



BSI Standards Publication

IT Security techniques — Digital signatures with appendix

Part 3: Discrete logarithm based mechanisms

National foreword

This British Standard is the UK implementation of [ISO/IEC 14888-3:2018](#). It supersedes [BS ISO/IEC 14888-3:2016](#), which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee IST/33, IT - Security techniques.

A list of organizations represented on this committee can be obtained on request to its secretary.

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© The British Standards Institution 2018
Published by BSI Standards Limited 2018

ISBN 978 0 539 01614 7

ICS 35.030; 35.040.01

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This British Standard was published under the authority of the Standards Policy and Strategy Committee on 30 November 2018.

Amendments/corrigenda issued since publication

Date	Text affected
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**IT Security techniques — Digital
signatures with appendix —**

**Part 3:
Discrete logarithm based mechanisms**

*Techniques de sécurité IT — Signatures numériques
avec appendice —*

Partie 3: Mécanismes basés sur un logarithme discret





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Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents) or the IEC list of patent declarations received (see <http://patents.iec.ch>).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 27, *IT Security techniques*.

This fourth edition cancels and replaces the third edition (ISO 14888-3:2016), of which it constitute a minor revision. The main changes compared to the previous edition are as follows:

- SM2 algorithm has been added to [6.12](#) and [F.14](#);
- Chinese IBS algorithm has been added to [7.4](#) and [F.15](#);
- numerical examples of KCDSA, ECDSA and EC-KCDSA have been added to [F.3.4](#), [F.6.6](#), [F.6.7](#), [F.6.8](#), [F.7.7](#), [F.7.8](#) and [F.7.9](#);
- several formulae and symbols have been corrected.

A list of all parts in the ISO/IEC 14888 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Digital signature mechanisms can be used to provide services such as entity authentication, data origin authentication, non-repudiation and data integrity. A digital signature mechanism satisfies the following requirements.

- Given either or both of the following two things:
 - the verification key, but not the signature key;
 - a set of signatures on a sequence of messages that an attacker has adaptively chosen,
- it should be computationally infeasible for the attacker to:
 - produce a valid signature on a new message;
 - in some circumstances, produce a new signature on a previously signed message; or
 - recover the signature key;
- it should be computationally infeasible, even for the signer, to find two different messages with the same signature.

NOTE 1 Computational feasibility depends on the specific security requirements and environment.

NOTE 2 In some applications, producing a new signature on a previously signed message without knowing the signature key is allowed. One example of such applications is a membership credential in an anonymous digital signature mechanism as specified in [ISO/IEC 20008](#).

Digital signature mechanisms are based on asymmetric cryptographic techniques and involve the following three basic operations:

- a process for generating pairs of keys, where each pair consists of a private signature key and the corresponding public verification key;
- a process that uses the signature key, called the signature process;
- a process that uses the verification key, called the verification process.

The following are the two types of digital signature mechanisms:

- when, for a given signature key, any two signatures produced for the same message are always identical, the mechanism is said to be deterministic (or non-randomized) (see [ISO/IEC 14888-1](#) for further details);
- when, for a given message and signature key, any two applications of the signature process produce (with high probability) two distinct signatures, the mechanism is said to be randomized (or non-deterministic).

The mechanisms specified in this document are all randomized.

Digital signature mechanisms can also be divided into the following two categories:

- when the whole message has to be stored and/or transmitted along with the signature, the mechanism is termed a "signature mechanism with appendix" (such mechanisms are the subject of the ISO/IEC 14888 series);
- when the whole message, or part of it, can be recovered from the signature, the mechanism is termed a "signature mechanism giving message recovery" (the ISO/IEC 9796 series specifies mechanisms in this category).

The verification of a digital signature requires access to the signing entity's verification key. It is, thus, essential for a verifier to be able to associate the correct verification key with the signing entity or more

precisely, with (parts of) the signing entity's identification data. This association between the signer's identification data and the signer's public verification key can either be guaranteed by an outside entity or mechanism, or the association can be somehow inherent in the verification key itself. In the former case, the scheme is said to be "certificate-based." In the latter case, the scheme is said to be "identity based." Typically, in an identity-based scheme, the verifier can calculate the signer's public verification key from the signer's identification data. The digital signature mechanisms specified in this document are classified into certificate-based and identity-based mechanisms.

NOTE 3 For certificate-based mechanisms, various PKI standards can be used as the basis of key management. For further information, see [ISO/IEC 9594-8](#) (also known as X.509), [ISO/IEC 11770-3](#) and [ISO/IEC 15945](#).

The security of a signature mechanism is based on an intractable computational problem, i.e. a problem for which, given current knowledge, finding a solution is computationally infeasible, such as the factorization problem and the discrete logarithm problem. This document specifies digital signature mechanisms with appendix based on the discrete logarithm problem, and [ISO/IEC 14888-2](#) specifies digital signature mechanisms with appendix based on the factorization problem.

NOTE 4 The first edition of the [ISO/IEC 14888](#) series grouped identity-based mechanisms into [ISO/IEC 14888-2](#) and certificate-based mechanisms into [ISO/IEC 14888-3](#), with both parts covering mechanisms based on both the discrete logarithm and the factorization problems. Since the second edition was published, the mechanisms have been reorganized. [ISO/IEC 14888-2](#) now contains integer factoring-based mechanisms, and this document now contains discrete logarithm based mechanisms.

This document includes 14 mechanisms: two of which (DSA and Pointcheval/Vaudenay algorithm) were in [ISO/IEC 14888-3:1998](#), three of which (EC-DSA, EC-KCDSA, and EC-GDSA) were from [ISO/IEC 15946-2:2002](#) and three of which (KCDSA, IBS-1 and IBS-2) were added in [ISO/IEC 14888-3:2006](#), four of which (SRA, EC-RDSA, EC-SDSA and EC-FSDSA) were added in ISO/IEC 14888-3:2006/Amd 1:2010, and two of which (SM2 and Chinese IBS) are added in this document.

The mechanisms specified in this document use a collision resistant hash-function to hash the message being signed (possibly in more than one part). [ISO/IEC 10118](#) specifies hash-functions.

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NOTE 5 The mechanisms of EC-DSA, EC-GDSA, EC-RDSA and EC-FSDSA may be vulnerable to a key substitution attack^[10]. The attack is realized if an adversary can find two distinct public keys and one signature such that the signature is valid for both public keys. There are several approaches of avoiding this attack and its possible impact on the security of a cryptographic system. For example, the public key corresponding to the private signing key can be added into the message to be signed.

[Annexes A, B](#) and [D](#) are normative elements; [Annexes C, E, F, G](#) and [H](#) are for information.

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IT Security techniques — Digital signatures with appendix —

Part 3: Discrete logarithm based mechanisms

1 Scope

This document specifies digital signature mechanisms with appendix whose security is based on the discrete logarithm problem.

This document provides

- a general description of a digital signature with appendix mechanism, and
- a variety of mechanisms that provide digital signatures with appendix.

For each mechanism, this document specifies

- the process of generating a pair of keys,
- the process of producing signatures, and
- the process of verifying signatures.

[Annex A](#) defines object identifiers assigned to the digital signature mechanisms specified in this document, and defines algorithm parameter structures.

[Annex B](#) defines conversion functions of FE2I, I2FE, FE2BS, BS2I, I2BS, I2OS and OS2I used in this document.

[Annex D](#) defines how to generate DSA domain parameters.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

[ISO/IEC 10118-3](#), *IT Security techniques — Hash-functions — Part 3: Dedicated hash-functions*

[ISO/IEC 14888-1](#), *Information technology — Security techniques — Digital signatures with appendix — Part 1: General*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in [ISO/IEC 14888-1](#) and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.1 finite commutative group

finite set E with the binary operation “ $*$ ” such that

- for all group elements $a, b \in E$, $a * b \in E$;
- for all group elements $a, b, c \in E$, $(a * b) * c = a * (b * c)$;
- there exists a group element $e \in E$ with $e * a = a$ for all $a \in E$, where e is called the identity element of the group;
- for all group elements $a \in E$, there exists a group element $b \in E$ with $b * a = e$;
- for all group elements $a, b \in E$, $a * b = b * a$

Note 1 to entry: In some cases, such as when E is the set of points on an elliptic curve, arithmetic in the finite set E is described with additive notation.

3.2 cyclic group

finite commutative group (3.1), E , of n elements that contains a group element $a \in E$, called the generator, of order n

3.3 elliptic curve group

cyclic group (3.2) defined on the points of an elliptic curve over a finite field

Note 1 to entry: Let $F = GF(r)$ denote the Galois field with cardinality, r , where either r is an odd prime, p , or r is equal to 2^m , for some positive integer, m .

An elliptic curve defined over F can be determined by an affine curve formula, either of the form $y^2 = x^3 + a_1x + a_2$ (when $r = p$ for some odd prime p) or of the form $y^2 + xy = x^3 + a_1x^2 + a_2$ (when $r = 2^m$ for some positive integer m), where the coefficients a_1 and a_2 are (appropriately chosen) elements of F . The corresponding elliptic curve E consists of a collection of certain affine points from $F \times F$ together with a special (non-affine) point “at infinity”.

An affine point P of E is one that can be represented as an ordered pair $(P_x, P_y) \in F \times F$, such that the selection of $x = P_x$ and $y = P_y$ satisfies the given affine curve formula when the indicated arithmetic is performed in the field, F .

Let “ $+$ ” denote the binary operation known as “elliptic-curve addition”, defined for (most) affine points of E by the well-known secant-and-tangent rules. Once the collection of affine points of E is augmented by 0_E , a special point of E “at infinity” that serves as the identity element for “ $+$ ” (but is not represented as an ordered pair), the set E together with the binary operation “ $+$ ” forms a finite, commutative, elliptic-curve group, E .

Note 2 to entry: The cardinality of the elliptic-curve group, E , is one more than the number of ordered pairs in $F \times F$ that satisfy the affine curve formula for E .

