

Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements

ICS 21.200

National foreword

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INTERNATIONAL STANDARD

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Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements

*Calcul de la capacité de charge des engrenages cylindriques à dentures
droite et hélicoïdale — Application aux engrenages grande vitesse et aux
engrenages d'exigences similaires*



Reference number
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 9084 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

Annexes A and B form a normative part of this International Standard. Annex C is for information only.

Introduction

Procedures for the calculation of the load capacity of general spur and helical gears with respect to pitting and bending strength appear in ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5. This International Standard is derived from ISO 6336-1, ISO 6336-2 and ISO 6336-3 by the use of specific methods and assumptions which are considered to be applicable to industrial gears. Its application requires the use of allowable stresses and material requirements which are to be found in ISO 6336-5.

Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements

1 Scope

The formulae specified in this International Standard are intended to establish a uniformly acceptable method for calculating the pitting resistance and bending strength capacity of high speed gears and gears of similar requirements with straight or helical teeth.

The rating formulae in this International Standard are not applicable to other types of gear tooth deterioration, such as plastic yielding, micropitting, scuffing, case crushing, welding and wear, and are not applicable under vibratory conditions where there may be an unpredictable profile breakdown. The bending strength formulae are applicable to fractures at the tooth fillet, but are not applicable to fractures on the tooth working profile surfaces, failure of the gear rim, or failures of the gear blank through web and hub. This International Standard does not apply to teeth finished by forging or sintering. It is not applicable to gears which have a poor contact pattern.

This International Standard provides a method by which different gear designs can be compared. It is not intended to assure the performance of assembled drive gear systems. It is not intended for use by the general engineering public. Instead, it is intended for use by the experienced gear designer who is capable of selecting reasonable values for the factors in these formulae based on knowledge of similar designs and awareness of the effects of the items discussed.

CAUTION — The user is cautioned that the calculated results of this International Standard should be confirmed by experience.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*.

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*¹⁾.

ISO 6336-1:1996, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*.

ISO 6336-2:1996, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting)*.

ISO 6336-3:1996, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength*.

ISO 6336-5:1996, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*.

¹⁾ This was corrected and reprinted in 1997.

3 Terms and definitions

For the purposes of this International Standard, the terms and definitions given in ISO 1122-1 apply. For the symbols, see Table 1.

Table 1 — Symbols and abbreviations used in this International Standard

Symbol	Description or term	Unit
a	centre distance ^a	mm
b	facewidth	mm
b_B	facewidth of an individual helix of a double helical gear	mm
B	total facewidth of a double helical gear including gap width	mm
c_γ	mean value of mesh stiffness per unit facewidth	N/(mm · μm)
c'	maximum tooth stiffness of one pair of teeth per unit facewidth (single stiffness)	N/(mm · μm)
$d_{a1,2}$	tip diameter of pinion (or wheel)	mm
$d_{b1,2}$	base diameter of pinion (or wheel)	mm
$d_{f1,2}$	root diameter of pinion (or wheel)	mm
d_i	internal diameter of pinion shaft	mm
$d_{w1,2}$	pitch diameter of pinion (or wheel)	mm
$d_{1,2}$	reference diameter of pinion (or wheel)	mm
$f_{f\alpha}$	profile form deviation (the value for the total profile deviation F_α may be used alternatively for this, if tolerances complying with ISO 1328-1 are used)	μm
f_{ma}	mesh misalignment due to manufacturing deviations	μm
f_{pb}	transverse base pitch deviation (the values of f_{pt} may be used for the calculations in accordance with ISO 6336-1, using tolerances complying with ISO 1328-1)	μm
f_{sh}	helix deviation due to elastic deflections	μm
$f_{H\beta}$	tooth alignment deviation	μm
g_α	path length of contact	mm
h	tooth depth	mm
h_{aP}	addendum of basic rack of cylindrical gear	mm
h_{fP}	dedendum of basic rack of cylindrical gear	mm
h_{Fe}	bending moment arm for load application at the outer point of single pair tooth contact	mm
l	bearing span	mm
m^*	relative individual gear mass per unit facewidth referenced to line of action	kg/mm
m_n	normal module	mm
m_{red}	reduced gear pair mass per unit facewidth referenced to line of action	kg/mm
m_t	transverse module	mm
$n_{1,2}$	rotation speed of pinion (or wheel)	min ⁻¹
n_{E1}	resonance speed of pinion	min ⁻¹
pr	protuberance of the tool	mm
p_{bn}	normal base pitch	mm
p_{bt}	transverse base pitch	mm
q	finishing stock allowance	mm
q_s	notch parameter	—

