

Measurement of fluid flow — Evaluation of uncertainties

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National foreword

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The UK participation in its preparation was entrusted by Technical Committee CPI/30, Measurement of fluid flow in closed conduits, to Subcommittee CPI/30/9, General topics, which has the responsibility to:

- aid enquirers to understand the text;
- present to the responsible international/European committee any enquiries on the interpretation, or proposals for change, and keep the UK interests informed;
- monitor related international and European developments and promulgate them in the UK.

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Cross-references

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, the ISO/TR title page, pages ii to iv, pages 1 to 64, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

Amendments issued since publication

Amd. No.	Date	Comments

This British Standard, having been prepared under the direction of the Sector Board for Materials and Chemicals, was published under the authority of the Standards Board and comes into effect on 15 June 1998

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TECHNICAL REPORT

ISO/TR 5168

First edition
1998-04-01

Measurement of fluid flow — Evaluation of uncertainties

Mesure de débit des fluides — Calcul de l'incertitude



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Descriptors: Fluid flow, pipe flow, flow measurement, rules of calculation, error analysis.

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

- type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;
- type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;
- type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard (“state of the art”, for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 5168, which is a Technical Report of type 1, was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, Subcommittee SC 9, *Uncertainties in flow measurement*.

This document is being issued as a type 1 Technical Report because no consensus could be reached between ISO TC 30/SC 9 and ISO/TAG 4, *Metrology*, concerning the harmonization of this document with the *Guide to the expression of uncertainty in measurement*, which is a basic document in the ISO/IEC Directives. A future revision of this Technical Report will align it with the *Guide*.

This first edition as a Technical Report cancels and replaces the first edition as an International Standard (ISO 5168:1978), which has been technically revised.

Annex A and Annex B form an integral part of this Technical Report. Annex C, Annex D, Annex E and Annex F are for information only.

Introduction

One of the first International Standards to specifically address the subject of uncertainty in measurement was ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement*, published in 1978. The extensive use of ISO 5168 in practical applications identified many improvements to its methods; these were incorporated into a draft revision of this International Standard, which in 1990 received an overwhelming vote in favour of its publication. However, this draft revision of ISO 5168 was withheld from publication for a number of years since, despite lengthy discussions, no consensus could be reached with the draft version of a document under development by a Working Group of ISO Technical Advisory Group 4, *Metrology* (ISO TAG 4/WG 3). The TAG 4 document, *Guide to the expression of uncertainty in measurement* (GUM), was published in late 1993 as a basic document in the ISO/IEC Directives. At a meeting of the ISO Management Board in May 1995 it was decided to publish the revision of ISO 5168, *Measurement of fluid flow — Evaluation of uncertainties*, as a Technical Report.

One of the major differences between ISO/TR 5168 and the GUM is in the definitions and terminology. In addition, a substantial difference exists with respect to the concepts to be used to define practical measurement processes. In this Technical Report a normal distribution of the measurement data is assumed and Student's *t*-factor is used to determine the uncertainty. The method used to propagate elemental uncertainties to the overall uncertainty is essentially identical to that used in the GUM.

This document is published as a type 1 Technical Report instead of an International Standard because it is not consistent with the GUM. A future revision of this Technical Report will align the two documents.

1 Scope

Whenever a measurement of flowrate (discharge) is made, the value obtained from the experimental data is simply the best possible estimate of the true flowrate. In practice, the true flowrate may be slightly greater or less than this value.

1.1 This Technical Report details step-by-step procedures for the evaluation of uncertainties in individual flow measurements arising from both random and systematic error sources and for the propagation of component uncertainties into the uncertainty of the test results. These procedures enable the following processes to be carried out:

- a) estimation of the accuracy of results derived from flowrate measurement;
- b) selection of a proper measuring method and devices to achieve a required level of accuracy of flowrate measurement;
- c) comparison of the results of measurement;
- d) identification of the sources of errors contributing to a total uncertainty;
- e) refinement of the results of measurement as data accumulate.

NOTE It is assumed that the measurement process is carefully controlled and that all calibration corrections have been applied.

1.2 This Technical Report describes the calculations required in order to arrive at an estimate of the interval within which the true value of the flowrate may be expected to lie. The principle of these calculations is applicable to any flow measurement method, whether the flow is in an open channel or in a closed conduit.

NOTE Although this Technical Report has been drafted taking mainly into account the sources of error due to the instrumentation, it should be emphasized that the errors due to the flow itself (velocity distribution, turbulence, etc.) and to its effect on the method and on the response of the instrument can be of great importance with certain methods of flow measurement (see 5.7). Where a particular device or technique is used, some simplifications may be possible or special reference may have to be made to specific sources of error not identified in this Technical Report. Therefore reference should be made to the "Uncertainty of measurement" clause of the appropriate International Standard dealing with that device or technique.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this Technical Report. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this Technical Report are encouraged to investigate the possibility of applying the most recent editions of the standards listed below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 5725-1:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions*.

ISO 5725-2:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 2: Basic method for the determination of repeatability and reproducibility of a standard measurement method.*

ISO 5725-3:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 3: Intermediate measures of the precision of a standard measurement method.*

ISO 5725-4:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 4: Basic methods for the determination of the trueness of a standard measurement method.*

ISO 5725-6:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 6: Use in practice of accuracy values.*

3 Definitions and symbols

For the purposes of this Technical Report, the following definitions and symbols apply.

3.1 Definitions

3.1.1 correction

value which must be added algebraically to the indicated value to obtain the corrected result. It is numerically the same as a known error, but of opposite sign

3.1.2 coverage

percentage frequency at which an interval estimate of a parameter contains the true value. That is, in repeated sampling when the uncertainty interval provides 95 % coverage for each sample, over the long run the intervals will contain the true value 95 % of the time

3.1.3 error

result of a measurement minus the (conventional) true value of the measurement. See Figure 1

NOTE The known parts of an error of measurement may be compensated by applying appropriate corrections. The error of the corrected result can be characterized by an uncertainty.