3.4 order (of a group element a)

least positive integer n such that $a^n = e$, where e is the identity element of the group, a^n is defined recursively such that $a^0 = e$ and $a^m = a * a^{m-1}$ ($m > 0$), and $*$ is the group operation

3.5 pairing

function which takes two elements, P and Q , from an elliptic curve group (3.3) over a finite field, G_1 , as input, and produces an element from another cyclic group (3.2) over a finite field, G_2 , as output, and which has the following two properties (where it is assumed that the cyclic groups, G_1 and G_2 have order q , for some prime q , and for any two elements P, Q , the output of the pairing function is written as $\langle P, Q \rangle$)

- Bilinearity: If P, P_1, P_2, Q, Q_1, Q_2 are elements of G_1 , and a is an integer satisfying $1 \leq a \leq q - 1$, then

$$\langle P_1 + P_2, Q \rangle = \langle P_1, Q \rangle * \langle P_2, Q \rangle,$$

$$\langle P, Q_1 + Q_2 \rangle = \langle P, Q_1 \rangle * \langle P, Q_2 \rangle, \text{ and}$$

$$\langle [a]P, Q \rangle = \langle P, [a]Q \rangle = \langle P, Q \rangle^a$$

— Non-degeneracy: If P is a non-identity element of G_1 , $\langle P, P \rangle \neq 1$

3.6

trusted key generation centre

KGC

trusted third party, which, in an identity-based signature mechanism, generates a private signature key for each signing entity

4 Symbols and abbreviated terms

$a \oplus b$	bitwise exclusive OR of a and b , where a and b are either bits or strings of bits of the same length, and in the latter case, the XOR operation is performed bit-wise
a_1, a_2	elliptic curve coefficients
$a \bmod n$	for an arbitrary integer a and a positive integer n , the unique integer remainder r , $0 \leq r \leq (n - 1)$, satisfying $r = a - bn$, for some integer b .
(A, B, C)	the coefficients of the signature formula, which, for the mechanisms specified in Clause 6 , defines how the signature is computed
NOTE 1 The signature formula is specified in 5.2.1 .	
D	a parameter which specifies the relationship between the signature key and the verification key
E	an elliptic curve defined by two elliptic curve coefficients, a_1 and a_2
$\mathcal{E}$	a finite commutative group; for the mechanisms based on a multiplicative group, the elements of $\mathcal{E}$ are in $\mathbb{Z}_p^*$; for the mechanisms based on an additive group of elliptic curve points, the elements of $\mathcal{E}$ are the points on an elliptic curve E over $GF(r)$
$\#E$	the cardinality of E ; for the mechanisms based on a multiplicative group $\mathbb{Z}_p^*$, $\#E$ is $p - 1$; for the mechanisms based on an additive group of elliptic curve points, $\#E$ is one more than the number of points on the elliptic curve E over $GF(r)$ [including 0_E (the point at infinity)]
F	a finite field
F_p	a finite field of order p
$\gcd(N_1, N_2)$	the greatest common divisor of integers N_1 and N_2
G	an element of order q in $\mathcal{E}$
$GF(r)$	the finite field of cardinality r , where r is a prime power
G_1	a cyclic group of prime order q ; elements of G_1 are points on an elliptic curve over $GF(r)$
G_2	a cyclic group of prime order q ; elements of G_2 are elements of a finite field $GF(r)$
H_1	a hash-function that converts a data string into an element in G_1
NOTE 2 The input data string is converted to an integer first, then the integer is converted to a point on E over $GF(r)$ by using the I2P function, specified in Annex C .	
h, H_2	hash-functions, i.e. one of the mechanisms specified in ISO/IEC 10118

ID	a data string containing an identifier of the signer, used in Mechanisms SM2, IBS-1, IBS-2 and Chinese IBS
m	an embedding degree (or extension degree)
$[n]P$	multiplication operation that takes a positive integer n and a point P on the curve E as input and produces as output another point Q on the curve E , where $Q = [n]P = P + P + \dots + P$ added $n - 1$ times. The operation satisfies $[0]P = 0_E$ (the point at infinity), and $[-n]P = [n](-P)$
P	a generator of G_1 which is used in Mechanisms IBS-1, IBS-2 and Chinese IBS
p	a prime number or a power of a prime number
q	a prime number that is a divisor of $\#E$ and the order of G_1 and G_2
r	the size of $GF(r)$; in the mechanisms based on an additive group of elliptic curve points, r is a prime power, p^m , for some prime $p \geq 2$ and integer $m \geq 1$.
T	the assignment
T_1	the first part of the assignment T
T_2	the second part of the assignment T
U	the KGC's master private key, generated as a randomly chosen integer, which is used in mechanisms IBS-1, IBS-2 and Chinese IBS
V	the KGC's master public key, an element of G_1 , which is used in mechanisms IBS-1, IBS-2 and Chinese IBS
Z_N^*	the set of integers i with $0 < i < N$ and $\gcd(i, N) = 1$, with arithmetic defined modulo N
Z_p^*	the set of integers i with $0 < i < p$ and p a prime number, which is a multiplicative group
α	the bit-length of the prime number (or prime power) p
β	the bit-length of the prime number q
γ	the output bit-length of hash-functions h and H_2
Π	pre-signature
Π_X	x-coordinate of Π in which $\Pi = (\Pi_X, \Pi_Y)$ is an elliptic curve point
Π_Y	y-coordinate of Π in which $\Pi = (\Pi_X, \Pi_Y)$ is an elliptic curve point
Π_a	first element of Π in which $\Pi = (\Pi_a, \Pi_b)$ is an element of an extension field of degree 2
Π_b	second element of Π in which $\Pi = (\Pi_a, \Pi_b)$ is an element of an extension field of degree 2
0_E	the point at infinity on the elliptic curve E
$< >$	a bilinear and non-degenerate pairing
$ $	$X Y$ is used to mean the result of the concatenation of data items X and Y in the order specified.

5 General model

5.1 Parameter generation process

5.1.1 Certificate-based mechanisms

5.1.1.1 Generation of domain parameters

For digital signature mechanisms based on discrete logarithms, the set of domain parameters includes the following parameters:

- E , a finite commutative group;
- q , a prime divisor of $\#E$;
- G , an element of order q in E .

In the group E , multiplicative notation is used. It is worthwhile to note that the particular signature mechanism chosen may place additional constraints on the choice of E , q , and G .

5.1.1.2 Generation of signature key and verification key

A signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The corresponding public verification key Y is an element of E and is computed as Formula (1):

$$Y = G^{X^D} \quad (1)$$

where D is a parameter defined by the mechanism to be used. The value of D is one of two values, -1 and 1 .

NOTE An implementation is still considered compliant if it excludes a few integers from consideration as possible X values. For example, the value 1 can be excluded because this value results in the user's verification key being the generator, G , which is easily detectable.

5.1.2 Identity-based mechanisms

5.1.2.1 Notation

Both identity-based mechanisms specified in [Clause 7](#) are based on the use of pairings over elliptic curve groups. To specify identity-based mechanisms, the additive group notation is used.

5.1.2.2 Generation of domain parameters

The set of domain parameters includes the following parameters:

- E , a finite commutative group;
- $GF(r)$, the Galois field of cardinality r ;
- G_1 , a cyclic group of prime order q ;
- G_2 , a cyclic group of prime order q ;
- P , a generator of G_1 ;
- q , a prime number — the cardinality of G_1 and G_2 ;
- $\langle \cdot, \cdot \rangle$, a bilinear and non-degenerate pairing.

5.1.2.3 Generation of master key

A master private key of a KGC is a secretly generated random or pseudo-random integer U such that $0 < U < q$. The corresponding master public key V is an element of G_1 and is computed as:

$$V = [U]P$$

5.1.2.4 Generation of signature key and verification key

A signature key of a signing entity is an element of G_1 and is computed by the KGC as Formula (2):

$$X = [U]Y \quad (2)$$

where

U is the KGC's master private key;

$Y = H_1(ID)$ is the public verification key

where

ID is an identity string for the KGC;

H_1 is a hash-function.

5.1.3 Parameter selection

5.1.3.1 Selecting parameter size

The bit-lengths of parameters for typical security levels are shown in [Table 1](#). The minimum recommended security level is 2^{112} .

NOTE 1 Security level means the number of steps in the best known attack on a cryptographic primitive. If 2^{112} steps are required in the best known attack on a hash-function, the security level of the hash-function is 2^{112} . For a comprehensive analysis of parameter sizes, see References [25] and [34].

It is not necessary to select α , β and γ having the same security level; the security level of an implemented signature scheme is the minimum of the security levels of the parameters.

Table 1 — Parameter sizes according to the security level

Security level	2^{80}	2^{112}	2^{128}	2^{192}	2^{256}
α	1 024	2 048	3 072	7 680	15 360
β	160	224	256	384	512
γ	160	224	256	384	512

It is recommended that the security level of 2^{80} should only be used for legacy applications.

NOTE 2 Not every mechanism specified in this document provides all of the levels of security specified in this table. For example, DSA in [6.1](#) only supports the security levels up to 2^{128} .

5.1.3.2 Selecting a hash-function

Selection of hash-functions shall be based on those standardized in [ISO/IEC 10118-3](#). That is, h and H_2 shall be one of the mechanisms specified in [ISO/IEC 10118-3](#), and H_1 converts a data string obtained by using one of the mechanisms specified in [ISO/IEC 10118-3](#) into an element in G_1 .

The hash-functions used in this document should be collision-resistant.

The security strength for the selected hash-function should meet or exceed the security strength of the parameters used in key generation. The relationship between the security levels of a hash-function and the key generation parameters is shown in [5.1.3.1](#).

Furthermore, implementations that verify digital signatures shall have a way of securely determining which hash-function was used by the signer. Otherwise, an attacker might be able to convince a verifier to use a different weaker, hash-function and thus, bypass the intended security level.

5.1.4 Validity of domain parameters and verification key

The signature verifier may require assurance that the domain parameters and public verification key are valid, otherwise, there is no assurance of meeting the intended security even if the signature verifies, and an adversary may be able to generate signatures that verify.

Assurance of the validity of domain parameters can be provided by one of the following:

- selection of valid domain parameters from a trusted published source, such as a standard;
- generation of valid domain parameters by a trusted third party, such as a CA or a KGC;
- validation of candidate domain parameters by a trusted third party, such as a CA or a KGC;
- for the signer, generation of valid domain parameters by the signer using a trusted system;
- validation of candidate domain parameters by the user (i.e. the signer or verifier).

Assurance of validity of a public verification key can be provided by one of the following:

- for the signer, generation of the public verification/private signature key pair using a trusted system;
- for the signer or verifier, validation of the public verification key by a trusted third party, such as a CA or a KGC;
- validation of the public verification key by the user (i.e. the signer or verifier).

NOTE 1 Validation of domain parameters and keys is required. However, how to achieve this is outside the scope of this document.

NOTE 2 The method of authenticating the signer is dependent on the real applications, which is out of the scope of this document.

5.2 Signature process

5.2.1 General

All of the signature mechanisms in this document make use of a randomizing value K , which is used (along with the message) to produce a witness R (the first part of the signature) and an assignment (T_1, T_2) . The signature for the message is the pair (R, S) where S (the second part of the signature) is computed as the solution of a signature formula.

In the certificate-based mechanisms, specified in [Clause 6](#), the signature formula is Formula (3):

$$AK + BX^D + C \equiv 0 \pmod{q} \quad (3)$$

where

- (A, B, C) is a permutation of (S, T_1, T_2) or $(S, T_1, S + T_2)$;
- X is the private signature key;
- D is a parameter depending on the particular mechanism.

