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TECHNICAL REPORT

Effect of Instrumental Bandpass Function and Measurement Interval on Spectral Quantities

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This Technical Report has been prepared by CIE Technical Committee 2-60 of Division 2 "Physical Measurement of Light and Radiation" and has been approved by the Board of Administration as well as by Division 2 of the Commission Internationale de l'Eclairage for study and application. The document reports on current knowledge and experience within the specific field of light and lighting described, and is intended to be used by the CIE membership and other interested parties. It should be noted, however, that the status of this document is advisory and not mandatory.

Ce rapport technique a été élaboré par le Comité Technique CIE 2-60 de la Division 2 "Mesures Physiques de la Lumière et des Radiations" et a été approuvé par le Bureau et Division 2 de la Commission Internationale de l'Eclairage, pour étude et emploi. Le document expose les connaissances et l'expérience actuelles dans le domaine particulier de la lumière et de l'éclairage décrit ici. Il est destiné à être utilisé par les membres de la CIE et par tous les intéressés. Il faut cependant noter que ce document est indicatif et non obligatoire.

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EFFECT OF INSTRUMENTAL BANDPASS FUNCTION AND MEASUREMENT INTERVAL ON SPECTRAL QUANTITIES

Summary

This report presents a detailed study of bandwidth correction applied to spectra encountered in radiometric, photometric and spectrophotometric measurements. The mechanism of broadening of data recorded by both scanning and array spectrometers, due to bandpass function effects, is discussed in detail, along with an historical study of methods currently applied. The report presents a bandwidth correction algorithm that can be used for any real bandpass function and provides both a step-by-step guide on how to apply that correction, and a discussion of the theoretical and experimental limits to its applicability.

The effect of noise and measurement interval in measurement of both the spectrum and the bandpass function is discussed and methods for identifying the significance of noise effects in the corrected spectrum are presented. The reduced effect of noise when calculating spectrally integrated quantities (such as photometric or colorimetric values) is also examined. Practical guidelines are given for the application of bandwidth correction, both for the determination of the bandpass function and for estimating whether the correction may lead to an improvement in spectral accuracy.

L'EFFET DE LA BANDE PASSANTE INSTRUMENTALE ET DU PAS DE MESURE DANS LE CALCUL DES QUANTITÉS SPECTRALES

Résumé

Ce rapport présente une étude détaillée de la correction pour la bande passante appliquée aux spectres rencontrés en radiométrie, en photométrie et en spectrophotométrie. On y discute en détail du mécanisme de l'étalement des données enregistrées tant pour les spectromètres à balayage que pour ceux incorporant des photoéléments en rangée, accompagné d'une étude historique des méthodes couramment employées. Le rapport présente un algorithme de correction pour la largeur de bande qui peut être utilisé pour n'importe quelle fonction de bande passante réelle, explique comment l'appliquer étape par étape, et discute des limites théoriques et expérimentales à son applicabilité.

L'effet du bruit et du pas de mesure sur le spectre et sur la fonction de bande passante est discuté et des méthodes sont présentées pour identifier la signification du niveau de bruit dans le spectre corrigé. On examine aussi l'effet réduit du bruit sur les quantités intégrées spectralement, telles que les valeurs photométriques et colorimétriques. On fournit un guide pratique pour l'application de la correction pour la bande passante tant pour la détermination de la fonction de bande passante que pour l'estimation du bénéfice que la correction peut apporter à la précision spectrale.

AUSWIRKUNG DER GERÄTEBANDPASSFUNKTION UND DES MESSINTERVALLS AUF SPEKTRALE GRÖSSEN

Zusammenfassung

Dieser Bericht beinhaltet eine detaillierte Studie zur Anwendung der Bandpasskorrektur auf Spektren, so wie sie sich bei radiometrischen, photometrischen und spektralphotometrischen Messungen ergeben. Der Mechanismus der Verbreiterung von sowohl mit scannenden Spektrometern als auch mit Arrayspektrometern aufgenommenen Daten durch den Effekt der Bandpassfunktion, wird, zusammen mit einer Studie über die Geschichte der Methoden, die zur Zeit angewendet werden, im Detail erörtert. Dieser Bericht stellt einen Bandpasskorrekturalgorithmus vor, der für jede real existierende Bandpassfunktion angewendet werden kann, und bietet sowohl eine Schritt-für-Schritt-Anleitung zur Anwendung dieser Korrektur als auch eine Erörterung der theoretischen und experimentellen Grenzen seiner Anwendbarkeit.

Es wird der Einfluss von Störungen und Rauschen, sowie des Messintervalls der Messung sowohl auf Spektrum als auch auf Bandpassfunktion erörtert, und es werden Methoden zur Erkennung der Signifikanz von Störungen und Rauscheffekten auf das korrigierte Spektrum gezeigt. Ebenso wird der verminderte Einfluss von Störungen und Rauschen auf die spektral integrierten Größen (wie z.B. photometrische oder farbmétrische Werte) untersucht. Es werden praktische Richtlinien zur Anwendung der Bandpasskorrektur angegeben, sowohl zur Bestimmung der Bandpassfunktion als auch zur Abschätzung, ob die Korrektur zu einer Verbesserung der spektralen Genauigkeit führt.

1 Introduction

1.1 Objectives

In recent years there has been increased interest in the uncertainties associated with instrumental measurement of spectrally-dependent optical properties of sources, detectors, and materials and the errors that spectral instruments can introduce. The bandpass function of the instrument can introduce appreciable errors when measuring test artefacts that change rapidly with wavelength, particularly when the instrument's calibration is performed using a reference source, detector, or material that has a very different spectral characteristics from those of the test artefact.

This report aims to be a practical guide for users making spectral measurements to understand the nature of the errors introduced by the instrumental bandpass function and to provide a practical procedure for correcting measured values to take account of the instrumental bandpass function. The methodology described here deals with both triangular and non-triangular bandpass functions.

Clause 3 is written as an introduction to the concepts of the report. It is aimed at an audience that wants a practical guide. The ideas presented in this clause are discussed in more detail, and with more theoretical rigour, in later clauses of the report. This introduces some repetition, but also means that the main ideas of the report are collated in a single location. It is recommended that readers start with Clause 3.

1.2 Scope

This report describes the effect of instrumental bandpass and measurement wavelength interval on spectral quantities and provides recommendations on suitable methods to minimize the effect of instrumental bandpass functions. The report concentrates on a correction method of the form commonly used, that of a weighted mean over surrounding values applied to the data and based on assumption of the form of the spectrum across the bandpass. The report also provides information relating to the determination of the bandpass function itself for both monochromators and spectrometers.

As this report was being compiled, a true deconvolution method, not dependent on assumptions about the spectrum, was being developed (Eichstaedt et al. 2013). This method offers the promise of better spectral correction than the weighted-mean methods described in this report, but further study is required and a detailed description is not included here. Weighted-mean methods are simple to apply and give good results for spectral integrals (e.g. as in colorimetry), which are a common application of bandpass correction.