3.1.4 estimate

value calculated from a sample of data as a substitute for an unknown population parameter
for example, the experimental standard deviation (s) is the estimate which describes the population standard deviation (σ)

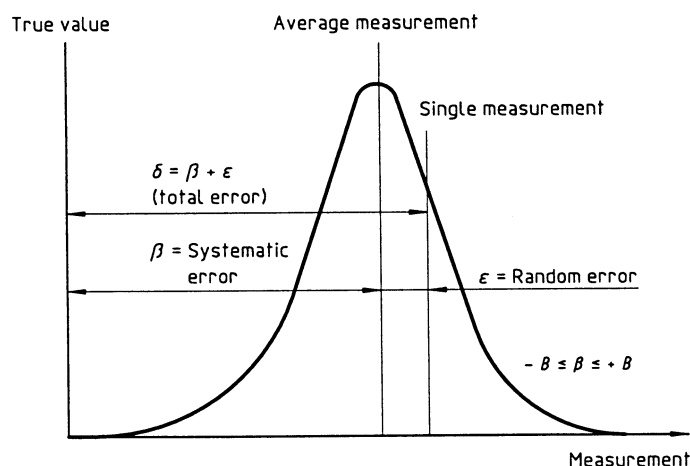


Figure 1 — Measurement error

3.1.5**fossilization**

creation of a fixed systematic error from a live random error when only a single calibration is relevant in the calibration process

3.1.6**influence [sensitivity] coefficient**

uncertainty propagated to the result due to unit uncertainty of the measurement (see subclause 7.4)

3.1.7**observed value**

value of a characteristic determined as the result of an observation or test

3.1.8**random error**

see Figure 2 and subclause 4.2

3.1.9**random uncertainty**

component of the uncertainty associated with the random error. See Figure 2

3.1.10**statistical quality control chart**

chart on which limits are drawn and on which are plotted values of any statistic computed from successive samples of a population

the statistics which are used (mean, range, percent defective, etc.) define the different kinds of control charts

3.1.11**systematic error**

see Figure 2 and Figure 3 and subclause 4.3

3.1.12**systematic uncertainty**

component of the uncertainty associated with the systematic error. See Figure 2

3.1.13**taylor's series**

power series to calculate the value of a function at a point in the neighbourhood of some reference point the series expresses the difference or differential between the new point and the reference point in terms of the successive derivatives of the function. Its form is:

$$f(x) - f(a) = \sum_{r=1}^{r=n-1} \frac{(x-a)^r}{r!} f'(a) + R_n$$

where $f'(a)$ denotes the value of the r th derivative of $f(x)$ at the reference point $x = a$. Commonly, if the series converges, the remainder R_n , is made infinitesimal by selecting an arbitrary number of terms and usually only the first term is used

3.1.14**uncertainty**

- 1) Half the uncertainty interval, for a symmetrical uncertainty interval.
- 2) The positive and negative components of a nonsymmetrical uncertainty interval, denoted by U^+ and U^- respectively.

3.1.15**uncertainty interval**

estimate characterizing the range of values within which the true value of a measurand is expected to lie

3.1.16

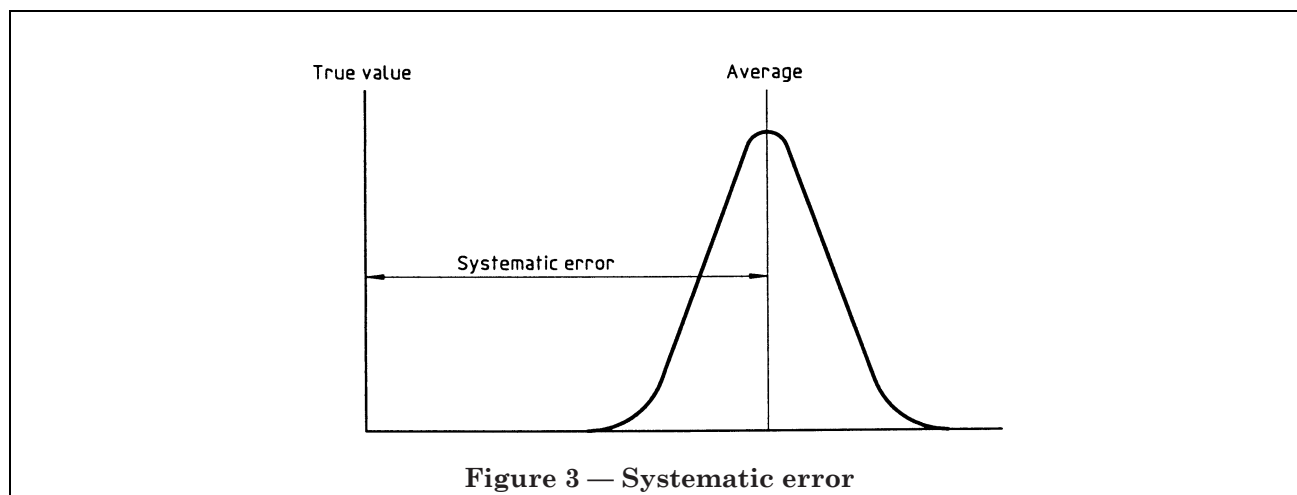
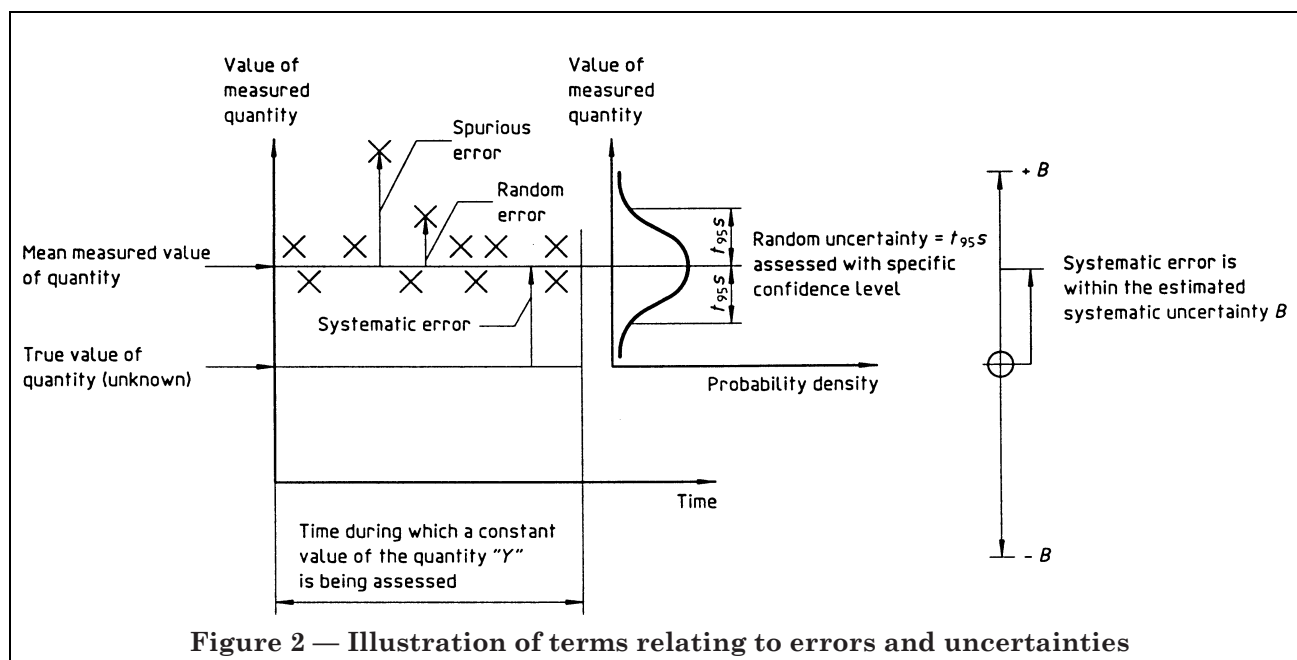
welch-satterthwaite method