In the identity-based mechanisms specified in [Clause 7](#), the signature formula is Formula (4):

$$[K]A + [U^D]B + C \equiv 0_E \text{ (in } G_1) \quad (4)$$

where

- (A, B, C) is a permutation of (S, T_1, T_2) or $(S, T_1, [Y^{-1}]S + T_2)$;
- U is the master private key;
- Y is the public verification key;
- D is a parameter depending on the particular mechanism.

The permutation will be specified or agreed upon when setting up the signature system.

The signature process and the formation of a signed message consist of the following eight stages (see [Figure 1](#)):

- producing the randomizer;
- producing the pre-signature;
- preparing the message for signing;
- computing the witness;
- computing the assignment (it is not necessary to compute the assignment in the identity-based mechanisms);
- computing the second part of the signature;
- constructing the appendix;
- constructing the signed message.

In this process, the signing entity makes use of its private signature key, its public verification key (optional) and the domain parameters.

5.2.2 Producing the randomizer

For each signature, the signing entity freshly generates a secret randomizer which is an integer K with $0 < K < q$. The output of this stage is K , which shall be kept secret and destroyed safely after use.

NOTE 1 The randomizer K can be considered as an ephemeral key.

NOTE 2 For the same rationale of [5.1.1.2](#), an implementation is still considered compliant if it excludes a few integers from consideration as possible K values.

5.2.3 Producing the pre-signature

The inputs to this stage are the randomizer K and optionally signature key X , with which the signing entity computes the pre-signature, Π , by using K and public parameters as input. In the certificate-based mechanisms specified in [Clause 6](#), it is computed as Formula (5):

$$\Pi = G^K \quad (5)$$

in E . In the identity-based mechanisms specified in [Clause 7](#), it is individually specified in the mechanisms. The output of this stage is the pre-signature, Π .

5.2.4 Preparing the message for signing

In the process of preparing the message, one of M_1 and M_2 becomes message M (with a prefix, optionally), the other becomes empty.

5.2.5 Computing the witness (the first part of the signature)

The variables to this stage are the pre-signature Π from 5.2.3 and M_1 from 5.2.4. The values of these variables are taken as inputs to the witness function. The output of the witness function is the witness R . The witness function is specified in the mechanisms.

5.2.6 Computing the assignment

The inputs to the assignment function are the first part of the signature, which is the witness R from 5.2.5, M_2 from 5.2.4, and, optionally, the verification key Y . The output of the assignment function is assignment $T = (T_1, T_2)$. In the certificate-based mechanisms specified in Clause 6, T_1 and T_2 are integers such that:

$$0 < |T_1| < q, 0 < |T_2| < q$$

In the identity-based mechanisms specified in Clause 7, T_1 and T_2 are elements of G_1 . It is not necessary to compute T in the identity-based mechanisms.

5.2.7 Computing the second part of the signature

The inputs to this stage are the randomizer K from 5.2.1, the signature key X , the assignment $T = (T_1, T_2)$ from 5.2.6, the permutation (A, B, C) of (S, T_1, T_2) , a variable D in 5.1.1.2 and domain parameter q as specified in 5.1.1.1 and 5.1.2.1.

In the certificate-based mechanisms, the signing entity forms the signature formula [See Formula (6)]:

$$AK + BX^D + C \equiv 0 \pmod{q} \quad (6)$$

and solves the signature formula for S , the second part of the signature, where $0 < S < q$.

In the identity-based mechanisms, the signing entity solves the signature formula for S , the second part of the signature such that $S \in G_1$. This solution satisfies the signature formula [see Formula (7)]:

$$[K]A + [U^D]B + C \equiv 0_E \text{ (in } G_1) \quad (7)$$

The pair (R, S) will be called the signature, Σ .

5.2.8 Constructing the appendix

The appendix is constructed from the signature and an optional text field, *text*, as $[(R, S), \text{text}]$. The text field could include a certificate that cryptographically ties the public verification key to the identification data of the signing entity.

As indicated in ISO/IEC 14888-1, depending on the application, there are different ways of forming the appendix and appending it to the message. The general requirement is that the verifier is able to relate the correct signature to the message. For successful verification, it is also essential that prior to the verification process, the verifier is able to associate the correct verification key with the signature.

5.2.9 Constructing the signed message

The signed message is obtained by the concatenation of message M and the appendix, i.e. $M \parallel [(R, S), \text{text}]$.

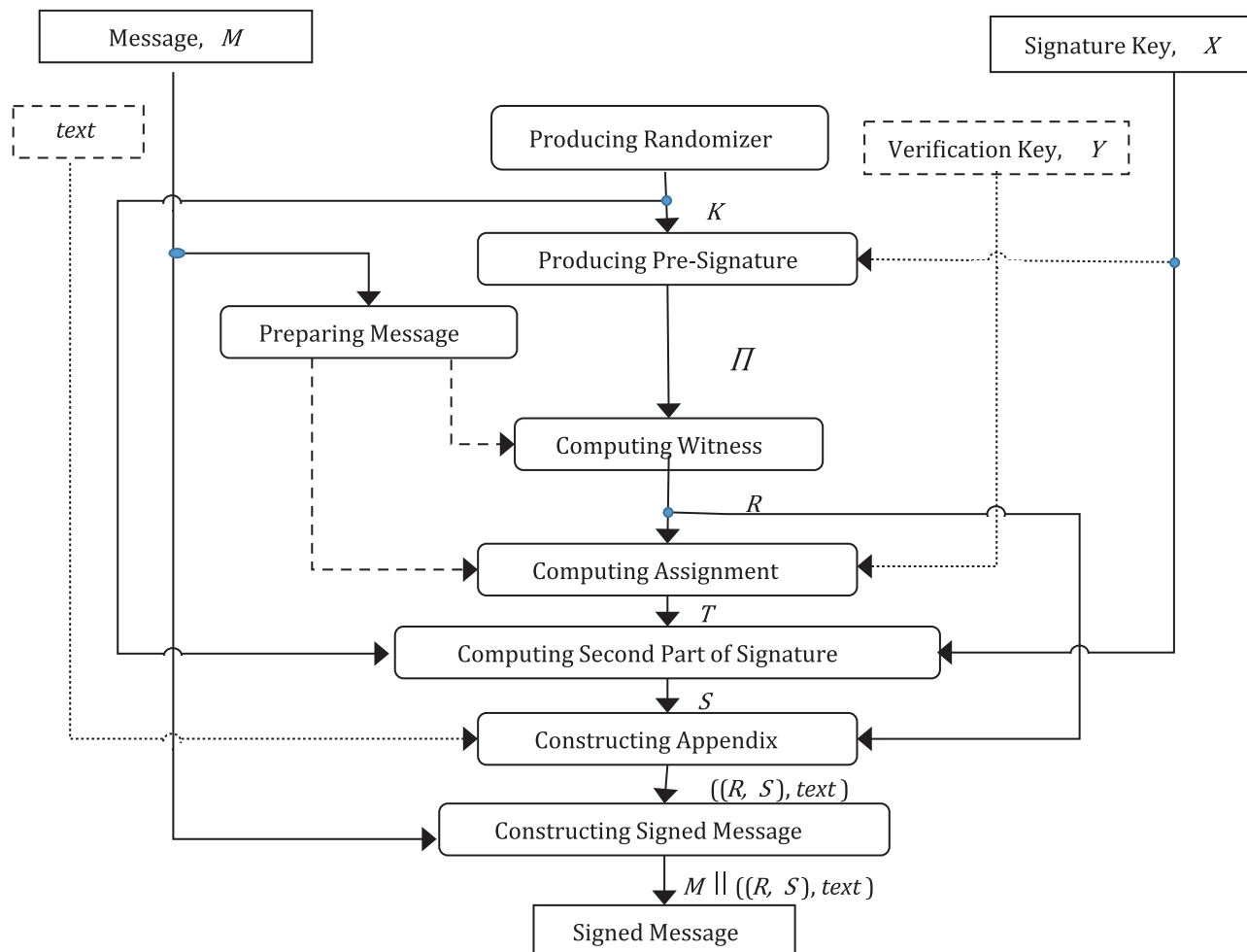


Figure 1 — Signature process with randomized witness (one of M_1 and M_2 is M , the other is empty)

5.3 Verification process

5.3.1 General

The verification process consists of the following six stages (see [Figure 2](#)):

- retrieving the witness;
- preparing message for verification;
- retrieving the assignment (it is optional to compute the assignment in the identity-based mechanisms);
- recomputing the pre-signature;
- recomputing the witness;
- verifying the witness.

In this process, the verifier makes use of the signer's verification key, KGC's master public key (only for the identity-based mechanisms specified in [Clause 7](#)) and the domain parameters.

5.3.2 Retrieving the witness

The verifier retrieves the signature (R, S) from the appendix, and divides it into the witness R and the second part of the signature S . Also, the verifier checks the range or the bit length of the signature elements, R and S , according to the rule specified by each signature process. If the predefined rule is violated, the signature shall be rejected.

5.3.3 Preparing message for verification

The verifier retrieves M from the signed message and divides the message into two parts M_1 and M_2 .

5.3.4 Retrieving the assignment

This stage is identical to 5.2.6. The inputs to the assignment function consist of the witness R from 5.3.2, M_2 from 5.3.3, and (optionally) the verification key Y . The assignment $T = (T_1, T_2)$ is recomputed as the output from the assignment function. In the identity-based mechanisms, it is not necessary to recompute T .

5.3.5 Recomputing the pre-signature

The inputs to this stage are the set of domain parameters, the verification key Y , the assignment $T = (T_1, T_2)$ from 5.3.4, the second part of the signature S from 5.3.2, and optionally R from 5.3.2. The verifier assigns to the coefficients (A, B, C) the values (S, T_1, T_2) according to the order specified by the signature function, and in the certificate-based mechanisms, computes the element Π' .

In the certificate-based mechanisms, Π' is computed in E as Formula (8):

$$\Pi' = Y^m G^n \quad (8)$$

where

$$m = -A^{-1}B \bmod q;$$

$$n = -A^{-1}C \bmod q.$$

In the identity-based mechanisms, it is individually specified in the mechanisms.

5.3.6 Recomputing the witness

The computations at this stage are the same as in 5.2.5. The verifier executes the witness function. The inputs are Π' from 5.3.5 and M_1 from 5.3.3. The output is the recomputed witness, R' .

In Mechanism IBS-2 specified in 7.2, the process of recomputing the witness is computing two verification functions, instead of computing R' .

5.3.7 Verifying the witness

The signature is verified if the recomputed witness, R' , from 5.3.6 is equal to R from 5.3.2.

In Mechanism IBS-2 specified in 7.2, the process of verifying the witness involves checking whether the two verification function values computed in 5.3.6 are identical, instead of verifying whether $R = R'$ holds.