This report also describes the effect of instrumental bandpass on spectrally integrated quantities, but it does not cover the propagation of errors and uncertainty from spectral measurement to spectrally integrated quantities.

Other related effects in spectrometer characterization include stray-light correction and wavelength accuracy. These are included only in as far as they affect the determination of the bandpass function. Full determination and correction of stray-light and wavelength errors are not described within this report.

2 Terminology

2.1 References for terminology

Where possible terms are used as described in the International Lighting Vocabulary (ILV) (CIE 2011a) and the International Vocabulary of Metrology (VIM) (JCGM 2012).

2.2 Instrumentation

The following terms are all used to describe instruments that select a particular narrow spectral region or which separate a broadband spectrum into its spectral components. In this document "spectrometer" is used to describe this class of instruments.

2.2.1 Spectrometer

Spectrometer is used in this report to describe the general class of instruments that measure spectral quantities of optical radiation over a continuous range of wavelengths, with individual measurements being made over a narrow range of wavelengths determined by an instrument's bandpass function. Different kinds of spectrometers are used to measure these spectral quantities for sources, detectors and materials. For example, for measuring the spectral quantities of light sources – spectral irradiance, spectral radiance, total spectral radiant flux – a spectroradiometer is used. On the other hand, for measuring spectral quantities related to materials – spectral transmittance, spectral reflectance – a spectrophotometer or spectroreflectometer is used. The term spectrometer is used throughout this document to refer to any such instrument; this includes the instrument itself plus any entrance optics or geometric components (apertures etc.) that influence the radiation entering the instrument.

2.2.2 Spectroradiometer

The International Lighting Vocabulary (CIE 2011a) defines a spectroradiometer as an

“instrument for measuring radiometric quantities in narrow wavelength intervals over a given spectral region”.

2.2.3 Spectrophotometer

The International Lighting Vocabulary (CIE 2011a) defines a spectrophotometer as an

“instrument for measuring the ratio of two values of a radiometric quantity at the same wavelength”.

2.2.4 Monochromator

The International Lighting Vocabulary (CIE 2011a) defines a monochromator as an

“optical device which isolates a single narrow spectral band of optical radiation from a broader spectral input, and for which the centre wavelength of the transmitted radiation can be altered”.

Monochromators are instruments that select a narrow spectral region from a source that emits radiation at a range of wavelengths. They can be used to select a single spectral line of a line source, or to make sequential spectral readings of a broadband source, usually through mechanical rotation of a dispersive element (such as a grating or prism). Due to the sequential mode of operation, the measurements are slow. However, an important advantage is that two monochromators can be arranged in series to reduce stray-light effects significantly. This type of instrument is called a *double monochromator*, and, since it has the best stray-light rejection, is considered the most accurate. Since this configuration acts like a double bandpass filter, the spectral rejection of stray light is nearly the square of that of a single monochromator. Spectrometers using a single dispersive element (including detector-array types and polychromators) are classified as a *single-monochromator type*.

Double monochromators can be *additive* or *subtractive*. In additive dispersion, the two gratings rotate in the same direction. The second monochromator further disperses the light from the first monochromator, which means that the spectrum that emerges at the exit slit has half the bandwidth, i.e. twice the resolution, as that emerging from the first monochromator. In subtractive dispersion, the two gratings rotate in opposite directions. Here the second monochromator recombines the light dispersed by the first monochromator, thereby improving the uniformity of the spectral distribution of the monochromator output. However, the bandwidth of the measured spectrum is the same as for a single monochromator i.e. twice that found for a comparable double monochromator in additive dispersion.

A monochromator is frequently used with the addition of an appropriate single-element detector, to record the spectrum, and appropriate input optics to define the radiometric quantity being measured. In this report the term “scanning spectrometer” is used to describe this combination.

2.2.5 Array spectrometer

An **array spectrometer** is also known as a *detector array spectrometer* or a *diode array spectrometer*. These spectrometers use a photodiode array or charge-coupled device (CCD) detector array (placed in the exit plane) as a detector and record all the spectral readings almost simultaneously.

Note that they are usually very susceptible to stray-light errors due to their compact design and broad spectral response. Where CCD or photodiode arrays have not been available (historically, or in some spectral regions today), *multi-channel spectrometers* or *spectrographs* have sometimes been used. A multi-channel spectrometer works as a series of single monochromators having the same input. Where these provide continuous spectral coverage they can be treated as array spectrometers for bandwidth correction purposes. Where they provide discrete wavelengths, then for bandwidth correction they can be treated as scanning instruments, but with no choice of measurement wavelength.

2.3 Instrumental passband and instrumental bandpass

When a spectrometer is set to measure a particular wavelength, it actually measures a narrow range of wavelengths around the chosen wavelength. The contiguous band of wavelengths in which a non-zero fraction of the incident light is selectively transmitted is known as the **instrumental passband**. The **instrumental bandpass** relates to this passband and is often defined by the bandpass function and/or bandwidth (see below). It is important to note that spectrometers can have non-zero responsivity across a very wide spectral region. In general the full spectral response to monochromatic radiation is dealt with in two regions – a central region of relatively strong response, which is the bandpass region, and a weaker response at wavelengths outside this region – the stray-light response.

It is not within the scope of this document to discuss the issue of stray light in detail. However, the existence of stray light cannot be ignored. It will be discussed in more detail in 4.2. It is necessary to define where the instrument passband ends and stray-light response begins. The approach to dealing with this issue is discussed in Clause 6. It should be noted that both stray light and the bandpass function can change over the spectral range of the instrument.

2.3.1 Bandpass function

The input spectrum $S(\lambda)$, or signal as a function of wavelength that would be recorded with a spectrometer of infinitely small resolution, is the product of a source spectrum, the detector response spectrum and the transmittance of the spectrometer, where the latter term includes the optical extent, transmission and reflection losses and grating efficiency. The range of wavelengths passed by a spectrometer with finite resolution at a given wavelength setting (or position in an array spectrometer) is determined by line spread function (LSF) (also known as the slit function), which is a convolution of the distribution of light over the entrance and exit apertures. The response of a spectrometer, the measured signal $M(\lambda)$, is the convolution of an input spectrum $S(\lambda)$ with the LSF $F(\lambda)$:

$$M(\lambda) = S(\lambda) * F(\lambda) \quad (1)$$

For a source over an extended range of wavelengths, the result of the convolution at a given wavelength λ_0 is given by the convolution integral

$$M(\lambda_0) = \int S(\lambda) f(\lambda_0 - \lambda) d\lambda \quad (2)$$

where the range of integration extends over that of the LSF. For a monochromatic source of wavelength λ_{line} , only that wavelength contributes to the integral. Treating the spectrum as a delta function with value 1 at the line wavelength λ_{line} and zero elsewhere, the measured spectrum becomes

$$M(\lambda_0) = f(\lambda_0 - \lambda_{\text{line}}); \quad (3)$$

the measured spectrum is indeed the input line spread over surrounding wavelengths, as it would appear in a normal measurement. If the LSF is normalized to have unit area, the total power in the measured spectrum is that of the input spectral line. The LSF is defined in the spectral interval $(\lambda_{\text{line}} - \Delta \leq \lambda \leq \lambda_{\text{line}} + \Delta)$, where Δ is the bandwidth, as defined in 2.3.2. The function is assumed to go to zero outside this spectral interval.

