the bending moment arm h_{Fe} relevant to load application at the outer point of single pair external gear tooth contact for method B is shown in Figure 3. The following equation uses the symbols which are illustrated in Figure 3:

$$Y_F = \frac{\frac{6 h_{Fe}}{m_n} \cos \alpha_{Fen}}{\left(\frac{S_{Fn}}{m_n}\right)^2 \cos \alpha_n} \quad \dots (11)$$

In order to evaluate precise values of h_{Fe} , S_{Fn} and α_{Fen} , it is first necessary to derive a value of θ which is reasonably accurate, usually after two or three iterations of equation (15). Determination of Y_F by graphical means is not recommended.

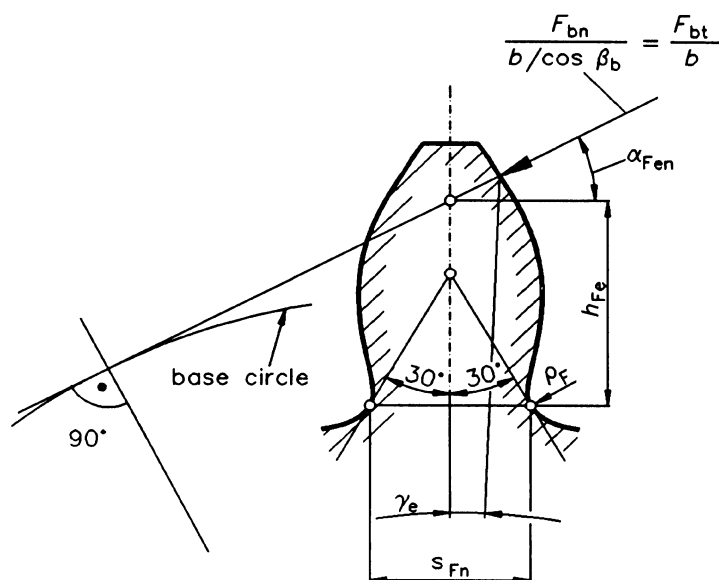


Figure 3 — Determination of the normal chordal dimensions of the tooth root critical section for Method B.

5.2.1 External gearing²⁾

First determine the auxiliary values for equation (11):

$$E = \frac{\pi}{4} m_n - h_{fp} \tan \alpha_n + \frac{s_{pr}}{\cos \alpha_n} - (1 - \sin \alpha_n) \frac{\rho_{fp}}{\cos \alpha_n} \quad \dots (12)$$

With

$$s_{pr} = pr - q \text{ (see Figure 2).}$$

$$s_{pr} = 0 \text{ when gears are not undercut.}$$

$$G = \frac{\rho_{fp}}{m_n} - \frac{h_{fp}}{m_n} + x \quad \dots (13)$$

$$H = \frac{2}{z_n} \left(\frac{\pi}{2} - \frac{E}{m_n} \right) - \frac{\pi}{3} \quad \dots (14)$$

$$\theta = \frac{2G}{z_n} \tan \theta - H \quad \dots (15)$$

The value $\theta = \pi/6$ may be used as a seed value in the iteration of the transcendental equation (15). Generally the function converges after two iterations.

a) Tooth-root normal chord S_{Fn} :

$$\frac{S_{Fn}}{m_n} = z_n \sin \left(\frac{\pi}{3} - \theta \right) + \sqrt{3} \left(\frac{G}{\cos \theta} - \frac{\rho_{fp}}{m_n} \right) \quad \dots (16)$$

b) Radius of root fillet ρ_F (Figure 3):

$$\frac{\rho_F}{m_n} = \frac{\rho_{fp}}{m_n} + \frac{2G^2}{\cos \theta (z_n \cos^2 \theta - 2G)} \quad \dots (17)$$

c) Bending moment arm h_{Fe} :

$$\beta_b = \arccos \sqrt{1 - (\sin \beta \cos \alpha_n)^2} = \arcsin (\sin \beta \cos \alpha_n) \quad \dots (18)$$

$$z_n = \frac{z}{\cos^2 \beta_b \cos \beta} \quad \dots (19)$$

Approximation:

$$z_n \approx \frac{z}{\cos^3 \beta} \quad \dots (20)$$

$$\epsilon_{\alpha n} = \frac{\epsilon_\alpha}{\cos^2 \beta_b} \quad \dots (21)$$

$$d_n = \frac{d}{\cos^2 \beta_b} = m_n z_n \quad \dots (22)$$

$$p_{bn} = \pi m_n \cos \alpha_n \quad \dots (23)$$

$$d_{bn} = d_n \cos \alpha_n \quad \dots (24)$$

²⁾ If the tip of the tooth has been rounded or chamfered, it is necessary to replace the tip diameter d_a in the calculation by d_{Na} the "effective tip diameter", d_{Na} is the diameter of a circle near the tip cylinder, containing limits of the usable gear flanks.

$$d_{an} = d_n + d_a - d \quad \dots (25)$$

$$d_{en} = 2 \frac{z}{|z|} \sqrt{\left[\sqrt{\left(\frac{d_{an}}{2} \right)^2 - \left(\frac{d_{bn}}{2} \right)^2} - \frac{\pi d \cos \beta \cos \alpha_n}{|z|} (\epsilon_{an} - 1) \right]^2 + \left(\frac{d_{bn}}{2} \right)^2} \quad \dots (26)$$

The number z is positive for external gears and negative for internal gears.

$$\alpha_{en} = \arccos \left(\frac{d_{bn}}{d_{en}} \right) \quad \dots (27)$$

$$\gamma_e = \frac{0,5 \pi + 2 \tan \alpha_n x}{z_n} + \text{inv} \alpha_n - \text{inv} \alpha_{en} \quad \dots (28)$$

$$\alpha_{Fen} = \alpha_{en} - \gamma_e = \tan \alpha_{en} - \text{inv} \alpha_n - \frac{0,5 \pi + 2 \tan \alpha_n x}{z_n} \quad \dots (29)$$

$$\frac{h_{Fe}}{m_n} = \frac{1}{2} \left[(\cos \gamma_e - \sin \gamma_e \tan \alpha_{Fen}) \frac{d_{en}}{m_n} - z_n \cos \left(\frac{\pi}{3} - \theta \right) - \frac{G}{\cos \theta} + \frac{\rho_{fP}}{m_n} \right] \quad \dots (30)$$

5.2.2 Internal gearing³⁾

It is assumed that the value of the form factor of a special rack can be substituted as an approximate value of the form factor of an internal gear. The profile of such a rack should be a version of the basic rack profile, so modified that it would generate the normal profile, including tip and root circles, of an exact counterpart gear of the internal gear. The load direction angle is α_n (see Figure 4). Equations are provided below for the determination of values to be substituted in equation (11).

a) Tooth-root normal chord S_{Fn2} :

$$\frac{S_{Fn2}}{m_n} = 2 \left[\frac{\pi}{4} + \frac{h_{fP2} - \rho_{fP2}}{m_n} \tan \alpha_n + \frac{\rho_{fP2} - s_{pr}}{m_n \cos \alpha_n} - \frac{\rho_{fP2}}{m_n} \cos \frac{\pi}{6} \right] \quad \dots (31)$$

b) Bending moment arm h_{Fe2} :

$$\frac{h_{Fe2}}{m_n} = \frac{d_{en2} - d_{fn2}}{2 m_n} - \left[\frac{\pi}{4} + \left(\frac{h_{fP2}}{m_n} - \frac{d_{en2} - d_{fn2}}{2 m_n} \right) \tan \alpha_n \right] \tan \alpha_n - \frac{\rho_{fP2}}{m_n} \left(1 - \sin \frac{\pi}{6} \right) \quad \dots (32)$$

where

d_{en2} is derived from equation (25) with parameters having 2 added to the subscripts

d_{fn2} derivation is analogous to that of d_{an} (equation (25)); $d_{fn2} - d_{f2} = d_{n2} - d_2$.

and:

$$h_{fP2} = \frac{d_{n2} - d_{fn2}}{2} \quad \dots (33)$$

³⁾ If the tip of the tooth has been rounded or chamfered, it is necessary to replace the tip diameter d_a in the calculation by d_{Na} the "effective tip diameter", d_{Na} is the diameter of a circle near the tip cylinder, containing limits of the usable gear flanks.