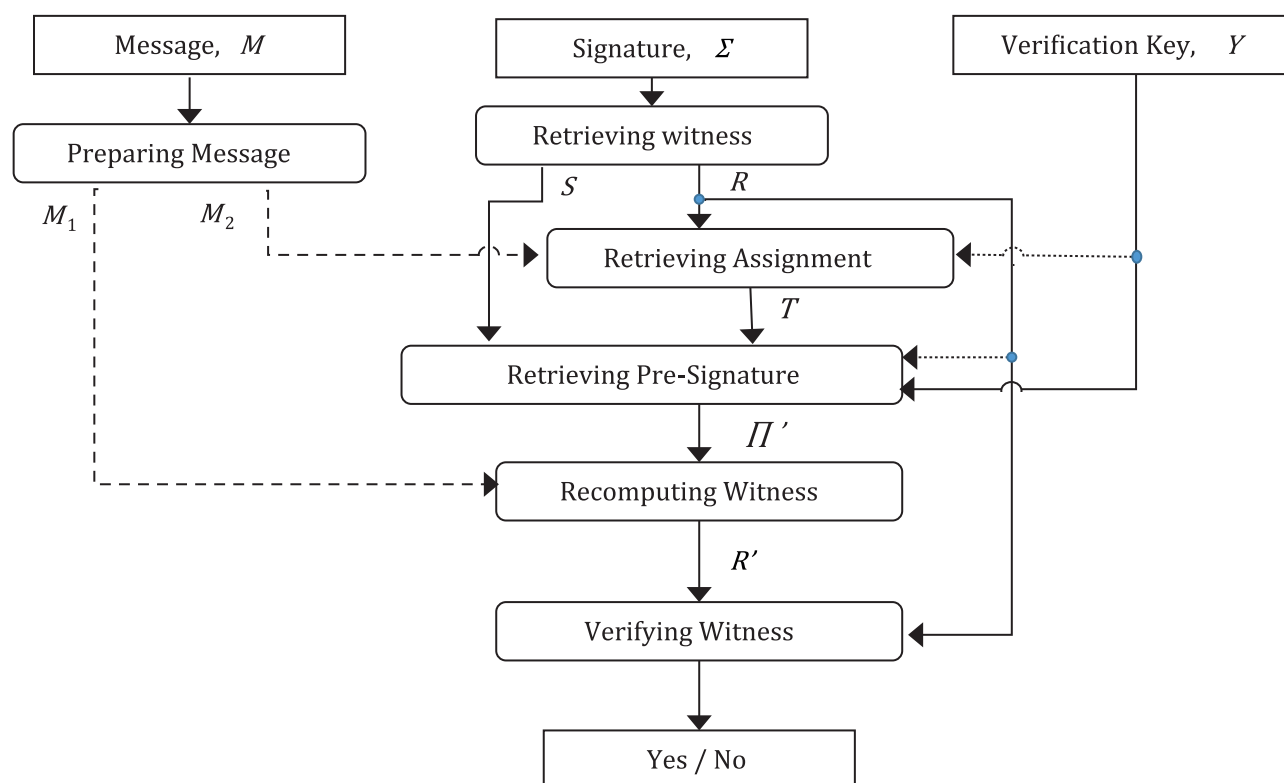


Figure 2 — Verification process with a randomized witness

6 Certificate-based mechanisms

6.1 General

[Clause 6](#) specifies eleven certificate-based mechanisms. These make use of arithmetic in the multiplicative group Z_p^* or the additive group of elliptic curve points. The mechanisms using arithmetic in the multiplicative group Z_p^* , are the Digital Signature Algorithm (DSA), the Korean Certificate-based Digital Signature Algorithm (KCDSA), the Pointcheval/Vaudenay algorithm, and the Schnorr Digital Signature Algorithm (SDSA). The mechanisms using arithmetic in the additive group of elliptic curve points are the Elliptic Curve DSA (EC-DSA), the Elliptic Curve KCDSA (EC-KCDSA), the Elliptic Curve German Digital Signature Algorithm (EC-GDSA), the Elliptic Curve Russian Digital Signature Algorithm (EC-RDSA), the Elliptic Curve Schnorr Digital Signature Algorithm (EC-SDSA), the Elliptic Curve Full Schnorr Digital Signature Algorithm (EC-FSDSA) and the SM2 algorithm.

In elliptic curve arithmetic, an elliptic curve point is represented as affine coordinates. That is, an elliptic curve point Π has two coordinates: x-coordinate, Π_x , and y-coordinate, Π_y . Elliptic curves for EC-DSA, EC-KCDSA, EC-GDSA, EC-RDSA, EC-SDSA, EC-FSDSA and SM2 are restricted to non-singular and non-supersingular curves.

The hash-function identifier can be used for binding the signature mechanism and the hash-function.

6.2 DSA

6.2.1 General

DSA (Digital Signature Algorithm) is a signature mechanism with $E = Z_p^*$, p a prime, and q a prime dividing $p - 1$. The parameter D of DSA is equal to 1. The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The witness function is defined by Formula (9):

$$R = \Pi \bmod q \quad (9)$$

where $\Pi = G^K \bmod p$ for some integer K .

The assignment function is defined by Formula (10):

$$(T_1, T_2) = (-R, -\text{BS2I}(\gamma, H)) \quad (10)$$

where $H = h(M)$ is the truncated hash-code of message M , converted to an integer according to the conversion rule given in [Annex B](#).

The coefficients (A, B, C) of the DSA signature formula are set as per Formula (11):

$$(A, B, C) = (S, T_1, T_2) \quad (11)$$

Thus, the signature formula becomes Formula (12):

$$SK - RX - \text{BS2I}(\gamma, H) \equiv 0 \pmod{q} \quad (12)$$

NOTE This mechanism is taken from Reference [17]. The notation here has been changed slightly from Reference [17] to conform with the notation used elsewhere in this document.

6.2.2 Parameters

p a prime, where $2^{\alpha-1} < p < 2^\alpha$.

q a prime divisor of $p - 1$, where $2^{\beta-1} < q < 2^\beta$.

G a generator of the subgroup of order q , such that $1 < G < p$.

Four choices for the pair (α, β) are allowed in DSA, which are (1 024, 160), (2 048, 224), (2 048, 256), and (3 072, 256). It is recommended that the security strength of the (α, β) pair and the γ be the same unless an agreement has been made between participating entities to use a stronger hash function.

The integers p , q , and G can be public and can be common to a group of users.

The parameters p , q and G are generated as specified in [Annex D](#). If compatibility with the NIST Federal standard is not required then the parameters p and q can be generated by using the prime generation techniques given in [ISO/IEC 18032](#).

It is recommended that all users check the proper generation of the DSA public parameters according to Reference [17].

NOTE If a signer were free to specifically choose the domain parameter q to facilitate a collision between hash values, then an attack against such an instance of DSA could be mounted in which the required collision on the underlying hash-function could be found with complexity of 2^{74} , 2^{101} , or 2^{114} (corresponding to $\gamma = 160$, 224, or 256, respectively), as opposed to the most secure instances, in which the complexity of finding a collision would be 2^{80} , 2^{112} , or 2^{128} . [39] The attack is, however, easily detectable. Furthermore, the attack cannot be mounted when domain parameters are generated as specified in Reference [17] which includes the method specified in [Annex D](#). If the use of an appropriate method for the generation of domain parameters cannot be verified, the attack can also be prevented by using a mechanism of the type specified in [6.3](#), [6.4](#) and [6.7](#).

It is recommended that digital signatures based on SHA-1 should only be used for legacy applications.

6.2.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is Formula (13):

$$Y = G^X \bmod p \quad (13)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.2.4 Signature process

6.2.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.2.4.2 Producing the pre-signature

The input to this stage is the randomizer K , and the signing entity computes Formula (14):

$$\Pi = G^K \bmod p \quad (14)$$

6.2.4.3 Preparing the message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.2.4.4 Computing the witness

The signing entity computes $R = \Pi \bmod q$ where the witness is simply a function of the pre-signature. Thus, see Formula (15):

$$R = (G^K \bmod p) \bmod q \quad (15)$$

6.2.4.5 Computing the assignment

The signing entity computes the hash-code; if the output bit length of the selected hash-function is larger than $\lceil \log_2 q \rceil$, H is set to the leftmost (most significant) $\lceil \log_2 q \rceil$ bits of $h(M_2)$. Otherwise, H is $h(M_2)$. Afterwards, H is converted to integer according to conversion rule, BS2I, in [Annex B](#). The assignment (T_1, T_2) is $(-R, -\text{BS2I}(\gamma, H))$.

6.2.4.6 Computing the second part of the signature

The signature is (R, S) where R is computed in [6.2.4.4](#) and [see Formula (16)]:

$$S = (K^{-1} (\text{BS2I}(\gamma, H) + XR)) \bmod q \quad (16)$$

where H is the leftmost (most significant) $\min(\beta, \gamma)$ bits of $h(M_2)$.

The value of $h(M_2)$ is an γ -bit string output of the appropriate hash-function in [6.2.2](#). For use in computing S , this string shall be converted to an integer.

It is required to check if $R = 0$ or $S = 0$. If either $R = 0$ or $S = 0$, a new value of K should be generated and the signature should be recalculated (it is extremely unlikely that $R = 0$ or $S = 0$ if signatures are generated properly).

6.2.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, $text$, $((R, S), text)$.

6.2.4.8 Constructing the signed message

A signed message is the concatenation of a message, M , and the appendix:

$$M||((R, S), text)$$

6.2.5 Verification process

6.2.5.1 General

Prior to verifying the signature of a signed message, it is necessary that the verifier has trusted copies of p , q , G and Y .

6.2.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier checks to see that $0 < R < q$ and $0 < S < q$. If either condition is violated, the signature shall be rejected.

6.2.5.3 Preparing the message for verification

The verifier retrieves $M_2 = M$ from the signed message. M_1 is empty.

6.2.5.4 Retrieving the assignment

This stage is identical to [6.2.4.5](#). The inputs to the assignment function consist of the witness R from [6.2.5.2](#) and M_2 from [6.2.5.3](#). The assignment $T = (T_1, T_2)$ is recomputed as output from the assignment function, [6.2.4.5](#).

6.2.5.5 Recomputing the pre-signature

The inputs to this stage are domain parameters, verification key Y , assignment $T = (T_1, T_2)$ from [6.2.5.4](#) and second part of the signature S from [6.2.5.2](#). The verifier obtains a recomputed value Π' of the pre-signature using Formula (17):

$$\Pi' = Y^{-S^{-1}T_1 \bmod q} G^{-S^{-1}T_2 \bmod q} \bmod p \quad (17)$$

6.2.5.6 Recomputing the witness

The computations at this stage are the same as in [6.2.4.4](#). The verifier executes the witness function. The input is Π' from [6.2.5.5](#). Note that M_1 is empty. The output is the recomputed witness R' such that $R' = \Pi' \bmod q$.