The bandpass function $B(\lambda) = b(\lambda - \lambda_0)$ is the LSF reversed in wavelength about the central point λ_0 , for which the convolution integral becomes

$$M(\lambda_0) = \int_{\lambda_0 - \Delta}^{\lambda_0 + \Delta} S(\lambda) b(\lambda - \lambda_0) d\lambda \quad (4)$$

Equation (4) becomes the starting point for the weighted-mean method for bandwidth correction defined in Clause 3. For discussion on the experimental determination of the bandpass function (and the effect of stray light) see Clause 6.

2.3.2 Bandwidth, passband, full-width at half maximum

In general one can think of the **bandwidth** as a metric of the **passband**. CIE Publication 63 (CIE 1984) defines bandwidth as the area under the bandpass curve divided by the height at the nominal centre wavelength (usually the centroid - see 2.4.2); such a definition considers the equivalent width for a rectangular passband. For a symmetric triangle or trapezoid bandpass shape, the bandwidth is equivalent to the full-width at half maximum (FWHM) commonly specified. In this report, the term bandwidth is defined as half the full base-width of the the bandpass function. This is denoted by the symbol Δ and used throughout this report. Note that this requires a definition of ‘goes to zero’; see Clauses 4.2 and 6 for how this is practically implemented. This definition is more useful for non-triangular (and indeed, non-symmetric) bandpass functions, and is required for the bandwidth correction algorithm used in this report.

For a symmetrical triangular bandpass function (common for practical good-quality monochromators) the half full base-width is equal to the equivalent rectangular bandwidth.

CIE 15:2004 (CIE 2004) recommends matching the ‘bandwidth’ and step interval when measuring the spectral properties of sources with rapidly varying spectral characteristics (note CIE 15:2004 assumes a symmetrical triangular bandpass function). See Clause 9 for a discussion of this point and the implications for a non-triangular bandpass function.

2.3.3 Reciprocal linear dispersion and slit width

With a monochromator, the slit function is determined by its reciprocal linear dispersion and the physical width of the entrance and exit slits. The **reciprocal linear dispersion**, D^{-1} , is a measure of the wavelength dispersion at the monochromator exit slit in nm/mm. This depends on the physical dimensions of the monochromator (focal length of the mirrors), the angles between the mirrors and the grating, and the grooves per mm of the grating.

The **slit width** is the size of the slits used to define the bandwidth – i.e. the exit and entrance slits for a single monochromator or an additive double monochromator and the entrance and middle slits for a subtractive double monochromator. With a multi-channel spectrometer that uses a detector array, the exit “slit” is the physical width of the individual elements in the array.

In the absence of optical aberrations within the instrument, the height of the slits or array elements does not affect the bandpass function, except in terms of the stray-light response. Where aberrations are present, these can lead to curved slit images and in this case the slit height or height of the array elements will affect the bandpass.

2.3.4 Spectrometer resolution

The resolution of a spectrometer refers to its ability to determine the natural width of individual lines and to distinguish neighbouring lines in a spectrum. Resolution is commonly specified as equivalent to the bandwidth given as full-width at half maximum. This follows from the convolution of two Gaussian functions of equal strength separated in wavelength by the bandwidth. Spectral structure on a scale smaller than the resolution cannot be directly distinguished in a measurement, even if measurements are made at wavelengths separated by less than the resolution, without applying a bandwidth correction scheme.

2.4 Instrumental wavelength accuracy

2.4.1 Nominal wavelength (of measurement)

A spectrometer is designed to measure a particular wavelength when the grating is rotated to a particular position or when reading the signal from a particular part of the detector array. This is referred to as the **nominal wavelength (of measurement)**, denoted here by λ_0 . Calibration of the wavelength scale may alter the value assigned to that wavelength.

2.4.2 Centroid wavelength

The centroid wavelength λ_c of a bandpass function, $b(\lambda - \lambda_0)$ is given by

$$\lambda_c = \frac{\int \lambda b(\lambda - \lambda_0) d\lambda}{\int b(\lambda - \lambda_0) d\lambda}. \quad (5)$$

It should be noted that the centroid wavelength may be different from the nominal wavelength, though with accurate wavelength scale calibration this difference should be small (see 2.4.3). The bandwidth function is usually normalized to unit area, for which the denominator of Equation (5) is unity.

2.4.3 Wavelength error

The difference between the centroid wavelength and the nominal wavelength is the wavelength error of the spectrometer, which can be reduced by a wavelength calibration. Provided the bandpass function has its zero-wavelength set to the centroid wavelength, it is recommended that the wavelength scale be calibrated and corrected before bandwidth correction, although the bandwidth correction described in this report can also provide some wavelength correction (see 6.6). If the bandpass centre is chosen to be other than the centroid wavelength, wavelength calibration and bandpass correction must consistently apply this definition. The assumption of a symmetric and triangular bandpass function is common; in actual practice, there are many instances where this is not the case. For example, the bandpass function of array spectrometers typically changes over the spectral range of the instruments and is neither symmetric nor triangular. Often the instrument manufacturer can provide more information on this topic and/or in some case has already adjusted the readings to correct for the instrument-specific bandpass function or asymmetry. Nevertheless, a careful wavelength calibration of the spectrometer using proper methods and means is strongly recommended (see 4.3). Any residual wavelength error following this wavelength correction can be dealt with by the bandwidth correction procedure described in this report.

2.5 Stray light

Stray light is unwanted radiation within the spectrometer caused by scatter from the grating or mirror and/or reflections from the slits, detector array, or instrument components. This unwanted radiation means that a spectrometer, at a given wavelength, responds to radiation of wavelengths other than those within the selected bandpass. Stray light is most prominent in single-grating instruments and, in particular, in array spectrometers. See 4.2 for a discussion of stray light.

2.6 Measurement interval

The measurement interval, also known as ‘sampling interval’ or ‘(measurement) wavelength step’, describes the spectral interval between the nominal wavelength of one measured value and the nominal wavelength of the next measured value. The symbol used for measurement interval is δ . In some more mathematical clauses of this report the term ‘wavelength step’ is used preferentially to reduce confusion with the mathematical concept of an interval. Elsewhere ‘measurement interval’ or ‘sampling interval’ is used. The three terms are considered synonymous in this document.

