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COMMISSION INTERNATIONALE DE L'ECLAIRAGE
INTERNATIONAL COMMISSION ON ILLUMINATION
INTERNATIONALE BELEUCHTUNGSKOMMISSION

TECHNICAL REPORT

DETERMINATION OF MEASUREMENT UNCERTAINTIES IN PHOTOMETRY

SUPPLEMENT 1: MODULES AND EXAMPLES FOR THE DETERMINATION OF MEASUREMENT UNCERTAINTIES

Part 2: Examples for Models with Individual Inputs

CIE 198-SP1.2:2011

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Quantities related to photometric and
other measurements
Calibration

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3. To provide guidance in the application of principles and procedures in the development of international and national standards in the fields of light and lighting.
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2. D'élaborer des normes et des méthodes de base pour la métrologie dans les domaines de la lumière et de l'éclairage.
3. De donner des directives pour l'application des principes et des méthodes d'élaboration de normes internationales et nationales dans les domaines de la lumière et de l'éclairage.
4. De préparer et publier des normes, rapports et autres textes, concernant toutes matières relatives à la science, la technologie et l'art dans les domaines de la lumière et de l'éclairage.
5. De maintenir une liaison et une collaboration technique avec les autres organisations internationales concernées par des sujets relatifs à la science, la technologie, la normalisation et l'art dans les domaines de la lumière et de l'éclairage.

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Tous les quatre ans, une Session plénière passe en revue le travail des Divisions et des Comités Techniques, en fait rapport et établit les projets de travaux pour l'avenir. La CIE est reconnue comme la plus haute autorité en ce qui concerne tous les aspects de la lumière et de l'éclairage. Elle occupe comme telle une position importante parmi les organisations internationales.

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1. Ein internationaler Mittelpunkt für Diskussionen aller Fragen auf dem Gebiet der Wissenschaft, Technik und Kunst der Lichttechnik und für den Informationsaustausch auf diesen Gebieten zwischen den einzelnen Ländern zu sein.
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3. Richtlinien für die Anwendung von Prinzipien und Vorgängen in der Entwicklung internationaler und nationaler Normen auf dem Gebiet der Lichttechnik zu erstellen.
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Die Arbeit der CIE wird in 7 Divisionen, jede mit etwa 20 Technischen Komitees, geleistet. Diese Arbeit betrifft Gebiete mit grundlegendem Inhalt bis zu allen Arten der Lichtanwendung. Die Normen und Technischen Berichte, die von diesen international zusammengesetzten Divisionen ausgearbeitet werden, sind von der ganzen Welt anerkannt.

Alle vier Jahre findet eine Session statt, in der die Arbeiten der Divisionen überprüft, berichtet und neue Pläne für die Zukunft ausgearbeitet werden. Die CIE wird als höchste Autorität für alle Aspekte des Lichtes und der Beleuchtung angesehen. Auf diese Weise unterhält sie eine bedeutende Stellung unter den internationalen Organisationen.

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This Technical Report has been prepared by CIE Technical Committee 2-43 of Division 2 "Physical Measurement of Light and Radiation" and has been approved by the Board of Administration of the Commission Internationale de l'Eclairage for study and application. The document reports on current knowledge and experience within the specific field of light and lighting described, and is intended to be used by the CIE membership and other interested parties. It should be noted, however, that the status of this document is advisory and not mandatory.

Ce rapport technique a été élaboré par le Comité Technique CIE 2-43 de la Division 2 "Mesures Physiques de la Lumière et des Radiations" et a été approuvé par le Conseil d'Administration de la Commission Internationale de l'Eclairage, pour étude et emploi. Le document expose les connaissances et l'expérience actuelles dans le domaine particulier de la lumière et de l'éclairage décrit ici. Il est destiné à être utilisé par les membres de la CIE et par tous les intéressés. Il faut cependant noter que ce document est indicatif et non obligatoire.

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The following members of TC 2-43 "Determination of measurement uncertainties in photometry" took part in the preparation of this Technical Report. The committee comes under Division 2 "Physical Measurement of Light and Radiation" and the terms of reference were: "To prepare a CIE recommendation as basis for the determination of measurement uncertainties valid for selected quantities used in photometry."

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MODULES AND EXAMPLES FOR THE DETERMINATION OF MEASUREMENT UNCERTAINTIES

Summary

This report supplements Technical Report CIE 198:2011 *Determination of Measurement Uncertainties in Photometry* for the determination of measurement uncertainties associated with the values of selected quantities in photometry. It is organised in a 1st part, which shows modules for the construction of measurement equations and three more parts with examples grouped with increasing complexity. The 2nd part (this part) deals with single output quantities determined from several different input quantities, while in the 3rd part examples¹ for two and more output quantities are presented which are determined from the same set of input quantities. Examples of quantities defined as integrals of distributions (spectral, angular, spatial) are presented in the 4th part.

MODULES ET EXEMPLES POUR LA DETERMINATION DES INCERTITUDES DE MESURES APPLIQUEE AUX GRANDEURS PHOTOMETRIQUES

Resume

Ce rapport apporte des compléments au rapport technique CIE 198:2011 *Détermination des incertitudes de mesure en photométrie* pour la détermination des incertitudes de mesure associées aux valeurs d'une sélection de grandeurs photométriques. Il se divise en quatre parties. La première partie, qui fait l'objet de ce document, indique les modules pour établir les equations de mesure. Les trois autres parties donnent des exemples de complexité croissante. La deuxième partie (cette partie) traite d'une grandeur de sortie unique obtenue à partir de plusieurs grandeurs en entrée. La troisième présente des exemples² de deux grandeurs de sortie, ou plus, obtenues à partir du même ensemble de données en entrée. Des exemples de grandeurs définies par l'intégrale d'une distribution (spectrale, angulaire ou spatiale) sont présentés dans la quatrième partie.

MODULE UND BEISPIELE ZUR BESTIMMUNG VON MESSUNSICHERHEITEN

Zusammenfassung

Dieser Bericht ergänzt den Technischen Bericht CIE 198:2011 *Determination of Measurement Uncertainties in Photometry* zur Bestimmung von Messunsicherheiten, die den Werten ausgewählter Größen in der Photometrie beigeordnet sind. Er ist gegliedert in einen 1. Teil, der Module zum Erzeugen von Messgleichungen zeigt und drei weitere Teile mit Beispielen, angeordnet nach wachsender Komplexität. Der 2. Teil (dieser Teil) behandelt einzelne Ausgangsgrößen, die aus mehreren verschiedenen Eingangsgrößen berechnet werden, während im 3. Teil Beispiele³ für zwei und mehr Ausgangsgrößen gegeben werden, die vom gleichen Satz Eingangsgrößen bestimmt werden. Beispiele von Größen, die als Integrale von Verteilungen (spektral, richtungsabhängig, räumlich) definiert sind, werden im 4. Teil zusammengefasst.

¹ Till to date just one example in part 3 and 4 respectively is available, further ones are to follow.

² A ce jour il y a seulement un exemple en partie 3 resp. 4. Autres exemples sont en phase de planification.

³ Z.Z. ist nur jeweils ein Beispiel in Teil 3 und Teil 4 vorhanden. Weitere Beispiele sind in Planung.

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Scope

This supplement to Technical Report CIE 198:2011 *Determination of Measurement Uncertainties in Photometry* for determination of measurement uncertainties associated with the values of selected quantities used in photometry is intended as a practical help and guidance and collects modules and examples. It is planned that CIE Division 2 will develop more modules and examples for current and newly created quantities in existing and future technical reports, respectively, which will be added as additional supplements.

Introduction

Part 1 shows a collection of small models referred as 1.xx, which are therein denoted as “modules”. They represent typical quantities (mechanical, electrical, optical etc.) contributing to the combined MUs⁴ associated with the values of photometric quantities. The modelling of a measurement is affected by the instrumentation, the measurement procedure and the reference standards used for traceability. Thus, the modules in this part have to be modified to fit the measurement procedures carried out in a laboratory.

Part 2 (this part) in all supplements shows examples of complete models for single photometric quantities, which are determined from a series of input quantities mainly of different kind. Several models for the evaluation of output quantities are shown starting with the substitution method and using the modules explained in supplements part 1. The combination of the MUs of input quantities with the related sensitivity coefficients is demonstrated in uncertainty budgets to obtain the values of the output quantities and the associated standard MUs. The modelling of a measurement is affected by the instrumentation, the measurement procedure and the reference standards used for traceability. Thus, the models given in the examples have to be modified to fit the measurement procedures carried out in each laboratory.

Part 3 in all supplements gives examples for solutions of systems of equations such as the calibration factors of a tristimulus head or curve fitting with more than one coefficient. Using these approximation functions the values of the quantities are calculated and the associated MUs are determined from the coefficients with correlated MUs. The procedure is explained and demonstrated with examples. It is more convenient to evaluate the values and associated MUs of input quantities with correlated MUs by using matrices instead of conventional mathematics and the examples are given this way. For a more general use of the MC method the covariance matrix of the input quantities has to be converted by Cholesky-decomposition. This procedure is also demonstrated for some cases.