method for estimating degrees of freedom of the result when combining experimental standard deviations with unequal degrees of freedom

3.1.17

working standard

standard, usually calibrated against a reference standard, which is used routinely to calibrate or check material measures or measuring instruments



3.2 Symbols

Symbol	Meaning
B	Systematic uncertainty of a symmetrical uncertainty interval.
	$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2}$
B_{ij}	Elemental systematic uncertainty. The j subscript indicates the category, i.e.: $j = 1$ calibration $= 2$ data acquisition $= 3$ data reduction $= 4$ method $= 5$ subjective or personal The i subscript is the number assigned to a given elemental source of error. If i is more than a single digit, a comma is used between i and j.
B^+, B^-	Positive and negative systematic uncertainties of a nonsymmetrical uncertainty interval.
overbar ($\bar{}$)	Mean value (of a variable).
M	Number of redundant instruments or tests.
N	Sample size.
s^2	Unbiased estimate of the variance, σ^2 .
s_{ij}	Estimate of the experimental standard deviation from one elemental source. The subscripts are the same as the elemental systematic uncertainties in B_{ij} .
	$s = \sqrt{\sum_j \sum_i s_{ij}^2}$
$s_{\bar{x}}$	Experimental standard deviation of the mean; equal to $\frac{s}{\sqrt{N}}$
S_{pooled}	$= \left[\frac{\sum_{i=1}^M \sum_{j=1}^N (x_{ij} - \bar{x}_i)^2}{M(N-1)} \right]^{1/2}$
	where
	$\bar{x}_i$ is the arithmetic mean of all x_i at the j th datum point.
t_{95}	Student's statistical parameter at the 95 % confidence level. The degrees of freedom, ν , of the sample estimate of the experimental standard deviation are needed to obtain the t values.
U^+, U^-	Positive and negative uncertainties of a nonsymmetrical uncertainty interval.
U_{ADD}	$= B + t_{95}s_{\bar{x}}$
U_{RSS}	$= \sqrt{B^2 + (t_{95}s_{\bar{x}})^2}$

$\bar{x}$ Arithmetic mean of the data values; x_i .

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$$

x_i Value of x at the i th datum point.

x_{ij} Value of x_i at the j th datum point.

$\bar{Y}$ Arithmetic mean of the n measurements of the variable Y .

Y_i A basic measurement.

β Systematic error, the fixed, or constant component of the total error, δ .

Δ Difference between measurements.

δ Total error.

ε Random error.

Θ_i Influence coefficient $\partial R / \partial Y_i$.

μ Population mean.

σ^2 Variance, the square of the standard deviation.

Subscripts

ADD Additive model.

RSS Root-sum-square model.

NOTE In ISO 5168:1978 and in many standards for flowrate measurement, e is used to indicate absolute uncertainty and E is used for relative uncertainty. In this Technical Report U' is used for relative uncertainty.

4 General principles of measurement uncertainty analysis

4.1 Nature of errors

All measurements have errors even after all known corrections and calibrations have been applied. The errors may be positive or negative and may be of a variable magnitude. Many errors vary with time. Some have very short periods while others vary daily, weekly, seasonally or yearly. Those which remain constant or apparently constant during the test are called systematic errors. The actual errors are rarely known; however, upper bounds on the errors can be estimated. The objective is to construct an uncertainty interval (or sometimes referred to as range) within which the true value will lie with a stated probability.

Errors are the differences between the measurements and the true value which is always unknown. The total measurement error, δ , is divided into two components: β , a fixed systematic error and a random error, ε , as shown in Figure 2. In some cases, the true value may be arbitrarily defined as the value that would be obtained by a specific metrology laboratory. Uncertainty is an estimate of the error which in most cases would not be exceeded. There are three types of error to be considered:

- a) random errors — see 4.2;
- b) systematic errors — see 4.3;
- c) spurious errors or mistakes (assumed to be identified and rejected prior to statistical analysis) — see 4.4.

It is rarely possible to give an absolute upper limit to the value of the error. It is, therefore, more practicable to give an interval within which the true value of the measured quantity can be expected to lie with a suitably high probability. This “uncertainty interval” is shown as $[\bar{x} - U, \bar{x} + U]$ in Figure 4 (the interval is twice the calculated uncertainty).

Since measurement systems are subject to two types of errors, systematic and random, it follows that an accurate measurement is one that has both small random and small systematic errors (see Figure 5).