Typically, spectra are measured (‘sampled’) at a wavelength step equal to the bandwidth. This condition allows structure on a scale of the resolution to be seen in the measured spectrum over the full measurement range. Such a condition accurately records the total power contribution from spectral lines, although a moderate mismatch of sampling interval and bandwidth can lead to large errors (see Clause 10).

Any spectrum can be broken down to a sum of periodic terms through the process of Fourier analysis. If the Fourier components are zero above a particular Fourier frequency (the band limit), the Shannon-Nyquist sampling theorem says that all the spectral information is contained in a set of measurements recorded at an interval of half the period of that frequency.

The terms “over-sampled” and “under-sampled” in sampling theory refer specifically to sampling with measurement intervals shorter or longer than the Nyquist criterion, independent of the bandwidth. The terms have commonly been used in radiometry and photometry to refer to measurement intervals significantly shorter and longer, respectively, than the bandwidth. In this report the terms “hyper-scanned” and “sparsely-scanned” are used instead to avoid confusion with the meanings within sampling theory. Sparsely scanned measurements may be appropriate for a source, detector or material where the spectrum changes slowly and smoothly with wavelength or where a physical model can be used. With an unknown artefact, especially one that may have sharp features in its spectrum, a sparsely scanned measurement risks losing important information between the recorded values.

2.7 Mathematical descriptions and symbols

2.7.1 Measurand, measurement and measured values

A **measurand** is a particular quantity subject to measurement (JCGM 2012). **Measurement** is the process by which a *measured value* is obtained to represent a *measurand*. For example, a user will take a measurement with a spectrometer to determine a **measured value** (a number) of the measurand, e.g. spectral irradiance.

We imagine the concept of a *true spectrum* – that is a continuous function that could be sampled using an ideal spectrometer, with infinitely narrow bandwidth and no noise, to obtain a *true signal* at each sampling wavelength. If the true spectrum were measured with an instrument of finite bandwidth (but still no noise), then we have the *measured spectrum* – that is, values sampled at the measurement interval from a continuous function where the true spectrum has been influenced by the bandpass function. The measured spectrum is the measurand. The measured spectrum $M(\lambda)$ is a combination of the true spectrum $S(\lambda)$ and the (normalized) bandpass function $b(\lambda - \lambda_0)$. The measured spectrum at wavelength λ_0 is given as

$$M(\lambda_0) = \int S(\lambda) b(\lambda - \lambda_0) d\lambda. \quad (6)$$

When a measurement is made at a particular wavelength using a real instrument, we obtain the *measured signal* (the measured value) at this wavelength. This may be influenced by noise.

From the measured signals we try to re-construct the true signals at these wavelengths, using a bandwidth correction algorithm. In practice we will never perfectly determine the true signal,

except in some very specific cases (linear or quadratic true spectra and no noise). Instead we obtain the *corrected signal* at these wavelengths.

These distinctions can be understood by applying a Fourier analysis to the complete measurement system, as described in Clause 8.

2.7.2 Symbols used in formulae in this report

Quantity	Symbol
Nominal wavelength of measurement. The wavelength to which a monochromator is 'set', or the wavelength assigned to each element of an array spectrometer. For wavelength-corrected instruments, this should be (very) close to the centroid wavelength of the bandpass function, λ_c .	λ_0
True spectrum. The continuous spectrum that would be obtained by measurement with a hypothetical perfect system with infinitely narrow bandpass at the nominal wavelength and no noise. Note, this includes factors such as the source spectral output, spectrometer transmittance, detector responsivity and (in the case of measurements on materials) the transmittance or reflectance of the material. It is likely that the true spectrum is a voltage (or current) spectral distribution; it is not the spectrum of the spectral quantity being measured, although it is related to this latter spectrum through the absolute calibration of the spectrometer.	$S(\lambda)$
Line spread function $F(\lambda)$. Measured spectrum of a spectrometer to a monochromatic source of wavelength λ_0 , about that wavelength.	$f(\lambda - \lambda_0)$
Bandpass function $B(\lambda)$. Line spread function with the wavelength scale reversed. $B_N(\lambda)$ is the bandpass function normalized to unit area.	$b(\lambda - \lambda_0)$
Bandwidth (BW). In this report, half the full base-width of the bandpass function. The bandpass function is assumed to go to zero for all wavelengths outside the range defined by the bandwidth about the wavelength λ_0 , i.e.: $B(\lambda) = 0$ for $\begin{cases} \lambda \leq \lambda_0 - \Delta \\ \lambda \geq \lambda_0 + \Delta \end{cases}$ For asymmetrical bandpass functions, one side of the function may go to zero within the range defined by the bandwidth. In this report the FWHM is not used as the bandwidth – see 2.3.2.	Δ
Measured spectrum. The continuous function representing the measurand, i.e. the true spectrum modified by the bandwidth and noise.	$M(\lambda)$
Measurement interval. The nominal wavelength separation between measurements i.e. the spectral distance (likely to be in nm) between successive measured values of $M(\lambda)$.	δ
True signals. The values of the true spectrum at each measurement wavelength. In this report, generally expressed as the true signal at the nominal wavelength and the true signals at immediate measurement intervals either side of the nominal wavelength, e.g. $S_0 = S(\lambda_0)$, $S_{-1\delta} = S(\lambda_0 - \delta)$, $S_{-2\delta} = S(\lambda_0 - 2\delta)$, etc.	$S_0, S_{\pm 1\delta}, S_{\pm 2\delta} \dots$
Measured signals. The values of the measured spectrum at each measurement wavelength. In this report, generally expressed as the measured signal at the nominal wavelength and the measured signals at immediate measurement intervals either side of the nominal wavelength, e.g. $M_0 = M(\lambda_0)$, $M_{-1\delta} = M(\lambda_0 - \delta)$, $M_{-2\delta} = M(\lambda_0 - 2\delta)$, etc. Measured signals may include noise.	$M_0, M_{\pm 1\delta}, M_{\pm 2\delta}$
Corrected signal. The result of applying a bandwidth correction algorithm, calculated at the wavelength of the nominal wavelength.	C_0

3 A practical introduction to bandwidth correction

3.1 Spectrometers for spectral measurements

3.1.1 Introduction

Spectrometers are used to determine various spectral quantities. These include the spectral irradiance, spectral radiance or spectral total radiant flux of sources (CIE 1984), the reflectance and transmittance of materials in different geometries (CIE 2004) and the spectral responsivity of detectors (CIE 2011b). A spectrometer consists of appropriate input optics, a spectrally dispersive element¹ and a detector. The spectrometer may be operated sequentially (that is each wavelength of interest is measured in turn) or simultaneously (an array of detector elements measures the full spectrum). The most common systems are scanning instruments, which operate sequentially, and array-spectrometers, which operate simultaneously.