c) Root fillet radius, ρ_{F2}

When the root fillet radius of an internal gear tooth, ρ_{F2} , is known, it shall be used for ρ_{FP2} . When it is not known, the following approximation may be used:

$$\rho_{F2} = \rho_{FP2} = 0,15 m_n \quad \dots (34)$$

This is to be validated.

$$\rho_{FP2} = \frac{c_p}{1 - \sin \alpha_n} = \frac{h_{f2} - h_{NF2}}{1 - \sin \alpha} = \frac{d_{NF2} - d_{f2}}{2(1 - \sin \alpha_n)} \quad \dots (35)$$

where

d_{NF2} represents the diameter of a circle near the tooth-roots, containing the limits of the usable flanks of an internal gear or the larger external gear of a mating pair. For internal gearing, diameters have a negative sign.

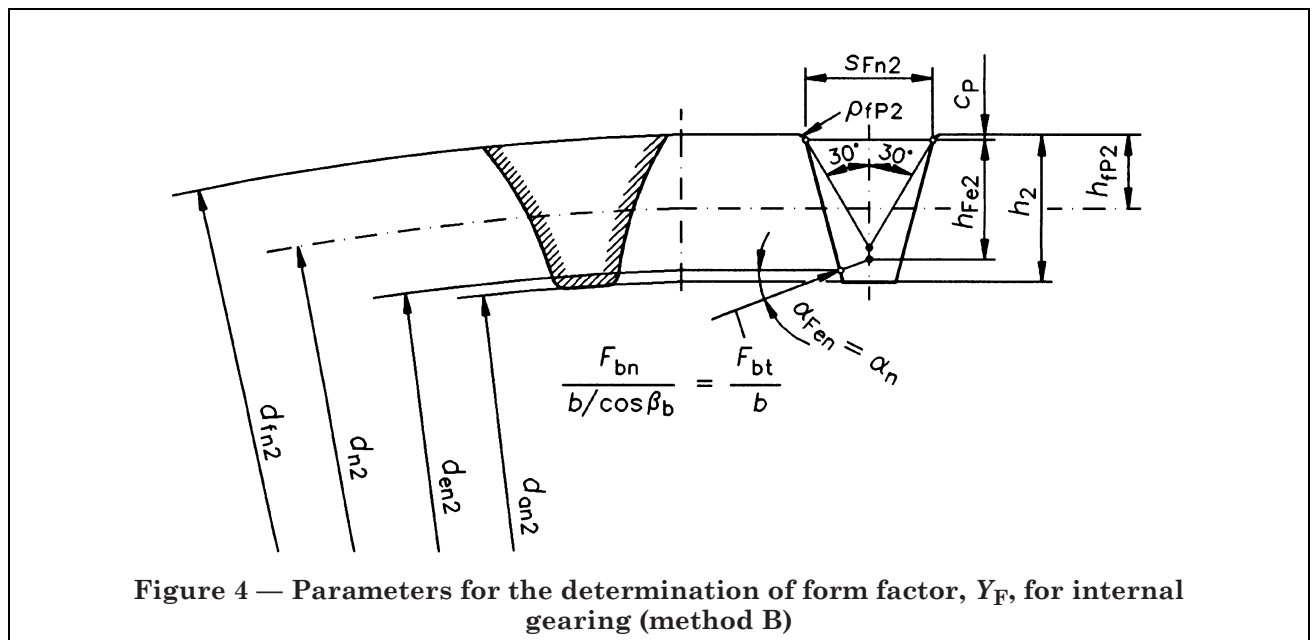


Figure 4 — Parameters for the determination of form factor, Y_F , for internal gearing (method B)

5.3 Form factor, Y_{Fa} : Method C

The determination of the normal chordal dimension, S_{Fn} , of the tooth-root critical section and the bending moment arm, h_{Fa} , relevant to load application at the external gear tooth tip for method C is shown in Figure 5.

Y_{Fa} shall be used together with Y_{ϵ} , and is only valid for gearing having $\epsilon_{\alpha n} < 2$.