6.2.5.7 Verifying the witness

The verifier compares the recomputed witness, R' from [6.2.5.6](#) to the value of R from [6.2.5.2](#). If $R' = R$, then the signature is verified.

6.3 KCDSA

6.3.1 General

KCDSA (Korean Certificate-based Digital Signature Algorithm) is a signature mechanism with $E = Z_p^*$, p a prime, and q a prime dividing $p - 1$. Verification key Y is $G^{X^{-1}}$; that is, the parameter D is -1 . The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The witness function is defined by Formula (18):

$$R = h(I2BS(\alpha, \Pi)) \quad (18)$$

If γ is greater than β , then the witness function is replaced by Formula (19):

$$R = I2BS(\beta, BS2I(\gamma, h(I2BS(\alpha, \Pi))) \bmod 2^\beta) \quad (19)$$

Domain parameters shall indicate the employed hash-function. The assignment function is defined by Formula (20):

$$(T_1, T_2) = (V, -1) \quad (20)$$

where $V = BS2I(\beta, R \oplus H) \bmod q$. The value H is the hash-code from the public key Y and message M .

The coefficients (A, B, C) of the KCDSA signature formula are set as Formula (21):

$$(A, B, C) = (T_2, S, T_1) \quad (21)$$

Thus, the signature formula becomes Formula (22):

$$-K + SX^{-1} + V \equiv 0 \pmod{q} \quad (22)$$

NOTE This mechanism is taken from Reference [36]. The notation here has been changed slightly from Reference [36] to conform with notation used elsewhere in this document.

6.3.2 Parameters

- p a prime, where $2^{\alpha-1} < p < 2^\alpha$
- q a prime divisor of $p - 1$, where $2^{\beta-1} < q < 2^\beta$
- F an integer such that $1 < F < p - 1$ and $F^{(p-1)/q} \bmod p > 1$
- G $F^{(p-1)/q} \bmod p$, an element of order q in Z_p^*
- l the input block size (in bits) of the selected hash-function h

Hash-function identifier or OID with specified hash-function.

Three choices of the triplet (α, β, h) are allowed in KCDSA, which are (2 048, 224, SHA-224), (3 072, 256, SHA-256), and (2 048, 224, SHA-256). Among these, (2 048, 224, SHA-224) and (3 072, 256, SHA-256) are recommended, while (2 048, 224, SHA-256) can be used in the case when only SHA-256 is available and SHA-224 is not.

The integers, p , q , G and l , can be public and can be common to a group of users.

6.3.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is -1 . The corresponding public verification key Y is Formula (23):

$$Y = G^{X^{-1}} \bmod p \quad (23)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.3.4 Signature process

6.3.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.3.4.2 Producing the pre-signature

The input to this stage is the randomizer K and the signing entity computes Formula (24):

$$\Pi = G^K \bmod p \quad (24)$$

6.3.4.3 Preparing the message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.3.4.4 Computing the witness

The signing entity computes witness $R = h(\text{I2BS}(\alpha, \Pi))$, where the output of h is the hash-code of the bit string of length α converted from the pre-signature Π . If γ is greater than β , then, the computation of witness is replaced by $R = \text{I2BS}(\beta, \text{BS2I}(\gamma, h(\text{I2BS}(\alpha, \Pi))) \bmod 2^\beta)$.

The conversion rules, I2BS and BS2I, are given in [Annex B](#).

6.3.4.5 Computing the assignment

The signing entity computes the assignment $(T_1, T_2) = (V, -1)$ where $V = \text{BS2I}(\beta, R \oplus H) \bmod q$, where $H = h(Y' || M_2)$ is the hash-code of the concatenation of $Y' = \text{I2BS}(l, Y \bmod 2^l)$ and message M_2 . The value of Y' is a bit string of length l . In computing V , the bit string $R \oplus H$ shall be converted to an integer before modulo reduction with respect to q .

If γ is greater than β , then the computation of H is replaced by $H = \text{I2BS}(\beta, \text{BS2I}(\gamma, h(Y' || M_2))) \bmod 2^\beta$.

NOTE Y' is a fixed value for a user, thus, this value can be kept as a user parameter.

6.3.4.6 Computing the second part of the signature

The signature is (R, S) where R is computed in [6.3.4.4](#) and [see Formula (25)]:

$$S = X(K - V) \bmod q \quad (25)$$

6.3.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, $text, ((R, S), text)$.

6.3.4.8 Constructing the signed message

A signed message is the concatenation of a message, M , and the appendix:

$$M||((R, S), \text{text})$$

6.3.5 Verification process

6.3.5.1 General

Prior to verifying the signature of a signed message, it is necessary that the verifier has trusted copies of p , q and G .

The verifier also acquires the necessary data items for the verification process: for example, the verification key Y (see [ISO/IEC 14888-1:2008](#), Clause 9 for additional required data items).

6.3.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier then checks whether the following conditions hold or not:

- $0 < S < q$;
- If the length of the value γ is not greater than β , the bit length of R is equal to the output bit length of the employed hash-function h ;
- If the length of the value γ is greater than β , the bit length of R is equal to β .

If any of the above condition does not hold the signature shall be rejected.

6.3.5.3 Preparing the message for verification

The verifier retrieves $M_2 = M$ from the signed message. M_1 is empty.

6.3.5.4 Retrieving the assignment

This stage is identical to [6.3.4.5](#). The inputs to the assignment function consist of the witness R from [6.3.5.2](#) and M_2 from [6.3.5.3](#). The assignment $T = (T_1, T_2)$ is recomputed as output from the assignment function, [6.3.4.5](#).

6.3.5.5 Recomputing the pre-signature

The inputs to this stage are domain parameters, verification key Y , assignment $T = (T_1, T_2)$ from [6.3.5.4](#) and second part of the signature S from [6.3.5.2](#). The verifier obtains a recomputed value Π' of the pre-signature using Formula (26):

$$\Pi' = Y^S \bmod q \cdot G^{T_1 \bmod q} \bmod p \quad (26)$$

6.3.5.6 Recomputing the witness

The computations at this stage are the same as in [6.3.4.4](#). The verifier executes the witness function. The input is Π' from [6.3.5.5](#). Note that M_1 is empty. The output is the recomputed witness R' .

6.3.5.7 Verifying the witness

The verifier compares the recomputed witness, R' from [6.3.5.6](#) to the value of R from [6.3.5.2](#). If $R' = R$, then the signature is valid.

6.4 Pointcheval/Vaudenay algorithm

6.4.1 General

The method of Pointcheval/Vaudenay is a variant of the DSA algorithm, with $E = Z_p^*$, p a prime, and q a prime divisor of $p - 1$. The parameter D is equal to 1. The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The witness is defined by Formula (27):

$$R = \Pi \bmod q \quad (27)$$

and the assignment function is defined by Formula (28):

$$(T_1, T_2) = (-R, -H) \quad (28)$$

where $H = h(\text{I2BS}(\beta, R) || M)$ is the hash-code of the concatenation of the witness R and the message M . Note that the computation of T_2 above requires the conversion of the hash-code to an integer. The conversion function is given in [Annex B](#).

The coefficients (A, B, C) of the Pointcheval/Vaudenay signature formula are set as Formula (29):

$$(A, B, C) = (S, T_1, T_2) \quad (29)$$

Thus, the signature formula becomes Formula (30):

$$SK - RX - H \equiv 0 \pmod{q} \quad (30)$$

NOTE This mechanism is based on the algorithm designed by D. Pointcheval and S. Vaudenay in Reference [31].

6.4.2 Parameters

p a prime

q a prime divisor of $p - 1$

F an integer such that $1 < F < p - 1$ and $F^{(p-1)/q} \bmod p > 1$

G $F^{(p-1)/q} \bmod p$

Hash-function identifier or OID with specified hash-function

It is recommended that all users check the proper generation of the public parameters.

6.4.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is Formula (31):

$$Y = G^X \bmod p \quad (31)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.4.4 Signature process

6.4.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.4.4.2 Producing the pre-signature

The input to this stage is the randomizer K and the signing entity computes Formula (32):

$$\Pi = G^K \bmod p \quad (32)$$

6.4.4.3 Preparing message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.4.4.4 Computing the witness

The signing entity computes $R = \Pi \bmod q$ where the witness is simply a function of the pre-signature. Thus, see Formula (33):

$$R = (G^K \bmod p) \bmod q \quad (33)$$

6.4.4.5 Computing the assignment

The signing entity computes the assignment $(T_1, T_2) = (-R, -\text{BS2I}(\gamma, H))$, where $H = h(\text{I2BS}(\beta, R) || M_2)$ is the hash-code of the concatenation of the witness and message M_2 . Before the concatenation, the witness shall be converted to a bit string of length β .

6.4.4.6 Computing the signature

The signature is (R, S) where R is computed in 6.4.4.4 and [see Formula (34)]:

$$S = K^{-1}(\text{BS2I}(\gamma, H) + XR) \bmod q \quad (34)$$

6.4.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, $text$, $((R, S), text)$.

6.4.4.8 Constructing the signed message

A signed message is the concatenation of a message, M , and the appendix:

$$M || ((R, S), text)$$

6.4.5 Verification process

6.4.5.1 General

Prior to verifying the signature of a signed message, it is necessary that the verifier has trusted copies of p , q and G .

The verifier also acquires the necessary data items for the verification process: for example, the verification key Y (see [ISO/IEC 14888-1:2008](#), Clause 9 for additional required data items).

6.4.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier then checks to see that $0 < R < q$ and $0 < S < q$. If either condition is violated, the signature shall be rejected.

6.4.5.3 Preparing the message for verification

The verifier retrieves $M_2 = M$ from the signed message. M_1 is empty.

6.4.5.4 Retrieving the assignment

This stage is identical to 6.4.4.5. The inputs to the assignment function consist of the witness R from 6.4.5.2 and M_2 from 6.4.5.3. The assignment $T = (T_1, T_2)$ is recomputed as output from the assignment function, 6.4.4.5.

6.4.5.5 Recomputing the pre-signature

The inputs to this stage are domain parameters, verification key Y , assignment $T = (T_1, T_2)$ from 6.4.5.4 and second part of the signature S from 6.4.5.2. The verifier obtains a recomputed value Π' of the pre-signature by computing it using Formula (35):

$$\Pi' = Y^{-S^{-1}T_1 \bmod q} G^{-S^{-1}T_2 \bmod q} \bmod p \quad (35)$$

6.4.5.6 Recomputing the witness

The computations at this stage are the same as in 6.4.4.4. The verifier executes the witness function. The inputs are Π' from 6.4.5.5. Note that M_1 is empty. The output is the recomputed witness R' .

6.4.5.7 Verifying the witness

The verifier compares the recomputed witness, R' , from 6.4.5.6 to the value of R from 6.4.5.2. If $R' = R$, then the signature is valid.

