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TECHNICAL REPORT

ABSOLUTE METHODS FOR REFLECTION MEASUREMENT

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Preface

This technical report has been prepared by the CIE Technical Committee on Materials (TC-2.3), and was a part of the program of the TC-2.3 Subcommittee on Standards and Techniques under the chairmanship of W. Erb of PTB. The prime author of the document was Dr. N. Ooba of the Electrotechnical Laboratory, Tanashi, Tokyo.

The list of members who have been on the Committee's roster during the period (1975-1979) are given below. Also listed, as consultants, are the members of the Subcommittee on Standards and Techniques who collaborate with the chairman in the preparation of the report.

It was not the intent of the Committee to compare various known methods of absolute reflectance measurement or to make a recommendation as to which method is preferred over the other. Such a recommendation would require extensive additional experimental work that could not be carried out at this time. The aim of the Committee was to describe the available methods known to the Committee and this is the best the Committee could do at this time.

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ABSOLUTE METHODS FOR REFLECTION MEASUREMENTSSummary

Measurements of reflectance or reflectance factor are made by comparison to appropriately chosen reference standards. In 1969 the CIE recommended that the perfect reflecting diffuser be taken as the ideal standard.

As there is no existing material which corresponds to the perfect diffuser, the problem is one of transforming measurements made with reference to real physical standards to those of absolute values relative to the perfect reflecting diffuser. The main interest of this report is the theory and procedures by which this conversion is made rather than the properties of the materials, the measuring instruments or the preparation of the samples. For this information the reader is requested to refer to the original papers.

The material has been classified into principles of measurement rather than quantities to be determined. These classifications are:

- Goniophotometric methods,
- Methods used in hemispherical irradiators,
- Methods using hemispherical or spheroidal mirrors,
- Methods using Kubelka-Munk theory,
- Methods based on the theory of integrating sphere.

The methods based on the theory of integrating sphere include: the two Taylor's methods, the Benford method, the Sharp-Little method, and the Double sphere method.

By compiling this information under a single cover, the practitioners of this type of measurement have thus an access to a ready reference on this complex subject.

METHODES ABSOLUES DE MESURES DE LA REFLEXIONRésumé

Les mesures de facteur de réflexion et de facteur de réflectance sont faites par comparaison avec des matériaux de référence convenablement choisis. En 1969, la CIE recommanda que le diffuseur parfait par réflexion soit pris comme référence idéale.

Puisqu'il n'existe pas de matériau qui corresponde au diffuseur parfait, le problème est celui de la transformation des mesures faites par référence à un étalon physique réel, en mesures relatives au diffuseur parfait par réflexion. L'intérêt principal de ce rapport est relatif à la théorie et aux méthodes par lesquelles cette conversion est faite, plutôt qu'aux propriétés des matériaux, aux instruments de mesure ou à la préparation des échantillons. En ce qui concerne ces derniers sujets le lecteur est prié de se référer aux publications originales.

Le sujet a été subdivisé selon les principes de mesure, plutôt que selon les grandeurs qui sont déterminées. Ces subdivisions sont:

Méthodes goniophotométriques,
Méthodes employant des diffuseurs hémisphériques,
Méthodes utilisant des miroirs sphériques ou sphéroïdaux,
Méthodes fondées sur la théorie de Kubelka et Munk,
Méthodes fondées sur la théorie des sphères d'intégration.

Les méthodes fondées sur la théorie des sphères d'intégration comprennent: les deux méthodes de Taylor, la méthode de Benford, la méthode Sharp-Little et la méthode de la double sphère.

En réunissant cette information dans un même document, les utilisateurs de ce type de mesures auront ainsi accès à une référence commode sur ce sujet complexe.

ABSOLUTE REFLEXIONS-MESSMETHODEN

Zusammenfassung

Bei Messungen des Reflexionsgrades oder des Reflexionsfaktors wird in der Praxis auf ausgewählte Reflexionsnormale bezogen. 1969 hat die CIE den vollkommen mattsweißen Körper als ideales Normal empfohlen.

Da es kein Material mit den Eigenschaften des vollkommen mattsweißen Körpers gibt, stellt sich die Aufgabe, die relativ zu verfügbaren Normalen erhaltenen Messergebnisse auf Absolutewerte, d.h. auf den vollkommen mattsweißen Körper zu beziehen. Das Hauptaugenmerk dieses Berichtes richtet sich auf die Theorie und die Methoden, mit denen absolute Reflexionswerte gemessen werden können; die Eigenschaften der Materialien, die Messgeräte und die Herstellung der Normale werden nur am Rande erwähnt. Für nähere Auskünfte sei der Leser auf die Originalarbeiten verwiesen.