Table 1 — Symbols and abbreviations used in this International Standard (continued)

Symbol	Description or term	Unit
s_{pr}	residual fillet undercut	mm
s_{Fn}	tooth-root chord at the critical section	mm
s_R	rim thickness	mm
u	gear ratio ^a $ u = z_2/z_1 \geq 1$	—
v	circumferential velocity (without subscript: at reference circle $\approx$ circumferential velocity at working pitch circle)	m/s
$x_{1,2}$	rack shift coefficient of pinion (or wheel)	—
y_β	running-in allowance (equivalent misalignment)	μm
z_n	virtual number of teeth of a helical gear	—
$z_{1,2}$	number of teeth of pinion (or wheel) ^a	—
C_a	tip relief	μm
C_B	basic rack factor	—
C_R	gear blank factor	—
E	modulus of elasticity, Young's modulus	N/mm^2
F_m	mean transverse load at the reference cylinder ($= F_t K_A K_v$)	N
F_t	(nominal) transverse tangential load at reference cylinder	N
$F_{t\text{eq}}$	equivalent tangential load at reference cylinder	N
F_β	total helix deviation	μm
$F_{\beta x}$	initial equivalent misalignment (before running-in)	μm
J^*	moment of inertia per unit facewidth	$\text{kg}\cdot\text{mm}^2/\text{mm}$
K_v	dynamic factor	—
K_A	application factor	—
$K_{F\alpha}$	transverse load factor (tooth-root stress)	—
$K_{F\beta}$	face load factor (tooth-root stress)	—
$K_{H\alpha}$	transverse load factor (contact stress)	—
$K_{H\beta}$	face load factor (contact stress)	—
K_γ	mesh load factor (takes into account the uneven distribution of the load between meshes for multiple transmission paths)	—
$M_{1,2}$	auxiliary values for the determination of $Z_{B,D}$	—
N	resonance ratio	—
N_L	number of cycles	—
P	transmitted power	kW
Ra	arithmetic mean roughness value (as specified in ISO 4287)	μm
Rz	mean peak-to-valley roughness (as specified in ISO 4287)	μm
S_F	safety factor (tooth breakage)	—
$S_{F\text{min}}$	minimum safety factor (tooth breakage)	—
S_H	safety factor (pitting)	—
$S_{H\text{min}}$	minimum safety factor (pitting)	—
$T_{1,2}$	nominal torque at the pinion (or wheel)	Nm
Y_F	form factor, for the influence on nominal tooth-root stress with load applied at the outer point of single pair tooth contact	—

Table 1 — Symbols and abbreviations used in this International Standard (continued)