Part 4 in all supplements deals with output quantities that are calculated from a series of input quantities mainly of the same kind, such as the values of distribution functions of spectrally or angularly resolved quantities. The output quantities are defined either by weighted integrals taken over that list of values or as the coefficients of a curve fitting. The weights may be specific functions such as the spectral luminous efficiency ($V(\lambda)$) tabulated at fixed wavelengths and used for the spectroradiometric determination of the values of photometric or colorimetric quantities, or functions (e.g. trigonometric) defined in a continuous range of arguments for goniophotometric or absolute integrating sphere methods for the luminous flux determination. Examples for the evaluation of combined MUs under these conditions are also presented.

All the examples show the values of the output quantities, presented with associated expanded MUs and an explanation on the meaning of that statement. The conversion from combined to expanded MU is achieved by multiplication with the coverage factor as function of the related effective DOF, and the evaluation is explained based on individual entries in the MU budget.

⁴ For abbreviations not explained in this document refer to the main document.

Part 2: Examples for Models with Individual Inputs

The following examples explain how to introduce the various conditions in a measurement process into the "model of evaluation" also called the "measurement equation". The examples 2.1 and 2.2 are of more principle importance. They are without the details of a real measurement and show in practice the relations explained in modules 1.1 to 1.3 (see Part 1 of Supplement 1). The structures are explained from the creation of the model to the derivatives and a recommended format for the MU budget. In general all information should be stated with standard MUs rather than using the relative presentation. This allows the variables to have zero values with zero MUs without any change of formats. For more convenience the contributions for the combined MU in the uncertainty budget are also presented in a relative format.

The examples 2.3 to 2.5 are focused on specific quantities based on the modules in Part 1 (temperature measurement with a Pt-1000 resistor, determination of the effective distance between source and photometer, and correction for non-Lambertian angular distribution).

The operational conditions and the handling of lamps and photometers are demonstrated in the examples 2.6 to 2.9. These examples prepare the models for the fundamental calibrations in the examples 2.10 to 2.15.

For all examples the solutions are determined three times. Following the GUM method an Excel sheet (i) and a Mathematica routine (ii) are performed. The PD of the output quantities is determined additionally (iii) by MC simulations, and each curve is calculated from 1 million tries.

The used Excel workbook and the Mathematica program are downloadable at <http://www.cie.co.at/index.php/DOWNLOADS/TC2-43>.

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2.1 Measurement / Calibration by Substitution Method

Example: The calibration of a DUT by the substitution method with a similar reference instrument was explained in module 1.2 and is demonstrated in this example. The example is valid in general, but is explained for the responsivity of photometers. No additional influences such as stray-light, ambient temperature, etc. are considered, except a drift of one signal offset.

Procedure: A stable light source illuminates sequentially the DUT photometer and the REF photometer both having similar properties. For the two devices readings are taken for signal and offset, respectively. This procedure is repeated 10 times and the values obtained are listed in Table 2.1. The two devices are operated with the same amplifier in a fixed range.

Table 2.1 - Photocurrent and offset for DUT and REF photometer

No.	U_{T1}/V	U_{T0}/V	U_{T10}/V	U_{R1}/V	U_{R0}/V
1	25,057	0,002	25,055	17,968 1	0,005 8
2	25,100	0,034	25,066	18,015 5	0,012 4
3	25,117	0,066	25,051	17,970 7	0,012 2
4	25,066	0,070	24,996	18,036 9	0,009 4
5	25,078	0,112	24,966	17,954 2	0,010 2
6	25,097	0,096	25,001	17,972 1	0,014 4
7	25,163	0,142	25,021	17,946 3	0,006 0
8	25,196	0,175	25,021	17,952 1	0,014 8
9	25,112	0,191	24,921	17,992 1	0,008 1
10	25,254	0,210	25,044	17,966 1	0,013 8
MVs	25,124 0	0,109 8	25,014 2	17,977 4	0,010 7
SDMs	0,019 7	0,021 9	0,014 2	0,009 2	0,001 1
CCs	0,770 8			-0,018 2	

Data: In the table the column entitled "No." counts the data sets of the 10 repetitions. The readings of the DUT for signal and offset are given in columns marked U_{T1} and U_{T0} , while the difference $U_{T1} - U_{T0}$ is shown in column U_{T10} . The columns headed U_{R1} and U_{R0} list the corresponding readings for the reference meter. The bottom lines of the columns give MVs with related SDMs.

The CCs from Eqn. (M.3)⁵ for the signal and for the offset of the two meters are shown in the last line. For the reference photometer the readings for signal and offset have a negligible CC $r(U_{R1}, U_{R0}) = -0,018\ 16$, which indicates a stable source. But the readings from the DUT show a significant drift of the offset and a strong correlation with the readings for the signal. The correlation is compensated, if the differences of signal and offset in column U_{T10} are taken for averaging.

$$\text{Model: } s_T = s_R \cdot U_{T10} / (U_{R1} - U_{R0}) \quad (2.1.1)$$

The luminous responsivity s_T of the DUT and the associated MU is calculated from the model with the non-correlated signals of the two photometers and the luminous responsivity s_R of the reference photometer.

Quantities: s_T output quantity: luminous responsivity of DUT photometer

⁵ References to equations and tables starting with "M." refer to equations in the main document.

$s_R = 1,254\ 3\ \text{nA/lx}$, luminous responsivity of the REF photometer certified with relative expanded $k=2$ MU $U_{\text{rel}}(s_R) = 0,43\ \%$, converted into absolute standard MU $u(s_R) = (1,254\ 3 \times 0,004\ 3/2 = 0,002\ 7)\ \text{nA/lx}$; Type B with DOF $\nu \rightarrow \infty$, no correlation.

$U_{T10} = 25,014\ 2\ \text{V}$, signal DUT minus offset (to compensate for correlation) Table 2.1, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta_{T10} = 0,001\ \text{V}$ and is taken as standard MU $u(U_{T10}) = 0,014\ 2\ \text{V}$; Type A with DOF $\nu = 9$, no correlation.

$U_{R1} = 17,977\ 4\ \text{V}$, signal REF photometer Table 2.1, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta = 0,000\ 1\ \text{V}$ and is taken as standard MU $u(U_{R1}) = 0,009\ 2\ \text{V}$; Type A with DOF $\nu = 9$, no correlation.

$U_{R0} = 0,010\ 7\ \text{V}$, signal offset REF photometer Table 2.1, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta = 0,000\ 1\ \text{V}$ and is taken as standard MU $u(U_{R0}) = 0,001\ 1\ \text{V}$; Type A with DOF $\nu = 9$, no correlation.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and calculated from the model in Eqn. (2.1.1) as partial derivatives with respect to the corresponding quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$\begin{aligned} c_1 &= \frac{\partial s_T}{\partial s_R} = \frac{s_T}{s_R} = 1,392\ 3; & c_3 &= \frac{\partial s_T}{\partial U_{R1}} = -\frac{s_T}{(U_{R1} - U_{R0})} = -0,097\ 2\ \frac{\text{nA}}{\text{lx V}} \\ c_2 &= \frac{\partial s_T}{\partial U_{T10}} = \frac{s_T}{U_{T10}} = 0,069\ 8\ \frac{\text{nA}}{\text{lx V}}; & c_4 &= \frac{\partial s_T}{\partial U_{R0}} = \frac{s_T}{(U_{R1} - U_{R0})} = 0,097\ 2\ \frac{\text{nA}}{\text{lx V}} \end{aligned} \quad (2.1.2)$$

The combined MU of the output quantity is evaluated from the contributions of the input quantities (see Eqn. (M.20)), which are products of MU and sensitivity coefficients. The combined MU of the output quantity is just the square root of the sum of squares of the MU contributions (see Eqn. (M.17)), which are sorted for decreasing contribution in the uncertainty budget. The headers in the table below explain the columns and the contributions are given in absolute and also in relative presentation for a better comparison.

Uncertainty budget for model: $s_T = s_R \cdot U_{T10} / (U_{R1} - U_{R0})$									
No	Quantity X_i	Symbol	Value x_i	Abs. stand. uncertainty $u(x_i)$	Type	DOF ν_i	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $u_{\text{rel},i}(y)$
1	ref. responsivity [nA/lx]	s_R	1,254 3	0,002 7	B	∞	1,392 3	0,003 8	0,002 2
2	DUT voltage [V]	U_{T10}	25,014	0,014 2	A	9	0,069 8	0,001 0	0,000 6
3	ref voltage [V]	U_{R1}	17,977	0,009 2	A	9	-0,097 2	-0,000 9	-0,000 5
4	ref. offset value [V]	U_{R0}	0,010 7	0,001 1	A	9	0,097 2	0,000 1	0,000 1
5	DUT responsivity [nA/lx]	s_T	1,746 3			1 422		0,004 0	0,002 3

NOTE The MU budget is calculated by an "Excel-sheet" and a "Mathematica program" both denoted "2.1". The PD of the model in Eqn. (2.1.1) is determined by a MC simulation and shown in Fig. 2.1. The shape of the PD is quite similar to the expected normal distribution.