The spectral quantities measured using a spectrometer are important in themselves, but many of them can also be combined in a weighted spectral integral to determine colorimetric and photometric quantities such as luminance. The uncertainties associated with such calculations, based on spectral measurements, may be lower than those obtained using more direct measurements of colorimetric or photometric quantities using a tri-stimulus colorimeter or photometer. This is because such filtered instruments fail to match the CIE-defined weighting functions perfectly and corrections must be made to account for differences between the spectral profile of the test and reference artefact. With correctly performed spectral measurements, the impact of differences between the test and reference artefact can be (almost) insignificant and the CIE-defined weighting functions can be used directly without correction. However in practice, spectrometers themselves, particularly array spectrometers, introduce their own errors into this determination. The two most significant effects are instrument bandpass – the subject of this document – and instrument stray light.

3.1.2 Calibration

A spectrometer system must be calibrated if it is to provide reliable measured values. There are two ways in which this is done – by comparison with a reference artefact or by responsivity calibration. Most commonly calibration is performed by comparison. The spectrometer is used to compare, for example, the radiant spectra of a test and a reference source, or the spectral responses of a test and a reference detector, or the spectral reflectances of a test and a reference tile. At a National Metrology Institute (NMI), such comparisons at the highest level occur between a primary reference artefact and test artefact – for example between a blackbody source of known temperature and a lamp. At other stages in the calibration chain this comparison is between a secondary reference artefact and test artefact, for example between two lamps that may be similar or different types.

Alternatively a responsivity calibration could involve artefacts of different quantities. For example the irradiance responsivity of the spectrometer may be determined by comparison with a detector of known irradiance responsivity using monochromatic radiation and then the spectrometer could be used to determine the irradiance produced by a reference source.

In either case, the instrument calibration may take place immediately before and/or after the test artefact measurement, or there may be a period of some days, weeks or months between the calibration and measurement. The calibration may be performed by a separate organization, perhaps the manufacturer of the instrument or a NMI. All these possibilities will have a slightly different implication for how bandpass should be corrected and by whom.

3.1.3 Measurement geometry

The different quantities relating to sources, materials or detectors are measured by configuring the spectrometer with appropriate input and output optics. For sources, the input optics will collect light from all (generally irradiance, flux, intensity measurements) or part (radiance measurements) of the source either directionally (radiance, irradiance, intensity) or

¹ Usually the spectrally dispersive element is a diffraction grating, though other options, including prisms, are used.

from all directions (flux) and the output optics will collect as much light from the spectrometer as possible onto a detector. In the case of array spectrometers, no output optics are needed because the detector array combines optical imaging and detection.

For measurements on detectors, the input optics are used to collect light from a stable source and the output optics are used to bring this light to the detector. For measurements on transmitting or reflecting specimens, spectrometers may be used either to provide monochromatic irradiation of the sample or to collect the light reflected from or transmitted through the sample at a selected monochromatic wavelength in an appropriate, defined geometry. In the former case the monochromator input optics collect light from a stable source and the output optics bring this light to the material sample in the required incident geometry. In the latter case the input optics collect light from the sample in the required geometry. In some situations (particularly for highest accuracy fluorescence measurements), the CIE recommended method is to use monochromatic irradiation of the sample and scanning (using a second monochromator) or simultaneous (using a detector array) monochromatic detection of the light collected from the sample (CIE 1988; CIE 2007) .

Sometimes the spectrometer will be sold as configured for one of these tasks. Often the spectrometer is sold with no input optics – either simply a slit, or with a fibre bundle connected to the entrance slit. In these circumstances, additional input optics must be carefully designed in order to yield a measurement device for the required spectroradiometric quantity. For example in the case of spectral irradiance measurements, the light source should never directly illuminate the entrance slit or the fibre-optic input of a spectrometer. The spectral throughput of a monochromator is sensitive to the (spatial and angular) uniformity of the illumination of the entrance slit and to the polarization of that light. As test and reference measurements will rarely have completely identical geometries and polarization states, it is essential that the input optics remove the sensitivity to both the polarization and the spatial and angular non-uniformity; in practice this means illuminating the entrance slit, and thus the dispersive element, uniformly with unpolarized light. This is often achieved with a small integrating sphere, the output of which is focussed onto the entrance slit or attached to the fibre. If this provides insufficient light, then other types of diffuser are used instead.

3.1.4 Bandwidth in a grating-based device

Most spectrometers use a grating as the wavelength dispersive element. A grating will disperse the spectrum such that a particular angular dispersion from the grating corresponds to a particular wavelength. In a simultaneous detection instrument, the dispersed spectrum is incident on an array detector, where each array element effectively acts as an exit slit at a different wavelength. In a sequential instrument, the rotation of the grating determines which wavelength is incident on the exit slit.

A monochromator is designed such that when monochromatic light fully illuminates its entrance slit and the grating is rotated to the correct angle for that monochromatic wavelength, an image of the entrance slit is formed on the exit slit. As the grating is rotated, this image moves and the throughput decreases. This means that for a monochromator with equal entrance and exit slit widths and unity magnification aberration-free optics, the bandpass function will be triangular, as the rectangular image of the entrance slit moves across the rectangular exit slit. (The bandpass function is trapezoidal for a perfect monochromator in which the entrance and exit slits are of different widths or the magnification of optics is not unity.) In practice, aberrations and imperfections mean the bandpass function is never perfectly triangular (or perfectly trapezoidal). See Clause 5 for further information on this point.

3.2 The effect of bandwidth on spectral measurements

Clause 7 gives some examples of the effect of bandwidth on spectral measurements. Where the spectrum being measured changes linearly with wavelength, and the bandpass function is symmetric about the nominal wavelength of measurement, the measured spectrum is the same as the true spectrum. The more non-linear is the behaviour of the true spectrum within a distance of two bandwidths about the nominal wavelength, the larger is the effect of bandwidth on the measurement.

Bandwidth has the effect of broadening and reducing 'peaks' in the true spectrum. The narrower the peak compared to the bandwidth, the greater this effect. If there are two spectral peaks within a bandwidth, then the measured spectrum will merge these peaks into a single feature.

Where an instrument is being used for comparison purposes (for example where the instrument is being calibrated using a reference artefact and then used to measure a test artefact), the effect of bandwidth on the spectrum of interest will depend on the degree of similarity between the spectra of the test and reference artefacts. In particular it will depend on the similarity (or otherwise) of the second derivative of these spectra. The worst-case scenarios are thus one where one artefact has a spectrum that increases non-linearly with wavelength, while the other artefact has a spectrum that decreases non-linearly with wavelength, or one where one artefact has a spectral feature (peak) within the measured region, but the other does not.

3.3 The effect of measurement interval for line sources

Discharge lamps, including HID lamps and tubular and compact fluorescent lamps, have spectra that are complex mixtures of multi-line and continuous radiation. The narrow spectral features mean that the full details of the true spectrum cannot be determined from a typical radiometric measurement. This sort of analysis requires the high resolution of a spectroscopic instrument. However, for many applications it is not necessary to determine the true spectrum. The user may only be interested in the power contained within a particular spectral band, e.g. the irradiance spectrum of a lamp integrated between 404 nm and 406 nm, or in integrated values weighted, e.g. by colour matching functions.