6.5 SDSA

6.5.1 General

SDSA (Schnorr Digital Signature Algorithm) is a signature mechanism with $E = Z_p^*$, p a prime, and q a prime dividing $p - 1$. The parameter D is equal to 1. The message is prepared such that M_1 is the message to be signed, i.e. $M_1 = M$, and M_2 is empty. The witness function is defined by setting R equal to a hash-code. The assignment function is defined by setting $T_1 = -1$ and T_2 equal to the negative of the integer which is obtained by converting R to an integer according to the conversion rule, BS2I, given in Annex B and then reducing modulo q .

The coefficients (A, B, C) of the SDSA signature formula are set as Formula (36):

$$(A, B, C) = (T_1, T_2, S) \quad (36)$$

Thus, the signature formula becomes Formula (37):

$$-K + T_2X + S \equiv 0 \pmod{q} \quad (37)$$

NOTE SDSA stands for Schnorr Digital Signature Algorithm. The mechanism is taken from References [32] and [33]. The notation here has been changed from References [32] and [33] to conform with the notation used in the ISO/IEC 14888 series.

6.5.2 Parameters

p a prime, where $2^{\alpha-1} < p < 2^\alpha$

q a prime divisor of $p - 1$, where $2^{\beta-1} < q < 2^\beta$

G a generator of the subgroup of order q , such that $1 < G < q$

Four choices for the pair (α, h) are allowed in SDSA, namely (1 024, SHA-1), (2 048, SHA-224), (2 048, SHA-256), and (3 072, SHA-256). Corresponding β should be selected according to α in [Table 1](#).

The integers p , q , and G can be public and can be common to a group of users.

The parameters p , q and G are generated as specified in [Annex D](#). The parameters p and q can be generated using the prime generation techniques given in [ISO/IEC 18032](#).

It is recommended that all users check the proper generation of the SDSA public parameters according to Reference [\[17\]](#).

It is recommended that digital signatures based on SHA-1 should only be used for legacy applications.

6.5.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is Formula (38):

$$Y = G^X \bmod p \quad (38)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.5.4 Signature process

6.5.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.5.4.2 Producing the pre-signature

The input to this stage is the randomizer K , and the signing entity computes Formula (39):

$$\Pi = G^K \bmod p \quad (39)$$

6.5.4.3 Preparing message for signing

The message is prepared such that M_1 is the message to be signed, i.e. $M_1 = M$, and M_2 is empty.

6.5.4.4 Computing the witness

The signing entity computes the witness R as the hash-code of the pre-signature Π and the message M_1 [see Formula (40)]:

$$R = h(\text{I2BS}(\alpha, \Pi) || M) \quad (40)$$

6.5.4.5 Computing the assignment

The value of the witness R is converted to an integer according to conversion rule, BS2I, in [Annex B](#) and then reducing modulo q . The assignment (T_1, T_2) is $(-1, -\text{BS2I}(\gamma, R) \bmod q)$.

6.5.4.6 Computing the second part of the signature

The signature is (R, S) where $S = (K + \text{BS2I}(\gamma, R)X) \bmod q$.

As an option, one may wish to check if $R = 0$ or $S = 0$. If either $R = 0$ or $S = 0$, a new value of K should be generated and the signature should be recalculated (it is extremely unlikely that $R = 0$ or $S = 0$ if signatures are generated properly).

6.5.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field *text*.

6.5.4.8 Constructing the signed message

A signed message is the concatenation of the message, M , and the appendix

$$M || ((R, S) || \text{text})$$

6.5.5 Verification process

6.5.5.1 General

The verifying entity acquires the necessary data items required for the verification process.

6.5.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier checks to see that R is a non-zero string within the range of the hash function and that $0 < S < q$.

6.5.5.3 Preparing message for verification

The verifier retrieves M from the signed message and divides the message into two parts M_1 and M_2 , such that $M_1 = M$ and M_2 is empty.

6.5.5.4 Retrieving the assignment

The input to the assignment function consists of the witness R from [6.5.5.2](#). The assignment is Formula (41):

$$T = (T_1, T_2) = (-1, -\text{BS2I}(\gamma, R) \bmod q) \quad (41)$$

6.5.5.5 Recomputing the pre-signature

The inputs to this stage are domain parameters, verification key Y , assignment $T = (T_1, T_2)$ from [6.5.5.4](#) and second part of the signature S from [6.5.5.2](#). The verifier obtains a recomputed value Π' of the pre-signature by computing it using Formula (42):

$$\Pi' = Y^{(T_2 \bmod q)} G^{(-ST_1 \bmod q)} \bmod p = Y^{(T_2 \bmod q)} G^{(S \bmod q)} \bmod p \quad (42)$$

6.5.5.6 Recomputing the witness

The computations at this stage are the same as in [6.5.4.4](#) and [6.5.4.5](#). The verifier executes the witness function. The input is Π' from [6.5.5.5](#) and M_1 from [6.5.5.3](#). The output is the recomputed witness R' which is the hash-code of the recomputed pre-signature Π' and the message M .

6.5.5.7 Verifying the witness

The verifier compares the recomputed witness, R' from [6.5.5.6](#) to the retrieved version of R from [6.5.5.2](#). If $R' = R$, then the signature is verified.

6.6 EC-DSA

6.6.1 General

EC-DSA (Elliptic Curve Digital Signature Algorithm) is an elliptic curve analogue of the DSA algorithm. The coefficients (A, B, C) of the EC-DSA signature formula are set as Formula (43):

$$(A, B, C) = (S, T_1, T_2) \quad (43)$$

where $(T_1, T_2) = (-R, -\text{BS2I}(\gamma, H))$ and $H = h(M)$ is the truncated hash-code of message M , converted to an integer according to the conversion rule given in [Annex B](#). The hash-function h is one of SHA-1, SHA-224, SHA-256, SHA-384 and SHA-512 described in [ISO/IEC 10118-3](#).

Verification key Y is $[X]G$; that is, the parameter D is equal to 1. The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The witness function is defined by Formula (44):

$$R = \text{FE2I}(r, \Pi_X) \bmod q \quad (44)$$

The conversion rule, FE2I, is given in [Annex B](#).

Thus the signature formula becomes Formula (45):

$$SK - RX - \text{BS2I}(\gamma, H) \equiv 0 \pmod{q} \quad (45)$$

NOTE This mechanism is based on the algorithm described in Reference [7].

6.6.2 Parameters

F	a finite field
E	an elliptic curve group over field F
$\#E$	the cardinality of E
q	a prime divisor of $\#E$
G	a point on the elliptic curve of order q

All these parameters can be public and can be common to a group of users. The security strength of the hash function used shall meet or exceed the security strength associated with the bit length of q .

It is recommended that all users check the proper generation of the EC-DSA public parameters according to Reference [7] or Reference [17].

It is recommended that digital signatures based on SHA-1 should only be used for legacy applications.

6.6.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is Formula (46):

$$Y = [X]G \quad (46)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.6.4 Signature process

6.6.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.6.4.2 Producing the pre-signature

The input to this stage is the randomizer K and the signing entity computes Formula (47):

$$\Pi = [K]G \quad (47)$$

6.6.4.3 Preparing message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.6.4.4 Computing the witness

The signing entity computes $R = \text{FE2I}(r, \Pi_X) \bmod q$.

6.6.4.5 Computing the assignment

The signing entity computes the hash-code; if the output bit length of the selected hash-function is larger than $\lceil \log_2 q \rceil$, H is set to the leftmost (most significant) $\lceil \log_2 q \rceil$ bits of $h(M_2)$. Otherwise, H is $h(M_2)$. Afterwards, H is converted to integer according to conversion rule, BS2I, in [Annex B](#). The assignment (T_1, T_2) is $(-R, -\text{BS2I}(\gamma, H))$.

6.6.4.6 Computing the second part of the signature

The signature is (R, S) where R is computed in [6.6.4.4](#), and [see Formula (48)]:

$$S = (K^{-1}(XR + \text{BS2I}(\gamma, H))) \bmod q \quad (48)$$

It is required to check if $R = 0$ or $S = 0$. If either $R = 0$ or $S = 0$, a new value of K should be generated and the signature should be recalculated (it is extremely unlikely that $R = 0$ or $S = 0$ if signatures are generated properly).

6.6.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, *text*, $((R, S), \text{text})$.

6.6.4.8 Constructing the signed message

A signed message is the concatenation of the message, M , and the appendix:

$$M || ((R, S), \text{text})$$

6.6.5 Verification process

6.6.5.1 General

The verifying entity acquires the necessary data items required for the verification process.

6.6.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier then first checks to see that $0 < R < q$ and $0 < S < q$; if either condition is violated the signature shall be rejected.

6.6.5.3 Preparing message for verification

The verifier retrieves M from the signed message and divides the message into two parts M_1 and M_2 . M_1 will be empty and $M_2 = M$.

6.6.5.4 Retrieving the assignment

This stage is identical to [6.6.4.5](#). The inputs to the assignment function consist of the witness R from [6.6.5.2](#) and M_2 from [6.6.5.3](#). The assignment $T = (T_1, T_2)$ is recomputed as output from the assignment function, [6.6.4.5](#).

6.6.5.5 Recomputing the pre-signature

The inputs to this stage are system parameters, verification key Y , assignment $T = (T_1, T_2)$ from [6.6.5.4](#) and second part of the signature S from [6.6.5.2](#). The verifier obtains a recomputed value Π' of the pre-signature by computing it using Formula (49):

$$\Pi' = [-S^{-1}T_1 \bmod q]Y + [-S^{-1}T_2 \bmod q]G \quad (49)$$

6.6.5.6 Recomputing the witness

The computations at this stage are the same as in [6.6.4.4](#). The verifier executes the witness function. The input is Π' from [6.6.5.5](#). The output is the recomputed witness R' .

6.6.5.7 Verifying the witness

The verifier compares the recomputed witness, R' from [6.6.5.6](#) to the retrieved version of R from [6.6.5.2](#). If $R' = R$, then the signature is verified.

6.7 EC-KCDSA

6.7.1 General

EC-KCDSA (Elliptic Curve Korean Certificate-based Digital Signature Algorithm) is a signature mechanism with verification key $Y = [X^{-1}]G$; that is, the parameter D is equal to -1 . The message is

prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The witness function is defined by Formula (50):

$$R = h(\text{FE2BS}(r, \Pi_X)) \quad (50)$$

If γ is greater than β , then the witness function is replaced by Formula (51):

$$R = \text{I2BS}(\beta', \text{BS2I}(\gamma, h(\text{FE2BS}(r, \Pi_X))) \bmod 2^{\beta'}) \quad (51)$$

where β' is $8 * \lceil \beta / 8 \rceil$.

Domain parameters shall indicate the employed hash-function. The assignment function is defined by Formula (52):

$$(T_1, T_2) = (V, -1) \quad (52)$$

where $V = \text{BS2I}(\beta', (R \oplus H)) \bmod q$. The value H is the hash-code from public key Y and message M .

