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TECHNICAL REPORT

Colorimetry, 4th Edition

CIE 015:2018

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Descriptor: Colorimetry
Colour of objects
Colour vision
Perception of colour

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3. Richtlinien für die Anwendung von Prinzipien und Vorgängen in der Entwicklung internationaler und nationaler Normen auf dem Gebiet der Lichttechnik zu erstellen.
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5. Liaison und technische Zusammenarbeit mit anderen internationalen Organisationen zu unterhalten, die mit Fragen der Wissenschaft, Technik, Normung und Kunst auf dem Gebiet der Lichttechnik zu tun haben.

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Alle vier Jahre findet eine Session statt, in der die Arbeiten der Divisionen berichtet und überprüft werden, sowie neue Pläne für die Zukunft ausgearbeitet werden. Die CIE wird als höchste Autorität für alle Aspekte des Lichtes und der Beleuchtung angesehen. Auf diese Weise unterhält sie eine bedeutende Stellung unter den internationalen Organisationen.

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This Technical Report has been prepared by CIE Technical Committee 1-85 of Division 1 "Colour and Vision" and has been approved by the Board of Administration as well as by Division 1 of the Commission Internationale de l'Eclairage. The document reports on current knowledge and experience within the specific field of light and lighting described, and is intended to be used by the CIE membership and other interested parties. It should be noted, however, that the status of this document is advisory and not mandatory.

Ce rapport technique a été élaboré par le Comité Technique CIE 1-85 de la Division 1 "Vision et Couleur" et a été approuvé par le Bureau et Division 1 de la Commission Internationale de l'Eclairage. Le document expose les connaissances et l'expérience actuelles dans le domaine particulier de la lumière et de l'éclairage décrit ici. Il est destiné à être utilisé par les membres de la CIE et par tous les intéressés. Il faut cependant noter que ce document est indicatif et non obligatoire.

Dieser Technische Bericht ist vom Technischen Komitee CIE 1-85 der Division 1 "Sehen und Farbe" ausgearbeitet und vom Vorstand sowie Division 1 der Commission Internationale de l'Eclairage gebilligt worden. Das Dokument berichtet über den derzeitigen Stand des Wissens und Erfahrung in dem behandelten Gebiet von Licht und Beleuchtung; es ist zur Verwendung durch CIE-Mitglieder und durch andere Interessierte bestimmt. Es sollte jedoch beachtet werden, dass das Dokument eine Empfehlung und keine Vorschrift ist.

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Sadly, during the preparation of the final draft of this Technical Report, the Chairman, Janos Schanda, passed away. The members of TC 1-85 wish to record their thanks for his leadership and acknowledge his great insight into the science of colour, as shown in small part by the work of this Technical Committee. Ellen Carter was subsequently appointed Chair in September 2015.

CONTENTS

Summary	vii
Résumé	vii
Zusammenfassung	vii
1 Scope	1
2 Preface	1
3 Recommendations concerning standard observer data	3
3.1 CIE 1931 standard colorimetric observer	3
3.2 CIE 1964 standard colorimetric observer	4
3.3 Cone-fundamental-based colorimetric observers	5
3.3.1 Cone-fundamental-based tristimulus functions	6
3.3.2 MacLeod–Boynton chromaticity diagrams	9
4 Recommendations concerning standard physical data of illuminants and sources	10
4.1 Recommendations concerning standard physical data of illuminants	10
4.1.1 CIE standard illuminants	10
4.1.2 Other D illuminants	12
4.1.3 Illuminant B	14
4.1.4 Illuminant C	14
4.1.5 Illuminant E	15
4.2 Artificial sources representative of illuminants	15
4.2.1 Source A	15
4.2.2 Source B	15
4.2.3 Source C	15
4.2.4 Source D65	15
4.2.5 Notes on artificial sources representative of illuminants	15
5 Recommendations concerning standard of reflectance	15
6 Recommendations concerning geometric conditions for colorimetry	16
6.1 Terms and definitions used in this clause	16
6.2 Recommendations concerning geometric conditions for colorimetry	17
6.3 Recommended nomenclature for directional irradiation	17
6.3.1 Forty-five degree directional geometry (45°x)	17
6.3.2 Forty-five degree annular geometry (45°a)	17
6.3.3 Zero degree directional geometry (0°)	17
6.3.4 Eight degree geometry (8°)	18
6.4 Recommended geometries for reflection measurements	18
6.4.1 Diffuse : eight degree geometry, specular component included (di:8°)	18
6.4.2 Diffuse : eight degree geometry, specular component excluded (de:8°)	18
6.4.3 Eight degree : diffuse geometry, specular component included (8°:di)	18
6.4.4 Eight degree : diffuse geometry, specular component excluded (8°:de)	18
6.4.5 Diffuse : diffuse geometry (d:d)	18
6.4.6 Alternative diffuse geometry (d:0°)	18
6.4.7 Forty-five degree annular : normal geometry (45°a:0°)	19
6.4.8 Normal : forty-five degree annular geometry (0°:45°a)	19
6.4.9 Forty-five degree directional : normal geometry (45°x:0°)	19
6.4.10 Normal : forty-five degree directional geometry (0°:45°x)	19
6.5 Recommended geometry for transmission measurements	20
6.5.1 Normal : normal geometry (0°:0°)	20
6.5.2 Diffuse : normal geometry, regular component included (di:0°)	20

6.5.3	Diffuse : normal geometry, regular component excluded (de:0°)	20
6.5.4	Normal : diffuse geometry, regular component included (0°:di)	20
6.5.5	Normal : diffuse geometry, regular component excluded (0°:de)	20
6.5.6	Diffuse : diffuse geometry (d:d)	20
7	Recommendations concerning the calculation of tristimulus values and chromaticity coordinates	21
7.1	Calculation of tristimulus values	21
7.1.1	Secondary light sources (reflecting or transmitting objects)	22
7.1.2	Illuminants and self-luminous objects	23
7.2	The use of abridged or truncated data	23
7.2.1	Abridged method for data available at intervals of a maximum 5 nm	23
7.2.2	Abridged method for data available at intervals larger than 5 nm	24
7.2.3	Extrapolation and interpolation	24
7.2.4	Weighting factors	25
7.2.5	Numerical procedures	25
7.2.6	Bandwidth of a spectrometer	26
7.3	Calculation of chromaticity coordinates	26
7.4	Equations representing relationships between colour stimuli	27
8	Recommendations concerning uniform colour spacing and colour differences	27
8.1	CIE 1976 uniform chromaticity scale (UCS) diagram	27
8.2	CIE 1976 approximately uniform colour spaces	28
8.2.1	CIE 1976 L*a*b* colour space; CIELAB colour space	28
8.2.2	CIE 1976 L*u*v* colour space; CIELUV colour space	30
8.2.3	Notes on CIE 1976 approximately uniform colour spaces	31
8.3	Improved industrial colour-difference evaluation	32
8.3.1	CIEDE2000 colour-difference formula	32
9	Miscellaneous colorimetric terms and formulae	34
9.1	Special metamerism indices	34
9.1.1	Special metamerism index: change in illuminant	35
9.1.2	Special metamerism index: change in observer	37
9.2	Assessment of the quality of a daylight simulator for colorimetry	38
9.3	The evaluation of whiteness	38
9.4	Correlated colour temperature	39
10	Colour appearance modelling and colour fidelity index	40
10.1	CIE colour appearance model CIECAM02	40
10.2	CIE 2017 Colour Fidelity Index	41
11	Tables	42
11.1	Spectral data of colorimetric observers	43
11.2	Tables representing the relative spectral power distributions of illuminants and other colorimetric data	51
Annex A	Historic recommendations, now obsolete, as well as references to non-CIE colour-difference formulae	77
A.1	Dominant wavelength and purity	77
A.2	Illuminant B and Source B	78
A.3	Illuminant C and Source C	78
A.4	CIE 1964 uniform colour space and colour-difference formula	79
A.5	CIE 1994 colour-difference formula (CIE94)	79
A.6	CMC(l:c) colour-difference formula	80
A.7	DIN99 colour-difference formulae	81

Annex B Definitions of the colour-matching functions, the CIE RGB system and the derivation of the CIE XYZ system from the CIE RGB system for the 1931 standard colorimetric observer.....	82
B.1 Determination of the $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ colour-matching functions.....	82
B.2 Derivation of the CIE XYZ trichromatic system from the CIE RGB trichromatic system	82
B.3 Definition of the colour-matching functions in the CIE 1964 trichromatic system.....	83
Annex C Reverse transformation from values L^* , a^* , b^* to tristimulus values X , Y , Z	89
Annex D Detailed description of the CAM02-UCS model including CIECAM02.....	90
D.1 The forward mode of CAM02-UCS including CIECAM02	90
D.2 The reverse mode of CAM02-UCS including CIECAM02	94
Annex E Information on the use of Planck's equation for standard air	98
References	99

COLORIMETRY

Summary

This publication provides the recommendations of the CIE concerning colorimetry. Specifically, it includes the use of the standard colorimetric observers and standard illuminants, the reference standard for reflectance, the illuminating and viewing conditions, the calculation of tristimulus values, chromaticity coordinates, colour space coordinates and colour differences and various other colorimetric practices and formulae. As a new feature, the publication also includes further details of advanced colorimetry, including colour appearance models, and new findings on cone-fundamental-based tristimulus functions, with appropriate references to other CIE publications. Also new illuminants for different LED types are introduced.

This publication is consistent with the fundamental data and procedures described in the CIE International Standards on colorimetry.

For further details of some of the phenomena discussed here the reader is directed to the appropriate CIE Technical Reports.

This report replaces CIE 15:2004 "Colorimetry, 3rd Edition".

COLORIMÉTRIE

Résumé

Cette publication donne les recommandations CIE concernant la colorimétrie. Plus précisément elle traite de l'emploi des observateurs de référence pour la colorimétrie et des illuminants normalisés, de la référence normalisée pour le facteur de réflexion, des conditions d'éclairage et d'observation, du calcul des composantes trichromatiques, des coordonnées trichromatiques, des espaces chromatiques et des écarts de couleur, ainsi que diverses autres formules et pratiques en colorimétrie. A titre de nouveauté, la publication comprend également des détails complémentaires sur la colorimétrie avancée, comprenant les modèles d'apparence chromatique, y compris les nouveaux résultats sur les fonctions trichromatiques basées sur les fondamentales de cônes et les références appropriées aux autres publications CIE. De nouveaux illuminants pour différents types de LED sont également présentés.

Cette publication est en accord avec les données fondamentales et les procédures décrites dans les normes internationales CIE en colorimétrie.

Pour des détails complémentaires relatifs aux questions traitées ici, le lecteur est invité à consulter les rapports techniques appropriés.

Ce document remplace la publication CIE 15:2004 "Colorimetry, 3^{ème} édition".

FARBMESSUNG

Zusammenfassung

Diese Publikation beinhaltet die Empfehlungen der CIE für die Farbmessung. Insbesondere beinhaltet sie die Anwendung der Normalbeobachter und der Normlichtarten, des Weißstandards, der Beleuchtungs- und Beobachtungsbedingungen, die Berechnung von Normfarbwerten, Normfarbwertanteilen, Farbraumkoordinaten und Farbabständen, und verschiedene andere farbmessische Praktiken und Formeln. Als neues Feature beinhaltet diese Publikation auch weitere Details, inklusive Farberscheinungsmodellen und neue Erkenntnissen zu auf Sehzapfen-Grundfunktionen basierenden Farbwertfunktionen, mit entsprechenden Referenzen zu anderen CIE-Publikationen. Ebenfalls werden neue Lichtarten für LEDs eingeführt.

Diese Publikation stimmt mit den grundlegenden Daten und Verfahren überein, die in den CIE-Normen zur Farbmessung beschrieben werden.

Weitere Einzelheiten einiger hier diskutierter Phänomene findet der Leser in geeigneten technischen Berichten der CIE.

Dieser Bericht ersetzt CIE 15:2004 "Colorimetry, 3rd Edition".

1 Scope

This report, which replaces CIE 15:2004 “Colorimetry, 3rd edition” (CIE, 2004b), is intended to provide a consistent and comprehensive account of the recommendations of the CIE for basic and advanced colorimetry. It summarizes basic colorimetric data and practices, and introduces colour appearance specifications.