Symbol	Description or term	Unit
$Y_{R\ rel\ T}$	relative surface factor	—
Y_S	stress correction factor	—
Y_X	size factor (tooth-root)	—
Y_β	helix angle factor (tooth-root)	—
$Y_{\delta\ rel\ T}$	relative notch sensitivity factor	—
Z_V	speed factor	—
$Z_{B,D}$	single pair tooth contact factors for the pinion (or wheel)	—
Z_E	elasticity factor	$\sqrt{N/mm^2}$
Z_H	zone factor	—
Z_L	lubricant factor	—
Z_R	roughness factor (pitting)	—
Z_W	work-hardening factor	—
Z_X	size factor (pitting)	—
Z_β	helix angle factor (pitting)	—
Z_ϵ	contact ratio factor (pitting)	—
α_n	normal pressure angle	°
α_t	transverse pressure angle	°
α_{wt}	transverse pressure angle at the pitch cylinder	°
α_P	pressure angle of the basic rack for cylindrical gears	°
β	helix angle (without subscript — at the reference cylinder)	°
β_b	base helix angle	°
ϵ_α	transverse contact ratio	—
$\epsilon_{\alpha n}$	transverse contact ratio of virtual spur gear pairs	—
ϵ_β	axial overlap ratio	—
ϵ_γ	total contact ratio ($\epsilon_\gamma = \epsilon_\alpha + \epsilon_\beta$)	—
κ_β	running-in factor (equivalent misalignment)	—
ρ_{fP}	root fillet radius of the basic rack for cylindrical gears	mm
ρ_F	tooth-root fillet radius at the critical section	mm
σ_F	tooth-root stress	N/mm ²
$\sigma_{F\ lim}$	nominal stress number (bending)	N/mm ²
σ_{FE}	allowable stress number (bending)	N/mm ²
σ_{FG}	tooth-root stress limit	N/mm ²
σ_{FP}	permissible bending stress	N/mm ²
σ_{F0}	nominal contact stress	N/mm ²
σ_H	calculated contact stress	N/mm ²
$\sigma_{H\ lim}$	allowable stress number (contact)	N/mm ²
σ_{HG}	modified allowable stress number ($= \sigma_{HP} S_{H\ min}$)	N/mm ²
σ_{HP}	permissible contact stress	N/mm ²
$\omega_{1,2}$	angular velocity of pinion (or wheel)	rad/s

^a For external gear pairs a , u , z_1 and z_2 are positive; for internal gear pairs a , u and z_2 are negative with z_1 positive.

4 Application

4.1 Design, specific applications

4.1.1 General

Gear designers shall recognize that requirements for different applications vary considerably. Use of the procedures of this International Standard for specific applications demands a careful appraisal of all applicable considerations, in particular:

- the allowable stress of the material and the number of load repetitions;
- the consequences of any percentage of failure (failure rate);
- the appropriate safety factor.

Design considerations to prevent fractures emanating from stress raisers in the tooth flank, tip chipping and failures of the gear blank through the web or hub should be analysed by general machine design methods.

Any variances according to the following shall be reported in the calculation statement.

- a) If a more refined method of calculation is desired or if compliance with the restrictions given in 4.1 is for any reason impractical, relevant factors may be evaluated according to the basic standard or another application standard.
- b) Factors derived from reliable experience or test data may be used instead of individual factors according to this International Standard. Concerning this, the criteria for Method A in ISO 6336-1:1996, 4.1.8.1, are applicable.

In other respects, rating calculations shall be strictly in accordance with this International Standard if stresses, safety factors, etc. are to be classified as being in accordance with this International Standard.

This International Standard recognizes all high speed gears and gears of similar requirements besides high speed, and special purpose gear units used in petroleum, chemical and gas industries. For these ISO 13691 may apply.

This International Standard is applicable when the wheel blank, shaft/hub connections, shafts, bearings, housings, threaded connections, foundations and couplings conform to the requirements regarding accuracy, load capacity and stiffness which form the basis for the calculation of the load capacity of gears.

Although the method described in this International Standard is mainly intended for recalculation purposes, by means of iteration it can also be used to determine the load capacities of gears. The iteration is accomplished by selecting a load and calculating the corresponding safety factor against pitting, S_{H1} , for the pinion. If S_{H1} is greater than $S_{H \min}$ the load is increased, if it is smaller than $S_{H \min}$ the load is reduced. This is done until the load chosen corresponds to $S_{H1} = S_{H \min}$. The same method is used for the wheel ($S_{H2} = S_{H \min}$) and also for the safety factors against tooth breakage, $S_{F1} = S_{F2} = S_{F \min}$.

4.1.2 Gear data

This International Standard is applicable within the following constraints.