4.2 Random error

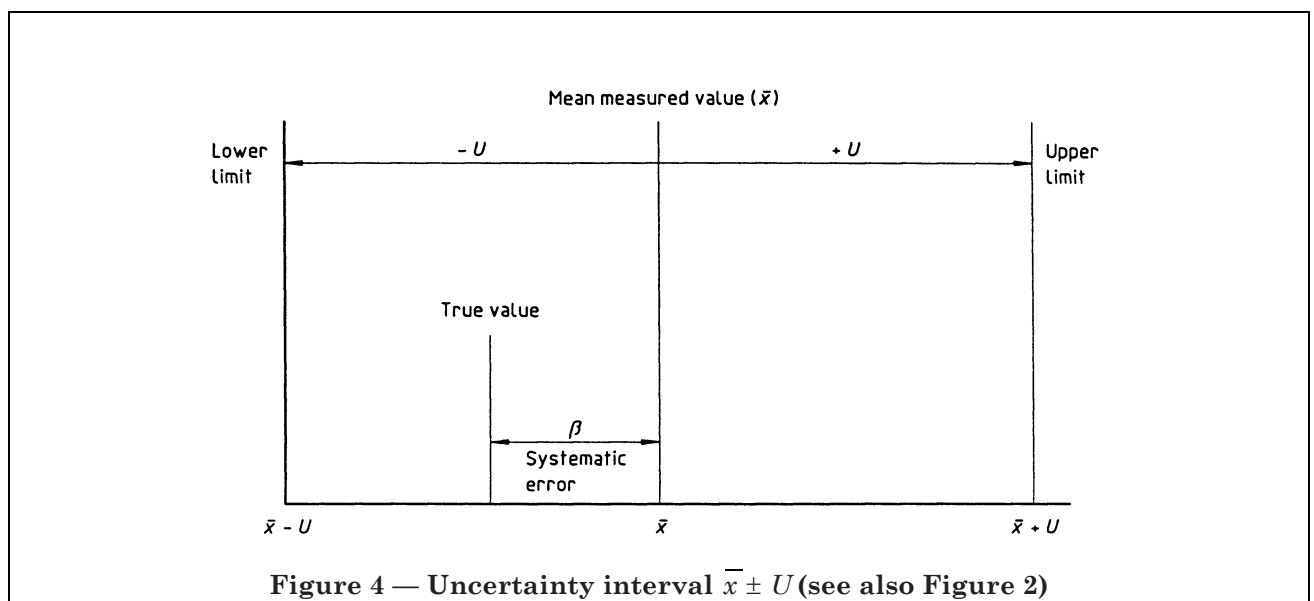
Random errors are caused by numerous, small, independent influences which prevent a measurement system from delivering the same reading when supplied with the same input value of the quantity being measured. The data points deviate from the mean in accordance with the laws of chance, such that the distribution usually approaches a normal distribution as the number of data points is increased. Random errors are sometimes referred to as precision errors. The standard deviation (σ) (see Figure 6) is used as a measure of the random error, ε . A large standard deviation means large scatter in the measurements. The statistic (s) is calculated from a sample to estimate the standard deviation and is called the experimental standard deviation.

$$s = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}} \quad \dots (1)$$

where

N is the number of measurements;

$\bar{x}$ is the average value of individual measurements x .