2 Preface

By general consent in all countries, the specification of basic standards for use in colorimetry is the province of the Commission Internationale de l'Eclairage (CIE). The first major recommendations regarding colorimetric standards were made by the CIE in 1931, and these formed the basis of modern colorimetry. The original recommendations of 1931 were reviewed from time to time by the CIE Colorimetry Committee and later by CIE Division 1, *Vision and Colour*. When necessary, changes were made. New recommendations were added to supplement those in existence or to broaden the scope of colorimetry in accordance with developments in practice and science.

The deliberations and recommendations made by the CIE Colorimetry Committee and Division 1 are recorded in the Proceedings of the various Sessions of the CIE. However, the access to these Proceedings has always been rather limited and much of the material published in the Proceedings is obsolete or inconsistent with current colorimetric practice. The recommendations, while few in number, also present an incoherent picture.

For these reasons, the CIE published in 1971 a special document on colorimetry to provide a consistent and comprehensive account of basic colorimetric recommendations. This document was not intended to be a textbook on colorimetry but rather a reference to the basic standards that govern modern colorimetry. The document was issued as CIE Publication No. 15 (CIE, 1971). New editions, Publications CIE 15.2 (published in 1986) and CIE 15:2004 (the third edition) were published with additional and updated material.

Since the publication of the third edition a number of colorimetry-related publications have been published by CIE, some of which are referred to later in the text. The technical reports include:

- CIE 166-2005 Cognitive Colour
- CIE 167-2005 Recommended Practice for Tabulating Spectral Data for Use in Colour Computations
- CIE 168:2005 Criteria for the Evaluation of Extended-Gamut Colour Encodings
- CIE 170-1:2006 Fundamental Chromaticity Diagram with Physiological Axes – Part 1
- CIE 176-2006 Geometric Tolerances for Colour Measurements
- CIE 177:2007 Colour Rendering of White LED Light Sources
- CIE 182:2007 Calibration methods and photoluminescent standards for total radiance factor measurements
- CIE 184:2009 Indoor Daylight Illuminants
- CIE 185:2009 Reappraisal of Colour Matching and Grassmann's Laws
- CIE 192:2010 Practical Daylight Sources for Colorimetry
- CIE 195:2011 Specification of Colour Appearance for Reflective Media and Self-Luminous Displays
- CIE 199:2011 Methods for Evaluating Colour Differences in Images
- CIE 204:2013 Methods for Re-defining CIE D Illuminants
- CIE 208: 2014 Effect of Stimulus Size on Colour Appearance
- CIE 211: 2014 Colour Appearance in Peripheral Vision

- CIE 214:2014 Effect of instrumental bandpass function and measurement interval on spectral quantities
- CIE 170-2:2015 Fundamental Chromaticity Diagram with Physiological Axes – Part 2: Spectral Luminous Efficiency Functions and Chromaticity Diagrams
- CIE 217:2016 Recommended Method for Evaluating the Performance of Colour-Difference Formulae
- CIE 224:2017 Colour Fidelity Index for Accurate Scientific Use

The most fundamental tables and formulae were summarized in a series of standards, published as joint ISO/CIE International Standards, known as the Colorimetry Series:

- ISO 11664-1:2007/CIE S 014-1:2006 Colorimetry — Part 1: CIE Standard Colorimetric Observers (ISO/CIE, 2007a)
- ISO 11664-2:2007/CIE S 014-2:2006 Colorimetry — Part 2: CIE Standard Illuminants for Colorimetry (ISO/CIE, 2007b)
- ISO 11664-3:2012/CIE S 014-3:2011 Colorimetry — Part 3: CIE Tristimulus Values (ISO/CIE, 2012)
- ISO 11664-4:2008/CIE S 014-4:2007 Colorimetry — Part 4: CIE 1976 $L^*a^*b^*$ Colour Space (ISO/CIE, 2008)
- ISO/CIE 11664-5:2016 Colorimetry — Part 5: CIE 1976 $L^*u^*v^*$ Colour Space and u' , v' Uniform Chromaticity Scale Diagram (ISO/CIE, 2016)
- ISO/CIE 11664-6:2014 Colorimetry — Part 6: CIEDE2000 Colour-Difference Formula (ISO/CIE, 2014)

NOTE The numbering of the joint ISO/CIE standards will be harmonized step by step after subsequent systematic reviews, such that all ISO/CIE standards will have a common number (as currently is already the case with Parts 5 and 6).

To help interested parties find their way around these publications, CIE decided to update from time to time the fundamental publication CIE 15, and CIE Division 1 decided at its meeting in Sun City, ZA, in 2011 to start the process to update CIE 15:2004.

This technical report is nevertheless not a textbook, but a summary of relevant CIE recommendations on colorimetry. For a number of recommendations that need detailed explanation this publication provides only a short summary, and interested readers are directed to the appropriate CIE documents.

It is anticipated that further amendments of colorimetric practice and further standards will be published in the future, thus one should consult the latest CIE list of publications on the CIE website (<http://www.cie.co.at/>).

The wording of the original recommendations has been altered to be consistent with modern nomenclature and in some cases the original recommendations have also been modified in content to bring them in line with present day thinking and practice. The versions given in this document should be considered current and they supersede all previous recommendations until such time as any amendments are published by the CIE.

The recommendations are divided into the following eight groups:

- recommendations concerning standard observer data and data of cone-fundamental-based colorimetric observers;
- recommendations concerning standard physical data of illuminants and sources;
- recommendations concerning the standard of reflectance;
- recommendations concerning geometric conditions for colorimetry;
- recommendations concerning the calculation of tristimulus values and chromaticity coordinates;
- recommendations concerning uniform colour spacing and colour differences;

- recommendations concerning miscellaneous colorimetric practices and formulae;
- recommendations concerning colour appearance description.

The definitive data relating to standard colorimetric observers and illuminants are those given at 1 nm intervals in the appropriate CIE International Standards (ISO/CIE, 2007a; ISO/CIE, 2007b). Abridged data, at 5 nm intervals, are given in the tables of Clause 11 in this document, and these may be used in those cases where calculation at the wider intervals is unlikely to lead to any significant error.

For all colorimetric calculations, wavelength in standard air (dry air at 15 °C and 101 325 Pa, containing 0,03 % by volume of carbon dioxide) should be used.

Items of mainly historic importance have been placed into Annex A and further explanatory material can be found in subsequent annexes.

3 Recommendations concerning standard observer data

The following recommendations are based on a number of original documents (CIE, 1931; CIE, 1959; CIE, 1963; CIE, 2006a; CIE, 2015).

3.1 CIE 1931 standard colorimetric observer

For correlation with visual colour matching of fields subtending between about 1° and about 4° at the eye of the observer, it is recommended that colorimetric specifications of colour stimuli should be based on the colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ defined in the joint ISO/CIE International Standard (ISO/CIE, 2007a) – by values given at 1 nm wavelength intervals from 360 nm to 830 nm.

These functions define the CIE 1931 standard colorimetric observer (in technical applications often written as the 2° standard colorimetric observer¹).

In the case where more coarsely sampled data will produce no significant calculation error, the data of Table 1 in this report may be used instead of the 1 nm interval data. The values given in Table 1 are values selected from the standard at 5 nm intervals from 380 nm to 780 nm, rounded to six decimal places.

NOTE 1 $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are the normalized tristimulus values of monochromatic radiation of equal radiant power relative to a set of reference stimuli [X], [Y], [Z]. The normalization is such that the maximum of $\bar{y}(\lambda)$ is unity and $\sum \bar{x}(\lambda) = \sum \bar{y}(\lambda) = \sum \bar{z}(\lambda)$. The functions are commonly used to obtain the tristimulus values X , Y , Z of colour stimuli (see 7.1 for details).

NOTE 2 The reference stimuli [X], [Y], [Z], which are derived from the real reference stimuli [R], [G], [B] of the original trichromatic system, were chosen for reasons of convenience in colorimetric computations. In particular, they were chosen such that $\bar{y}(\lambda)$ is identical with $V(\lambda)$, the spectral luminous efficiency function for photopic vision, which defines the CIE standard photometric observer for photopic vision (ISO/CIE, 2004) (see Annex B for details).

NOTE 3 The values of $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ given in Table 1 agree closely with those defined originally in 1931. Three minor changes have been introduced: at $\lambda = 775$ nm the new values of $\bar{x}(\lambda)$ is 0,000 059 instead of 0,0000; at $\lambda = 555$ nm $\bar{y}(\lambda)$ is 1,0000 instead of 1,0002; and at $\lambda = 740$ nm $\bar{y}(\lambda)$ is 0,000 249 instead of 0,0003. These changes are considered insignificant in most colorimetric calculations. The ISO/CIE International Standard (ISO/CIE, 2007a) contains these corrected values.

NOTE 4 If the colour-matching functions of Table 1 are required at closer intervals, the values at 1 nm intervals given in the ISO/CIE International Standard (ISO/CIE, 2007a) should be used. When wavelength intervals smaller than 1 nm are required, linear interpolation of the values given in the ISO/CIE International Standard should be used.

¹ A 2° visual field represents a diameter of about 17 mm at a viewing distance of 0,5 m.

NOTE 5 The chromaticity coordinates (see 7.3) $x(\lambda)$, $y(\lambda)$, $z(\lambda)$ of the spectral stimuli are calculated as

$$\begin{aligned}x(\lambda) &= \frac{\bar{x}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)} \\y(\lambda) &= \frac{\bar{y}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)} \\z(\lambda) &= \frac{\bar{z}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}\end{aligned}\quad (3.1)$$

Chromaticity coordinates $x(\lambda)$ and $y(\lambda)$ selected at 5 nm intervals (from 380 nm to 780 nm) are given to the precision of five decimal places in Table 1.

NOTE 6 The chromaticity coordinates x_E , y_E , z_E of the equi-energy illuminant as derived from the sums $\sum \bar{x}(\lambda)$, $\sum \bar{y}(\lambda)$, $\sum \bar{z}(\lambda)$ of Table 1 in the ISO/CIE International Standard (with function values given at 1 nm intervals from 360 nm to 830 nm) (ISO/CIE, 2007a) are

$$x_E = 0,333\ 314$$

$$y_E = 0,333\ 288$$

$$z_E = 0,333\ 398$$

When derived from the sums $\sum \bar{x}(\lambda)$, $\sum \bar{y}(\lambda)$, $\sum \bar{z}(\lambda)$ of Table 1 in the present report (with function values selected at 5 nm intervals from 380 nm to 780 nm, rounded to six decimal places) the chromaticity coordinates of the equi-energy illuminant are

$$x_E = 0,333\ 334$$

$$y_E = 0,333\ 331 \quad (3.2)$$

$$z_E = 0,333\ 335$$

As the original 1931 tables were defined at 5 nm intervals, and these values were retained except for the minor corrections mentioned in NOTE 3 above, the present Table 1 agrees with the original intent that the tristimulus values of the equi-energy illuminant should be closely equal. (The equi-energy illuminant – Illuminant E (see 4.1.5) – is radiation for which the spectral power distribution is constant.)

3.2 CIE 1964 standard colorimetric observer

For correlation with visual colour matching of fields of angular subtense larger than 4° at the eye of the observer, it is recommended that colorimetric specifications of colour stimuli should be based on the colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ defined in the joint ISO/CIE International Standard (ISO/CIE, 2007a) by values given at 1 nm wavelength intervals from 360 nm to 830 nm.

These functions define the CIE 1964 standard colorimetric observer (in technical applications often written as the 10° standard colorimetric observer²).

In the case where more coarsely sampled data will produce no significant calculation error, Table 2 in this report may be used instead of the 1 nm interval data. The values given in Table 2 are values selected from the standard at 5 nm intervals from 380 nm to 780 nm, rounded to six decimal places.

NOTE 1 $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ are the normalized tristimulus values of monochromatic radiation of equal radiant power relative to a set of reference stimuli $[X_{10}]$, $[Y_{10}]$, $[Z_{10}]$. The normalization is such that the maximum of $\bar{y}_{10}(\lambda)$ is unity and $\sum \bar{x}_{10}(\lambda) = \sum \bar{y}_{10}(\lambda) = \sum \bar{z}_{10}(\lambda)$. The functions

² A 10° visual field represents a diameter of about 87 mm at a viewing distance of 0,5 m.

are commonly used to obtain the tristimulus values X_{10} , Y_{10} , Z_{10} of colour stimuli (see 7.1 for details).