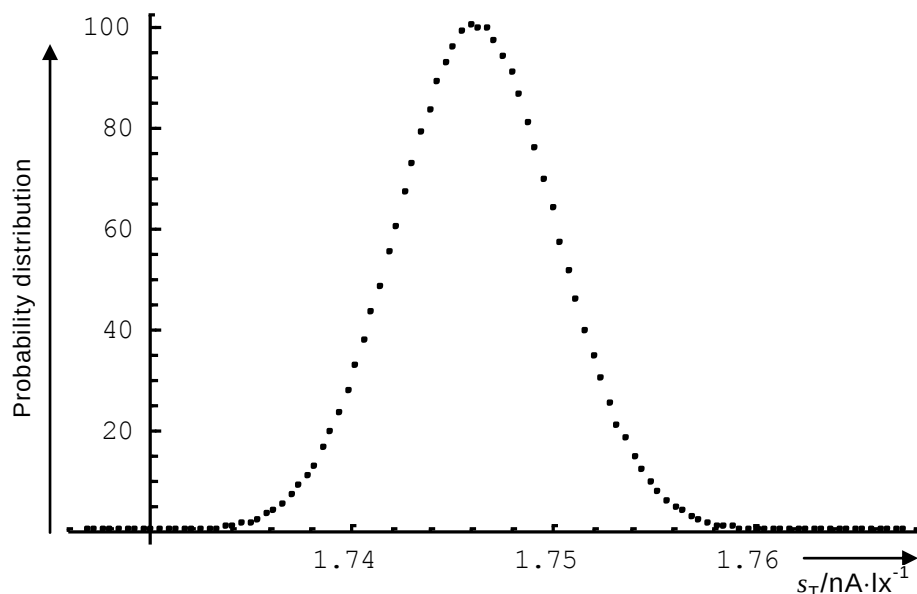


Fig. 2.1 - PD of the luminous responsivity value from model Eqn. (2.1.1)

Degrees of freedom: The MU budget for the calculation of the DUT responsivity has three contributions of Type A evaluation with DOF $\nu = 9$. Therefore, the Welch-Satterthwaite formula of Eqn. (M.36) can be simplified. The effective DOF is sufficiently high to use $k = 2$.

$$\nu_{\text{eff}} = u^4(s_T) / \sum_{i=1}^n \frac{u_i^4(y)}{\nu_i} = \frac{0,004\,0^4}{\frac{0,001\,0^4}{9} + \frac{0,000\,9^4}{9} + \frac{0,000\,1^4}{9}} = 1422 \gg 100 \quad (2.1.3)$$

$$k = 2 \cdot \sqrt{1 + 3,3/\nu_{\text{eff}}} = 2,0$$

Expanded uncertainty: The coverage factor for a probability of 95,45 % is $k = 2$ due to the large DOF. The expanded MU is determined as product and rounded to two decimal places.

$$U = k \cdot u(s_T) = 2 \times 0,004\,0 \text{ nA/lx} = 0,008\,0 \text{ nA/lx} \quad (2.1.4)$$

Reported result: The calibration factor s_T of the photometer is **$(1,746\,3 \pm 0,008\,0) \text{ nA/lx}$** .

The expanded MU is determined as the standard MU multiplied by the coverage factor $k = 2$, which corresponds to a coverage probability of approximately 95 %. The standard MU has been determined in accordance with the „Guide to the expression of uncertainty in measurement“.

NOTE The result could also be presented in a relative or 2 percentage presentations e.g.:

- The calibration factor s_T was determined as $1,746\,3 \times (1 \pm 0,004\,6) \text{ nA/lx}$
- The calibration factor s_T was determined as $1,746\,3 \text{ nA/lx} \pm 0,46 \%$.

2.2 Stabilisation by Monitor Technique

Example: Calibration of a photometer by a reference photometer with similar properties, and using a third photometer as monitor for stabilisation as explained in module 1.3. (Influences such as ambient temperature, stray-light, etc. are not considered in this example.)

Procedure: A light source with limited stability illuminates sequentially the test photometer (DUT) and the reference photometer. A third photometer, the monitor, with proven stability measures simultaneously with each of the two other photometers the same representative part of the emitted light of the source. Readings of signals and offsets are taken for all devices, and this procedure is repeated 10 times. The signals corrected for the offsets are listed in Table 2.2 (offsets are so small, that their contribution to the MUs is negligible.). All three photometers are operated in fixed ranges, to simplify the equations.

Table 2.2 - Photometer signals for monitor technique

No.	U_T / V	U_{TM} / V	q_T	U_R / V	U_{RM} / V	q_R
1	28,077	16,824	1,668 9	23,554	16,066	1,466 1
2	28,338	16,988	1,668 1	23,971	16,375	1,463 9
3	27,909	16,711	1,670 1	24,409	16,695	1,462 1
4	26,391	15,833	1,666 8	23,201	15,785	1,469 8
5	27,527	16,600	1,658 3	23,616	16,112	1,465 7
6	27,063	16,226	1,667 9	24,253	16,535	1,466 8
7	27,120	16,260	1,667 9	24,533	16,698	1,469 2
8	27,436	16,446	1,668 2	24,903	16,997	1,465 1
9	26,856	16,075	1,670 7	23,310	15,851	1,470 6
10	27,261	16,389	1,663 4	23,707	16,167	1,466 4
MVs	27,398	16,435	1,667 0	23,946	16,328	1,466 6
SDMs	0,187	0,112	0,001 2	0,178	0,126	0,000 8
CCs	0,995		0,064	0,998		

Data: The first column in Table 2.2 entitled "No." counts the data set. Columns marked U_T and U_{TM} list 10 readings of photometer signals for test (index "T") and monitor meter (index "M") and their ratio is given in column q_T . The columns headed U_R , U_{RM} and q_R show readings from the reference meter (index "R"), from the monitor and their ratio. At the bottom of the columns MV and SDM are given and the last line shows CCs evaluated from photometer and monitor readings. The CC for the columns with the ratios is shown with a negligible value of $r(q_T, q_R) = 0,064$.

The luminous responsivity s_T of the DUT is determined from the responsivity s_R of the reference and either from the MVs of the signals plus related SDMs plus correlations, or from the uncorrelated MVs of the ratios plus the related SDMs. The two possibilities are referred as two models marked by "I" and "II" and presented below.

$$\begin{aligned} \text{Model I: } s_T &= s_R \cdot \frac{U_T}{U_{TM}} \cdot \frac{U_R}{U_{RM}} \\ \text{Model II: } s_T &= s_R \cdot \frac{q_T}{q_R}; \quad q_T = \frac{U_T}{U_{TM}}; \quad q_R = \frac{U_R}{U_{RM}} \end{aligned} \quad (2.2.1)$$

Quantities for models I and II: s_T quantity: luminous responsivity of DUT photometer

- s_R = 1.2543 nA/lx, luminous responsivity of the REF photometer certified with relative expanded $k=2$ MU $U_{rel}(s_R) = 0,43 \%$, converted into absolute standard MU $u(s_R) = (1,254 \ 3 \times 0,004 \ 3/2 = 0,002 \ 7)$ nA/lx; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- U_R = 23,946 V, signal REF photometer Table 2.2, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta_R = 0,001$ V and is taken as standard MU $u(U_R) = 0,178$ V; Type A with DOF $\nu=9$, correlated to U_{RM} .
- U_{RM} = 16,328 V, signal monitor for REF photometer Table 2.2, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta_{RM} = 0,001$ V and is taken as standard MU $u(U_{RM}) = 0,126$ V; Type A with DOF $\nu=9$, correlated to U_R .
- U_T = 27,398 V, signal DUT photometer Table 2.2, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta_T = 0,001$ V and is taken as standard MU $u(U_T) = 0,187$ V; Type A with DOF $\nu=9$, correlated to U_{TM} .
- U_{TM} = 16,435 V, signal monitor for DUT photometer Table 2.2, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta_{TM} = 0,001$ V and is taken as standard MU $u(U_{TM}) = 0,112$ V; Type A with DOF $\nu=9$, correlated to U_T .
- $r(U_R, U_{RM}) = 0,998$, CC, Table 2.2 for the signals of the monitor and the REF photometer (see Eqn. (M.3)): strong correlation.
- $r(U_T, U_{TM}) = 0,995$, CC, Table 2.2 for the signals of the monitor and the DUT photometer (see Eqn. (M.3)): strong correlation.
- q_R = 1,466 6, ratio of signal REF photometer to monitor, Table 2.2, MV of $n=10$ independent readings, the SDM is significantly larger than the resolutions $\delta_R, \delta_{RM} = 0,001$ V and is taken as standard MU $u(q_R) = 0,000 \ 8$; Type A with DOF $\nu=9$, no correlation.
- q_T = 1,667 0, ratio of signal DUT photometer to monitor, Table 2.2, MV of $n=10$ independent readings, the SDM is significantly larger than the resolutions $\delta_T, \delta_{TM} = 0,001$ V and is taken as standard MU $u(q_T) = 0,001 \ 2$; Type A with DOF $\nu=9$, no correlation.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and are calculated from the model in Eqn. (2.2.1) as partial derivatives with respect to the corresponding quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$\begin{aligned}
 c_1 &= \frac{\partial s_T}{\partial U_{RM}} = \frac{s_T}{U_{RM}} = 0,087 \ 3 \frac{\text{nA}}{\text{lx V}}; & c_2 &= \frac{\partial s_T}{\partial U_R} = -\frac{s_T}{U_R} = -0,059 \ 5 \frac{\text{nA}}{\text{lx V}}; & c_3 &= \frac{\partial s_T}{\partial U_T} = \frac{s_T}{U_T} = 0,052 \ 0 \frac{\text{nA}}{\text{lx V}} \\
 c_4 &= \frac{\partial s_T}{\partial U_{TM}} = -\frac{s_T}{U_{TM}} = -0,086 \ 8 \frac{\text{nA}}{\text{lx V}}; & c_5 &= \frac{\partial s_T}{\partial s_R} = \frac{s_T}{s_R} = 1,136 \ 7
 \end{aligned} \quad (2.2.2)$$