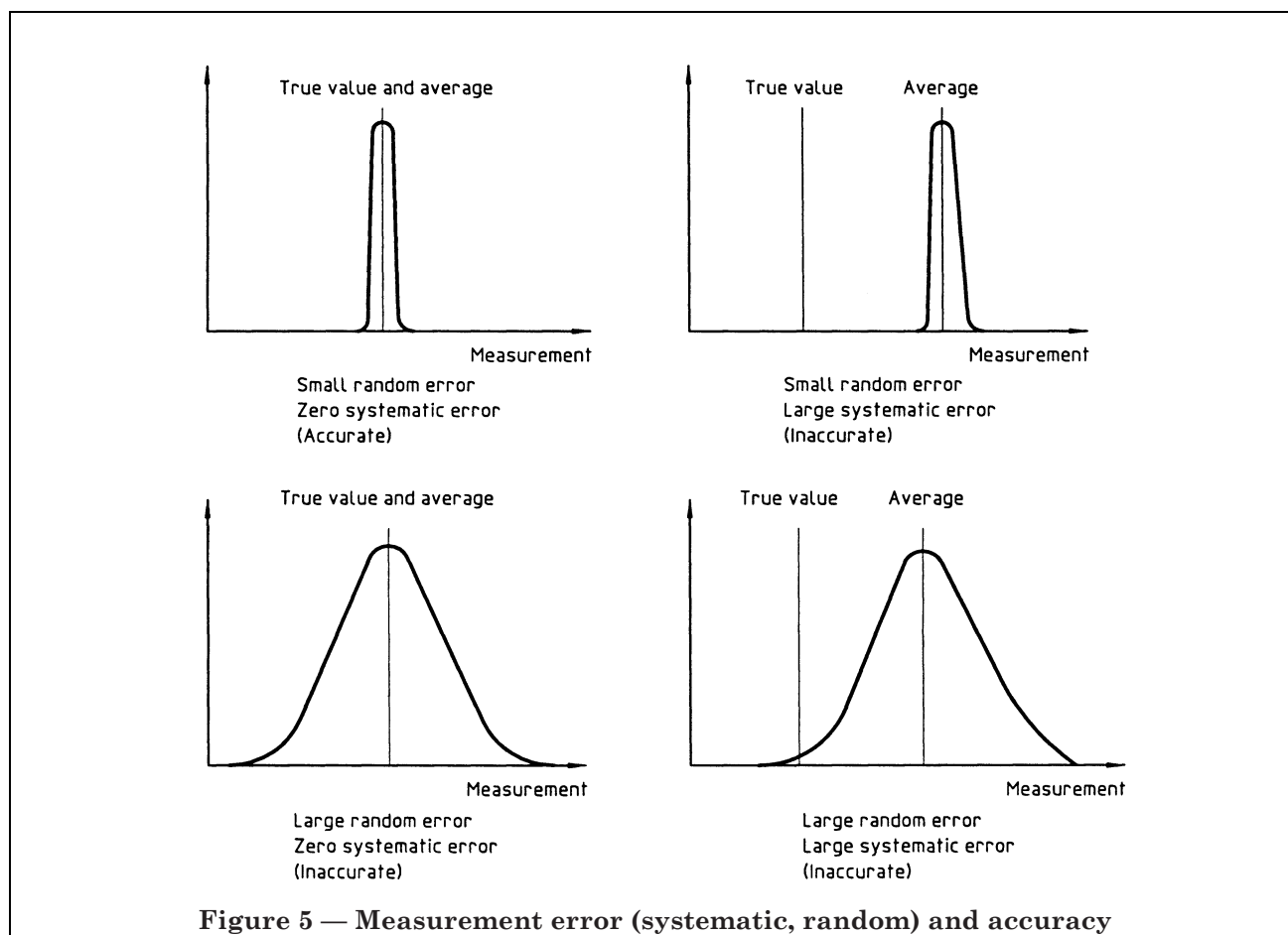


Figure 5 — Measurement error (systematic, random) and accuracy

For the normal distribution, the interval $\bar{x} \pm t_{95} s/\sqrt{N}$ will include the true mean, μ , approximately 95 % of the time. The random uncertainty of the mean is $t_{95} s/\sqrt{N}$. When the sample size is small, it is necessary to use the Student's t value at the 95 % level. For sample sizes equal to or greater than 30, two experimental standard deviations ($2s$) are used as an estimate of the random uncertainty in an individual measurement. This is explained in Annex A.

The random uncertainty can be reduced by making as many measurements as possible and using the arithmetic mean value, since the standard deviation of the mean of N independent measurements is $\sqrt{N}$ times smaller than the standard deviation of the measurements themselves.

$$\sigma_{\text{average}} = \frac{\sigma_{\text{individual}}}{\sqrt{N}} \quad \dots (2)$$

and, analogously

$$s_{\bar{x}} = \frac{s}{\sqrt{N}} \quad \dots (3)$$

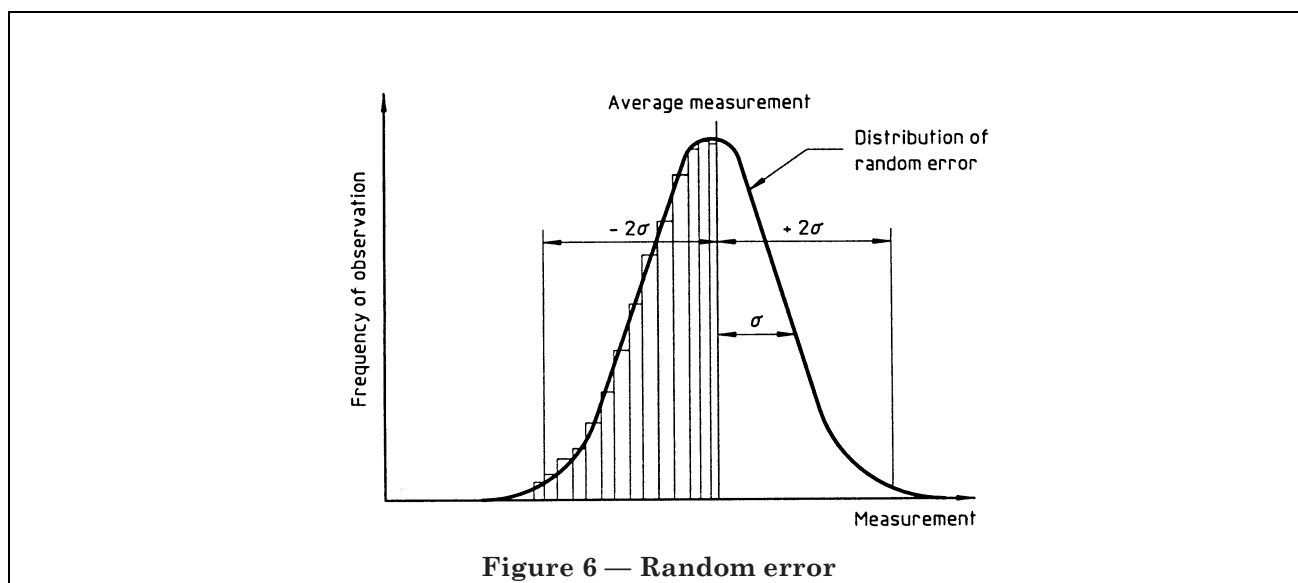


Figure 6 — Random error

4.3 Systematic error

The second component of the total error is the systematic error, β . At each flow level this error is constant for the duration of the test (Figure 1). In repeated measurements of a given sample, each measurement has the same systematic error. The systematic error can be determined only when the measurements are compared with the true value of the quantity measured and this is rarely possible. Systematic errors are sometimes referred to as biases.

Every effort shall be made to identify and account for all significant systematic errors. These may arise from (1) imperfect calibration corrections, (2) imperfect instrumentation installation, (3) imperfect data reduction, and may include (4) method errors, and (5) human errors. As the true systematic error is never known, an upper limit, B , is used in the uncertainty analysis.

In most cases, the systematic error, β , is equally likely to be plus or minus about the measurement. That is, it is not known if the systematic error is positive or negative, and the systematic uncertainty reflects this as $\pm B$. The systematic uncertainty, B , is estimated as an upper limit of the systematic error, β .

4.4 Spurious errors

Spurious errors are errors, such as human mistakes or instrument malfunction, which invalidate a measurement; for example, the transposing of numbers in recording data or the presence of pockets of air in leads from a water line to a manometer. Such errors cannot be treated with statistical analysis and the measurement should be discarded. Every effort should be made to eliminate spurious errors to properly control the measurement process.

To ensure control, all measurements should be monitored with statistical quality control charts. Drifts, trends and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are required for effective control. It is assumed herein that these precautions are observed and that the measurement process is under control; if not, the methods described are invalid.

After all obvious mistakes have been corrected or removed, there may remain a few observations which are suspicious solely because of their magnitude.

For errors of this nature, the statistical outlier tests given in Annex B should be used. These tests assume the observations are normally distributed. It is necessary to recalculate the experimental standard deviation of the distribution of observations whenever a datum is discarded as a result of the outlier test. It should also be emphasized that outliers should not be discarded unless there is an independent technical reason for believing that spurious errors may exist: data should not lightly be thrown away.

4.5 Combining elemental uncertainties

The test objective, test duration and the number of calibrations related to the test affect the classification of uncertainties into systematic and random components. Guidelines are presented in clause 6.