NOTE 2 The reference stimuli $[X_{10}]$, $[Y_{10}]$, $[Z_{10}]$, which are derived from the real reference stimuli $[R_{10}]$, $[G_{10}]$, $[B_{10}]$ of the original trichromatic system, were chosen for reasons of convenience in colorimetric computations (see Annex B for details).

NOTE 3 If the colour-matching functions of Table 2 are required at closer intervals, the values at 1 nm intervals given in the ISO/CIE International Standard (ISO/CIE, 2007a) should be used. When wavelength intervals smaller than 1 nm are required, linear interpolation of the values given in the ISO/CIE International Standard should be used.

NOTE 4 The chromaticity coordinates (see 7.3) $x_{10}(\lambda)$, $y_{10}(\lambda)$, $z_{10}(\lambda)$ of the spectral stimuli are calculated as

$$\begin{aligned} x_{10}(\lambda) &= \frac{\bar{x}_{10}(\lambda)}{\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)} \\ y_{10}(\lambda) &= \frac{\bar{y}_{10}(\lambda)}{\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)} \\ z_{10}(\lambda) &= \frac{\bar{z}_{10}(\lambda)}{\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)} \end{aligned} \quad (3.3)$$

Chromaticity coordinates $x_{10}(\lambda)$ and $y_{10}(\lambda)$ selected at 5 nm intervals (from 380 nm to 780 nm) are given to the precision of five decimal places in Table 2.

NOTE 5 The chromaticity coordinates $x_{10,E}$, $y_{10,E}$, $z_{10,E}$ of the equi-energy illuminant derived from the sums $\Sigma \bar{x}_{10}(\lambda)$, $\Sigma \bar{y}_{10}(\lambda)$, $\Sigma \bar{z}_{10}(\lambda)$ of Table 2 in the ISO/CIE International Standard (with function values given at 1 nm intervals from 360 nm to 830 nm) (ISO/CIE, 2007a) are

$$x_{10,E} = 0,333\ 296$$

$$y_{10,E} = 0,333\ 335$$

$$z_{10,E} = 0,333\ 369$$

When derived from the sums $\Sigma \bar{x}_{10}(\lambda)$, $\Sigma \bar{y}_{10}(\lambda)$, $\Sigma \bar{z}_{10}(\lambda)$ of Table 2 in the present report (with function values selected at 5 nm intervals from 380 nm to 780 nm, rounded to six decimal places), the chromaticity coordinates of the equi-energy illuminant are

$$x_{10,E} = 0,333\ 298$$

$$y_{10,E} = 0,333\ 336 \quad (3.4)$$

$$z_{10,E} = 0,333\ 366$$

NOTE 6 The large-field colour-matching data as defined by the CIE 1964 standard colorimetric observer are intended to apply to matches where the luminance and the relative spectral power distributions of the matched stimuli are such that no participation of the rod receptors of the visual mechanism is to be expected. This condition of observation is important, as "rod intrusion" may upset the colour-match predictions of the standard observer. For daylight, possible participation of rod vision in colour matches is likely to diminish progressively above $10\text{ cd}\cdot\text{m}^{-2}$ and be entirely absent above about $200\text{ cd}\cdot\text{m}^{-2}$. For further information, see Aguilar & Stiles, 1954; Adelson, 1982; Sharpe et al., 1989.

The CIE 1931 and 1964 standard colorimetric observers were both derived from trichromatic systems based on real red [R], green [G] and blue [B] reference stimuli, see Annex B.

3.3 Cone-fundamental-based colorimetric observers

The CIE 1931 colour-matching functions could only be defined after calibration with the 1924 $V(\lambda)$ function (CIE, 1924). This function had its problems because, in the short wavelength region, the values were a compromise due to large differences between the results of various experiments. Hence, in the following decades, several studies were aimed at developing a colorimetric system that accorded more closely with the known colorimetric and physiological

properties of the human eye. Eventually, in 1991, CIE set up a Technical Committee, TC 1-36, with the task to “Establish a (cone-) fundamental (-based) chromaticity diagram of which the coordinates correspond to physiologically significant axes”.

In Part 1 of the report by TC 1-36, published as CIE 170-1 (CIE, 2006a), the main concern is the concept of “cone fundamentals”, which are defined as the relative spectral sensitivity functions of the cone receptors as measured in the corneal plane.

First, the cone fundamentals for the field size of 10° are derived from the 10° colour-matching functions of Stiles and Burch (1959) by following the ideas proposed by Stockman and Sharpe (2000), applying König’s hypothesis³ and using novel data on the spectral sensitivity functions of dichromats and normal trichromats. By further correcting these cone fundamentals for the absorption of the ocular media and the macular pigment, and also taking into account the optical densities of the cone visual pigments, all for the field size of 10°, the low-density absorbance functions of the long-wave sensitive (L-), middle-wave sensitive (M-) and short-wave sensitive (S-) cone visual pigments are derived. Ultimately, by incorporating the age-dependent absorptions of the ocular media, the field-size-dependent absorption of the macula, and the field-size-dependent optical densities of the three cone visual pigments, a procedure for calculating the cone fundamentals as a function of age and field size is obtained. For a detailed outline of the concept, see CIE 170-1 (CIE, 2006a).

In CIE 170-1 the cone fundamentals for the average colorimetric observer at the particular age of 32 years (the average age of the Stiles–Burch 1959 observer group) and field sizes of 2° and 10° are derived both in terms of quanta (log units) and in terms of energy (linear units). Tabulations at 5 nm, 1 nm and 0,1 nm intervals from 390 nm to 830 nm – with the quanta- and energy-based values given to the precision of six, optionally nine significant figures – are available online at <http://files.cie.co.at/CFs.zip> ⁴.

Part 2 of the report by TC 1-36, published as CIE 170-2 (CIE, 2015), describes in detail two colorimetric constructs that both refer to the concept of cone fundamentals:

- Cone-fundamental-based tristimulus functions;
- MacLeod–Boynton chromaticity diagrams.

NOTE The principal colorimetric database of the cone fundamentals for the 10° field size are the 10° colour-matching functions of Stiles and Burch (1959).

3.3.1 Cone-fundamental-based tristimulus functions

In order to bridge the gap between the cone-fundamental framework and current CIE practice, and allow for comparison with the traditional CIE procedures, transformations from cone fundamentals into cone-fundamental-based tristimulus values and associated cone-fundamental-based chromaticity coordinates are defined in line with the principles of the CIE XYZ concept (Wold and Valberg, 1999; Schanda, 2007).

In particular, for an observer aged 32 years and a field size of 2°, it is recommended that XYZ cone-fundamental-based colorimetric specifications of colour stimuli should be referred to the cone-fundamental-based tristimulus functions for 2° field size $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ defined as

³ König’s hypothesis postulates that dichromatic vision is a reduced form of trichromatic vision where one cone system is missing and the two others are left unchanged in spectral sensitivity.

⁴ Note that the energy value of the 10° cone fundamental for 390 nm and the energy values of the 2° cone fundamentals published in CIE 170-1 (in tables with 5 nm intervals) are slightly different from those published on the CIE server at the given link. This is due to rounding errors in the calculation of the CIE 170-1 values. For any further reference the values on the CIE server should be taken. CIE 170-1 will be corrected in due course accordingly.

$$\begin{aligned}
\bar{x}_F(\lambda) &= 1,947\,354\,69 \bar{l}(\lambda) - 1,414\,451\,23 \bar{m}(\lambda) + 0,364\,763\,27 \bar{s}(\lambda) \\
\bar{y}_F(\lambda) &= 0,689\,902\,72 \bar{l}(\lambda) + 0,348\,321\,89 \bar{m}(\lambda) \\
\bar{z}_F(\lambda) &= 1,934\,853\,43 \bar{s}(\lambda)
\end{aligned}
\tag{3.5}$$

where $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, $\bar{s}(\lambda)$ are the cone fundamentals for age 32 years and the field size of 2° , as given in terms of energy (to the precision of nine significant figures) at 1 nm intervals (see <http://files.cie.co.at/CFs.zip>⁵).

Similarly, for an observer aged 32 years and a field size of 10° , it is recommended that XYZ cone-fundamental-based colorimetric specifications of colour stimuli should be referred to the cone-fundamental-based tristimulus functions for 10° field size $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ defined as

$$\begin{aligned}
\bar{x}_{F,10}(\lambda) &= 1,939\,864\,43 \bar{l}_{10}(\lambda) - 1,346\,643\,59 \bar{m}_{10}(\lambda) + 0,430\,449\,35 \bar{s}_{10}(\lambda) \\
\bar{y}_{F,10}(\lambda) &= 0,692\,839\,32 \bar{l}_{10}(\lambda) + 0,349\,675\,67 \bar{m}_{10}(\lambda) \\
\bar{z}_{F,10}(\lambda) &= 2,146\,879\,45 \bar{s}_{10}(\lambda)
\end{aligned}
\tag{3.6}$$

where $\bar{l}_{10}(\lambda)$, $\bar{m}_{10}(\lambda)$, $\bar{s}_{10}(\lambda)$ are the cone fundamentals for age 32 years and the field size of 10° , as given in terms of energy (to the precision of nine significant figures) at 1 nm intervals (see <http://files.cie.co.at/CFs.zip>⁵).

The values of $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ and $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ – formally termed cone-fundamental-based spectral tristimulus values – are tabulated to the precision of seven significant figures (at 1 nm intervals from 390 nm to 830 nm) in Table 10.7 and Table 10.8 in CIE 170-2 (CIE, 2015). These tables constitute the numeric bases of corresponding CIE 2015 XYZ cone-fundamental-based colorimetric systems. The particular values selected at 5 nm intervals from 390 nm to 780 nm and rounded to six decimal places are given in Table 3 and Table 4 of the present report.

NOTE 1 $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ and $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ are the normalized cone-fundamental-based tristimulus values of monochromatic radiation of equal radiant power relative to the sets of reference stimuli $[X_F]$, $[Y_F]$, $[Z_F]$ and $[X_{F,10}]$, $[Y_{F,10}]$, $[Z_{F,10}]$, respectively. The normalizations are such that the maximum of $\bar{y}_F(\lambda)$, as well as of $\bar{y}_{F,10}(\lambda)$, is unity, and $\sum \bar{x}_F(\lambda) = \sum \bar{y}_F(\lambda) = \sum \bar{z}_F(\lambda)$ and $\sum \bar{x}_{F,10}(\lambda) = \sum \bar{y}_{F,10}(\lambda) = \sum \bar{z}_{F,10}(\lambda)$.

NOTE 2 $\bar{y}_F(\lambda)$ and $\bar{y}_{F,10}(\lambda)$ are identical with $V_F(\lambda)$ and $V_{F,10}(\lambda)$, the cone-fundamental-based spectral luminous efficiency functions for 2° and 10° field size, respectively, recommended in CIE 170-2 (CIE, 2015).

NOTE 3 Table 3 and Table 4 are not renormalized tables for 5 nm wavelength intervals, but tables obtained by selecting every 5th value of the 1 nm-interval tables. This produces a slight deviation from the ideal criteria, $\sum \bar{x}_F(\lambda) = \sum \bar{y}_F(\lambda) = \sum \bar{z}_F(\lambda)$ and $\sum \bar{x}_{F,10}(\lambda) = \sum \bar{y}_{F,10}(\lambda) = \sum \bar{z}_{F,10}(\lambda)$, but for most practical purposes these deviations are negligible compared to other colorimetric uncertainties and are of the same magnitude as observed for the CIE standard colorimetric observers.

NOTE 4 The reason why $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ and $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ are tabulated from 390 nm instead of 380 nm (as in the cases of the CIE 1931 standard colorimetric observer and

⁵ Note that the energy values of the 2° cone fundamentals and the energy value of the 10° cone fundamental for 390 nm published in CIE 170-1 (in tables with 5 nm intervals) are slightly different from those published on the CIE server at the given link. This is due to rounding errors in the calculation of the CIE 170-1 values. For any further reference the values on the CIE server should be taken. CIE 170-1 will be corrected in due course accordingly.

the CIE 1964 standard colorimetric observer) is that 390 nm is the shortest wavelength for which the underlying cone fundamentals are defined (see CIE 170-1 and CIE 170-2). At the time of publication of this edition of the Colorimetry report, CIE has set up a Technical Committee with the partial task to extrapolate the cone fundamentals in order to define them (at 1 nm wavelength intervals) from 380 nm (or 360 nm) to 830 nm.