Uncertainty budget for model I: $s_T = s_R \cdot \frac{U_T}{U_{TM}} \cdot \frac{U_R}{U_{RM}}$									
No	Quantity X_i	Symbol	Value x_i	Abs. stand. uncertainty $u(x_i)$	Type	DOF ν_i	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $u_{rel,i}(y)$
1	reference monitor [V]	U_{RM}	16,328	0,126 0	A	9	0,087 3	0,011 0	0,007 7
2	reference voltage [V]	U_R	23,946	0,177 9	A	9	-0,059 5	-0,010 6	-0,007 4
3	DUT voltage [V]	U_T	27,398	0,186 8	A	9	0,052 0	0,009 7	0,006 8
4	DUT monitor [V]	U_{TM}	16,435	0,112 0	A	9	-0,086 8	-0,009 7	-0,006 8
5	ref. responsivity [nA/lx]	s_R	1,254 3	0,002 7	B	∞	1,136 7	0,003 1	0,002 2
6	intermediate uncertainty							0,020 8	0,014 6
7	Correlation [1]	$r(U_R, U_{RM})$	0,997 8					-0,015 3	-0,010 7
8	Correlation [1]	$r(U_T, U_{TM})$	0,994 7					-0,013 7	-0,009 6
9	DUT responsivity [nA/lx]	s_T	1,425 8			37	$u(s_T)$	0,003 3	0,002 3

Degrees of freedom: The MU budget for the calculation of the DUT responsivity has four contributions of Type A evaluation with DOF $\nu = 9$. Therefore, the Welch-Satterthwaite formula of Eqn. (M.36) can be simplified and the combined uncertainty (line 6) without the effect of correlation being taken. The effective DOF is sufficiently high to use $k = 2$.

$$v_{\text{eff}}(s_T) = u^4(s_T) \left/ \sum_{i=1}^n \frac{u_i^4(y)}{\nu_i} \right. = \frac{0,021^4}{\frac{0,011^4}{9} + \frac{0,011^4}{9} + \frac{0,010^4}{9} + \frac{0,010^4}{9}} = 37 \quad (2.2.3)$$

$$k = 2 \cdot \sqrt{1 + 3,3/v_{\text{eff}}} = 2,09$$

It should be noted, that the MU without the reduction by the correlation has to be used for this approximation. In practice model II will be more appropriate to the extremely strong correlation of this data set.

For Model I the correlation between the readings of the monitor photometer and the two other photometers is extremely strong and consequently the reduction in the MU associated with the responsivity of the DUT is also extreme. The same results for the DUT responsivity and associated MU are achieved with model II using the statistical characteristics of the ratios in the Table 2.2 above. The related sensitivity coefficients and the MU budget are given below.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and are calculated from the model in Eqn. (2.2.1) as partial derivatives with respect to the corresponding quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$c_1 = \frac{\partial s_T}{\partial s_R} = \frac{s_T}{s_R} = 1,136 6; \quad c_2 = \frac{\partial s_T}{\partial q_T} = \frac{s_T}{q_T} = 0,855 24 \frac{\text{nA}}{\text{lx}}; \quad c_3 = \frac{\partial s_T}{\partial q_R} = -\frac{s_T}{q_R} = -0,972 11 \frac{\text{nA}}{\text{lx}} \quad (2.2.4)$$

Uncertainty budget for model II $s_T = s_R \cdot \frac{q_T}{q_R}$									
No	Quantity X_i	Symbol	Value x_i	Abs. stand. uncertainty $u(x_i)$	Type	DOF ν_i	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $u_{rel,i}(y)$
1	ref. responsivity [nA/lx]	s_R	1,254 3	0,002 7	B	∞	1,136 6	0,003 1	0,002 2
2	DUT ratio [1]	q_T	1,667 0	0,001 2	A	9	0,855 2	0,001 0	0,000 7
3	ref. ratio [1]	q_R	1,466 6	0,000 8	A	9	-0,972 1	-0,000 8	-0,000 5
4	DUT responsivity [nA/lx]	s_T	1,425 7			724		0,003 3	0,002 3

NOTE The MU budget is calculated by an “Excel-sheet” and a “Mathematica program” both denoted “2.2”. A MC simulation of the model in Eqn. (2.2.1) is shown in Fig. 2.2. The shape of the PD is quite similar to the expected normal distribution.

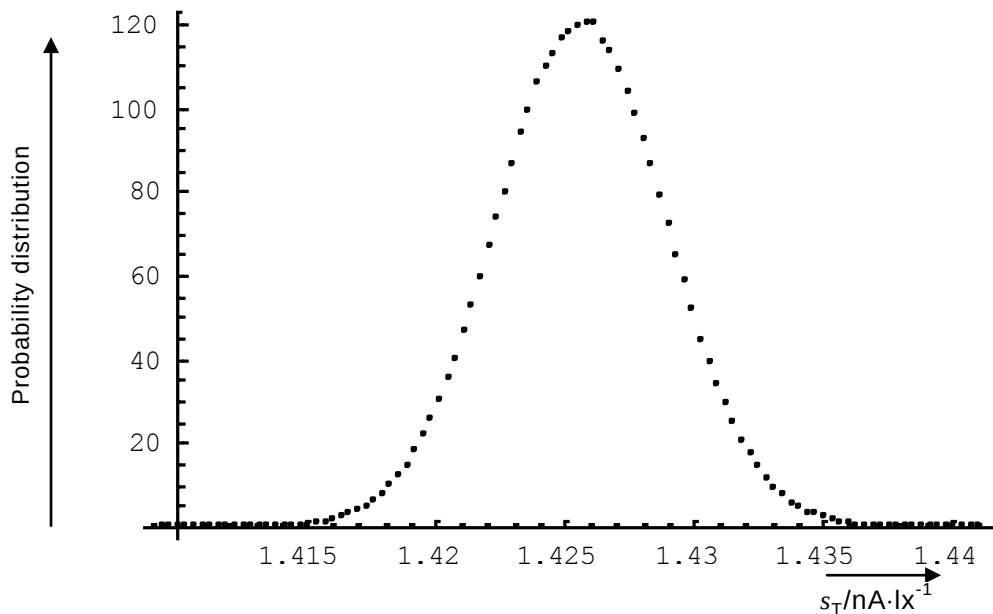


Fig. 2.2 - PD of the luminous responsivity values from model II Eqn. (2.2.1)

Degrees of freedom: The MU budget for the calculation of the DUT responsivity has two contributions of Type A evaluation with DOF $\nu = 9$. Therefore, the Welch-Satterthwaite formula of Eqn. (M.36) can be simplified. The effective DOF is sufficiently high to use $k = 2$.

$$\nu_{\text{eff}} = u^4(s_T) / \sum_{i=1}^n \frac{u_i^4(y)}{v_i} = \frac{0,003\,3^4}{\frac{0,0010^4}{9} + \frac{0,000\,8^4}{9}} = 724 > 100; \quad k = 2 \cdot \sqrt{1 + 3,3/\nu_{\text{eff}}} = 2,00 \quad (2.2.5)$$

Expanded uncertainty: The coverage factor for a probability of 95,45 % is $k = 2$ and so the relative expanded MU is determined as

$$U_{\text{rel}}(s_T) = \frac{k \cdot u(s_T)}{s_T} = 2 \times 0,003\,3/1,425\,7 = 0,004\,6 \quad (2.2.6)$$

Reported result: The luminous responsivity of the photometer is **1,425 7 × (1 ± 0,004 6) nA/lx**.

The relative expanded MU of the measured value is stated as the relative standard MU multiplied by the coverage factor $k = 2$, which corresponds to a coverage probability of approximately 95 %. The standard MU has been determined in accordance with the „Guide to the expression of uncertainty in measurement“.

2.3 Measurement of Temperatures with Pt-1000-Resistor

Example: The properties of many devices depend on the ambient temperature. A platinum resistor of e.g. 1000 Ω resistance mounted in close thermal contact to the device acts as a temperature sensor with well defined parameters of the nearly linear characteristic curve. The resistance is determined with an electric circuit shown in Fig. 2.3.1. The resistance of the wires is not considered in this example.

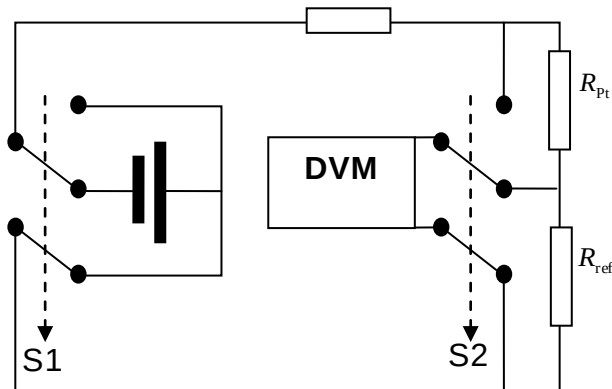


Fig. 2.3.1 - Electric circuit for the measurement of the resistance of a Pt-1000 temperature sensor

Procedure: The value of the Pt-1000 resistor R_{Pt} is measured by a substitution method. The electric circuit consists of a battery as power supply, two resistors in series with the Pt-1000 resistor, a voltmeter DVM and two switches S1 and S2. The resistor R_{ref} acts as reference with a certified resistance. The current in the circuit is limited to about 1 mA by the second resistor that ensures a negligible self-heating of the Pt-1000.