After all elemental error sources have been identified and classified as calibration, data acquisition, data reduction, methodic and subjective error sources and elemental standard deviations and systematic uncertainties estimated for each error source, a method for combining these elemental components into the experimental standard deviation and systematic uncertainty of the measurement is needed. The root-sum-square or quadrature combination is recommended.

$$s = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} s_{ij}^2} \quad \dots (4)$$

$$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2} \quad \dots (5)$$

4.6 Uncertainty of measurements

The measurement uncertainty analysis will be completed when:

- a) the systematic uncertainties and standard deviations of the measure have been propagated to uncertainty in the test result, keeping systematic and random components separate;
- b) if small samples are involved, an estimate of the degrees of freedom of the experimental standard deviation of the test result has been calculated from the Welch-Satterthwaite formula (see Annex A);
- c) the random and systematic uncertainties are combined into a single number to express a reasonable value for the overall uncertainty.

For simplicity of presentation, a single number, U , is needed to express a reasonable limit of error. The single number, some combination of the systematic and random uncertainties, must have a simple interpretation (e.g. the largest error reasonably expected), and be useful without complex explanation. For example, the true value of the measurement is expected to lie within the interval

$$[\bar{x} - U, \bar{x} + U] \quad \dots (6)$$

Since systematic uncertainties include those based on judgement and not on data, there is no way of combining systematic and random uncertainties to produce a single uncertainty figure with a statistically rigorous confidence level. However, since it is accepted that a single figure for the uncertainty of a measurement is often required, two alternative methods of combination are permitted:

- 1) linear addition:

$$U_{\text{ADD}} = B + t_{95}s_{\bar{x}} \quad \dots (7)$$

- 2) root-sum-square combination:

$$U_{\text{RSS}} = \sqrt{B^2 + (t_{95}s_{\bar{x}})^2} \quad \dots (8)$$

where B is the systematic uncertainty from equation (5) and $s_{\bar{x}}$ is the experimental standard deviation of the mean [equations (4) and (3)]. If large samples ($N > 30$) are used to calculate s , the value 2,0 may be used for t_{95} for simplicity. If small samples ($N \leq 30$) are used to calculate s , the methods in Annex A are required. There are two situations where it is possible to use a statistical confidence level for the uncertainty interval:

- a) if the systematic uncertainty is based on interlaboratory comparisons (see ISO 5725); and
- b) if the systematic uncertainty is judged to be negligible compared to the random uncertainty. Here the uncertainty interval is the test result $\pm t_{95}s_{\bar{x}}$, which is at the 95 % confidence level.

Typically U_{RSS} is considered to have coverage of approximately 95 %, and U_{ADD} is considered to have coverage between 95 % and 99 %.

4.7 Propagation of measurement uncertainties to test result uncertainties

If the test result is a function of several measurements, the experimental standard deviations and systematic uncertainties of the measurements must be combined or propagated to the test result using sensitivity factors, Θ , that relate the measurement to the test result (see 7.4). Small sample methods are given in Annex A.

In general, for m measurements, the experimental standard deviation and systematic uncertainty of the test result are obtained as follows:

$$s_R = \sqrt{\sum_{\text{all } m} (\Theta_m s_m)^2} \quad \dots (9)$$

$$B_R = \sqrt{\sum_{\text{all } m} (\Theta_m B_m)^2} \quad \dots (10)$$

The overall uncertainty for the test result is formed in the same manner as described for the measurement in 4.6.

4.8 Uncertainty analysis before and after measurement

Uncertainty analysis before measurement allows corrective action to be taken prior to the test to reduce uncertainties when they are too large or when the difference to be detected in the test is the same size or smaller than the predicted uncertainty. Uncertainty analysis before the test identifies the most cost-effective corrective action and the most accurate measurement method.

The before-measurement uncertainty analysis is based on data and information that exists before the test, such as calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analysis, expert opinions and, if necessary, special tests. With complex tests, there may be alternatives to evaluate prior to the test such as different test designs, instrumentation arrangements, alternative calculation procedures and concomitant variables. Corrective action resulting from this before-measurement analysis may include:

- a) improvements to instrumentation if the uncertainties are unacceptably high;
- b) selection of a different measurement or calibration method;
- c) repeated testing and/or increased sample sizes if the random uncertainties are unacceptably high. The experimental standard deviation of the mean is reduced as the number of samples used to calculate the mean is increased;
- d) instead of repeated testing, the test duration may be extended in order to average the output scatter (noise) of the flowmeter, resulting in a smaller random error per observation and hence a smaller random uncertainty;

NOTE For example, ultrasonic and vortex shedding meters may have to be calibrated against a master meter, allowing longer test times than allowed by a compact prover.

- e) rotating flowmeters normally generate an output showing a periodic cycle superimposed on an average meter factor. In this case the test duration shall be matched to an integer multiple of half or full periodic intervals in order to obtain the shortest test times.

After-measurement analysis is based on the actual measurement data. It is required to establish the final uncertainty. It is also used to confirm the before-measurement estimates and/or to identify data validity problems. When redundant instrumentation or calculation methods are available, the individual uncertainties should be compared for consistency with each other and with the before-measurement uncertainty analysis. If the uncertainty intervals do not overlap, a problem is indicated. The after-measurement random uncertainties should be compared with the before-measurement predictions.

5 Identification and classification of elemental measurement error sources

5.1 Summary of procedure

Make a complete list of every possible source of measurement error for all measurements that affect the end test result. For convenience, group them by some or all of the following categories:

- a) calibration,
- b) data acquisition,
- c) data reduction,
- d) errors of method and
- e) subjective or personal.

Within each category, there may be systematic and/or random error.

5.2 Systematic versus random

Systematic errors are those which remain constant in the process of measurement for a given value of flowrate.

Typical examples of systematic errors of flowrate measurements are:

- a) errors from a single flowmeter calibration;
- b) errors of determination of the constants in the working formula of a measuring method;
- c) errors due to truncating instead of rounding off the results of measurement.

Where the value and sign of a systematic error are known, it is assumed to be corrected (the correction being equal in value and opposite in sign to the systematic error). Inaccuracy of the correction results in a residual systematic uncertainty.

Random errors are those that produce variation (not predictable) in repeated measurements of the same quantity.

Typical random errors associated with flowrate measurement are those caused by inaccurate reading of the scale of a measuring instrument or by the scatter of the output signal of an instrument.