NOTE 5 Theoretically the coefficients of the linear combinations that determine the cone-fundamental-based tristimulus functions are applicable at 1 nm intervals. Due to normalization issues, the coefficients determined at wavelength intervals other than 1 nm will be different from the coefficients of Equations (3.5) and (3.6). In case the highest precision is required, the coefficients should be calculated by means of a specially designed computer program provided by CIE Division 1 (under development by TC 1-97). At the time of publication of this edition of the Colorimetry document, the beta version of the program (for Windows, macOS and Linux) is being tested.

NOTE 6 For an observer aged 32 years and a field size of 2°, the cone-fundamental-based chromaticity coordinates (see 7.3) $x_F(\lambda)$, $y_F(\lambda)$, $z_F(\lambda)$ of the spectral stimuli are calculated as

$$\begin{aligned} x_F(\lambda) &= \frac{\bar{x}_F(\lambda)}{\bar{x}_F(\lambda) + \bar{y}_F(\lambda) + \bar{z}_F(\lambda)} \\ y_F(\lambda) &= \frac{\bar{y}_F(\lambda)}{\bar{x}_F(\lambda) + \bar{y}_F(\lambda) + \bar{z}_F(\lambda)} \\ z_F(\lambda) &= \frac{\bar{z}_F(\lambda)}{\bar{x}_F(\lambda) + \bar{y}_F(\lambda) + \bar{z}_F(\lambda)} \end{aligned} \quad (3.7)$$

Similarly, for observers aged 32 years and a field size of 10°, the cone-fundamental-based chromaticity coordinates $x_{F,10}(\lambda)$, $y_{F,10}(\lambda)$, $z_{F,10}(\lambda)$ of the spectral stimuli are given by

$$\begin{aligned} x_{F,10}(\lambda) &= \frac{\bar{x}_{F,10}(\lambda)}{\bar{x}_{F,10}(\lambda) + \bar{y}_{F,10}(\lambda) + \bar{z}_{F,10}(\lambda)} \\ y_{F,10}(\lambda) &= \frac{\bar{y}_{F,10}(\lambda)}{\bar{x}_{F,10}(\lambda) + \bar{y}_{F,10}(\lambda) + \bar{z}_{F,10}(\lambda)} \\ z_{F,10}(\lambda) &= \frac{\bar{z}_{F,10}(\lambda)}{\bar{x}_{F,10}(\lambda) + \bar{y}_{F,10}(\lambda) + \bar{z}_{F,10}(\lambda)} \end{aligned} \quad (3.8)$$

Chromaticity coordinates $x_F(\lambda)$, $y_F(\lambda)$, and $x_{F,10}(\lambda)$, $y_{F,10}(\lambda)$ selected at 5 nm intervals from 390 nm to 780 nm are given to the precision of five decimal places in Table 3 and Table 4.

NOTE 7 The cone-fundamental-based chromaticity coordinates $x_{F,E}$, $y_{F,E}$, $z_{F,E}$ of the equi-energy illuminant derived from the sums $\Sigma \bar{x}_F(\lambda)$, $\Sigma \bar{y}_F(\lambda)$, $\Sigma \bar{z}_F(\lambda)$ of Table 10.7 in CIE 170-2 (with function values given at 1 nm intervals from 390 nm to 830 nm, to the precision of seven significant figures) (CIE, 2015) are

$$x_{F,E} = 0,333\ 33$$

$$y_{F,E} = 0,333\ 33$$

$$z_{F,E} = 0,333\ 33$$

When derived from the sums $\Sigma \bar{x}_F(\lambda)$, $\Sigma \bar{y}_F(\lambda)$, $\Sigma \bar{z}_F(\lambda)$ of Table 3 in the present report (with function values selected at 5 nm intervals from 390 nm to 780 nm, rounded to six decimal places), the cone-fundamental-based chromaticity coordinates of the equi-energy illuminant are

$$x_{F,E} = 0,333\ 31$$

$$y_{F,E} = 0,333\ 30 \quad (3.9)$$

$$z_{F,E} = 0,333\ 39$$

The cone-fundamental-based chromaticity coordinates $x_{F,10,E}$, $y_{F,10,E}$, $z_{F,10,E}$ of the equi-energy illuminant derived from the sums $\Sigma \bar{x}_{F,10}(\lambda)$, $\Sigma \bar{y}_{F,10}(\lambda)$, $\Sigma \bar{z}_{F,10}(\lambda)$ of Table 10.8 in CIE 170-2 (with function values given at 1 nm intervals from 390 nm to 830 nm, to the precision of seven significant figures) (CIE, 2015) are

$$x_{F,10,E} = 0,333\ 33$$

$$y_{F,10,E} = 0,333\ 33$$

$$z_{F,10,E} = 0,333\ 33$$

When derived from the sums $\Sigma \bar{x}_{F,10}(\lambda)$, $\Sigma \bar{y}_{F,10}(\lambda)$, $\Sigma \bar{z}_{F,10}(\lambda)$ of Table 4 in the present report (with function values selected at 5 nm intervals from 390 nm to 780 nm, rounded to six decimal places), the cone-fundamental-based chromaticity coordinates of the equi-energy illuminant are

$$x_{F,10,E} = 0,333\ 32$$

$$y_{F,10,E} = 0,333\ 31$$

$$z_{F,10,E} = 0,333\ 37$$

(3.10)

NOTE 8 For observer ages different from 32 years or field sizes different from 2° or 10°, the coefficients of the linear combinations transforming the cone fundamentals $\bar{l}_{\text{field size, age}}(\lambda)$, $\bar{m}_{\text{field size, age}}(\lambda)$, $\bar{s}_{\text{field size, age}}(\lambda)$ into corresponding cone-fundamental-based tristimulus functions $\bar{x}_{F, \text{field size, age}}(\lambda)$, $\bar{y}_{F, \text{field size, age}}(\lambda)$, $\bar{z}_{F, \text{field size, age}}(\lambda)$ will be different from the coefficients of Equations (3.5) and (3.6). In such cases, the cone-fundamental-based tristimulus functions and the associated cone-fundamental-based spectral chromaticity coordinates should be calculated by means of a specially designed computer program provided by CIE Division 1 (under development by TC 1-97). At the time of publication of this edition of the Colorimetry document, the beta version of the program (for Windows, macOS and Linux) is being tested.

3.3.2 MacLeod–Boynton chromaticity diagrams

An LMS tristimulus space (also called LMS cone-excitation space) is a three-dimensional vector space in which the coordinate axes represent the normalized excitations of the long-wave-sensitive (L), the middle-wave-sensitive (M), and the short-wave-sensitive (S) cones. In particular, under the hypothesis that only the responses of the L and M cones contribute to luminous intensity as determined by flicker photometry, a Cartesian LMS tristimulus space can be constructed in which colour stimuli of equal flicker-photometric luminous intensities are represented in a plane that (1) is parallel to the S-axis and (2) intersects the L- and the M-axes at numerically equal values. The projective counterpart of this plane defines a two-dimensional colorimetric representation known as a MacLeod–Boynton chromaticity diagram.

The corresponding MacLeod–Boynton chromaticity coordinates, l_{MB} , m_{MB} , s_{MB} , are defined as the ratios

$$\begin{aligned} l_{MB} &= \frac{L_{MB}}{L_{MB} + M_{MB}} \\ m_{MB} &= \frac{M_{MB}}{L_{MB} + M_{MB}} \\ s_{MB} &= \frac{S_{MB}}{L_{MB} + M_{MB}} \end{aligned} \quad (3.11)$$

where L_{MB} , M_{MB} , S_{MB} are the appropriately normalized cone excitations, formally named MacLeod–Boynton tristimulus values. In the MacLeod–Boynton chromaticity diagram, l_{MB} is the abscissa, and s_{MB} is the ordinate (MacLeod and Boynton, 1979).

In CIE 170-2 (CIE, 2015), the normalizing constants of the MacLeod–Boynton tristimulus values and the corresponding tables of the spectral MacLeod–Boynton chromaticity coordinates $l_{MB}(\lambda)$, $m_{MB}(\lambda)$, $s_{MB}(\lambda)$ are given for the particular cases of an observer aged 32 years and field sizes of 2° and 10°. Included in the report is also a further outline of the underlying physiologically-based colorimetric concept.

NOTE For a given colour stimulus, the values of l_{MB} , m_{MB} , s_{MB} differ with observer age and field size. Age- and field-size-parameterized spectral MacLeod–Boynton chromaticity coordinates, $l_{MB, \text{field size, age}}(\lambda)$, $m_{MB, \text{field size, age}}(\lambda)$, $s_{MB, \text{field size, age}}(\lambda)$, can be calculated by means of a specially designed computer program provided by CIE Division 1 (under development by TC 1-97). At the time of publication of this edition of the Colorimetry document, the beta version of the program (for Windows, macOS and Linux) is being tested.

4 Recommendations concerning standard physical data of illuminants and sources

4.1 Recommendations concerning standard physical data of illuminants

It is recommended that the following illuminants, defined by their relative spectral power distributions given in Table 1 of the ISO/CIE International Standard (ISO/CIE, 2007b), be used for general colorimetry.

This recommendation deviates from earlier recommendations regarding CIE sources A, B, C (CIE, 1931; CIE, 1951). A distinction is made between *illuminant* and *source*. The term *source* refers to a physical emitter of light, such as a lamp or the sun and the sky. The term *illuminant* refers to a specific spectral power distribution, not necessarily provided directly by a source, and not necessarily realizable as a source. The present recommendation first defines illuminants by relative spectral power distributions and then discusses some artificial sources representative of illuminants. The definition of these sources is considered secondary, as it is conceivable that new developments in materials will bring about improved sources that represent the illuminants more accurately and are more suitable for laboratory use. At present, no recommendation has been made for a source representing standard illuminant D65. The original recommendations regarding standard illuminant D65 (originally called D₆₅) and other illuminants D representing daylight of different correlated colour temperatures are given in CIE Proceedings (CIE, 1963; CIE, 1967).

In 1968, the Comité International des Poids et Mesures modified the "International Practical Temperature Scale, 1948 (amended 1960)" and the value of the radiation constant c_2 was set equal to $1,4388 \cdot 10^{-2}$ m·K. This modification has affected the colour temperature or correlated colour temperature of the CIE illuminants and sources. It was further revised to "International Temperature Scale 1990" (ITS-90), but the c_2 value is unchanged (Consultative Committee for Thermometry, 1989).

4.1.1 CIE standard illuminants

4.1.1.1 CIE standard illuminant A

CIE standard illuminant A is intended to represent typical tungsten-filament lighting. Its relative spectral power distribution is that of a Planckian radiator at a temperature of approximately 2 855,5 K. CIE standard illuminant A is used in many calibration procedures, and it is the only illuminant for which the corresponding source can also be realized with reasonable accuracy. This recommendation is retained despite the diminishing use of incandescent lamps in general lighting applications.

The relative spectral power distribution of CIE standard illuminant A, $S_A(\lambda)$, is defined by the equation

$$S_A(\lambda) = 100 \left(\frac{560}{\lambda} \right)^5 \times \frac{\exp \frac{1,435 \times 10^7}{2\,848 \times 560} - 1}{\exp \frac{1,435 \times 10^7}{2\,848 \lambda} - 1} \quad (4.1)$$

where λ is the wavelength in nanometres and the numerical values in the two exponential terms are definitive constants originating from the first definition of Illuminant A in 1931. This spectral power distribution is normalized to the value 100 (exactly) at the wavelength 560 nm (exactly).

Equation (4.1) is equivalent to and can be derived from the expression

$$S(\lambda) = 100 M_{e,\lambda}(\lambda, T) / M_{e,\lambda}(560, T) \quad (4.2)$$

where

$$M_{e,\lambda}(\lambda, T) = c_1 \lambda^{-5} [\exp(c_2 / \lambda T) - 1]^{-1} \quad (4.3)$$

and λ is the wavelength (in nanometres).