Der Bericht gliedert sich nach dem Prinzip der Messverfahren wie folgt:

goniophotometrische Methoden,
Methoden mit halbkugeligen Strahlern,
Methoden mit halbkugeligen oder sphärischen Spiegeln,
Methoden, die auf der Kubelka-Munk Theorie beruhen.

Methoden, die auf der Theorie integrierender Kugeln aufbauen: Taylorsche Methoden, Benfordsche Methode, Sharp-Little Methode und Doppelkugelmethode.

Durch die Zusammenstellung der Wirkungsweise der einzelnen Methoden in einem Bericht erhält der auf diesem Gebiet arbeitende Praktiker einen Zugang zu einer geschlossenen Übersicht über ein komplexes Thema.

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1. Introduction

Measurements of diffusely reflecting characteristics of materials are of great importance as the basic measurements of illuminating engineering, and have become more and more requisite for many other fields of science, technology and industry that are concerned not only with human vision but also with various effects of every wavelength region of optical radiation on substances. In general, measurements of such characteristics as reflectance or radiance factor are made by means of reflectometers in comparison with those of reference standards appropriately chosen.

In connection with this, CIE recommended in 1931¹ that, in colorimetric measurements of opaque materials, smoked magnesium oxide was to be taken as the reference standard, but this is superseded by an ideal standard, i.e. the perfect reflecting diffuser since 1969.² In fact, there is no existing material corresponding to the perfect diffuser with sufficient approximation, so that the characteristics of the reference standards must be measured in absolute scale in order to correct the measured values of the sample to be tested when a high accuracy is required.

The purpose of this report is to survey the methods that are used for measuring the absolute* reflection values of such reference standards, by outlining the papers which have so far been published on this subject.

*Measurements that relate the reflection values of a material to that of the perfect reflecting diffuser are often called "absolute" measurements.

The characteristic under investigation is expressed as the ratio of the reflected radiant flux from the sample to be measured and the flux incident on it, but generally direct measurements of these two fluxes are not practicable. Therefore the relation is converted into a new relation between quantities which are practically measurable with a accuracy required. The main interests are then directed in this report to the theory and procedure in which the above conversion is made, and accordingly descriptions on other subjects such as the preparations or the properties of the samples and such as measuring instruments are mostly omitted. The reader who wants to study these subjects and the details of principles as well is requested to refer to the original papers. As regards the properties and the preparations of reference standards, several references are added to the reference list.

For the purpose of surveying the methods, it will be more convenient to classify them according to the principles of measurements rather than to classify according to the quantities to be determined, because a group of different quantities is sometimes measured on the basis of the same principle. Therefore, in this report the following classification is made:

- Goniphotometric methods
- Methods using hemispherical irradiators
- Methods using hemispherical or spheroidal mirrors
- Methods based on Kubelka-Munk theory
- Methods based on the theory of integrating sphere.

The formulas presented in this report are those given by the authors quoted. For reasons of consistency, however, changes were sometimes made in terminology and in the use of symbols. It is a matter of course that the reported theories are often controversial because they are based on assumptions and approximations. It was not the aim of this report to make refined studies on the theories but relate on them only as far as to seem necessary to understand the experimental procedures. None of the formulas or methods are warranted by CIE.

2. Terminology

In principle the terminology in this report refers to the CIE Vocabulary published in 1970.³ Some additional remarks are as follows: (1) Radiant and luminous quantities are used interchangeably hereinafter; papers published in early dates are often concerned only with luminous quantities, so that terms such as luminous flux or luminance factor are used here for these papers, whereas recent papers are mostly concerned with spectral quantities, sometimes including UV or IR regions, for which terms such as radiant flux or radiance factor are suitably used. Luminous quantities differ from radiant quantities only in the respect that they are to be measured with detectors equivalent to the CIE standard photometric observer, and this difference is not essential as far as this report is concerned.

(2) The term 'reflectance factor' is defined as the ratio of the radiant flux reflected in the directions delimited by a given cone with apex at a point of the surface under test to that reflected in the same directions by a perfect reflecting diffuser

identically irradiated. If the solid angle of the cone approaches zero, or 2π sr, the reflectance factor approaches radiance factor, or reflectance, respectively. In the strict sense, values measured with actual instruments are to be expressed as reflectance factor together with the indication of the cone, but in general, the term radiance factor or reflectance is used for simplicity, hence in this report. However, it should be noted that these terms must be used after due consideration for the deviations of actual geometry used in the measurement from the ideal geometry.