The coefficients (A, B, C) of the EC-KCDSA signature formula are set as Formula (53):

$$(A, B, C) = (T_2, S, T_1) \quad (53)$$

Thus, the signature formula becomes Formula (54):

$$-K + SX^{-1} + V \equiv 0 \pmod{q} \quad (54)$$

NOTE This mechanism is taken from Reference [37]. The notation here has been changed slightly from Reference [37] to conform with notation used elsewhere in this document.

6.7.2 Parameters

l the input block size (in bits) of the selected hash-function h

F a finite field

E an elliptic curve group over field F

$\#E$ the cardinality of E

q a prime divisor of $\#E$

G a point on the elliptic curve of order q

Hash-function identifier or OID with specified hash-function

All these parameters can be public and can be common to a group of users.

It is recommended that all users check the proper generation of the EC-KCDSA public parameters according to Reference [37].

6.7.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is -1 . The corresponding public verification key Y is Formula (55):

$$Y = [X^{-1}]G \quad (55)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.7.4 Signature process

6.7.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.7.4.2 Producing the pre-signature

The input to this stage is the randomizer K and the signing entity computes Formula (56):

$$\Pi = [K]G \quad (56)$$

6.7.4.3 Preparing the message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.7.4.4 Computing the witness

The signing entity computes $R = h(\text{FE2BS}(r, \Pi_X))$, where output of h is the hash-code of Π_X . If γ is greater than β , then the computation of witness is replaced by $R = \text{I2BS}(\beta', \text{BS2I}(\gamma, h(\text{FE2BS}(r, \Pi_X)))) \bmod 2^{\beta'}$.

The conversion rules, FE2BS, I2BS and BS2I, are given in [Annex B](#).

6.7.4.5 Computing the assignment

The signing entity computes the assignment $(T_1, T_2) = (V, -1)$ where $V = \text{BS2I}(\beta', R \oplus H) \bmod q$, where $H = h(Y' \parallel M_2)$ is the hash-code of the concatenation of Y' and message M_2 . The value of Y' is the leftmost l bits of a bit sequence $\text{FE2BS}(r, Y_X) \parallel \text{FE2BS}(r, Y_Y)$. If l is greater than the length of the sequence, 0 is padded as much as needed after the sequence. In computing V , the bit string $R \oplus H$ shall be converted to an integer before modulo reduction with respect to q .

If γ is greater than β , then the computation of H is replaced by $H = \text{I2BS}(\beta', \text{BS2I}(\gamma, h(Y' \parallel M_2))) \bmod 2^{\beta'}$.

NOTE Y' is a fixed value for a user, thus, this value can be kept as a user parameter.

6.7.4.6 Computing the second part of the signature

The signature is (R, S) where R is computed in [6.7.4.4](#) and [see Formula (57)]:

$$S = X(K - V) \bmod q \quad (57)$$

6.7.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, $text, ((R, S), text)$.

6.7.4.8 Constructing the signed message

A signed message is the concatenation of a message, M , and the appendix:

$$M \parallel ((R, S), \text{text})$$

6.7.5 Verification process

6.7.5.1 General

Prior to verifying the signature of a signed message, it is necessary that the verifier has trusted copies of E , q and G .

The verifier also acquires the necessary data items for the verification process: for example, the verification key Y (see [ISO/IEC 14888-1:2008](#), Clause 9 for additional required data items).

6.7.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier then checks whether the following conditions hold or not:

- $0 < S < q$;
- if the length of the value γ is not greater than β , the bit length of R is equal to the output bit length of the employed hash-function h ;
- if the length of the value γ is greater than β , the bit length of R is equal to β' .

If any of the above condition does not hold the signature shall be rejected.

6.7.5.3 Preparing the message for verification

The verifier retrieves $M_2 = M$ from the signed message. M_1 is empty.

6.7.5.4 Retrieving the assignment

This stage is identical to [6.7.4.5](#). The inputs to the assignment function consist of the witness R from [6.7.5.2](#) and M_2 from [6.7.5.3](#). The assignment $T = (T_1, T_2)$ is recomputed as output from the assignment function, [6.7.4.5](#).

6.7.5.5 Recomputing the pre-signature

The inputs to this stage are domain parameters, verification key Y , assignment $T = (T_1, T_2)$ from [6.7.5.4](#) and the second part of the signature S from [6.7.5.2](#). The verifier obtains a recomputed value Π' of the pre-signature using Formula (58):

$$\Pi' = [S \bmod q]Y + [T_1 \bmod q]G \quad (58)$$

6.7.5.6 Recomputing the witness

The computations at this stage are the same as in [6.7.4.4](#). The verifier executes the witness function. The input is Π' from [6.7.5.5](#). Note that M_1 is empty. The output is the recomputed witness R' .

6.7.5.7 Verifying the witness

The verifier compares the recomputed witness R' from [6.7.5.6](#) to the value of R from [6.7.5.2](#). If $R' = R$, then the signature is valid.

6.8 EC-GDSA

6.8.1 General

EC-GDSA (Elliptic Curve German Digital Signature Algorithm) is a signature mechanism with verification key $Y = [X^{-1}]G$; that is, the parameter D is equal to -1 . The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The witness function is defined by Formula (59):

$$R = \text{FE2I}(r, \Pi_X) \bmod q \quad (59)$$

The coefficients (A, B, C) of the EC-GDSA signature formula are set as Formula (60):

$$(A, B, C) = (T_1, S, T_2) \quad (60)$$

where

$$(T_1, T_2) = (-R, H);$$

H is the hash-code of the message M .

Thus, the signature formula becomes Formula (61):

$$-RK + SX^{-1} + H \equiv 0 \pmod{q} \quad (61)$$

NOTE EC-GDSA stands for Elliptic Curve German Digital Signature Algorithm[16].

6.8.2 Parameters

F a finite field

E an elliptic curve group over field F

$\#E$ the cardinality of E

q a prime divisor of $\#E$

G a point on the elliptic curve of order q

Hash-function identifier or OID with specified hash-function

All these parameters can be public and can be common to a group of users.

It is recommended that all users check the proper generation of the public parameters.

6.8.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is -1 . Thus, the corresponding public verification key Y is Formula (62):

$$Y = [X^{-1}]G \quad (62)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.8.4 Signature process

6.8.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.8.4.2 Producing the pre-signature

The input to this stage is the randomizer K and the signing entity computes Formula (63):

$$\Pi = [K]G \quad (63)$$

6.8.4.3 Preparing message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.8.4.4 Computing the witness

The signing entity computes $R = \text{FE2I}(r, \Pi_X) \bmod q$ where the witness is simply a function of the pre-signature.

6.8.4.5 Computing the assignment

The signing entity computes the assignment $(T_1, T_2) = (-R, \text{BS2I}(\gamma, H))$ where H is the hash-code of the message M_2 .

6.8.4.6 Computing the second part of the signature

The signature is (R, S) where R is computed in [6.8.4.4](#) and [see Formula (64)]:

$$S = X(KR - \text{BS2I}(\gamma, H)) \bmod q \quad (64)$$

6.8.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, $text$, $((R, S), text)$.

6.8.4.8 Constructing the signed message

A signed message is the concatenation of the message, M , and the appendix:

$$M || ((R, S), text)$$

6.8.5 Verification process

6.8.5.1 General

The verifying entity acquires the necessary data items required for the verification process.

6.8.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier then first checks to see that $0 < R < q$ and $0 < S < q$; if either condition is violated the signature shall be rejected.

6.8.5.3 Preparing message for verification

The verifier retrieves M from the signed message and divides the message into two parts M_1 and M_2 . M_1 will be empty and $M_2 = M$.

6.8.5.4 Retrieving the assignment

This stage is identical to 6.8.4.5. The inputs to the assignment function consist of the witness R from 6.8.5.2 and M_2 from 6.8.5.3. The assignment $T = (T_1, T_2)$ is recomputed as output from the assignment function, 6.8.4.5.

6.8.5.5 Recomputing the pre-signature

The inputs to this stage are system parameters, verification key Y , assignment $T = (T_1, T_2)$ from 6.8.5.4 and second part of the signature S from 6.8.5.2. The verifier obtains a recomputed value Π' of the pre-signature by computing it using Formula (65):

$$\Pi' = [(-T_1)^{-1}S \bmod q]Y + [(-T_1)^{-1}T_2 \bmod q]G \quad (65)$$

6.8.5.6 Recomputing the witness

The computations at this stage are the same as in 6.8.4.4. The verifier executes the witness function. The input is Π' from 6.8.5.5. The output is the recomputed witness R' .

6.8.5.7 Verifying the witness

The verifier compares the recomputed witness, R' from 6.8.5.6 to the retrieved version of R from 6.8.5.2. If $R' = R$, then the signature is verified.

6.9 EC-RDSA

6.9.1 General

EC-RDSA (Elliptic Curve Russian Digital Signature Algorithm) is a signature mechanism with verification key $Y = [X]G$; that is, the parameter D is equal to 1. The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$. The coefficients (A, B, C) of the EC-RDSA signature formula are set as Formula (66):

$$(A, B, C) = (T_1, T_2, -S) \quad (66)$$

where $(T_1, T_2) = (H, R)$ and $H = h(M)$ is the hash-code of message M , converted to an integer as described in 6.9.4.5.

The witness function is defined by Formula (67):

$$R = \text{FE2I}(r, \Pi_X) \bmod q \quad (67)$$

Thus the signature formula becomes Formula (68):

$$HK + RX - S \equiv 0 \pmod{q} \quad (68)$$

NOTE EC-RDSA stands for Elliptic Curve Russian Digital Signature Algorithm. The mechanism is taken from Reference [21]. The notation here has been changed from Reference [22] to conform with the notation used in ISO/IEC 14888 (all parts).

6.9.2 Parameters

p a prime

E an elliptic curve group over the field $GF(p)$

$\#E$ the cardinality of E

q a prime divisor of $\#E$

G a point on the elliptic curve of order q

Hash-function identifier or OID with specified hash-function.

All these parameters can be public and can be common to a group of users.

6.9.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is Formula (69):

$$Y = [X]G \quad (69)$$

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

NOTE Reference [21] does not completely specify the process of generation of a user's secret signature key X .

6.9.4 Signature process

6.9.4.1 Producing the randomizer

The signing entity generates a random or pseudo-random integer K such that $0 < K < q$.

6.9.4.2 Producing the pre-signature

The input to this stage is the randomizer K , and the signing entity computes Formula (70):

$$\Pi = [K]G \quad (70)$$

6.9.4.3 Preparing message for signing

The message is prepared such that M_1 is empty and M_2 is the message to be signed, i.e. $M_2 = M$.

6.9.4.4 Computing the witness

The signing entity computes $R = \text{FE2I}(r, \Pi_X) \bmod q$.