NOTE CIE standard illuminant A was originally defined in 1931 as the relative spectral power distribution of a Planckian radiator of temperature

$$T_{\text{CIE 1931}} = 2\,848 \text{ K}, \quad (4.4)$$

the value of the second radiation constant c_2 then being taken as

$$c_2, \text{ CIE 1931} = 1,435\,0 \times 10^{-2} \text{ m}\cdot\text{K}. \quad (4.5)$$

To keep the Illuminant A values consistent from 1931, these old values of T and c_2 are used.

Then the ratio c_2 / T is given by

$$c_2 / T = 1,435 \times 10^7 / 2\,848 \text{ nm}. \quad (4.6)$$

Since the numerical value of the first radiation constant, c_1 , cancels out of Equation (4.2), this definition of CIE standard illuminant A involves no assumptions about the numerical values of c_1 , c_2 , and T other than the ratio defined in Equation (4.6). (See ISO/CIE 2007b).

CIE standard illuminant A is defined over the spectral region from 300 nm to 830 nm in the ISO/CIE International Standard (ISO/CIE, 2007b) to six significant figures. Should higher precision be needed, values calculated using Equation (4.1) should be used. Clause 11.2 contains abridged tables that can be used in most practical calculations. On the use of these tables, see 7.2.

NOTE 1 The definition reproduced here is in accordance with the values of Illuminant A originally published in 1931 (ISO/CIE, 2007b).

NOTE 2 Even though Equation (4.1) is based on Planck's equation considering wavelengths in vacuum, the wavelengths are to be taken as being in standard air (dry air at 15 °C and 101 325 Pa, containing 0,03 % by volume of carbon dioxide). This makes CIE standard illuminant A compatible with other CIE colorimetric and photometric data.

NOTE 3 The wavelength range for standard illuminants is 300 nm to 830 nm because, for evaluating photoluminescent material, the ultraviolet spectral range is relevant. For most colorimetric investigations the restricted wavelength range, 380 nm to 780 nm, can be used.

NOTE 4 Terminology in this publication follows the traditional terminology used in colorimetry. A separate publication deals with the current "uncertainty" description of stating measurement results and recommends methods for calculating colorimetric uncertainties, see (CIE, 2011a).

4.1.1.2 CIE standard illuminant D65

CIE standard illuminant D65 has a relative spectral power distribution representing a phase of daylight with a correlated colour temperature of approximately 6 500 K (also called the nominal correlated colour temperature of the daylight illuminant), symbol: $S_{\text{D65}}(\lambda)$. The official values of $S_{\text{D65}}(\lambda)$ are as given in the CIE standard on illuminants for colorimetry (ISO/CIE, 2007b).

NOTE 1 The correlated colour temperatures are affected by the numerical value of the radiation constant c_2 . In accordance with the *International Practical Temperature Scale, 1948, amended 1960* which was in use at the time when the procedure for calculating daylight illuminants was adopted by the CIE, the value of c_2 was equal to $1,438\,0 \times 10^{-2} \text{ m}\cdot\text{K}$. With this value, the correlated colour temperature of illuminant D65 is approximately equal to 6 500 K. The change of c_2 to the value of $1,438\,8 \times 10^{-2} \text{ m}\cdot\text{K}$ (International Practical Temperature Scale, 1968) increases the correlated colour temperatures of illuminant D65 by the factor 1,4388/1,4380. Thus the correlated colour temperature increases by approximately 4 K.

NOTE 2 Regarding the definition of correlated colour temperature (T_{cp}) of an illuminant: see 9.4.

NOTE 3 Clause 11.2 provides abridged tables that can be used in many practical calculations. On the use of these tables see 7.2. The values of the relative spectral power distribution of CIE standard illuminant D65 given in Table 5 at 5 nm intervals are consistent with the values from 300 nm to 830 nm at 1 nm intervals and with six significant figures as can be found in the

ISO/CIE International Standard (ISO/CIE, 2007b). They have been taken from the tables of the standard.

NOTE 4 If values at wavelengths other than those printed in Table 1 of the ISO/CIE International Standard (ISO/CIE, 2007b) are needed, linear interpolation should be used (see also NOTE 5).

NOTE 5 CIE standard illuminant D65 contains power in the UV range, as does natural outdoor daylight. CIE published a document describing indoor daylight for colorimetry and indoor daylight illuminants (CIE, 2009); details of this new method of calculation are given in 4.1.2.1.

NOTE 6 In 2013, CIE published a method for re-defining CIE D illuminants having a smooth spectral power distribution (CIE, 2013); details of this new method of calculation are given in 4.1.2.2.

4.1.2 Other D illuminants

It is recommended that, in the interest of standardization, D65 be used whenever possible. When D65 cannot be used, it is recommended that one of the daylight illuminants D50, D55, or D75 defined in Table 5 be used. When none of these daylight illuminants can be used, a daylight illuminant at a nominal correlated colour temperature (T_{cp}) can be calculated using the following equations. These equations will result in an illuminant whose correlated colour temperature is approximately equal to the nominal value, but not exactly so.

(a) Chromaticity

The 1931 (x , y) chromaticity coordinates of the daylight (D) illuminant to be defined must satisfy the following relation:

$$y_D = -3,000 x_D^2 + 2,870 x_D - 0,275 \quad (4.7)$$

with x_D being within the range of 0,250 to 0,380. The correlated colour temperature, T_{cp} , of daylight D is related to x_D by the following formulae based on normals to the Planckian locus on the CIE 1960 uniform-chromaticity-scale diagram (see A.4):

(i) for correlated colour temperatures from approximately 4 000 K up to and including 7 000 K:

$$x_D = \frac{-4,6070 \times 10^9}{T_{cp}^3} + \frac{2,9678 \times 10^6}{T_{cp}^2} + \frac{0,09911 \times 10^3}{T_{cp}} + 0,244063 \quad (4.8)$$

where T_{cp} is the correlated colour temperature in kelvin (K)."

(ii) for correlated colour temperatures from greater than 7 000 K to approximately 25 000 K:

$$x_D = \frac{-2,0064 \times 10^9}{T_{cp}^3} + \frac{1,9018 \times 10^6}{T_{cp}^2} + \frac{0,24748 \times 10^3}{T_{cp}} + 0,237040 \quad (4.9)$$

where T_{cp} is the correlated colour temperature in kelvin (K)."

(b) Relative spectral power distribution

The relative spectral power distribution $S_D(\lambda)$ of daylight D is to be calculated from

$$S_D(\lambda) = S_0(\lambda) + M_1 S_1(\lambda) + M_2 S_2(\lambda) \quad (4.10)$$

where $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$ are functions of wavelength, λ , given in Table 6, and M_1 , M_2 are factors whose values are related to the chromaticity coordinates x_D , y_D as follows:

$$M_1 = \frac{-1,3515 - 1,7703 x_D + 5,9114 y_D}{0,0241 + 0,2562 x_D - 0,7341 y_D} \quad (4.11)$$

$$M_2 = \frac{0,0300 - 31,4424 x_D + 30,0717 y_D}{0,0241 + 0,2562 x_D - 0,7341 y_D}$$

- NOTE 1 Seasonal and geographical variations in the spectral power distribution of daylight occur, particularly in the ultraviolet spectral region, but this recommendation should be used pending the availability of further information on these variations.
- NOTE 2 The spectral power distributions of daylight illuminants D produced by this recommendation are based on experimental observations over the wavelength range 330 nm to 700 nm, and on extrapolation in the wavelength ranges 300 nm to 330 nm and 700 nm to 830 nm (Judd et al., 1964). The extrapolated values are believed to be accurate enough for colorimetric purposes, but should not be used for other purposes.
- NOTE 3 If values at wavelengths other than those given in Table 6 are needed, linear interpolation should be used (see also 4.1.2.2).
- NOTE 4 The relative spectral power distributions of the D illuminants given in Table 5 (that of standard illuminant D65 also given in the ISO/CIE International Standard on illuminants for colorimetry (ISO/CIE, 2007b)) were derived by the procedure given above with some intermediate rounding and with some adjustments for changes in the International Temperature Scale. Thus, for historical reasons, the tabulated values are slightly different from the calculated values. For the time being, the tabulated values are the official data. Eventually these data may be superseded by the method described in 4.1.2.2.
- NOTE 5 When samples exhibiting luminescence excited by ultraviolet radiation and seen outdoors are involved, one of the D illuminants defined in these recommendations should always be used to represent daylight. For indoor viewing, one of the indoor daylight illuminants is recommended.
- NOTE 6 The method to calculate the values for the relative spectral power distributions of illuminants D50, D55, D65, and D75, as given in Table 5, is as follows:
1. Multiply the nominal correlated colour temperature (5 000 K, 5 500 K, 6 500 K or 7 500 K) by the ratio 1,4388/1,4380.
 2. Calculate x_D and y_D using Equations (4.8) or (4.9) and (4.7)
 3. Calculate M_1 and M_2 using Equations (4.11).
 4. Round M_1 and M_2 to three decimal places.
 5. Calculate $S_D(\lambda)$ for every 5 nm by $S_D(\lambda) = S_0(\lambda) + M_1 \cdot S_1(\lambda) + M_2 \cdot S_2(\lambda)$ using values of $S_0(\lambda)$, $S_1(\lambda)$ and $S_2(\lambda)$ from Table 6.
 6. Interpolate the 5 nm values of $S(\lambda)$ linearly to obtain values at intermediate wavelengths.
- This method should also be followed to calculate the relative spectral power distribution of illuminants at other nominal correlated colour temperatures.

4.1.2.1 Indoor daylight illuminants

The above discussed daylight illuminants that contain ultraviolet radiation in proportions as found in natural outdoor daylight are suitable for colorimetric calculations where the colour samples are seen outdoors. A paper sample whitened with optical brightener will look less bright indoors than outdoors, since the optical brightener will not be excited to the same extent by the indoor daylight. To be able to evaluate samples (paper, plastic, textile, etc.) that have been treated with optical brighteners, CIE introduced two indoor daylight illuminants ID50 and ID65 (see Table 7), with nominal correlated temperature of 5 000 K and 6 500 K, respectively (CIE, 2009). These illuminants consider average glazing ultraviolet absorption.

Thus CIE recommended the definition of indoor daylight illuminant spectra as the product of a CIE daylight illuminant and the spectral transmittance of average window glass, the transmittance of which is given in Table 8, at 5 nm intervals between 300 nm and 780 nm.

- NOTE 1 When samples that exhibit luminescence excited by ultraviolet radiation are seen indoors, one of the indoor D illuminants defined in this clause should be used to represent daylight. For outdoor viewing, one of the outdoor daylight illuminants is recommended.
- NOTE 2 As a general practice, if indoor daylight is used in the calculation, this should be noted; if no such notation is included, the use of a classical (outdoor) illuminant is presumed.

4.1.2.2 Smoothed D illuminants

A recurring problem with the CIE daylight illuminants is that the original spectral measurements were performed at 10 nm intervals and that all previous CIE recommendations prescribe, in case of the daylight spectra, linear interpolation between these fixed points (for any other smooth spectral functions, Sprague interpolation is recommended (CIE 2005a)). In Appendix C of the 2004 edition of CIE 15 (CIE, 2004b), it is stated that the interpolation of daylight spectra be such that the data should be differentiable twice.

The task of TC 1-74 was to analyse further the method described in Appendix C of the 2004 version of CIE 15, and also to seek alternative methods to obtain even smoother curves for daylight spectra. It was a requirement that any new smoothing routine should not provide tristimulus values that are meaningfully different from those obtained using the classical daylight spectra. CIE TC 1-74 analysed the method of calculating daylight spectra as described in 4.1.2, retaining the 10 nm fixed points, but using Lagrange, Sprague and spline interpolations instead of the linear interpolation. Details of the method are summarized in CIE 204:2013 “Methods for re-defining CIE D illuminants” (CIE, 2013). Further information can be found in the articles by Kránicz and Schanda (2000) and Sève and Duval (2003).

As in the real daylight emission spectra, the dips in the spectra are due to the absorption by different gaseous substances in the atmosphere, and these are much narrower absorption bands than manifested in the CIE daylight illuminant spectra with 10 nm wavelength interval. Thus, for colorimetric purposes a considerable smoothing of the spectra will not change the tristimulus values appreciably.