The shape of the measured spectrum for a spectral line is that of the LSF sampled at the measurement wavelength interval. Clause 10 demonstrates how the integral value is affected by both the sampling interval and the position of the samples with respect to the line centre. Depending on the shape of the LSF, large errors in the integral may occur if the measurement interval is comparable to but not exactly matched to the bandwidth. The conclusion is that where possible, line spectra to be integrated should be hyper-sampled, using a measurement interval of order 1/3 of the bandwidth or less. The smaller the measurement interval, the smaller is the possible error in the integral.

3.4 Determining the instrumental bandpass function

In order to understand the effect of bandpass and as a prerequisite to applying a bandwidth correction, it is important to measure the bandpass function of the instrument to be used. This measurement is made by observing the instrument's response to monochromatic radiation and is described in detail in Clause 6.

Ideally the instrument is wavelength-corrected using the centroid wavelength of the measured bandpass function as the nominal wavelength, but this is not absolutely essential if a bandwidth correction is to be applied, provided the same criterion for the reference value (offset from the centroid wavelength) is chosen for wavelength calibration and bandwidth correction (see 4.3).

The method for determining the bandpass function is defined in detail in Clause 6. Most commonly low-pressure gas emission lines (particularly those of mercury, neon and argon) are used to determine the bandpass function¹. Recommended practice is that the source physically fills the field-of-view of the spectrometer, that is, that the entrance slit and the instrument's acceptance angle are both fully and uniformly illuminated. Usually this requires a diffuser between the source and the entrance slit of a monochromator or fibre input. Most important is that the bandpass function be determined with the same illumination conditions as those for subsequent spectral measurements.

Since both scanning instruments and array spectrometers may have a different bandwidth for different wavelengths across the spectrum, it is important that measurements are made with several different line sources.

¹ These sources are also used for wavelength scale correction.

With a scanning monochromator, a bandpass function should be determined with as small a step size as possible; the lower limit of the step size is defined by the stepping resolution of the grating rotation stage. With a multi-channel spectrometer there may be insufficient 'pixels per bandwidth' to determine the bandpass function accurately. In this case it may be possible to model the bandpass function. Clause 6.7 suggests how this may be done. An alternative is to numerically interpolate between the measured pixels with a function such as a natural cubic spline.

Bandwidth correction assumes there is a finite spectral range to the bandpass function and that outside this finite range, the bandpass function is zero. With real experimental data this is never achieved because of stray light (see 4.2). It is necessary to produce an artificial cut-off and force values beyond this cut-off to zero (see 6.7).

3.5 Applying a bandwidth correction

In Clause 11, the weighted-mean bandpass correction is derived in terms of finite-difference derivatives of the measured spectrum. This derivation extends previous weighted-mean methods (also discussed in Clause 11). These methods replace a measured value with a weighted mean of surrounding values. No bandwidth correction algorithm can provide perfect correction (see 3.6 and 3.7 below), but the recommended method is suitable for many situations.

The correction algorithm can deal with non-triangular bandpass functions. Where the bandpass function is triangular, simpler expressions can be used (see Equations (21) and (22)).

3.5.1 Step 1: Determine the bandpass function and its moments

The correction coefficients are determined from the bandpass function, $B(\lambda)$, and its moments, defined as

$$I_n = \int \lambda^n B(\lambda) d\lambda, \quad (7)$$

for $n=0,1,\dots$. Therefore the first step is to measure the bandpass function as described in Clause 6. It is advisable to make and average multiple measurements and to make measurements with a narrow wavelength step, in order to obtain a good evaluation of the bandpass function. It is worth noting that the bandwidth correction coefficients determined from this method can be applied to all measurements made with the same bandpass function. Therefore the bandpass function can be calculated once for a spectrometer and applied to all measurements made using it (assuming the instrument does not change, e.g. on transportation, with temperature or over time, that the slit widths are not changed, and that the bandpass function does not vary as a function of wavelength).

From this process we obtain the normalized bandpass function $B_N(\lambda)$, defined over the spectral interval $(-\Delta \leq \lambda \leq \Delta)$ and with the value zero outside of this interval. The values of the measured bandpass function are scaled so that the zeroth moment (the area under the bandpass function) has the value unity.

$$I_0 = \int B_N(\lambda) d\lambda = 1. \quad (8)$$

The first moment provides the centroid wavelength of the bandpass function

$$I_1 = \int \lambda B_N(\lambda) d\lambda. \quad (9)$$

If the measurements of multiple spectral lines have been used to correct the wavelength scale of the spectrometer, then this means that this first moment will be close to zero. If the instrument is used 'as is' without wavelength correction, then this moment may not be zero, especially for asymmetric bandpass functions.

In order to calculate the bandwidth correction algorithm, (up to) the first four moments I_1 , I_2 , I_3 and I_4 must be calculated (numerically) from experimental data by approximating the integral by a sum. Where data are equi-spaced and relatively close together, this can be done as follows:

$$I_n \approx \sum_i \lambda_i^n B_N(\lambda_i) \delta_{bp}. \quad (10)$$

where δ_{bp} is the wavelength interval for the bandpass function measurements, which is generally much smaller than that for the subsequent spectral measurements (δ).

If the line spread function itself has been used to calculate the moments, the signs of the odd moments should be reversed (positive to negative and vice versa). The signs of the even moments are unchanged. Alternatively the line spread function can be converted into the bandpass function by reversing the wavelength scale; in this case the signs of the moments are unchanged.

For symmetrical triangular bandpass functions, the moments can be calculated by analytical solution of the integral equations. For symmetrical triangular bandpass functions with bandwidth Δ this gives

$$I_1 = 0 \quad I_2 = \frac{\Delta^2}{6} \quad I_3 = 0 \quad I_4 = \frac{\Delta^4}{15}. \quad (11)$$

3.5.2 Step 2: Determine the correction coefficients

The correction coefficients are calculated from the moments.

$$\begin{aligned} A_0 &= 1 \\ A_1 &= -I_1 \\ A_2 &= \frac{-I_2}{2} + I_1^2 \\ A_3 &= \frac{-I_3}{6} + I_1 I_2 - I_1^3 \\ A_4 &= \frac{-I_4}{24} + \frac{I_1 I_3}{3} + \frac{I_2^2}{4} - \frac{3I_1^2 I_2}{2} + I_1^4 \end{aligned} \quad (12)$$

Note that these correction coefficients do not depend on the spectrum being measured; they can be determined once for the instrument and can then be used to calculate the correction for any spectra measured using that instrument (provided the bandpass function does not change between measurements or with wavelength).