The DVM (linear response, resolution $\delta = 0,000\,01\text{ V}$, no calibration needed) with the switch S2 measures sequentially the voltages y_{Pt} and y_{ref} across the Pt-1000 and the reference resistor, respectively. Each voltage is determined as the difference of two readings with the polarity reversed by switch S1, which compensates for possible thermoelectric offsets. Finally for both resistors the related MVs $\bar{y}_{Pt}$, $\bar{y}_{ref}$ of 10 differences are taken. The relation $R_{Pt} = R_{ref} \cdot \bar{y}_{Pt} / \bar{y}_{ref}$ between these values is known from the substitution methods explained in module 1.2.

The characteristic curve $R_D(T_D)$ of a platinum resistor as a function of the device temperature T_D is standardised with coefficients R_0, a, b valid for temperatures within the range $0 \leq T_D \leq 100\text{ }^\circ\text{C}$. Since 2008 the Platinum resistors are grouped in 4 classes {AA, A, B, C} (see IEC 60751) such that calculated temperatures are within a tolerance interval ΔT around the value determined from the inverse $T_D(R_D)$ of the characteristic curve $R_D(T_D)$. The tolerance interval increases with higher temperatures as a linear function $U(\Delta T) = U_0(\Delta T) + U_1(\Delta T) \cdot T_D$ and is symmetric like an expanded MU. The value of the device temperature T_D is evaluated from the resistance $R_D(T_D)$. As example a Pt-1000 sensor of class A is taken with the following characteristics:

$$\begin{aligned}
 R_D(T_D) &= R_0 \cdot \left(1 + a \cdot (T_D - \Delta T) + b \cdot (T_D - \Delta T)^2\right) \\
 T_D(R_D) &= \frac{a}{2b} \left(-1 + \sqrt{1 - \frac{4b}{a^2} \left(1 - \frac{R_D(T_D)}{R_0}\right)} \right) + \Delta T \\
 U(\Delta T) &= U_1(\Delta T) \cdot T_D + U_0(\Delta T) = 0,4^\circ\text{C} \\
 \Delta T &= 0^\circ\text{C}; \quad u(\Delta T) = U(\Delta T)/k = 0,02^\circ\text{C}
 \end{aligned}
 \quad \left\{ \begin{array}{l} R_0 = 1000 \, \Omega \\ a = 3,9083 \cdot 10^{-3} \, 1/^\circ\text{C} \\ b = -5,775 \cdot 10^{-7} \, 1/^\circ\text{C}^2 \end{array} \right. \quad (2.3.1)$$

$$\left\{ \begin{array}{l} k = 2; \\ U_0(\Delta T) = 0,30^\circ\text{C} \\ U_1(\Delta T) = 0,004 \end{array} \right.$$

$$\text{Model: } R_D(T_D) = R_{\text{ref}} \frac{\bar{y}_{\text{Pt}}}{\bar{y}_{\text{ref}}}; \quad T_D = \frac{a}{2b} \left(-1 + \sqrt{1 - \frac{4b}{a^2} \left(1 - \frac{R_D(T_D)}{R_0}\right)} \right) + \Delta T \quad (2.3.2)$$

Quantities: $R_D(T_D)$ intermediate quantity: resistance of Pt-1000 sensor
 T_D output quantity: device temperature calculated from Pt-1000 sensor.

$R_{\text{ref}} = 1\,100,00 \, \Omega$, resistance of the REF resistor certified with relative expanded $k = 2$ MU
 $U(R_{\text{ref}}) = 0,16 \, \%$, converted into absolute standard MU $u(R_{\text{ref}}) = (1\,100,00 \times 0,001\,6/2 = 0,88) \, \Omega$; Type B with DOF $\nu \rightarrow \infty$, no correlation.
 $\bar{y}_{\text{Pt}} = 0,7828 \, \text{V}$, voltage difference across the Pt-1000 sensor, MV of $n = 10$ independent readings, the SDM is significantly larger than the resolution $\delta_{\text{Pt}} = 0,00001 \, \text{V}$ and is taken as standard MU $u(\bar{y}_{\text{Pt}}) = 0,0008 \, \text{V}$; Type A with DOF $\nu = 9$, no correlation.
 $\bar{y}_{\text{ref}} = 0,7891 \, \text{V}$, voltage difference across the reference resistor, MV of $n = 10$ independent readings, the SDM is significantly larger than the resolution $\delta_{\text{ref}} = 0,00001 \, \text{V}$ and is taken as standard MU $u(\bar{y}_{\text{ref}}) = 0,0007 \, \text{V}$; Type A with DOF $\nu = 9$, no correlation.
 $R_0 = 1\,000 \, \Omega$, nominal value of a platinum resistor standardised as temperature sensor with guaranteed tolerance interval; no MU, no correlation.
 $a = 3,9083 \cdot 10^{-3} \, ^\circ\text{C}^{-1}$, nominal value of the 1st coefficient of the temperature curve for a platinum resistor standardised as temperature sensor with guaranteed tolerance interval; no MU, no correlation.
 $b = -5,775 \cdot 10^{-7} \, ^\circ\text{C}^{-2}$, nominal value of 2nd coefficient of the temperature curve for a platinum resistor standardised as temperature sensor with guaranteed tolerance interval; no MU, no correlation.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and are calculated from the model in Eqn. (2.3.2) as partial derivatives with respect to the regarded quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$\begin{aligned}
 c_1 &= \frac{\partial R_D}{\partial \bar{y}_{\text{Pt}}} = \frac{R_D}{\bar{y}_{\text{Pt}}} = 1394 \frac{\Omega}{\text{V}}; & c_2 &= \frac{\partial R_D}{\partial \bar{y}_{\text{ref}}} = -\frac{R_D}{\bar{y}_{\text{ref}}} = -1382,9 \frac{\Omega}{\text{V}}; & c_3 &= \frac{\partial R_D}{\partial R_{\text{ref}}} = \frac{R_D}{R_{\text{ref}}} = 0,99202 \\
 c_5 &= \frac{\partial T_D}{\partial R_D} = \frac{1}{a \cdot R_0} \frac{1}{\sqrt{1 - \frac{4b}{a^2} \left(1 - \frac{R_D}{R_0}\right)}} = 0,25765 \frac{^\circ\text{C}}{\Omega}; & c_6 &= \frac{\partial T_D}{\partial \Delta T} = 1
 \end{aligned} \quad (2.3.3)$$

Lines 1 to 4 in the upper part of the MU budget demonstrate the MU for the determination of the resistance using the substitution method, while the lines 5 to 10 in the lower part deal with the determination of the temperature and the associated MU.

Uncertainty budget for model: $R_D(T_D) = R_{\text{ref}} \frac{\bar{y}_{\text{pt}}}{\bar{y}_{\text{ref}}}$; $T_D = \frac{a}{2b} \left(-1 + \sqrt{1 - \frac{4b}{a^2} \left(1 - \frac{R_D(T_D)}{R_0} \right)} \right) + \Delta T$									
No	Quantity X_i	Symbol	Value X_i	Abs. stand. uncertainty $u(x_i)$	Type	DOF ν_i	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $u_{\text{rel},i}(y)$
1	mean voltage sensor [V]	$\bar{y}_{\text{pt}}$	0,782 8	0,000 8	A	9	1 394	1,115 2	0,001 0
2	mean voltage reference [V]	$\bar{y}_{\text{ref}}$	0,789 1	0,000 7	A	9	-1 383	-0,968 0	-0,000 9
3	reference resistance [Ω]	R_{ref}	1 100,00	0,880 0	B	∞	0,992	0,873 0	0,000 8
4	sensor resistance [Ω]	$R_D(T_D)$	1 091,2			31		1,7	0,001 6
5	sensor resistance [Ω]	$R_D(T_D)$	1 091,2	1,7	A	31	0,257 6	0,438 0	0,018 7
6	tolerance interval [$^{\circ}\text{C}$]	ΔT	0,0	0,20	B	∞	1,000 0	0,200 0	0,008 5
7	nominal resistance [Ω]	R_0	1 000,0	0,0					
8	characteristic constant [$1/^{\circ}\text{C}$]	a	$3,908\ 3 \times 10^{-3}$	0,0					
9	characteristic constant [$1/^{\circ}\text{C}^2$]	b	$-5,775 \times 10^{-7}$	0,0					
10	device temperature [$^{\circ}\text{C}$]	T_D	23,42			45		0,48	0,020

NOTE The MU budget is calculated by an “Excel-sheet” and a “Mathematica program” both denoted “2.3”. A MC simulation of the model in Eqn. (2.3.2) is shown in Fig. 2.3.2. The shape of the PD is quite similar to the expected normal distribution.