Smoothing using a LOWESS (Locally Weighted Scatter Plot Smoothing) filter technique was used, and the results of different amounts of smoothing were tested on a database of reflection spectra that included 100 000 samples (Kosztján and Schanda, 2013). A smoothing routine was finally selected where, when applied to the over 100 000 samples, the maximum CIELAB colour difference (see 8.2.1.3) between using the D65 illuminant and the smoothed D65 illuminant was smaller than 0,3 and the average colour difference was below 0,1.

Based on these observations, TC 1-74 recommended to test in practice the use of the smoothed $S_{0,s}(\lambda)$, $S_{1,s}(\lambda)$ and $S_{2,s}(\lambda)$ functions, published in CIE 184 on Indoor daylight illuminants (CIE, 2009). The publication also contains tables for smoothed (outdoor) daylight illuminants, sD50, sD55, sD60, sD65, sD70, sD75, and smoothed indoor daylight illuminants, sID50 and sID65, with 1 nm steps between 300 nm and 830 nm (for the sID illuminants only between 314 nm and 780 nm).

4.1.3 Illuminant B

This illuminant is intended to represent direct sunlight with a correlated colour temperature of approximately 4 900 K.

NOTE The use of this illuminant is deprecated. However, for a description of illuminant B and source B see Annex A.2.

4.1.4 Illuminant C

This illuminant is intended to represent average daylight with a correlated colour temperature of approximately 6 800 K.

NOTE 1 Illuminant C does not have the status of a CIE standard but its relative spectral power distribution, tristimulus values and chromaticity coordinates are given in Table 5 and Table 9.1 and Table 9.2, as many practical measurement instruments and calculations still use this illuminant.

NOTE 2 The tristimulus values and chromaticity coordinates of the illuminant C and of D50, D55, D75, calculated from the values given in Table 6, are shown in Table 9.1 and Table 9.2. The reproduced tristimulus values are those of the perfect reflecting diffuser irradiated by the respective illuminant (see 7.1).

4.1.5 Illuminant E

In many practical colorimetric calculations the reference chromaticity coordinates are $x = 0,3333$, $y = 0,3333$, which are obtained if the tristimulus integration is made using a spectrum that has a wavelength-independent spectral power distribution.

For an illuminant that is defined by the equi-energy spectrum, i.e. a spectrum where the relative spectral power distribution $S(\lambda) = 1$ for all λ , CIE endorses the term "CIE illuminant E."

For many practical colorimetric calculations, it is sufficient to have a constant spectral power distribution between 380 nm and 780 nm.

4.2 Artificial sources representative of illuminants

It is recommended that the following artificial sources should be used if it is desired to realize the illuminants defined in 4.1 for actual laboratory inspection.

4.2.1 Source A

CIE standard illuminant A is to be realized by a gas-filled tungsten filament lamp operating at a correlated colour temperature of 2 855,5 K (this value produces the same spectral distribution as the defined illuminant A with the value of the second radiation constant fixed to $c_2 = h \cdot c \cdot k^{-1} = 1,438\,776\,877\,5 \times 10^{-2} \text{ m} \cdot \text{K}$ where h is Planck's constant, c is the speed of light in vacuum, and k is the Boltzmann constant – see also 4.1.1.1). If the source is also to be used in the ultraviolet spectral region, a lamp having an envelope or window made of fused-quartz or silica must be used because glass absorbs the ultraviolet component of the radiation from the filament.

4.2.2 Source B

See Annex A.2.

4.2.3 Source C

See Annex A.3.

4.2.4 Source D65

At present no artificial source is recommended to realize CIE standard illuminant D65 or any other illuminant D of different correlated colour temperature. It is expected that new developments in light sources and filters will eventually offer sufficient basis for a CIE recommendation. Meanwhile the CIE has agreed on a formula to describe the quality of a daylight simulator for colorimetry, see 9.2.

4.2.5 Notes on artificial sources representative of illuminants

NOTE 1 The artificial sources defined above and recommended as representative sources for CIE illuminants are named "CIE sources for colorimetry".

NOTE 2 Whenever the highest accuracy of the spectral power distribution of a standard is required, it is advisable to make a spectroradiometric characterization of the actual source used, because the relative spectral power distribution of the source may not exactly coincide at all wavelengths with that defining the corresponding illuminant.

NOTE 3 The spectrum of a D65 simulator fluorescent lamp is reproduced in Table 10.3 as FL3.15 (see also 9.2).

5 Recommendations concerning standard of reflectance

The fundamental property in the measurement of colour is the colour stimulus function, $\varphi_\lambda(\lambda)$ (CIE, 2011b). For lamps and displays, the colour stimulus function is based on the emitted spectral radiant flux. For material objects, the relative colour stimulus function is given by the product of the incident spectral radiant flux and either the spectral transmittance of the optically transparent material or the spectral reflectance of the optically opaque material. There is an intermediate case in which the material is partially transmitting and partially reflecting and

includes a diffuse component. Such materials are classified as translucent and the proper colour stimulus function is determined from the observing intent.

Light from a source of light, light from a source that has been transmitted by a transparent or translucent material, and light that has been reflected from a material surface may be quantified by use of the methods of radiometry or photometry that take into consideration the phenomena where the material surface and the interior structure may change the geometric distribution of the spectral radiant flux.

The reference standard for reflectance is based on the theoretical concept known as the perfect reflecting diffuser (CIE, 1986; CIE, 2011b). It is defined as the ideal Lambertian (geometrically isotropic) diffuser with a reflectance equal to unity at all wavelengths.

To measure actual reflectance, one must integrate all the flux in the hemisphere above the surface of the material. There are three methods for deriving the absolute reflectance of a material (CIE, 1979a); the most commonly used method involves the use of an integrating sphere to capture all the flux reflected from the material. There are a number of materials identified that are suitable for use as physical reflectance standards (CIE, 1979b; ASTM, 2015) as they are close approximations to the perfect reflecting diffuser. For real measurements, the various material standards such as pressed barium sulphate or polytetrafluoroethylene (PTFE) must be characterized in terms of the perfect reflecting diffuser for the required geometry, using one of the absolute methods, and then those standards are used to “transfer” this realization of the perfect reflecting diffuser onto a more durable material, such as a synthetic opal glass, a porcelain or ceramic plaque, or a quantity of a white powder, such as specially refined barium sulphate (BaSO_4), polytetrafluoroethylene (PTFE) or polytetrachloroethylene (PTCE).

Measurements based on the comparison of the reflected spectral flux from a material to the reflected spectral flux from a reference material are termed reflectance factor. Since the perfect reflecting diffuser is both spectrally and geometrically isotropic, sampling the reflected flux in one direction in the upper hemisphere should be sufficient to capture the total reflectance. For real materials, even for close approximations of the perfect reflecting diffuser, the reflectance and reflectance factor will not be equal, and standards of reflectance factor will be valid only for one specific geometry of influx and efflux.

6 Recommendations concerning geometric conditions for colorimetry

Colorimetric specifications are derived from spectral or tristimulus measurements. The measured values depend on the illumination and viewing geometry, called “geometric conditions” or simply “geometry”. The degree of correlation between measured values and visual appraisals depends on the degree to which the geometric conditions of measurement simulate the geometric conditions of viewing.

6.1 Terms and definitions used in this clause

6.1.1

reference plane, <in photometric or radiometric measurements>

plane in which the surface of a sample or standard is placed during measurements

Note 1 to entry: For reflection measurements, the geometry is defined with respect to the reference plane. For transmission measurements, there is a reference plane for the incident light and a second reference plane, displaced by the sample thickness, for the transmitted light.

[SOURCE: CIE DIS 017:2016, Term 17-25-110]

6.1.2

sampling aperture

area of the reference plane on which measurements are made

Note 1 to entry: The sampling aperture is delimited by the area illuminated or by the area over which the receiver senses flux, whichever is smaller. If the illuminated area is the larger, the area measured is said to be “overfilled”; if it is the smaller, the area measured is said to be “underfilled”.

[SOURCE: CIE DIS 017:2016, Term 17-25-114]

6.1.3**influx geometry
irradiation geometry
incidence geometry**

angular distribution of the radiation incident on the sample being measured with respect to the centre of the sampling aperture

[SOURCE: CIE DIS 017:2016, Term 17-25-092]

6.1.4**efflux geometry**

angular distribution of the receiver responsivity with respect to the centre of the sampling aperture on the sample being measured

[SOURCE: CIE DIS 017:2016, Term 17-25-093]

6.1.5**influx area**

portion of the sample area that is illuminated by the influx geometry

6.1.6**efflux area**

portion of the sample area from which reflected light is collected by the efflux geometry

6.2 Recommendations concerning geometric conditions for colorimetry

- The use of the term “viewing” with reference to measuring instruments is deprecated because it blurs the important distinction between instrumental measurements and visual observation.
- The influx geometry and efflux geometry together specify the geometric nature of the measurement for reflectance and transmittance measurements.
- It is recommended that the geometry used when determining the colorimetric specification of diffusely reflecting samples corresponds to one of the irradiating (influx) and reflection/transmission (efflux) conditions (CIE, 2006b).
- Specific recommendations on how to use the geometries described below together with the standards described in the previous clause, and how to perform the required measurements of the optical properties of reflectance and transmittance can be found in CIE 130 (CIE, 1998).

6.3 Recommended nomenclature for directional irradiation**6.3.1 Forty-five degree directional geometry (45°x)**

Irradiation of reflecting materials at 45° to the normal, at one azimuth angle. This single geometry emphasizes texture and directionality of the specimen. The “x” in the symbol indicates that the azimuthal direction of the incident beam is in the x direction on the reference plane.

6.3.2 Forty-five degree annular geometry (45°a)

In measuring the colours of reflecting samples with 45° illumination, the effects of texture and directionality are minimized by irradiating at 45° to the normal, from all azimuthal directions, simultaneously. This irradiation (influx) geometry may be achieved by the use of a small source and an elliptic ring reflector or other aspheric optics. This geometry is sometimes approximated by the use of a number of light sources in a ring or a number of fibre bundles illuminated by a single source and terminated in a ring. Such an approximation to annular geometry is called *circumferential geometry*, symbol 45°c.

6.3.3 Zero degree directional geometry (0°)

Irradiation of reflecting materials at the normal of the specimen surface.

6.3.4 Eight degree geometry (8°)

Irradiation of reflecting materials at 8° to the normal, at one azimuth angle. It is used to substitute the zero-degree directional geometry in many practical applications, as it permits, in reflection measurements, to differentiate between specular (regular) component included and specular component excluded measurements.

6.4 Recommended geometries for reflection measurements

6.4.1 Diffuse : eight degree geometry, specular component included (di:8°)

For di:8° geometry, it is recommended that the sample be irradiated by an integrating sphere, that the influx area be at least 2 mm greater in diameter than the efflux area, that the area of the sampling aperture be uniformly irradiated with a variation from the centre to the edge of less than 10 %, that it be irradiated uniformly from all directions within the hemisphere bounded by its plane, that the responsivity of the receiver be uniform over the area of the sampling aperture, that the axis of the reflected (efflux) beam be $8^\circ \pm 0,5^\circ$ off the normal to the centre of the specimen, and that radiation reflected at the sampling aperture be evaluated uniformly at all directions within a cone of half angle of $2^\circ \pm 0,25^\circ$ about the axis of the collection (efflux) beam.

6.4.2 Diffuse : eight degree geometry, specular component excluded (de:8°)

It is recommended that all specifications for di:8° be met, except that there should be no radiation reflected in the direction of the receiver by a plane first-surface mirror at the sampling aperture and that there should be no rays specularly reflected within 1° of such rays, as an allowance for instrumental scattering of stray light or misalignment. This is best accomplished by providing a port in the wall of the integrating sphere at -8° off the normal to the specimen. The opening in the sphere wall should have the size of a $4^\circ \pm 0,25^\circ$ half-angle cone, centred on the sampling aperture.

6.4.3 Eight degree : diffuse geometry, specular component included (8°:di)

It is recommended that the conditions for di:8° be met, but with the light path reversed, so the sampling aperture is irradiated under 8° and flux reflected at the sampling aperture is collected at all angles in the hemisphere bounded by the reference plane. The sample port should be underfilled with radiation and the irradiated area should be uniform in irradiance from the centre to the edge of the spot with a variation of less than 10 %. The influx beam should be conical with a half angle of $2^\circ \pm 0,25^\circ$ about the axis of the incidence (influx) beam.