Equation (36) uses the symbols as illustrated in Figure 5:

$$Y_{Fa} = \frac{6 \frac{h_{Fa}}{m_n} \cos \alpha_{Fan}}{\left(\frac{S_{Fn}}{m_n} \right)^2 \cos \alpha_n} \quad \dots (36)$$

The parameters required for the calculation of Y_{Fa} can be determined using the iterative method explained in 5.2.

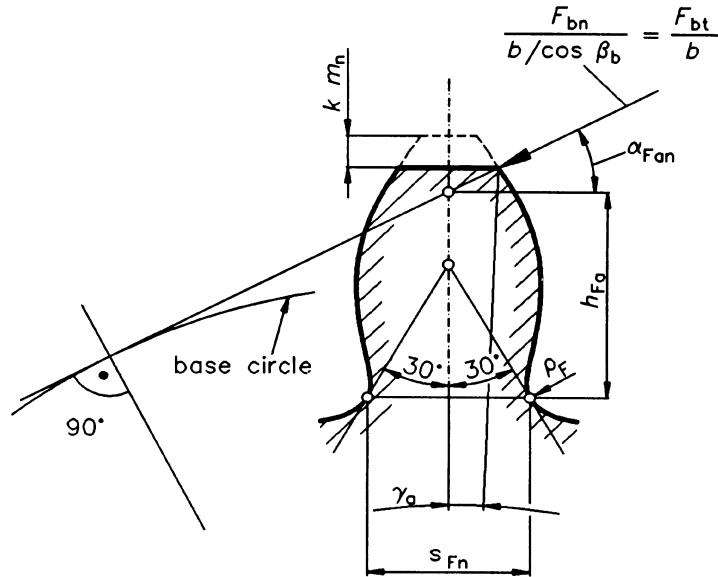


Figure 5 — Determination of the normal chordal dimensions of the tooth root critical section for Method C

5.3.1 External gearing

5.3.1.1 Graphical values

The factor Y_{Fa} can be read from Figure 9 to Figure 16 for a series of common basic rack profiles as a function of the virtual number of teeth z_n and the rack shift coefficient x . The graphs have been calculated using equations provided in 5.3.1.2. Figure 9 to Figure 16 are relevant to gear wheels without addendum reduction and without tip chamfer. The bending moment arm values h_{Fa} for gears having truncated addenda or chamfered tips are slightly smaller than those used for the curves. Because of this, the curve values tend to err on the side of safety.

Use Figure 8 for graphical values of z_n .

5.3.1.2 Determination by calculation

Determine the following values to be substituted in equation (36):

- Tooth-root normal chord, S_{Fn} : obtain from equation (16) with values from equations (12) to (15).
- Root fillet radius, ρ_F : obtain from equation (17) with values from equations (12) to (15).
- Bending moment arm, h_{Fa} :

$$\alpha_{an} = \arccos \left(\frac{d_{bn}}{d_{an}} \right) = \arccos \left[\frac{\cos \alpha_n}{1 + \frac{(d_a - d)}{m_n z_n}} \right] \quad \dots (37)$$

$$\gamma_a = \frac{0,5 \pi + 2 \tan \alpha_n x}{z_n} + \text{inv} \alpha_n - \text{inv} \alpha_{an} \quad \dots (38)$$

$$\alpha_{Fan} = \alpha_{an} - \gamma_a = \tan \alpha_{an} - \text{inv} \alpha_n - \frac{0,5 \pi + 2 \tan \alpha_n x}{z_n} \quad \dots (39)$$

$$\frac{h_{Fa}}{m_n} = \frac{1}{2} \left[(\cos \gamma_a - \sin \gamma_a \tan \alpha_{Fan}) \frac{d_{an}}{m_n} - z_n \cos \left(\frac{\pi}{3} - \theta \right) - \frac{G}{\cos \theta} + \frac{\rho_F}{m_n} \right]$$

$$\frac{h_{Fa}}{m_n} = 0,5 z_n \left[\frac{\cos \alpha_n}{\cos \alpha_{Fan}} - \cos \left(\frac{\pi}{3} - \theta \right) \right] + 0,5 \left(\frac{p_{fp}}{m_n} - \frac{G}{\cos \theta} \right)$$

where

- z_n is the virtual number of teeth; see equations (19) and (20);
- G see equation (13);
- θ see equation (15);
- d_{an} see equation (25);
- d_{bn} see equation (24);
- d_n see equation (22).