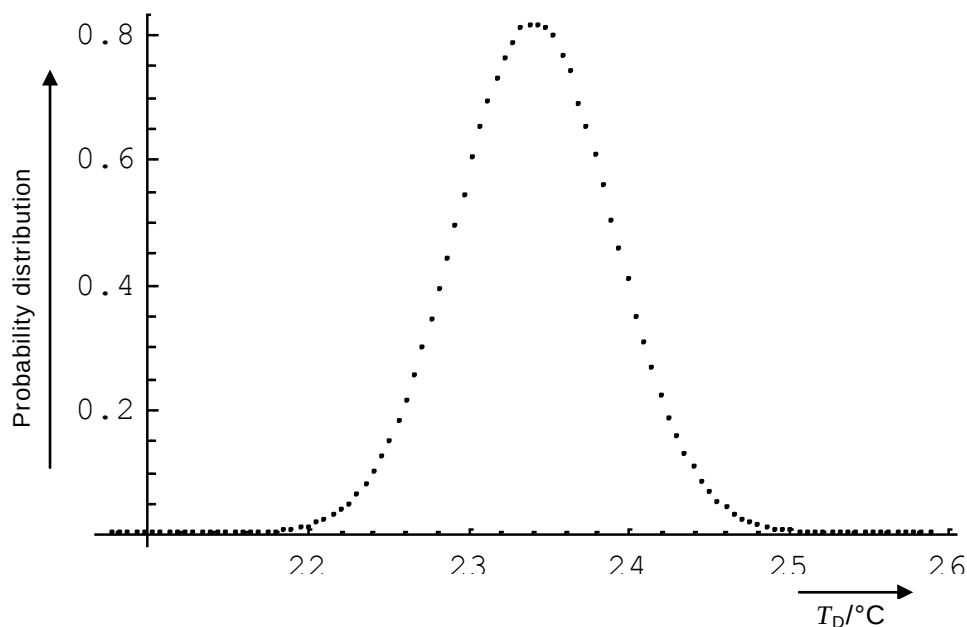


Fig. 2.3.2 - PD of the temperature value from model Eqn. (2.3.2)

Degrees of freedom: The upper part of the MU budget for the calculation of the DUT responsivity has two contributions of Type A evaluation with DOF $\nu=9$. Therefore, the Welch-Satterthwaite formula of Eqn. (M.36) can be simplified. The resulting effective DOF $\nu_{\text{eff}}(R_D) = 31$ is the only contribution with limited DOF in the lower part of the MU budget for the calculation of the device temperature and the Welch-Satterthwaite formula can be simplified again. The effective DOF $\nu_{\text{eff}}(T_D) = 45$ is sufficiently high to use $k = 2,07 \approx 2$.

$$v_{\text{eff}} = u^4(y) / \sum_{i=1}^n \frac{u_i^4(y)}{v_i}; \quad \left\{ \begin{array}{l} v_{\text{eff}}(R_D) = \frac{1,7^4}{\frac{1,1^4}{9} + \frac{0,97^4}{9}} \cong 31 \\ v_{\text{eff}}(T_D) = v_{\text{eff}}(R_D) \cdot \frac{0,48^4}{0,44^4} \cong 45 \end{array} \right\} \quad k = 2 \cdot \sqrt{1 + 3,3/v_{\text{eff}}} \approx 2 \quad (2.3.4)$$

Expanded uncertainty: The coverage factor for a probability of 95,45 % is rounded to $k = 2$ and so the expanded MU is determined as

$$U = k \cdot u(T_D) = 2 \times 0,48 \text{ °C} = 0,96 \text{ °C} \quad (2.3.5)$$

Reported result: The temperature T_D of the device is **(23,42 ± 0,96) °C.**

The expanded MU of the measured value is stated as the standard MU multiplied by the coverage factor $k = 2$, which corresponds to a coverage probability of approximately 95 %. The standard MU has been determined in accordance with the „Guide to the expression of uncertainty in measurement“.

2.4 Measurement the Effective Distance

Example: The effective distance between the filament of a luminous intensity lamp and the aperture of a photometer is determined from the photometer signals taken at two positions with precisely known distance in between. If the initial distance is large enough for a valid inverse square law (see module 1.5), then from the ratio of the photocurrents at the two (unknown) distances the effective initial distance is calculated. The influences of stray-light, a non-Lambertian angular distribution, any instability of the source or the effects of "field of view limitation" of the photometer are not considered in this example.

Procedure: A source with the property of a luminous intensity standard (nearly Lambertian angular distribution) illuminates a photometer at a distance d_0 . The offset-corrected photometer signal y_0 is determined as the MV plus the SDM from $n=10$ independent readings. A second series of similar readings y_1 is taken at a distance increased by the dislocation Δd . The effective distance follows from the ratio $y_0/y_1 = ((d_0 + \Delta d)/d_0)^2$ of the photometer signals solved for the effective distance d_0 .

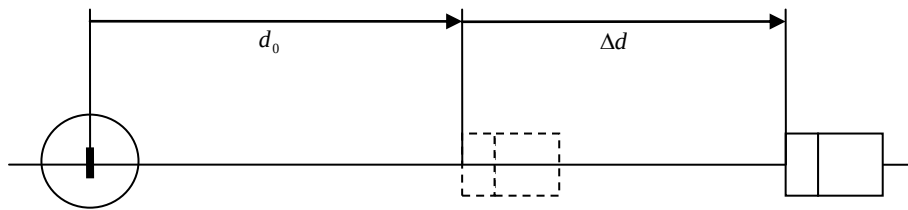


Fig. 2.4.1 - Effective distance between lamp and photometer determined from the distance between two photometer positions

$$\text{Model: } d_0 = \Delta d / (\sqrt{y_0/y_1} - 1) \quad (2.4.1)$$

Quantities: d_0 output quantity: effective distance between source and photometer.

Δd = 0,500 0 m, length of REF meter certified with standard MU $u(\Delta d) = 0,000\ 5$ m; Type B with DOF $\nu \rightarrow \infty$, no correlation.

y_0 = 8,000 0 V, signal photometer, MV of $n=10$ independent readings at the initial distance d_0 , the SDM is significantly larger than the resolution $\delta_0 = 0,000\ 01$ V and is taken as standard MU $u(y_0) = 0,000\ 8$ V, Type A with DOF $\nu=9$, no correlation.

y_1 = 6,000 0 V, signal photometer, MV of $n=10$ independent readings at the enlarged distance $d_0 + \Delta d$, the SDM is significantly larger than the resolution $\delta_1 = 0,000\ 01$ V and is taken as standard MU $u(y_1) = 0,000\ 6$ V, Type A with DOF $\nu=9$, no correlation.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and are calculated from the model in Eqn. (2.4.1) as partial derivatives with respect to the corresponding quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$\begin{aligned} c_1 = \frac{\partial d_0}{\partial \Delta d} &= \frac{1}{\sqrt{y_0/y_1} - 1} = 6,464; & c_2 = \frac{\partial d_0}{\partial y_0} &= \frac{\Delta d}{2\sqrt{y_0/y_1}(y_0 + y_1) - 4y_0} = -1,508 \frac{\text{m}}{\text{V}} \\ c_3 = \frac{\partial d_0}{\partial y_1} &= \frac{\Delta d \sqrt{y_0/y_1}}{2y_1(\sqrt{y_0/y_1} - 1)^2} = 2,010 \frac{\text{m}}{\text{V}} \end{aligned} \quad (2.4.2)$$

Uncertainty budget for model: $d_0 = \Delta d / (\sqrt{y_0/y_1} - 1)$									
No	Quantity X_i	Symbol	Value x_i	Abs. stand. uncertainty $u(x_i)$	Type	DOF ν_i	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $u_{rel,i}(y)$
1	dislocation [m]	Δd	0,500 0	0,0005	B	∞	+6,464 1	+0,003 2	0,001 0
2	initial photometer signal [V]	y_0	8,000 0	0,0008	A	9	-1,507 8	-0,001 2	-0,000 4
3	dislocated photometer [V]	y_1	6,000 0	0,0006	A	9	+2,010 4	+0,001 2	0,000 4
4	effective distance [m]	d_0	3,232 1			365		0,003 6	0,001 1

NOTE The MU budget is calculated by an "Excel-sheet" and a "Mathematica program" both denoted "2.3". A MC simulation of the model in Eqn. (2.4.1) is shown in Fig. 2.4.2. The shape of the PD is significantly different than the expected normal distribution. Obviously the dominant contribution from the RPD is not yet smeared out.