- a) Types of gear
 - external and internal, involute spur, helical and double helical gears;
 - for double helical gears, it is assumed that the total tangential load is evenly distributed between the two helices; if this is not the case (e.g. due to externally applied axial forces), this shall be taken into account; the two helices are treated as two single helical gears in parallel;
 - planetary and other gear trains with multiple transmission paths.

b) Range of speeds

n_1 more than or equal to $3\,600\text{ min}^{-1}$ (synchronous speed of two-pole motor at 60 Hz current frequency); it is also applicable for gears of high accuracy needed for special requirements at lower speeds.

c) Gear accuracy

accuracy grade 6 or better according to ISO 1328-1 (affects K_v , $K_{H\alpha}$, $K_{H\beta}$ and $K_{F\beta}$).

d) Range of the transverse contact ratios of virtual spur gear pairs

$1,2 < \epsilon_\alpha < 2,5$ (affects c' , c_γ , K_v , $K_{H\beta}$, $K_{F\alpha}$, $K_{H\alpha}$ and $K_{F\beta}$).

e) Range of helix angles

β less than or equal to 30° (affects c' , c_γ , K_v , $K_{H\beta}$ and $K_{F\beta}$).

f) Basic racks

no restriction²⁾, but see d).

4.1.3 Pinion and pinion shaft

This International Standard is applicable to pinions integral with shafts or bored pinions mounted symmetrically between their bearings. It is assumed that the bored pinions will be mounted on solid shafts or on hollow shafts with $d_i/d_{shi} < 0,5$ (this affects $K_{H\beta}$ and $K_{F\beta}$).

4.1.4 Wheel blank, wheel rim

This International Standard is applicable when s_R , the thickness of the wheel rim under the tooth roots of internal and external gears, is $> 3,5 m_n$.

4.1.5 Materials

These include steel materials (affects Z_E , $\sigma_{H\lim}$, σ_{FE} , K_v , $K_{H\beta}$, $K_{F\beta}$, $K_{H\alpha}$ and $K_{F\alpha}$). For materials and their abbreviations used in this International Standard, see Table 2. For information on other materials, see ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5.

Table 2 — Materials

Material	Abbreviation
Through-hardening steel, alloy or carbon, through hardened ($\sigma_B \geq 800\text{ N/mm}^2$)	V
Case-hardened steel, case hardened	Eh
Steel, flame- or induction-hardened	IF
Nitriding steel, nitrided	NT (nitr.)
Through-hardening and case-hardening steel, nitrided	NV (nitr.)
Through-hardening and case-hardening steel, nitrocarburized	NV (nitrocar.)

4.1.6 Lubrication

The calculation procedures are valid subject to the condition that the gears are spray lubricated at all times of operation with a lubricant approved by the manufacturer/designer of the gears and the lubricant is sprayed at a tem-

2) For all practical purposes, it may be assumed that the proportions of the basic rack of the tool are equal to those of the basic rack of the gear.

perature and rate which ensures that temperatures assumed for purposes of calculations are not exceeded (affects lubricant film formation, i.e. factors Z_L , Z_V and Z_R).

4.2 Safety factors

It is necessary to distinguish between the safety factor relative to pitting, S_H , and the safety factor relative to tooth breakage, S_F .

For a given application, adequate gear load capacity is demonstrated by the computed values of S_H and S_F being equal to or greater than the values $S_{H \min}$ and $S_{F \min}$, respectively.

Choice of the value of a safety factor should be based on the degree of confidence in the reliability of the available data and the consequences of possible failures.

Important factors to be considered are the following:

- the allowable stress numbers used in the calculation are valid for a given probability of failure (the material values in ISO 6336-5 are valid for 1 % probability of damage);
- the specified quality and the effectiveness of quality control at all stages of manufacture;
- the accuracy of specification of the service duty and external conditions;
- tooth breakage is often considered to be a greater hazard than pitting.

Therefore, the chosen value for $S_{F \min}$ should to be greater than the value chosen for $S_{H \min}$. It is recommended that the minimum values of the safety factors should be agreed upon between the purchaser and the manufacturer.