The effect of random errors on the random uncertainty may be reduced by averaging multiple results of the same value of the quantity.

The preliminary decision to determine if a given elemental source contributes to systematic uncertainty, random uncertainty or both, is made by adopting the following recommendation: the uncertainty of a measurement should be put into one of two categories depending on how the uncertainty is derived. The value of a random uncertainty is derived by a statistical analysis of repeated measurements, while that of a systematic uncertainty is estimated by nonstatistical methods.

This recommendation avoids a complex decision and keeps the statistical estimates separate from the judgement estimates as long as possible. The decision is preliminary and will be reviewed after consideration of the defined measurement process.

5.3 Categorization of elemental error sources

Possible error sources can be divided arbitrarily into three to five categories:

- a) calibration (see 5.4);
- b) data acquisition (see 5.5);
- c) data reduction (see 5.6);
- d) method-related (see 5.7);
- e) subjective or personal (see 5.8).

The size and complexity of the measurement uncertainty analysis may lead to the use of any or all of these categories.

For example, metrological maintenance (calibration, verification, certification) of flowmeters, flowrate measurements and processing of the data are done by different personnel. To control the possible sources of error, it is advisable to relate them to the stages of preparation, measurement and processing of the data.

In such cases, it is advisable to classify error sources into:

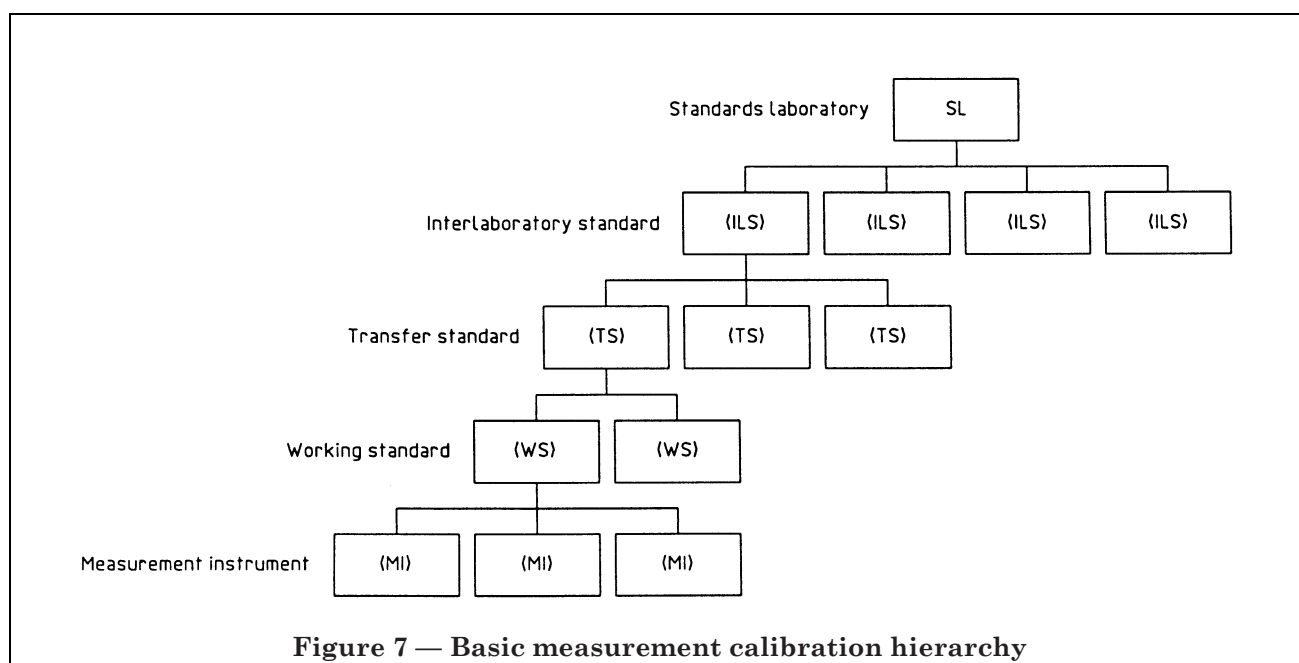
- a) calibration error sources (see 5.4);

- b) errors of measurement or data acquisition error sources (see 5.5);
- c) errors of processing the measurement data or data reduction error sources (see 5.6).

5.4 Calibration error sources

The major purpose of the calibration process is to determine systematic errors in order to eliminate them. Thus, the calibration process exchanges the large systematic uncertainty of an uncalibrated or poorly calibrated instrument for the smaller combination of the systematic uncertainty of the reference instrument and the random uncertainty of the comparison. This exchange of uncertainties is fundamental and is the basis of the notion that the uncertainty of the standard should be substantially less than that of the test instrument.

Figure 7 shows a typical transducer calibration hierarchy. Each calibration in this hierarchy constitutes an error source, with which is associated a pair of elemental uncertainties — the systematic uncertainty and the experimental standard deviation of the process. It should be noted that, from one step to another, these elemental uncertainties, as listed in Table 1, may be cumulative or independent. For example, B_{21} may include B_{11} . The second digit of the subscript indicates the category, i.e. 1 indicates calibration.



5.5 Data acquisition error sources

Figure 8 illustrates some of the error sources associated with a typical pressure data acquisition system. Data are acquired by measuring the electrical output resulting from pressure applied to a strain gauge type pressure-measurement instrument. Other error sources, such as probe errors, installation effects and environmental effects, also may be present. The effects of these error sources should be determined by performing overall system calibrations, comparing known applied pressures with measured values. However, should it not be possible to do this, then it is necessary to estimate each of the elemental uncertainties and combine them to determine the overall uncertainty.