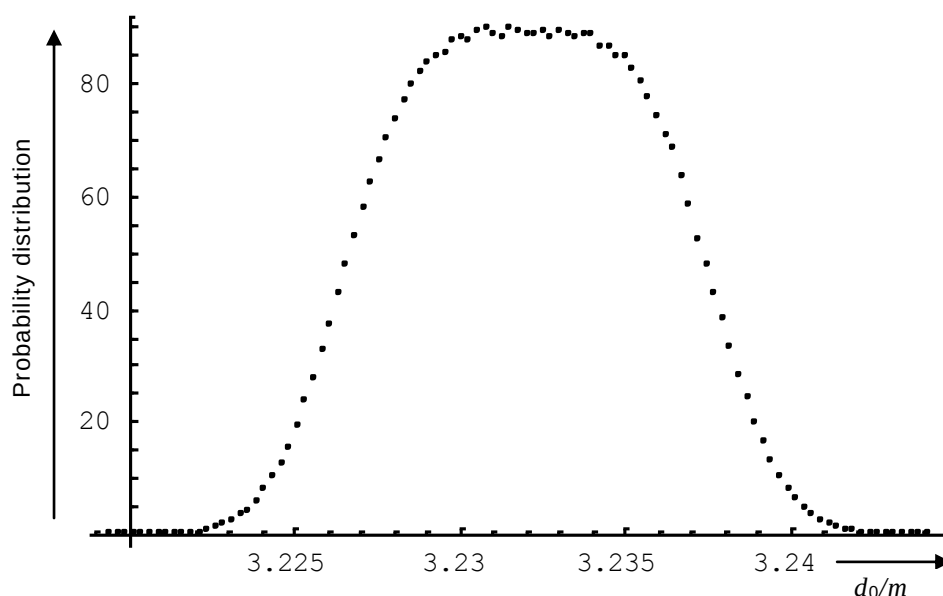


Fig. 2.4.2 - PD of the effective distance values from model Eqn. (2.4.1)

Degrees of freedom: The MU budget for the calculation of the effective distance has two contributions of Type A evaluation with DOF $\nu=9$. Therefore, the sum in the Welch-Satterthwaite formula of Eqn. (M.36) can be simplified. The effective DOF is sufficiently high to use $k=2$.

$$\nu_{\text{eff}} = u^4(y) / \sum_{i=1}^n \frac{u_i^4(y)}{\nu_i} = \frac{0,003\,6^4}{\frac{0,001\,2^4}{9} + \frac{0,001\,2^4}{9}} = 365 \gg 100 \quad (2.4.3)$$

$$k = 2 \cdot \sqrt{1 + 3.3/\nu_{\text{eff}}} = 2,00$$

Expanded uncertainty: The coverage factor for a probability of 95,45 % is rounded to $k=2$ and so the expanded MU is determined as

$$U(d_0) = k \cdot u(d_0) = 2 \times 0,003\,6\,\text{m} = 0,007\,2\,\text{m} \quad (2.4.4)$$

Reported result: The effective distance d_0 was determined as

(3,232 1 ± 0,007 2) m.

The expanded MU of the measured value is stated as the standard MU multiplied by a coverage factor $k = 2$, which corresponds to a coverage probability of approximately 95 %. The standard MU has been determined in accordance with the „Guide to the expression of uncertainty in measurement“.

2.5 Correction for non-Lambertian Distribution

Explanation: CIE-Publ. 127 defines for "Averaged LED Intensity" geometries for two specified distances, which are measured from the tip of the LED package to the entrance window of the photometer, with an area of 100 mm². Provided the light is emitted as a narrow beam and the emitting centre of the LED is located at the tip, then the value of the Averaged LED Intensity would be smaller than a value measured with a photometer that has a very small input aperture. The ratio of these two values is unity for large distances and Lambertian distributions. Decreasing distance and narrow beams reduce the value of the ratio, as explained in the module 1.6.

Procedure: Assume, the intensity distribution of an LED can be approximated by the angular distribution function $I(\vartheta) = I_0 \cos^g(\vartheta)$ and values are measured in the direction of the optical axis I_0 and for increasing angles $I(\vartheta)$ up to angle $\vartheta_{0,5}$, at which the intensity is one-half the maximum intensity. These measurements are carried out with a large distance between LED and photometer. The contour exponent $g = \log[0,5] / \log[\cos \vartheta_{0,5}]$ is calculated from the angle. The additional values for distance d and the radii of the source r_s and the photometer r_p are determined as characteristics of the instrumentation. Module 1.6 explains the origin of the factor v_{focus} , which converts a maximum luminous intensity I_{max} into an average luminous intensity value of such as the Averaged LED intensity I_{LED} .

$$\begin{aligned} \text{Model: } I(\vartheta) &= I_0 \cos^g(\vartheta); \\ v_{\text{focus}} &= \left[1 - \frac{g+3}{4} \left(\frac{r_p}{d} \right)^2 - \left(\frac{r_s}{d} \right)^2 \right]; \quad \text{for } I_{\text{LED,B}} = I_{\text{max}} \cdot v_{\text{focus}} \\ g &= \log[0,5] / \log[\cos \vartheta_{0,5}]; \quad \frac{\partial g}{\partial \vartheta_{0,5}} = g \cdot \tan \vartheta_{0,5} / \log[\cos \vartheta_{0,5}] \end{aligned} \quad (2.5.1)$$

Quantities: v_{focus} output quantity: conversion factor from the maximum LED intensity in the direction of the optical axis to the "Averaged LED Intensity"

$\vartheta_{0,5}$ = 6°, angle of the LED angular distribution from $n = 20$ independent readings for different rotations about optical axis, the SDM is significantly larger than the resolution $\delta(\vartheta_{0,5}) = 0,02^\circ$ and is taken as standard MU $u(\vartheta_{0,5}) = 0,2^\circ$; Type A with DOF $\nu = 19$, no correlation.

g = $\log[0,5] / \log[\cos \vartheta_{0,5}] = 126,2$, contour exponent; the partial derivative $\partial g / \partial \vartheta_{0,5} = g \cdot \tan \vartheta_{0,5} / \log[\cos \vartheta_{0,5}] = -2\,414,4$ multiplied with the MU of the angle (converted into radian instead of degrees) gives the associated standard MU $u(g) = |g \cdot u(\vartheta_{0,5}) \cdot \tan \vartheta_{0,5} / \log[\cos \vartheta_{0,5}]| = 8,43$; Type A with DOF $\nu = 19$, no correlation.

d = 100,0 mm, distance between the tip of the LED and the effective location of the photometer entrance window, estimated MU $u(d) = 1,0$ mm; Type B with DOF $\nu \rightarrow \infty$, no correlation.

r_p = 5,64 mm, effective radius of the 100 mm² photometer area with the associated standard MU $u(r_p) = 0,1$ mm; Type B with DOF $\nu \rightarrow \infty$, no correlation.

r_s = 2,5 mm, effective radius of the circular area of an LED package; the area acts as a lens with (nearly) constant luminance. The variations between LED types is covered by an enlarged standard MU $u(r_s) = 1,5$ mm; Type B with DOF $\nu \rightarrow \infty$, no correlation.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and are calculated from the model in Eqn. (2.5.1) as partial derivatives with respect to the regarded quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$\begin{aligned} c_1 &= \frac{\partial v_{\text{focus}}}{\partial g} = \left(\frac{r_p}{2d} \right)^2 = -0,000\,8; & c_2 &= \frac{\partial v_{\text{focus}}}{\partial r_p} = \frac{g+3}{2d} \left(\frac{r_p}{d} \right) = -0,036\,4 \text{ mm}^{-1} \\ c_3 &= \frac{\partial v_{\text{focus}}}{\partial d} = \frac{g+3}{2d} \left(\frac{r_p}{d} \right)^2 + \frac{2}{d} \left(\frac{r_s}{d} \right)^2 = +0,002\,1 \text{ mm}^{-1}; & c_4 &= \frac{\partial v_{\text{focus}}}{\partial r_s} = \frac{2}{d} \left(\frac{r_s}{d} \right) = -0,000\,5 \text{ mm}^{-1} \end{aligned} \quad (2.5.2)$$

Uncertainty budget for model: $v_{\text{focus}} = \left[1 - \frac{g+3}{4} \left(\frac{r_p}{d} \right)^2 - \left(\frac{r_s}{d} \right)^2 \right]$									
No	Quantity X_i	Symbol	Value x_i	Abs. stand. uncertainty $u(x_i)$	Type	DOF ν_i	Absolute sensitivity c_i	Absolute contribution $u_i(y)$	Relative contribution $u_{\text{rel},i}(y)$
1	contour exponent [1]	g	126,2	8,43	A	19	-0,000 8	-0,006 7	-0,007 5
2	radius photometer area [mm]	r_p	5,64	0,10	B	∞	-0,036 4	-0,003 6	-0,004 1
3	distance [mm]	d	100,0	1,0	B	∞	0,002 1	0,002 1	0,002 3
4	radius source [mm]	r_s	2,50	1,50	B	∞	-0,000 5	-0,000 8	-0,000 8
5	conversion factor [1]	v_{focus}	0,897			38		0,008 0	0,008 9

NOTE The MU budget is calculated by an “Excel-sheet” and a “Mathematica program” both denoted “2.5”. A MC simulation of the model in Eqn. (2.5.1) is shown in Fig. 2.5. The shape of the PD is quite similar to the expected normal distribution.