For calculation of the actual safety factor, see 6.1.5 (S_H , pitting) and 7.1.4 (S_F , tooth breakage).

4.3 Input data

The following data shall be available for the calculations:

a) gear data:

a , z_1 , z_2 , m_n , d_1 , d_{a1} , d_{a2} ³⁾, b , x_1 , x_2 , α_n , β , ϵ_α , ϵ_β , basic rack profile;

b) design and manufacturing data:

C_{a1} , C_{a2} , Ra_1 , Ra_2 , Rz_1 , Rz_2 ;

materials, material hardnesses and heat treatment details; material quality grades, gear accuracy grades, bearing span, gear dimensions, polar or mass moments of inertia of pinion and wheel and when applicable, profile and helix modification;

c) operating data:

P or T or F_t , n_1 , v_1 , working characteristics of driving and driven machines.

Requisite geometrical data can be calculated according to national standards.

Information to be exchanged between the manufacturer and purchaser should include data specifying material preferences, lubrication, safety factor and externally applied forces due to vibrations and overloads (application factor).

3) When tooth tips are chamfered or rounded, substitute $d_{N1,2}$ for $d_{a1,2}$.

4.4 Numerical equations

The units listed in Clause 3 shall be used in all calculations. Information which will facilitate the use of this International Standard is provided in annex C of ISO 6336-1:1996.

5 Influence factors

5.1 General

The influence factors K_v , $K_{H\alpha}$, $K_{H\beta}$, $K_{F\alpha}$ and $K_{F\beta}$ are all dependent on the tooth load. Initially this is the applied load (nominal tangential load multiplied by the application factor).

These factors are also interdependent and shall therefore be calculated successively as follows:

- K_v with the applied tangential load $F_t K_A$ (equivalent load, multiple mesh trains with $F_t K_A K_\gamma^4$);
- $K_{H\beta}$ or $K_{F\beta}$ with the recalculated load $F_t K_A K_v$.

5.2 Nominal tangential load, F_t , nominal torque, T , nominal power, P

The nominal tangential load, F_t , is determined in the transverse plane at the reference cylinder. It is based on the input torque to the driven machine. This is the torque corresponding to the heaviest regular working condition. Alternatively, the nominal torque of the prime mover can be used as a basis if it corresponds to the torque requirement of the driven machine, or some other suitable basis can be chosen.

$$F_t = \frac{2\,000\,T_{1,2}}{d_{1,2}} = \frac{19\,098 \times 1\,000P}{d_{1,2}n_{1,2}} = \frac{1\,000P}{v} \quad (1)$$

$$T_{1,2} = \frac{F_t d_{1,2}}{2\,000} = \frac{1\,000P}{\omega_{1,2}} = \frac{9\,549P}{n_{1,2}} \quad (2)$$

$$P = \frac{F_t v}{1\,000} = \frac{T_{1,2} \omega_{1,2}}{1\,000} = \frac{T_{1,2} n_{1,2}}{9\,549} \quad (3)$$

$$v = \frac{d_{1,2} \omega_{1,2}}{2\,000} = \frac{d_{1,2} n_{1,2}}{19\,098} \quad (4)$$

$$\omega_{1,2} = \frac{\pi n_{1,2}}{30} = \frac{2\,000v}{d_{1,2}} = \frac{n_{1,2}}{9\,549} \quad (5)$$

5.3 Non-uniform load, non-uniform torque, non-uniform power

When the transmitted load is not uniform, consideration should be given not only to the peak load and its anticipated number of cycles, but also to intermediate loads and their numbers of cycles. This type of load is classed as a *duty cycle* and may be represented by a load spectrum. In such cases, the cumulative fatigue effect of the duty cycle is considered in rating the gearset. A method of calculating the effect of the loads under this condition is given in ISO/TR 10495.

4) The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems, or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by inserting a distribution factor K_γ to follow K_A , as appropriate, to adjust the average tangential load per mesh as necessary.