If the bandpass function is triangular (or nominally triangular) then these coefficients can be calculated analytically, giving:

$$A_1 = 0, \quad A_2 = \frac{-\Delta^2}{12}, \quad A_3 = 0, \quad A_4 = \frac{\Delta^4}{240}. \quad (13)$$

3.5.3 Step 3: Calculate the correction

The finite-difference approximations to the derivatives are calculated for the nominal wavelength λ_0 from the measured signal at this wavelength, M_0 , and the measured signals either side at wavelengths $\lambda_0 \pm n\delta$, $\lambda_0 \pm 2n\delta$, ..., where n is the integer multiple of measurement intervals used for calculating the derivatives. The measured signals used to calculate the correction are denoted as $M_{\pm n}$, $M_{\pm 2n}$, ... For the common practice of measurements recorded at an interval equal to the bandwidth, n has the value 1 (nearest

neighbours), but for hyper-sampled data, n should be chosen so that derivatives are calculated using measured values that span the bandwidth. Clause 9.3 and the example in Clause 15.3 show that noise in hyper-sampled data can be greatly increased by choosing closely spaced points for the correction

The first and second derivatives are given here in two forms. For almost all spectra there will be no practical difference between these two. Note that the finite-difference approximations to the derivatives expressed below require two measured values below and two above the value that is to be corrected. This can cause a problem when applying the bandwidth correction to the measurement points at the extremes of the wavelength range considered. There are alternative expressions using only longer or only shorter wavelengths to estimate the local derivative that can be used in this case, but often it is simpler to make measurements at an additional two wavelengths either side of the spectral region of interest, so that the expressions below can be used.

$$\begin{aligned}
 M' &\approx \frac{1}{12n\delta}(M_{-2n} - 8M_{-n} + 8M_n - M_{2n}) \approx \frac{1}{2n\delta}(-M_{-n} + M_n), \\
 M'' &\approx \frac{1}{12(n\delta)^2}(-M_{-2n} + 16M_{-n} - 30M_0 + 16M_n - M_{2n}) \approx \frac{1}{(n\delta)^2}(M_{-n} - 2M_0 + M_n), \\
 M''' &\approx \frac{1}{2(n\delta)^3}(-M_{-2n} + 2M_{-n} - 2M_n + M_{2n}), \\
 M^{iv} &\approx \frac{1}{(n\delta)^4}(M_{-2n} - 4M_{-n} + 6M_0 - 4M_n + M_{2n}).
 \end{aligned} \tag{14}$$

The corrected signal C_0 at wavelength λ_0 is given by the formula

$$C_0 = A_0M(\lambda_0) + A_1M'(\lambda_0) + A_2M''(\lambda_0) + A_3M'''(\lambda_0) + A_4M^{iv}(\lambda_0) + \dots, \tag{15}$$

which can be truncated after a particular derivative order. The contribution of the third-order term is typically small. (Note the correction coefficients A_i are as obtained from Equations (12) in step 1 above.)

Explicit expressions for the corrected value in terms of the surrounding values can be obtained by substituting the derivative expressions of Equation (14) into Equation (15).

Truncated at the second derivative, the corrected signal C_0 at wavelength λ_0 is given by the three-point expression

$$C_0 = \left[\frac{I_1^2}{(n\delta)^2} + \frac{I_1}{2n\delta} - \frac{I_2}{2(n\delta)^2} \right] M_{-n} + \left[1 - \frac{2I_1^2}{(n\delta)^2} + \frac{I_2}{(n\delta)^2} \right] M_0 + \left[\frac{I_1^2}{(n\delta)^2} - \frac{I_1}{2n\delta} - \frac{I_2}{2(n\delta)^2} \right] M_{+n}. \tag{16}$$

Truncation at the fourth derivative yields the five-point correction

$$C_0 = a_{-2}M_{-2n} + a_{-1}M_{-n} + a_0M_0 + a_1M_n + a_2M_{2n}, \tag{17}$$

where the coefficients a_k are determined from the correction coefficients (Equations (12)):

$$\begin{aligned}
a_0 &= \left[A_0 - \frac{5A_2}{2(n\delta)^2} + \frac{6A_4}{(n\delta)^4} \right], \\
a_{\pm 1} &= \left[\pm \frac{2A_1}{3n\delta} + \frac{4A_2}{3(n\delta)^2} \mp \frac{A_3}{(n\delta)^3} - \frac{4A_4}{(n\delta)^4} \right], \\
a_{\pm 2} &= \left[\mp \frac{A_1}{12n\delta} - \frac{A_2}{12(n\delta)^2} \pm \frac{A_3}{2(n\delta)^3} + \frac{A_4}{(n\delta)^4} \right].
\end{aligned} \tag{18}$$

3.5.4 Step 4: Test the correction

It is valuable to compare the corrected signals obtained as a result of Equation (15) by truncating at the second- and fourth-derivative term (or equivalently by Equations (16) or (17), respectively). If the spectral profiles obtained are 'similar', but with the profile using the fourth derivative being slightly more 'spiky' than that truncated at the second derivative, this is an indication that the correction is of good quality. If the spectral profile obtained using the fourth-derivative correction is particularly noisy in comparison with that using the second-derivative correction, then this is an indication that experimental noise is a problem.

3.5.5 Simplification for triangular bandpass functions

For a triangular bandpass function the moments and correction coefficients can be calculated analytically (see 11.5 for more details). In this case the corrected signals using the derivative approach and adjacent measurements are given by

$$C_0 = M_0 - \frac{\Delta^2}{12} M_0'' + \frac{\Delta^4}{240} M_0^{iv} + \dots \tag{18}$$

The second-order corrected signals for a triangular bandpass function can be calculated as

$$C_0 = \left[-\frac{\Delta^2}{12\delta^2} \right] M_{-1} + \left[1 + \frac{\Delta^2}{6\delta^2} \right] M_0 + \left[-\frac{\Delta^2}{12\delta^2} \right] M_{+1} \tag{19}$$

which, when the step size, δ , is equal to the bandwidth, Δ , simplifies further to

$$C_0 = \frac{-1}{12} M_{-1} + \frac{14}{12} M_0 + \frac{-1}{12} M_{+1}. \tag{20}$$

Similarly, the fourth-order correction for a triangular bandpass function when the step size is equal to the bandwidth can also be simplified to

$$C_0 = (2M_{-2} - 23M_{-1} + 222M_0 - 23M_{+1} + 2M_{+2}) / 180. \tag{21}$$

3.6 Limits to bandwidth correction for noise-free measurements

The bandwidth correction described in the previous clause is a powerful tool for correcting the measured spectral signals. However it is important to understand the limitations of such a correction. One limitation is due to the presence of experimental noise in the measured values; this is discussed in the next clause. It is important to understand that even in the absence of noise there is a limit to how well the true spectral signals can be recovered from the measured signals.