6.4.4 Eight degree : diffuse geometry, specular component excluded (8°:de)

It is recommended that the geometric conditions for de:8° be met, but with the light path reversed. The sample port should be underfilled with radiation and the irradiated area should be uniform in irradiance from centre to edge of the spot with a variation of less than 10 %. The specular reflection component should be excluded providing a port in the sphere wall at -8° off the normal with a half angle cone of $4^\circ \pm 0,25^\circ$.

6.4.5 Diffuse : diffuse geometry (d:d)

It is recommended that for irradiation the specifications for di:8° be met, and flux reflected at the sampling aperture is collected at all angles in the hemisphere bounded by the reference plane. In this geometry the sampling aperture can be neither underfilled nor overfilled. In practice, this geometry is difficult to produce.

6.4.6 Alternative diffuse geometry (d:0°)

An alternative diffuse geometry is the strict specular excluded geometry defined when the efflux direction is along the specimen normal. Because the efflux is along the normal, there is the potential for first surface reflection off the receiving optical surface (inter-reflection stray light). It will be necessary to "tilt" the optics slightly to avoid this error. Additionally, the angular size of the efflux port must be at least 2° larger than the half angle of the efflux beam.

6.4.7 Forty-five degree annular : normal geometry (45°a:0°)

It is recommended that the sampling aperture be irradiated uniformly from all directions between two circular cones with their axes on the normal to the sampling aperture and apices at the centre of the sampling aperture, the smaller cone having a half angle of $43^\circ \pm 0,25^\circ$ and the larger of $47^\circ \pm 0,25^\circ$. It is recommended that the receiver uniformly collects and evaluates all radiation reflected within a cone with its axis on the normal ($0^\circ \pm 0,5^\circ$) to the sampling aperture, apex at the centre of the sampling aperture, and a half angle of $2^\circ \pm 0,25^\circ$. As with the d:0° geometry, it may be necessary to “tilt” the receiving optics to prevent inter-reflection stray light.

6.4.8 Normal : forty-five degree annular geometry (0°:45°a)

It is recommended that the angular and spatial conditions for 45°a:0° be met, with the light path reversed, so the sampling aperture is irradiated normally and reflected radiation is collected within an annulus centred at $45^\circ \pm 0,5^\circ$ to the normal. In this geometry, the influx optics may need to be tilted to avoid inter-reflection stray light.

6.4.9 Forty-five degree directional : normal geometry (45°x:0°)

It is recommended that the angular and spatial conditions for 45°a:0° be met with the exception that irradiation is only at one azimuth angle; this excludes the specular component, but emphasizes texture and directionality. The “x” in the symbol indicates that the azimuthal direction of the incident beam is in the x direction on the reference plane.

6.4.10 Normal : forty-five degree directional geometry (0°:45°x)

It is recommended that the angular and spatial conditions for 45°x:0° be met, with the light path reversed, so the sampling aperture is irradiated normally and reflected radiation is collected at one azimuth angle at 45° to the normal.

NOTE 1 Conditions according to 6.4.1, 6.4.2, 6.4.6, 6.4.7, 6.4.8, 6.4.9 and 6.4.10 are used for the measurement of the reflectance factor, $R(\lambda)$. For directional measurement with a sufficiently small angular spread, the values of reflectance factor become identical to values of radiance factor. For condition 6.4.3, for measurement with an ideal sphere, where all the ports of the integrating sphere have zero size and the reflectance of the sphere wall is unity, reflectance is measured. In the limit of narrow angles, the 45°x:0° geometry gives the radiance factor $\beta_{45:0}$, the 0°:45°x geometry gives the radiance factor $\beta_{0:45}$. Under the ideal conditions, the di:8° geometry gives the radiance factor $\beta_{di:8}$, which approximates the radiance factor $\beta_{d:0}$, and the 8°:di geometry approximates the total reflectance ρ .

NOTE 2 It is important that the particular irradiating and measuring conditions used should be specified even if they are within the range of one of these recommended conditions. Measurements of some types of samples (for example retro-reflective materials) may require a different geometry or tolerances.

NOTE 3 When integrating spheres are used, they should be fitted with white-coated baffles to prevent light passing directly between the sample and the sphere wall irradiated or measured. When the regular component of reflection is to be included, the sphere efficiency for that part of the sphere wall that receives the regularly reflected radiation component should be of the highest value possible. The total area of the ports of the integrating sphere should not exceed 10 % of the internal reflecting sphere area.

NOTE 4 It should be noted that diffusing samples may scatter radiation in directions approximately parallel to their surfaces. Such radiation should be included in the measurement of diffuse reflectance.

NOTE 5 When integrating spheres are used for measuring photoluminescent material, the spectral power distribution of the irradiating system is altered by the reflected and emitted power from the sample (CIE, 1988). The use of the 45°a:0°, 45°x:0°, 0°:45°a, or 0°:45°x geometry is therefore preferable (see CIE, 2007; Gundlach and Mallwitz, 1976; Alman and Billmeyer, 1976).

NOTE 6 The notations used here are shown with the degree symbol, °, included. Many electronic displays used in instrumentation do not have this symbol in their character set. It is accepted that 0°:45°a and 0:45a indicate the same measurement geometry.

NOTE 7 The use of the former “slash” notation of *illumination* / *viewing* for describing a measurement geometry has been deprecated. The new notation is the ISO standard *Influx* : *Efflux* notation as recommended in CIE 176 (CIE, 2006a). The recommendations of CIE 176 formalize procedures for practical colorimetric calculations.

6.5 Recommended geometry for transmission measurements

6.5.1 Normal : normal geometry ($0^\circ:0^\circ$)

It is recommended that the irradiating (influx) and measuring (efflux) geometry be of identical right-circular conic form, with their axes on the normal ($0^\circ \pm 0,5^\circ$) to the centre of the sampling aperture, and a half angle of $5^\circ \pm 0,25^\circ$, that the surface and angular irradiation of the sampling aperture be uniform, and that the surface and angular responsivity of the receiver be uniform.

6.5.2 Diffuse : normal geometry, regular component included ($di:0^\circ$)

It is recommended that the sampling aperture be uniformly irradiated from all directions in the hemisphere bounded by the first reference plane and that the measuring (efflux) beam be as specified for the $0^\circ:0^\circ$ geometry.

6.5.3 Diffuse : normal geometry, regular component excluded ($de:0^\circ$)

It is recommended that the geometry be that specified for $di:0^\circ$ except that, with the sampling aperture open (i.e. no sample in place), there be no rays directed toward the receiver and no rays within 1° of such rays, as measured at the centre of the sampling aperture.

6.5.4 Normal : diffuse geometry, regular component included ($0^\circ:di$)

It is recommended that the geometry be the reverse of that specified for $di:0^\circ$ geometry.

6.5.5 Normal : diffuse geometry, regular component excluded ($0^\circ:de$)

It is recommended that the geometry be the reverse of that specified for $de:0^\circ$ geometry.

6.5.6 Diffuse : diffuse geometry ($d:d$)

It is recommended that the sampling aperture be uniformly irradiated at all angles within the hemisphere bounded by the first reference plane and that the transmitted flux be uniformly evaluated at all directions within the hemisphere bounded by the second reference plane.

NOTE 1 With all the above geometries transmittance is measured except for those where the regular component is excluded; in that case the transmittance factor is measured.

NOTE 2 It is important that the particular irradiating and collecting conditions used are specified, even if they are within the range of one of the conditions recommended here. Measurements of some types of samples may require a different geometry or tolerances.

NOTE 3 Integrating spheres should be fitted with white-coated baffles to prevent radiation passing directly from source to sample or reference in the case of diffuse irradiation or directly from sample or reference to detector in the case of diffuse collection. The total area of the ports of the integrating sphere should not exceed 10 % of the internal reflecting sphere area.

NOTE 4 The construction of an instrument for normal : normal measurements should be such that the irradiating (influx) and collecting (efflux) beams are equal whether there is a sample in place or not.

NOTE 5 It should be noted that diffusing samples may scatter radiation in directions approximately parallel to their surfaces, and such radiation should be included in the measurement of diffuse transmittance.

NOTE 6 Multiple reflections between the sample and the incident beam optics may cause measurement errors if the incident beam is normal to the sample surface. These can be eliminated by slightly tilting the sample.

NOTE 7 The use of the former "slash" notation for *illumination* / *viewing* has been deprecated for describing instrument measurement geometries. The preferred *influx* : *efflux* notation is the one utilized by the ISO (2009), the ASTM (2011) and the CIE (2006a).

7 Recommendations concerning the calculation of tristimulus values and chromaticity coordinates

The following recommendations formalize procedures for practical colorimetric calculations.

7.1 Calculation of tristimulus values

The joint ISO/CIE International Standard on CIE tristimulus values (ISO/CIE, 2012) recommends that the CIE tristimulus values of a colour stimulus should be obtained by multiplying at each wavelength the value of the colour stimulus function, $\varphi_\lambda(\lambda)$, by that of each of the CIE colour-matching functions, and integrating each set of products over the wavelength range corresponding to the entire visible spectrum, 360 nm to 830 nm. The integration can be carried out by numerical summation at wavelength intervals, $\Delta\lambda$, equal to 1 nm:

$$\begin{aligned} X &= k \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{x}(\lambda) \Delta\lambda & X_{10} &= k_{10} \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{x}_{10}(\lambda) \Delta\lambda \\ Y &= k \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{y}(\lambda) \Delta\lambda & Y_{10} &= k_{10} \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{y}_{10}(\lambda) \Delta\lambda \\ Z &= k \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{z}(\lambda) \Delta\lambda & Z_{10} &= k_{10} \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{z}_{10}(\lambda) \Delta\lambda \end{aligned} \quad (7.1)$$

Similarly, in the case of the cone-fundamental-based tristimulus values, it is recommended that the integration be carried out over the wavelength range 390 nm to 830 nm by numerical summation at wavelength intervals, $\Delta\lambda$, equal to 1 nm according to the equations

$$\begin{aligned} X_F &= k_F \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{x}_F(\lambda) \Delta\lambda & X_{F,10} &= k_{F,10} \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{x}_{F,10}(\lambda) \Delta\lambda \\ Y_F &= k_F \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{y}_F(\lambda) \Delta\lambda & Y_{F,10} &= k_{F,10} \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{y}_{F,10}(\lambda) \Delta\lambda \\ Z_F &= k_F \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{z}_F(\lambda) \Delta\lambda & Z_{F,10} &= k_{F,10} \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{z}_{F,10}(\lambda) \Delta\lambda \end{aligned} \quad (7.2)$$

In the above equations $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ and X , Y , Z , and $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ and X_{10} , Y_{10} , Z_{10} are colour-matching functions and tristimulus values referring to the CIE 1931 standard colorimetric observer and the CIE 1964 standard colorimetric observer as defined in (ISO/CIE, 2007a), respectively; $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ and X_F , Y_F , Z_F , and $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ and $X_{F,10}$, $Y_{F,10}$, $Z_{F,10}$ are cone-fundamental-based tristimulus functions and cone-fundamental-based tristimulus values referring to the CIE 2006 2° cone-fundamental-based colorimetric observer and the CIE 2006 10° cone-fundamental-based colorimetric observer as defined in (CIE, 2006a), respectively; $\varphi_{\lambda}(\lambda)$ is the colour stimulus function (see 7.1.1); and k , k_{10} , k_F , $k_{F,10}$ are normalizing constants defined below.

The colour-matching functions that define the CIE standard colorimetric observers are tabulated at 1 nm intervals from 360 nm to 830 nm in (ISO/CIE, 2007a). All rigorous calculations should use these 1 nm tables. For most practical purposes, the summation may be approximated by using wavelength intervals equal to 5 nm over the wavelength range 380 nm to 780 nm. Values of the CIE colour-matching functions at 5 nm intervals suitable for use in summation over this range of wavelengths are given in Table 1 and Table 2. When measurements have been made at smaller intervals than 5 nm, the appropriate values from the tables in the standards should be used (see 7.2.1).