5.3.2 Internal gearing

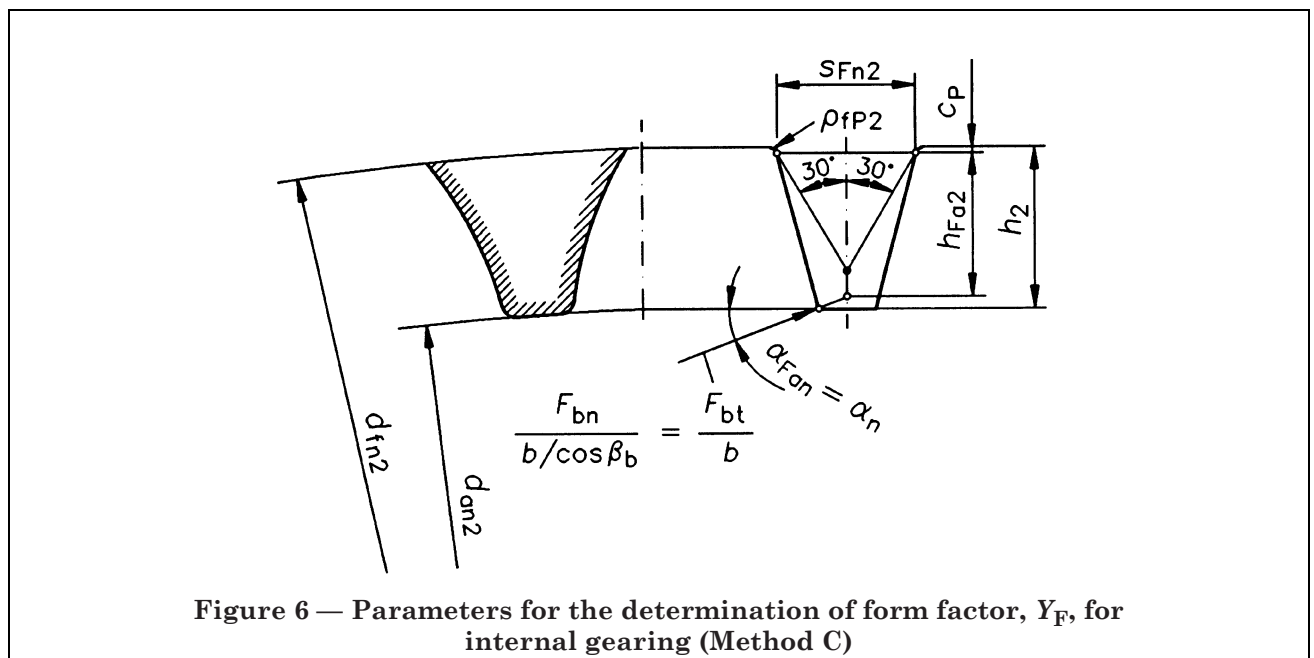


Figure 6 — Parameters for the determination of form factor, Y_F , for internal gearing (Method C)

It is assumed that the value of the form factor of a special rack can be substituted as an approximate value of the form factor of an internal gear. The profile of such a rack should be a version of the basic rack profile, so modified that it would generate the normal profile, including tip and root circles, of an exact counterpart gear of the internal gear. The tip load angle is α_n , see Figure 6.

5.3.2.1 Graphical values

The factor Y_{Fa} can be read from Figure 9 to Figure 16 in which charts for a number of common basic rack profiles as a function of the virtual number of teeth z_n and the profile shift coefficient x are provided. Corresponding to equation (34), the value of Y_{Fa} for internal gears is given in each figure as an approximation. Equation (34) shall be validated in this case.

The numerical values were calculated using the equations in 5.3.2.2 for (unmodified) teeth with the height of the basic rack profile (see 5.3.1.1).

5.3.2.2 Determination by calculation

Determine the following values to be substituted in equation (36).

- a) Tooth-root normal chord, S_{Fn2} : this is obtained from equation (31).