Any given spectrum can be described as the sum of a series of sine waves of successively higher frequencies (shorter periods) through the process of Fourier analysis where low and high frequencies correspond to spectral features that vary slowly and rapidly, respectively, with respect to wavelength. The measurement process does not record all of these sine components equally. As the period of the sine wave approaches the bandwidth, the system

response drops to a low value (zero in the case of a triangular response function) and recovery of the information becomes more difficult. In addition, the process of sampling the spectrum at discrete wavelengths can distort the measurement. Thus, in practice, even with perfect knowledge of the bandpass function and perfect noise-free measurements, it is not possible to obtain full correction of rapidly changing spectra (spectral features that are within a bandwidth). For line sources, the only information that can be determined is the power within a spectral band (see 3.3 above).

In 9.3 the ability of the correction algorithm to recover the frequency components of a spectrum is derived. The quality of correction takes the form of a 'modulation transfer function' (MTF) (see Figure 1). This graph, for a triangular response function and not including sampling effects, shows the ability of the correction algorithm to recover the true spectrum from the measured spectrum for different sine wave 'frequencies', where 'frequency' is expressed in units of inverse bandwidth. A peak amplitude of 1 indicates perfect correction. In the absence of noise, the fourth-order (5-point) correction, always performs better than the second-order (3-point) correction, and recovery is 'excellent' (amplitude recovery of 99,9 %) for frequencies below 0,2 (spectral features with a width of at least 5 times the bandwidth).

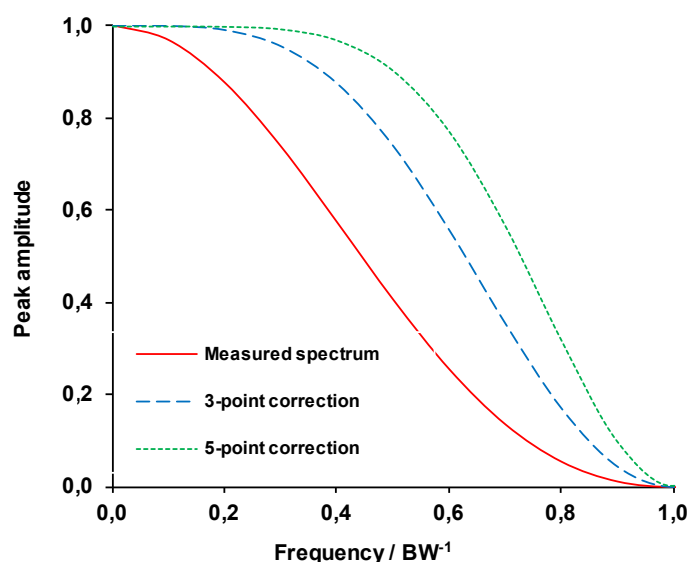


Figure 1 – Modulation transfer function analysis for measured and corrected signals, showing the peak amplitude recovery for periodic spectra of different frequencies

3.7 Limits to bandwidth correction in the presence of measurement noise

In a noise-free situation it is always beneficial to apply the correction. When there is measurement noise this is not always the case. When the measurement is obtained from the true spectrum, the bandpass has the effect of smoothing the spectrum – flattening out peaks, filling in troughs. Therefore to correct for bandwidth, the correction algorithm must rebuild the peaks and troughs, making the signal more 'spiky'. The effect of increasing the 'spikiness' means that spurious information contained in the measured signals due to random experimental noise will be translated into peaks and troughs by the correction algorithm. The correction algorithm relies on derivatives, and therefore differences between measurements spread over the bandwidth. The less the measured signal changes from one measurement wavelength to the next, the more sensitive will the correction algorithm be to measurement noise. This means that when the wavelength step used for correction is small (i.e. the changes in true signal between each correction step are small), noise will be far more significant.

Noise introduced by the correction process is correlated at the correction wavelengths. The significance of the increased noise¹ depends on the application. Where the aim is to determine quantities through spectral integration (photometric and colorimetric quantities, hazard irradiance, colour temperature, etc.), then this additional noise will be 'averaged out' both by accounting for the correlations and by the integration process (see 3.8 below) and usually it is still best to apply the correction. However, where the aim is to obtain spectral recovery of the true signals, then this noise can make the recovered spectrum appear worse than the measured spectrum, particularly for hyper-sampled data if the wavelength step for correction is too small; correlated effects are only taken into account when the measured values are combined.

Guidelines for determining the effect of bandpass correction are given in Clause 12. In a real measurement situation it is not possible to determine the impact of correction as the true spectrum is not known. However, by comparing the measurement and the second- and fourth-order corrected signals, it is possible to see whether the correction leads to an increase in noise or an improvement in the spectral shape. Applying a further convolution of the spectrum and the LSF can also give a guide to the effectiveness of correction, as described in Clause 12. Note that noise in the measurements to determine the bandpass function is much less critical. In the case of the bandpass function it is the moments that are used to determine the correction, unlike the situation for the measured signals where it is the derivatives that are used. The moments are integrals and integrals reduce noise while derivatives increase it.

3.8 Implication of bandwidth correction for the calculation of integrated quantities

Clause 13 looks at the impact of bandwidth and its correction on integrated quantities (quantities such as colorimetric or photometric values calculated from radiometric values, erythral irradiance, etc.).

Bandwidth has the effect of widening spectral features and thus effectively 'moves' signal from one wavelength to another. As a result integrated quantities based on a weighting function will be affected by bandwidth, and the effect can be significant if the bandwidth is wide. The bandwidth correction algorithm can be very successful at reducing this effect.

When the measured spectrum is noisy, the correction amplifies this noise. However, in determining the integrated quantities this noise amplification is smoothed out by the integration process. Therefore, even in the presence of noise, it is appropriate to apply the bandwidth correction² prior to calculating integrated quantities as the benefit of reducing the effect of bandwidth outweighs the additional random noise for integrated quantities.

3.9 When should bandwidth correction be applied and when not?

In general we would like to apply a correction when this improves the measured spectrum and not when it makes it worse. If the aim is spectrum recovery, then this depends on noise level, the shape of the true spectrum and the relation between the bandwidth and the wavelength interval. If the true spectrum is known to have very fine spectral features (e.g. a line source), then bandwidth correction will not be beneficial in the region of those features. Alternatively, if the true spectrum changes only slowly (e.g. such that a linear approximation can be applied within the bandwidth) with wavelength, then the measured spectrum is likely to be already an accurate representation of the source and any bandwidth correction may amplify any measurement noise. This is especially true in a comparison measurement (a test artefact measured by comparison to a reference artefact) where the test and reference artefacts have similar spectral profiles.

Bandwidth correction is most useful for relatively broad spectral features (e.g. LED peaks) and for situations where the test and reference artefacts have different spectral shapes (e.g. a

¹ Note, here 'noise' is assumed to be a spectrally random effect. Spectrally systematic (correlated) effects are the same with and without bandwidth correction. Random noise in the measured values becomes partly correlated in the corrected values.

² Note that for some applications the weighting functions have the bandwidth correction 'built in'. This is true for ASTM E308 (See 11.2.3). In this case additional bandwidth correction should not be performed.