The cone-fundamental-based tristimulus functions for 2° and 10° field size are tabulated at 1 nm intervals in CIE 170-2 (CIE, 2015). All rigorous calculations should use these 1 nm tables. For many practical purposes, the summation may be approximated by using wavelength intervals equal to 5 nm over the wavelength range 390 nm to 780 nm. The values selected at 5 nm intervals over this range are given rounded to six decimal places in Table 3 and Table 4. In calculations carried out by numerical integration at wavelength intervals of 5 nm, these tables provide equality of the cone-fundamental-based chromaticity coordinates of Illuminant E to the

precision of four decimal places (see NOTE 6 in 3.3.1). Properly normalized cone-fundamental-based tristimulus functions (i.e. normalized such that the cone-fundamental-based chromaticity coordinates of Illuminant E each equal 1/3 to an accuracy of at least five decimal places) can be calculated at user-defined wavelength intervals, $\Delta\lambda$, equal to 0,1 nm, 0,2 nm, ..., 5,0 nm by means of a specially designed computer program provided by CIE Division 1 (under development by TC 1-97; at the time of publication of this edition of the Colorimetry document, the beta version of the program (for Windows, macOS and Linux) is being tested.) (see 7.2.1).

Tristimulus values are usually evaluated on a relative basis. In such cases, the relative colour stimulus function, $\phi(\lambda)$, may be used instead of the colour stimulus function, $\phi_\lambda(\lambda)$, but it is essential that, for stimuli that will be considered together, all the spectral distributions involved be assessed on the same relative basis. The tristimulus values obtained are then relative in the sense that all the values involved may be multiplied by the same single arbitrary constant. In certain cases, however, k , k_{10} , k_F , and $k_{F,10}$ must be chosen according to agreed conventions – these are explained in 7.1.1 and 7.1.2.

Tristimulus values evaluated on a relative basis may need to be supplemented by the value of a suitable absolute photometric quantity.

7.1.1 Secondary light sources (reflecting or transmitting objects)

For reflecting or transmitting object colours, the colour stimulus function, $\phi_\lambda(\lambda)$, is replaced by the relative colour stimulus function, $\phi(\lambda)$, evaluated as

$$\phi(\lambda) = R(\lambda) \cdot S(\lambda) \quad \text{or} \quad \phi(\lambda) = \beta(\lambda) \cdot S(\lambda) \quad \text{or} \quad \phi(\lambda) = \rho(\lambda) \cdot S(\lambda) \quad \text{or} \quad \phi(\lambda) = \tau(\lambda) \cdot S(\lambda)$$

where

$R(\lambda)$ is the spectral reflectance factor of the specimen (preferably evaluated for one of the geometric conditions given in 6.4).

$\beta(\lambda)$ is the spectral radiance factor of the specimen (preferably evaluated for one of the geometric conditions given in 6.4).

$\rho(\lambda)$ is the spectral reflectance of the specimen (preferably evaluated for one of the geometric conditions given in 6.4).

$\tau(\lambda)$ is the spectral transmittance of the specimen (preferably evaluated for one of the geometric conditions given in 6.5).

$S(\lambda)$ is the relative spectral power distribution of the illuminant (which, in formal context, should be one of the CIE standard illuminants; see 4.1).

In these cases, the constants, k , k_{10} , k_F , and $k_{F,10}$, are chosen such that $Y = Y_{10} = Y_F = Y_{F,10} = 100$ for objects for which $R(\lambda)$ (or $\beta(\lambda)$ or $\rho(\lambda)$ or $\tau(\lambda)$) equals 1 for all wavelengths, implying that

$$k = 100 / \sum_{\lambda} S(\lambda) \bar{y}(\lambda) \Delta\lambda$$

$$k_{10} = 100 / \sum_{\lambda} S(\lambda) \bar{y}_{10}(\lambda) \Delta\lambda$$

and

$$k_F = 100 / \sum_{\lambda} S(\lambda) \bar{y}_F(\lambda) \Delta\lambda$$

$$k_{F,10} = 100 / \sum_{\lambda} S(\lambda) \bar{y}_{F,10}(\lambda) \Delta\lambda .$$

For other objects, the values of Y , Y_{10} , Y_F and $Y_{F,10}$, are then equal to the percentage values of luminous reflectance factor (in the case of $R(\lambda)$), or luminance factor (in the case of $\beta(\lambda)$), or luminous reflectance (in the case of $\rho(\lambda)$), or luminous transmittance (in the case of $\tau(\lambda)$). This

is because the functions $\bar{y}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{y}_F(\lambda)$ and $\bar{y}_{F,10}(\lambda)$ are identical to the CIE spectral luminous efficiency function for photopic vision, $V(\lambda)$ (ISO/CIE, 2004), the spectral luminous efficiency function of the CIE 10° photopic photometric observer, $V_{10}(\lambda)$ (CIE, 2005b), the cone-fundamental-based spectral luminous efficiency function for 2° field size, $V_F(\lambda)$ (CIE, 2015), and the cone-fundamental-based spectral luminous efficiency function for 10° field size, $V_{F,10}(\lambda)$ (CIE, 2015), respectively.

The luminous reflectance factor is the ratio of the luminous flux reflected by the specimen to the luminous flux reflected by the perfect reflecting diffuser or perfect non-diffusing reflector under the same spectral and geometric conditions. The luminance factor is the ratio of the luminance of the surface element, in the given direction, to that of the perfect reflecting or transmitting diffuser identically illuminated and viewed. The luminous reflectance or luminous transmittance, the values of which resemble those of (radiant) reflectance or (radiant) transmittance, is the ratio of the reflected or transmitted luminous flux to the incident luminous flux in the given spectral and geometric conditions.

7.1.2 Illuminants and self-luminous objects

For self-luminous objects and illuminants, the constants k , k_{10} , k_F and $k_{F,10}$ are usually chosen on the grounds of convenience and often set to one. If, however, the value of Y , Y_{10} , Y_F or $Y_{F,10}$ is required to be numerically equal to the absolute value of a photometric quantity, the corresponding constant must be set equal to the numerical value of the maximum luminous efficacy as determined with respect to the spectral luminous efficiency function concerned (i.e. $k = K_m = 683,002 \text{ lm} \cdot \text{W}^{-1}$; $k_{10} = K_{m,10} = 683,599 \text{ lm} \cdot \text{W}^{-1}$; $k_F = K_{F,m} = 683,358 \text{ lm} \cdot \text{W}^{-1}$ and $k_{F,10} = K_{F,m,10} = 683,144 \text{ lm} \cdot \text{W}^{-1}$), and the colour stimulus function, $\phi_\lambda(\lambda)$, must be equal to the spectral distribution of the radiometric quantity corresponding to the photometric quantity under consideration.

7.2 The use of abridged or truncated data

The use of the tables published in the ISO/CIE International Standard on CIE standard colorimetric observers (ISO/CIE, 2007a) requires that the colour stimulus function, $\phi_\lambda(\lambda)$, or the relative colour stimulus function, $\phi(\lambda)$, be known at 1 nm wavelength intervals over the wavelength range 360 nm to 830 nm. In practical applications, all the required data may not be available because the measurement was made at intervals larger than 1 nm, or unequal wavelength intervals were used, or data at the spectral extremes were omitted (truncation). Often it is possible to predict the unmeasured data. It is recognized that calculation from predicted, rather than from measured, data, or from abridged or truncated data, may be inexact, but the user can often be satisfied that the accuracy is sufficient for the intended use of the tristimulus values. It is important to use the same wavelength interval and range throughout for any set of calculations in which data for different colours are to be compared precisely. In particular, it should be noted that the exact values obtained for the perfect diffuser for a given illuminant depend on the wavelength interval and range used.

7.2.1 Abridged method for data available at intervals of a maximum 5 nm

For a wavelength interval, $\Delta\lambda$, equal to 5 nm:

If it is demonstrated that the resulting errors are insignificant for the purpose of the user, tristimulus values X , Y , Z and X_{10} , Y_{10} , Z_{10} should be calculated by numerical summation from 380 nm to 780 nm at wavelength intervals, $\Delta\lambda$, equal to 5 nm according to Equations (7.1) and (7.3), using colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ and $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$, respectively, defined with seven significant figures in the ISO/CIE International Standard (ISO/CIE, 2007a) (see 7.2.6 for bandwidth requirements). Alternatively, if it is demonstrated that the achieved accuracy is sufficient for the intended use of the tristimulus values, the colour-matching function values selected at 5 nm wavelength intervals from 380 nm to 780 nm and rounded to six decimal places can be used. These abridged sets of function values for $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ and $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ are given in Table 1 and Table 2, respectively.

If it is demonstrated that the resulting errors are insignificant for the purpose of the user, the cone-fundamental-based tristimulus values, X_F , Y_F , Z_F and $X_{F,10}$, $Y_{F,10}$, $Z_{F,10}$, should be calculated by numerical summation from 390 nm to 780 nm at wavelength intervals, $\Delta\lambda$, equal to 5 nm according to Equations (7.2) and (7.4), using cone-fundamental-based tristimulus functions with properly normalized values given to the precision of seven significant figures. The properly normalized (renormalized) values at 5 nm intervals can be determined by means of a specially designed computer program provided by CIE Division 1 (under development by TC 1-97; at the time of publication of this edition of the Colorimetry document, the beta version of the program (for Windows, MacOS and Linux) is being tested.). Alternatively, if it is demonstrated that the achieved accuracy is sufficient for the intended use of the tristimulus values, the functions $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ and $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ defined with seven significant figures in CIE 170-2 (CIE, 2015) can be applied, using the function values selected at 5 nm wavelength interval from 390 nm to 780 nm and rounded to six decimal places. These abridged sets of function values for $\bar{x}_F(\lambda)$, $\bar{y}_F(\lambda)$, $\bar{z}_F(\lambda)$ and $\bar{x}_{F,10}(\lambda)$, $\bar{y}_{F,10}(\lambda)$, $\bar{z}_{F,10}(\lambda)$ are given in Table 3 and Table 4, respectively.

For wavelength intervals, $\Delta\lambda$, less than 5 nm:

If the values of the colour stimulus function or relative colour stimulus function are provided at wavelength intervals of 2 nm, 3 nm or 4 nm, the same method as for intervals of 5 nm should be used, subject to the same conditions.

If the wavelength interval is less than 5 nm but not an integer multiple of 1 nm, either the colour-matching functions and the illuminant, or the colour stimulus data must be interpolated so that they match (see 7.2.3 for guidance on this).

In the particular case of cone-fundamental-based colour specifications, properly normalized cone-fundamental-based tristimulus functions with values given at intervals of any integer multiple of 0,1 nm less than or equal to 5 nm (i.e. $\Delta\lambda = n \times 0,1$ nm, $n = 1, 2, \dots, 50$) can be determined by means of the above-mentioned computer program under development by TC 1-97.

NOTE 1 Some publications give the colour-matching functions of the official ISO/CIE International Standard (ISO/CIE, 2007a) with values rounded to four significant figures. These rounded values may be used provided that it is demonstrated that the resulting errors are insignificant for the purpose of the user.

NOTE 2 Some CCD array spectrometers record data at unequal wavelength intervals. In this case, $\Delta\lambda$ in Equations (7.1) – (7.4) will vary.

7.2.2 Abridged method for data available at intervals larger than 5 nm

No official CIE recommendation covers methods for spectra where the data are available at intervals larger than 5 nm, as e.g. 10 nm or 20 nm. The results of research relating to computations of CIE tristimulus values are presented by Li et al. (2016a). The methods described in ASTM E308 might be used (ASTM, 2008 or ASTM, 2017).

7.2.3 Extrapolation and interpolation

Extrapolation of measured data may cause errors and should be used only if it can be demonstrated that the resulting errors are insignificant for the purpose of the user. The previous edition of this document (CIE, 2004b) recommended not extrapolating data, but stated that missing values beyond the truncated wavelength range can be obtained by the use of the nearest available information as a rough approximation. CIE Publication 167 (CIE, 2005a) recommended that the missing values could be obtained by linear extrapolation. After extensive comparison of extrapolation methods reported in (Wang et al., 2017), it was found that the second-order extrapolation method gives better results than the use of nearest available data points or linear extrapolation methods.

If the wavelengths of the measured data do not match exactly those of the colour-matching functions, one or the other need to be interpolated so that they match. For reflectance and transmittance data, and for light sources having smooth spectral distribution curves, either the measured data or the colour-matching functions may be interpolated. For interpolation of