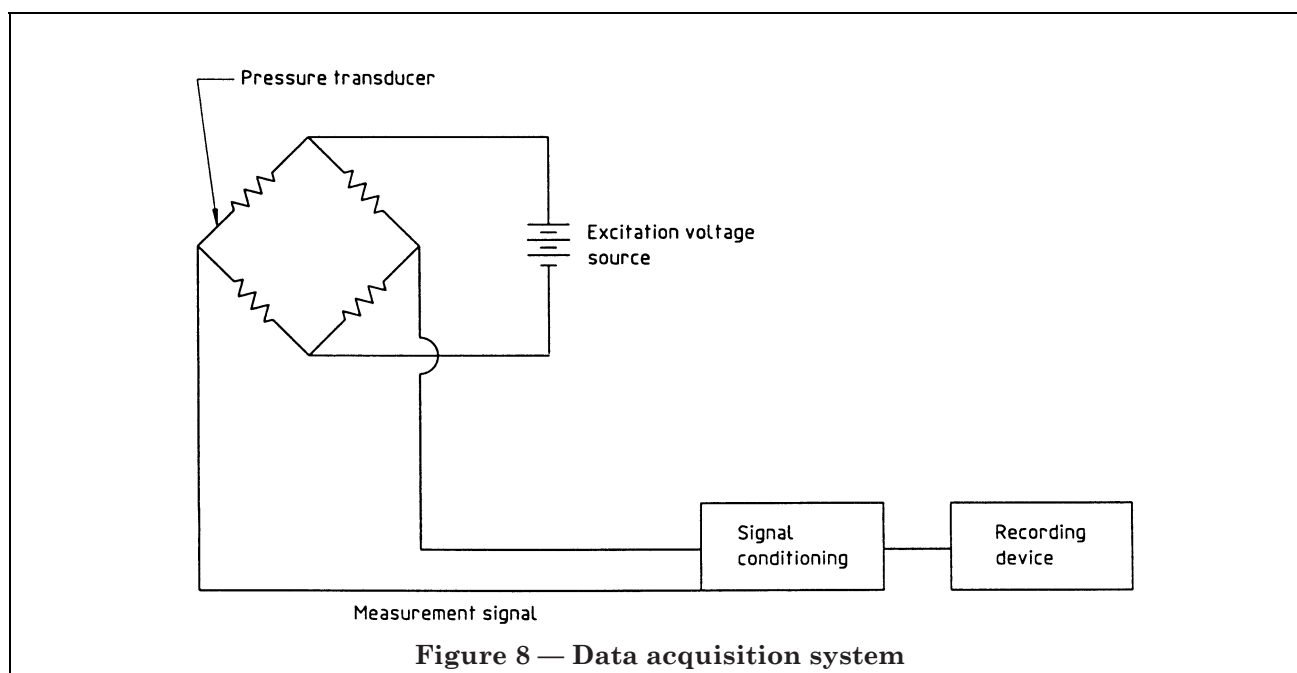
Some of the data acquisition error sources are listed in Table 2. Symbols for the elemental systematic uncertainties and the experimental standard deviations and for the degrees of freedom are shown. Note these elemental uncertainties are independent, not cumulative.

Table 1 — Calibration hierarchy error sources

Calibration	Systematic uncertainty	Experimental standard deviation	Degrees of freedom
SL — ILS	B_{11}	s_{11}	v_{11}
ILS — TS	B_{21}	s_{21}	v_{21}
TS — WS	B_{31}	s_{31}	v_{31}
WS — MI	B_{41}	s_{41}	v_{41}

Table 2 — Data acquisition error sources

Error source	Systematic uncertainty	Experimental standard deviation	Degrees of freedom
Excitation voltage	B_{12}	s_{12}	v_{12}
Signal conditioning	B_{22}	s_{22}	v_{22}
Recording device	B_{32}	s_{32}	v_{32}
Pressure transducer	B_{42}	s_{42}	v_{42}
Probe errors	B_{52}	s_{52}	v_{52}
Environmental effects	B_{62}	s_{62}	v_{62}
Spatial averaging	B_{72}	s_{72}	v_{72}



5.6 Data reduction error sources

Computations on raw data produce output in the required engineering units. Typical errors in this process stem from curve fits and computational resolution. Uncertainties associated with these error sources are often negligible.

Symbols for the data reduction error sources are listed in Table 3.

Table 3 — Data reduction error sources

Calibration	Systematic uncertainty	Experimental standard deviation	Degrees of freedom
Curve fit	B_{13}	s_{13}	v_{13}
Computational resolution	B_{23}	s_{23}	v_{23}

5.7 Method error sources

Errors of method are those associated with a particular measurement procedure (principles of use of instruments) and also with the uncertainty of constants used in calculations.

Some examples are errors from indirect methods of flowrate measurement associated with physical inaccuracy of the relationship between the measured quantity and flowrate, or with inaccuracy of the constants in the relationship. These inaccuracies may be due, for instance, to the fact that the flow conditions prevailing during the measurement are not identical to the conditions in which the calibration has been carried out or for which a standardized discharge coefficient has been established. In certain methods of flow measurement (differential pressure devices for instance), these sources of error arising from the flow conditions are covered by the uncertainty associated with the discharge coefficient, as far as the installation conditions prescribed in the standard are satisfied; if they are not, that standard does not apply. In other methods (velocity-area method for instance), the uncertainty arising from the flow conditions is identified as a component of the total uncertainty; it shall be evaluated by the user in each case and combined with the other elemental uncertainties.

5.8 Subjective error sources

Subjective error sources are caused by personal characteristics of the operators who calibrate flowmeters, perform measurements and process the data. These can include reading errors and miscalculations.

6 Estimation and presentation of elemental uncertainties

6.1 Summary of procedure

Obtain an estimate of each elemental uncertainty. If the data is available to estimate the experimental standard deviation, classify the uncertainty as a random uncertainty. Otherwise, classify it as a systematic uncertainty.

Review the test objective, test duration and number of calibrations that will affect the test result. Make the final classification of elemental uncertainties for each measurement. If an error increases the scatter in the measurement result in the defined test, it is a random error; otherwise, it is a systematic error.

6.2 Calculation of experimental standard deviation

There are many ways to calculate the experimental standard deviation.

- a) If the parameter to be measured can be held constant, a number of repeated measurements can be used to evaluate equation (1), repeated here:

$$s = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}} \quad \dots (11)$$

- b) If there are M redundant instruments or M redundant measurements and the parameter to be measured can be held constant to take N repeat readings, the following pooled estimate of the experimental standard deviation for individual readings can be used:

$$s_{\text{pooled}} = \left[\frac{\sum_{i=1}^M \sum_{j=1}^N (x_{ij} - \bar{x}_i)^2}{M(N-1)} \right]^{1/2} \quad \dots (12)$$

NOTE Here ij is used differently from elsewhere in the text.