6.9.4.5 Computing the assignment

The signing entity computes $H = h(M_2)$. H is then converted to an integer according to conversion rule BS2I in Annex B. If H is equal to 0 mod q , then H is set to 1. The assignment (T_1, T_2) is $(\text{BS2I}(\gamma, H), R)$, if $\text{BS2I}(\gamma, H) \neq 0 \pmod{q}$, or $(1, R)$ otherwise.

6.9.4.6 Computing the second part of the signature

The signature is (R, S) where R is computed as given in 6.9.4.4 and [see Formula (71)]:

$$S = RX + KH \bmod q \quad (71)$$

The signer should check whether $R = 0$ or $S = 0$. If either $R = 0$ or $S = 0$, a new value of K should be generated and the signature should be recalculated.

6.9.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field *text*, i.e. it will equal $((R, S) || \text{text})$.

6.9.4.8 Constructing the signed message

A signed message is the concatenation of the message M and the appendix:

$$M || ((R, S) || \text{text})$$

6.9.5 Verification process

6.9.5.1 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier then checks whether $0 < R < q$ and $0 < S < q$; if either condition does not hold, the signature shall be rejected.

6.9.5.2 Preparing message for verification

The verifier retrieves M from the signed message and divides the message into two parts M_1 and M_2 . M_1 will be empty and $M_2 = M$.

6.9.5.3 Retrieving the assignment

This stage is identical to [6.9.4.5](#). The inputs to the assignment function consist of the witness R from [6.9.5.1](#) and M_2 from [6.9.5.2](#). The assignment $T = (T_1, T_2)$ is recomputed as the output from the assignment function given in [6.9.4.5](#).

6.9.5.4 Recomputing the pre-signature

The inputs to this stage are system parameters, the verification key Y , the assignment $T = (T_1, T_2)$ from [6.9.5.3](#), and the second part of the signature S from [6.9.5.1](#). The verifier obtains a recomputed value Π' of the pre-signature by computing it using Formula (72):

$$\Pi' = [-T_1^{-1}T_2 \bmod q]Y + [T_1^{-1}S \bmod q]G \quad (72)$$

6.9.5.5 Recomputing the witness

The computations at this stage are the same as in [6.9.4.4](#). The verifier executes the witness function. The input is Π' from [6.9.5.4](#). The output is the recomputed witness, R' .

6.9.5.6 Verifying the witness

The verifier compares the recomputed witness, R' from [6.9.5.5](#) to the retrieved version of R from [6.9.5.1](#). If $R' = R$, then the signature is verified.

6.10 EC-SDSA

6.10.1 General

EC-SDSA (Elliptic Curve Schnorr Digital Signature Algorithm) is a signature mechanism with verification key $Y = [X]G$; that is, the parameter D is equal to 1. The message is prepared such that M_2 is empty and $M_1 = M$ the message to be signed. The witness R is computed as a hash-code of the message M and a random pre-signature $\Pi = [K]G$, by one of two methods, either:

- *normal* $R = h(\text{FE2BS}(r, \Pi_X) \parallel \text{FE2BS}(r, \Pi_Y) \parallel M)$; or
- *optimized* $R = h(\text{FE2BS}(r, \Pi_X) \parallel M)$.

The first method generates the witness by hashing the concatenation of the x-coordinate of Π , the y-coordinate of Π and the message M . The second method omits the y-coordinate from the hash calculation and thereby improves performance.

The second method is an optimized variant of EC-SDSA (see Reference [30]).

The coefficients (A, B, C) of the EC-SDSA signature formula are set as Formula (73):

$$(A, B, C) = (T_1, T_2, S) \quad (73)$$

where $(T_1, T_2) = (-1, -\text{BS2I}(\gamma, R) \bmod q)$.

Thus, the signature formula becomes Formula (74):

$$-K + T_2X + S \equiv 0 \pmod{q} \quad (74)$$

NOTE EC-SDSA stands for Elliptic Curve Schnorr Digital Signature Algorithm. The mechanism is taken from Reference [33]. The notation here has been changed from Reference [33] to conform with the notation used in the [ISO/IEC 14888](#) series.

6.10.2 Parameters

- F a finite field
- E elliptic curve group over the field F
- $\#E$ cardinality of E
- q prime divisor of $\#E$
- G a point of order q on the elliptic curve

Hash-function identifier or OID with specified hash-function

All these parameters can be public and can be common to a group of users.

It is recommended that all users check the proper generation of the public parameters according to Reference [7].

6.10.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is $Y = [X]G$.

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.10.4 Signature process

6.10.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.10.4.2 Producing the pre-signature

The input to this stage is the randomizer K , and the signing entity computes $\Pi = [K]G$.

6.10.4.3 Preparing message for signing

The message is prepared such that M_1 is the message to be signed, i.e. $M_1 = M$, and M_2 is empty.

6.10.4.4 Computing the witness

The signing entity computes $R = h(\text{FE2BS}(r, \Pi_X) || \text{FE2BS}(r, \Pi_Y) || M)$.

For the optimized variant of EC-SDSA, the signing entity instead computes $R = h(\text{FE2BS}(r, \Pi_X) || M)$.

6.10.4.5 Computing the assignment

The value of the witness R is converted to an integer according to conversion rule, BS2I, in [Annex B](#) and then reducing modulo q .

The assignment (T_1, T_2) is $(-1, -\text{BS2I}(\gamma, R) \bmod q)$.

6.10.4.6 Computing the second part of the signature

The signature is (R, S) where $S = (K + \text{BS2I}(\gamma, R)X) \bmod q$.

As an option, one may wish to check if $R = 0$ or $S = 0$. If either $R = 0$ or $S = 0$, a new value of K should be generated and the signature should be recalculated (it is extremely unlikely that $R = 0$ or $S = 0$ if signatures are generated properly).

6.10.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, *text*.

6.10.4.8 Constructing the signed message

A signed message is the concatenation of the message, M , and the appendix:

$$M || ((R, S) || \text{text})$$

6.10.5 Verification process

6.10.5.1 General

The verifying entity acquires the necessary data items required for the verification process.

6.10.5.2 Retrieving the witness

The verifier retrieves the witness R and the second part of the signature S from the appendix. The verifier first checks to see that R is a non-zero string within the range of the hash function and $0 < S < q$; if either condition is violated the signature shall be rejected.

6.10.5.3 Preparing message for verification

The verifier retrieves M from the signed message and divides the message into two parts M_1 and M_2 , such that $M_1 = M$ and M_2 is empty.

6.10.5.4 Retrieving the assignment

The input to the assignment function consists of the witness R from 6.10.5.2. The assignment $T = (T_1, T_2) = (-1, -\text{BS2I}(\gamma, R) \bmod q)$.

6.10.5.5 Recomputing the pre-signature

The inputs to this stage are system parameters, verification key Y , assignment $T = (T_1, T_2)$ from 6.10.5.4 and second part of the signature S from 6.10.5.2. The verifier obtains a recomputed value Π' of the pre-signature by computing it using the formula:

$$\Pi' = [-ST_1 \bmod q]G + [T_2 \bmod q]Y = [S \bmod q]G + [T_2 \bmod q]Y$$

6.10.5.6 Recomputing the witness

The computations at this stage are the same as in 6.10.4.4 and 6.10.4.5. The verifier executes the witness function from 6.10.4.4. The input is Π' from 6.10.5.5. The output is the recomputed witness R' which is the hash-code of the recomputed pre-signature Π' and the message M .

6.10.5.7 Verifying the witness

The verifier compares the recomputed witness, R' from 6.10.5.6 to the retrieved version of R from 6.10.5.2. If $R' = R$, then the signature is verified.

6.11 EC-FSDSA

6.11.1 General

EC-FSDSA (Elliptic Curve Full Schnorr Digital Signature Algorithm) is a signature mechanism with verification key $Y = [X]G$; that is, the parameter D is equal to 1. The message is prepared such that M_1 is empty and $M_2 = M$ the message to be signed. The witness R is computed as Formula (75):

$$R = \text{FE2BS}(r, \Pi_X) || \text{FE2BS}(r, \Pi_Y) \quad (75)$$

The coefficients (A, B, C) of the EC-FSDSA signature formula are set as Formula (76):

$$(A, B, C) = (T_1, T_2, S) \quad (76)$$

where $T = (T_1, T_2) = (-1, -\text{BS2I}(\gamma, h(R || M)) \bmod q)$.

Thus, the signature formula becomes Formula (77):

$$-K + T_2X + S \equiv 0 \pmod{q} \quad (77)$$

NOTE EC-FSDSA stands for Elliptic Curve Full Schnorr Digital Signature Algorithm. The mechanism is taken from Reference [33]. The notation here has been changed from Reference [33] to conform with the notation used in the ISO/IEC 14888 series.

6.11.2 Parameters

- F a finite field
- E an elliptic curve group over the field F
- $\#E$ the cardinality of E

q a prime divisor of $\#E$

G a point of order q on the elliptic curve E

Hash function identifier or OID with specified hash function

All these parameters can be public and can be common to a group of users.

It is recommended that all users check the proper generation of the public parameters according to Reference [7].

6.11.3 Generation of signature key and verification key

The signature key of a signing entity is a secretly generated random or pseudo-random integer X such that $0 < X < q$. The parameter D is 1. The corresponding public verification key Y is $Y = [X]G$.

A user's secret signature key X and public verification key Y are normally fixed for a period of time. The signature key X shall be kept secret.

6.11.4 Signature process

6.11.4.1 Producing the randomizer

The signing entity computes a random or pseudo-random integer K such that $0 < K < q$.

6.11.4.2 Producing the pre-signature

The input to this stage is the randomizer K , and the signing entity computes $\Pi = [K]G$.

6.11.4.3 Preparing message for signing

The message is prepared such that M_2 is the message to be signed, i.e. $M_2 = M$, and M_1 is empty.

6.11.4.4 Computing the witness

The signing entity computes $R = \text{FE2BS}(r, \Pi_X) || \text{FE2BS}(r, \Pi_Y)$.

6.11.4.5 Computing the assignment

The signing entity computes the hash code $h(R || M)$. Afterwards, the hash code is converted to an integer according to conversion rule, BS2I, in [Annex B](#) and then reduced modulo q . The assignment (T_1, T_2) is $(-1, -\text{BS2I}(\gamma, h(R || M)) \bmod q)$.

6.11.4.6 Computing the second part of the signature

The signature is (R, S) where $S = (K + \text{BS2I}(\gamma, h(R || M)))X \bmod q$.

As an option, one may wish to check if $R = 0$ or $S = 0$. If either $R = 0$ or $S = 0$, a new value of K should be generated and the signature should be recalculated. (It is extremely unlikely that $R = 0$ or $S = 0$ if signatures are generated properly).

6.11.4.7 Constructing the appendix

The appendix will be the concatenation of (R, S) and an optional text field, *text*.