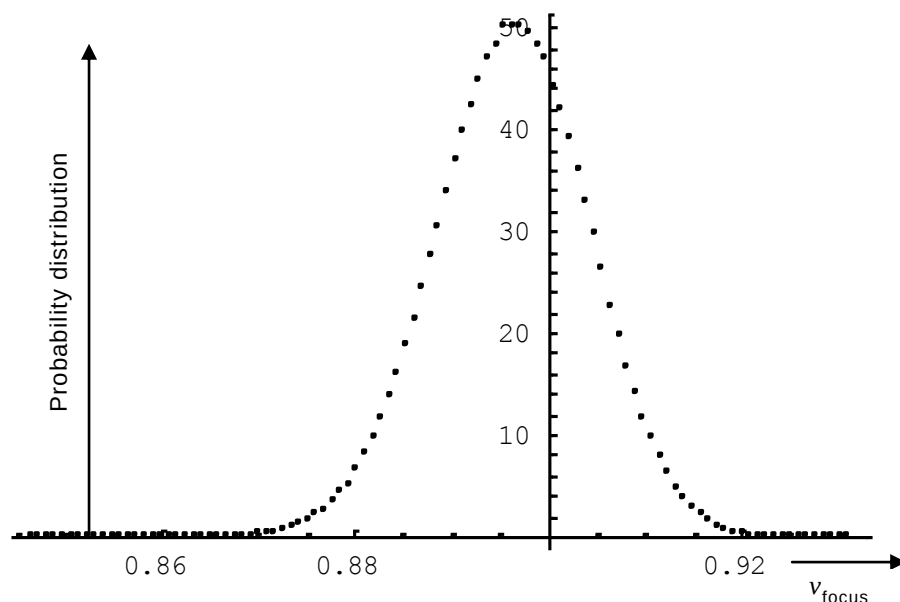


Fig. 2.5 - PD of the conversion factor value from model Eqn. (2.5.1)

Degrees of freedom: The MU budget for the calculation of the conversion factor has only one contribution of Type A evaluation with DOF $\nu=19$. Therefore, the sum in the Welch-Satterthwaite formula of Eqn. (M.36) can be simplified. The effective DOF is sufficiently high to use $k=2$.

$$\nu_{\text{eff}} = u^4(v_{\text{focus}}) / \sum_{i=1}^n \frac{u_i^4(y)}{\nu_i} = 19 \frac{0,008\,0^4}{0,006\,7^4} = 38; \quad k = 2 \cdot \sqrt{1 + 3,3/\nu_{\text{eff}}} = 2,09 \quad (2.5.3)$$

Expanded uncertainty: The coverage factor for a probability of 95,45 % is rounded to $k = 2$ and so the expanded MU is determined as $U(v_{\text{focus}}) = k \cdot u(v_{\text{focus}}) = 0,016$.

Reported result: The conversion factor was determined as $v_{\text{focus}} = (0,897 \pm 0,016)$.

The expanded MU of the measured value is stated as the standard MU multiplied by a coverage factor $k = 2$, which corresponds to a coverage probability of approximately 95 %. The standard MU has been determined in accordance with the „Guide to the expression of uncertainty in measurement“.

NOTE This result means that the luminous intensity in the direction of the optical axis of the LED is approximately 10 % higher than the value of the averaged intensity. This value is additionally modified, due to the incorrect assumption that the centre of the light emission (centre of gravity) would be exactly at the tip of the LED. Usually the distance to the photometer is larger, which also increases the value of the luminous intensity on the axis. This demonstrates the importance of the CIE definition of the Averaged LED Intensity as an agreed basis for a comparison of LEDs but the value is not useful for calculations of illuminance at arbitrary distances.

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2.6 Measurement of Lamp Voltages

Example: The measurement of a lamp voltage is discussed in module 1.10 and now demonstrated here by two MU budgets:

- High accuracy calibration, resulting in an output value with minimum associated MU for precisely specified conditions, and
- testing under practical operational conditions within specified tolerance intervals, resulting in an output value with increased associated MU.

The corrections for an offset voltage and a limited resolution are taken into account. The DVM calibration factor is considered including the dependence on ambient temperature and ageing. The example deals with the measurement of a lamp voltage in a 2-pole socket and a correction for the resistance of the contact between lamp cap and socket is also considered.

Procedure I: A lamp is powered with constant DC current and the operation is monitored by the lamp voltage U_L . A voltage $\bar{U}_{L1}$ is measured at the contacts of a 2-pole-socket as the MV of (independent) readings of a DVM, and the offset voltage $\bar{U}_{L0}$ is determined from additional readings after the current is switched off. Both types of readings are taken with a resolution $\pm \delta U_L$ and at an ambient temperature T_{al} . The voltage drop across the contacts of the cap and the socket is the product of the nominal lamp current J_{LA} and the resistor value $R_{L, cap}$, which was measured earlier.

The difference between nominal value and effective value of the lamp current is negligible and an averaged value of the resistance can be taken for correction. The DVM is operated in fixed range with a calibration factor c_L , which was determined with an ambient temperature T_{al0} a period of time Δt_L ago and the two relative coefficients for ageing $\gamma_{L, rel}$ and temperature $\alpha_{L, rel}$ are known from former measurements.

Table 2.6. Voltage and offset.

No.	U_{L1} / V	U_{L0} / V
1	25,000 4	0,002 2
2	25,023 9	0,002 1
3	25,006 3	0,002 1
4	25,020 1	0,002 1
5	25,017 7	0,002 0
6	25,018 5	0,002 0
7	25,010 6	0,002 1
8	25,023 1	0,002 1
9	25,006 7	0,002 0
10	25,013 4	0,002 1
MV	25,014 1	0,002 1
SDM	0,002 5	0,000 0

Data: The first column in Table 2.6 is entitled "No." and counts the data sets. The columns marked " U_{L1} " and " U_{L0} " hold $n=10$ readings, and using Eqn. (M.1) the MV and the SDM are shown in the two bottom lines of the related columns.

Model I:

$$U_{L4} = c_L \cdot (1 + \alpha_{L,rel} \cdot (T_{aL} - T_{aL0}) - \gamma_{L,rel} \cdot \Delta t_L) \cdot (\bar{U}_{L1} - \bar{U}_{L0})$$

$$U_{L2} = U_{L4} - J_{LA} \cdot R_{L,cap}$$
(2.6.1)

Quantities for model I: U_{L4} output quantity: lamp voltage for 4-pole operation.

- c_L = 1,000, calibration factor for the DVM certified with expanded $k=2$ MU $U(c_L) = 0,000\ 12$, converted into standard MU $u(c_L) = U(c_L)/2 = 0,000\ 060$; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- $\bar{U}_{L1}$ = 25,014 1 V, DVM indications Table 2.6, MV of $n=10$ independent readings, the SDM is significantly larger than the resolution $\delta_{L1} = 0,000\ 1$ V and is taken as MU $u(\bar{U}_{L1}) = 0,002\ 5$ V; Type A with DOF $\nu = 9$, no correlation.
- $\bar{U}_{L0}$ = 0,002 1 V, DVM indication offset Table 2.6, MV of $n=10$ independent readings, the SDM $s(\bar{U}_{L0}) = 0,000\ 01$ V is smaller than resolution $\delta_{L0} = 0,000\ 1$ V of DVM and due to Eqn. (M.6) the MU is $u(\bar{U}_{L0}) = (0,000\ 1/\sqrt{3} = 0,000\ 058)$ V; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- $\alpha_{L,rel}$ = $2,5 \times 10^{-5}$ K⁻¹, relative temperature coefficient of the DVM stated in the technical documents with associated standard MU $u(\alpha_{L,rel}) = 1 \times 10^{-5}$ K⁻¹; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- $\gamma_{L,rel}$ = $1,5 \times 10^{-5}$ 1/year, relative ageing coefficient of the DVM stated in the technical documents with associated standard MU $u(\gamma_{L,rel}) = 10^{-5}$ 1/year; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- T_{aL} = 23,00°C, ambient temperature measured with resolution $\delta_{aL} = 0,5^\circ\text{C}$, converted into standard MU $u(T_{aL}) = (0,5/\sqrt{3} = 0,29)^\circ\text{C}$; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- $R_{L,cap}$ = 0,002 09 Ω , contact resistance for the 2-pole socket within the interval $\pm 0,000\ 52$ Ω with RPD, converted into the MU $u(R_{L,cap}) = (0,000\ 52/\sqrt{3} = 0,000\ 30)$ Ω ; Type B with DOF $\nu \rightarrow \infty$, no correlation.
- Δt_L = 0,6 year, time since calibration of the DVM, no MU due to documentation, no correlation.
- T_{aL0} = 22°C, nominal value of ambient temperature during the calibration of the DVM; no MU, no correlation.
- J_{LA} = 3,821 0 A, nominal value of the current for the lamp; no MU, no correlation.

Sensitivity coefficients c_i are explained in Eqn. (M.12) and are calculated from the two models in Eqn. (2.6.1) as partial derivatives with respect to the corresponding quantity. They should be stated with the correct sign, which is needed in case of correlated input quantities.

$$c_1 = \frac{\partial U_{L4}}{\partial \bar{U}_{L1}} = -c_6 = -\frac{\partial U_{L4}}{\partial \bar{U}_{L0}} = c_L \cdot (1 + \alpha_{L,rel} \cdot (T_{aL} - T_{aL0}) - \gamma_{L,rel} \cdot \Delta t_L) = 1,000\ 02$$

$$c_2 = \frac{\partial U_{L4}}{\partial c_L} = (1 + \alpha_{L,rel} \cdot (T_{aL} - T_{aL0}) - \gamma_{L,rel} \cdot \Delta t_L) \cdot (\bar{U}_{L1} - \bar{U}_{L0}) = 25,012\ 4\ \text{V}$$

$$c_3 = \frac{\partial U_{L4}}{\partial \alpha_{L,rel}} = c_L \cdot (T_{aL} - T_{aL0}) \cdot (\bar{U}_{L1} - \bar{U}_{L0}) = 25,012\ \text{V} \cdot ^\circ\text{C}$$

$$c_4 = \frac{\partial U_{L4}}{\partial T_{aL}} = c_L \cdot \alpha_{L,rel} \cdot (\bar{U}_{L1} - \bar{U}_{L0}) = 0,000\ 625\ 3\ \frac{\text{V}}{^\circ\text{C}}$$

$$c_5 = \frac{\partial U_{L4}}{\partial \gamma_{L,rel}} = -c_L \cdot \Delta t_L \cdot (\bar{U}_{L1} - \bar{U}_{L0}) = -15,007\ 2\ \text{V} \cdot \text{year}$$
(2.6.2)