(3) The "perfect (reflecting) diffuser" is the ideal uniform diffuser with a reflectance equal to 1. It must be distinguished from the 'uniform diffuser' which has only the same radiance in all directions.

(4) Values of radiance factor or reflectance are strongly dependent on the geometry of irradiation and measurement. The following geometries are most important in the measurements of reflection standards.*

- | | |
|--------------------|----------------------|
| (1) normal/45° | (abbreviation, 0/45) |
| (2) 45°/normal | (abbreviation, 45/0) |
| (3) diffuse/normal | (abbreviation, d/0) |

*Conditions from (1) to (4) are recommended by CIE for the colorimetric specifications of opaque specimens, and in 0/45 and 0/d conditions specimens with mixed reflection should not be measured with strictly normal irradiation.⁴

- (4) normal/diffuse (abbreviation, 0/d)
 (5) diffuse/diffuse (abbreviation, d/d).

The first term indicates the geometry of irradiation, while the second term indicates that of measurement. The conditions (1), (2) and (3) give the radiance (luminance) factors abbreviated as $\beta_{0/45}$, $\beta_{45/0}$ and $\beta_{d/0}$ respectively, whereas conditions (4) and (5) give the reflectances abbreviated as $\rho_{0/d}$ and $\rho_{d/d}$, respectively.

3. Goniophotometric Methods

The absolute reflectance measurement is the measurement of the ratio of the total reflected luminous flux ϕ_r to the incident liminous flux ϕ_i :

$$\rho = \phi_r / \phi_i .$$

Let the luminous intensity of the reflecting surface be $I(\theta, \xi)$, where θ and ξ are the angle with surface normal and the azimuth angle respectively, then the reflected flux ϕ_r is expressed by

$$\phi_r = \int_0^{2\pi} d\xi \int_0^{\pi/2} I(\theta, \xi) \sin \theta d\theta .$$

Let the luminance factor of the sample in (θ, ξ) direction be $\beta(\theta, \xi)$, and the luminous intensity in the normal direction of the perfect diffuser identically illuminated by I_0 , then

$$I(\theta, \xi) = \beta(\theta, \xi) I_0 \cos \theta$$

and the incident flux ϕ_i is πI_0 , so that

$$\begin{aligned} \phi_r &= I_0 \int_0^{2\pi} d\xi \int_0^{\pi/2} \beta(\theta, \xi) \cos \theta \sin \theta d\theta, \text{ and} \\ \rho &= \frac{1}{\pi} \int_0^{2\pi} d\xi \int_0^{\pi/2} \beta(\theta, \xi) \cos \theta \sin \theta d\theta. \end{aligned} \quad (1)$$

From these equations it is seen that the goniophotometric measurement of $I(\theta, \xi)$ or $\beta(\theta, \xi)$ is the most fundamental method among various methods for absolute reflection measurements, since it can be applied to any sample having arbitrary angular distribution of I or β at least in principle, although it contains several difficulties in practical performance.*

As the materials usually used as reflectance standards approximate to uniform diffusers, I and β are dependent only on θ and not on ξ for normal incidence, so that the above equation (1) is simplified in this case as follows;

$$\rho = 2 \int_0^{\pi/2} \beta(\theta) \sin \theta \cos \theta d\theta. \quad (2)$$

*Apart from the absolute measurement, the goniophotometric method is indispensable to obtain the angular distribution of the reflected flux that is necessary to to analyze errors in all reflection measurements.

In the measurement of β , the most important cases are those under the conditions 0/45, 45/0 and d/0, because these conditions are the illuminating and viewing conditions for colorimetry recommended by the CIE,⁴ and the absolute value of β measured under one of these conditions is often used as the reference for β at other viewing angles under the same illuminating condition. Besides, the value $\beta_{d/0}$ gives the reflectance $\rho_{0/d}$ according to the reciprocity law.*

The luminous flux reflected from the sample to the directions in the vicinity of the direction of incidence cannot be measured, then its amount must be estimated when the diffuse reflectance is calculated. The error due to the uncertainty in this estimation must be examined, but it is usually small for approximately uniform diffusers.

Luminance factor $\beta(\theta, \xi)$ at a point on a sample is defined as $L(\theta, \xi)/L_0$, where $L(\theta, \xi)$ and L_0 are the luminance of the sample and that of the perfect reflecting diffuser identically illuminated.

*Erb⁵ compared the reflection values of pressed BaSO_4 tablets measured for four geometries 45/0, 0/45, d/0 and 0/d, and found that the values for the last two geometries showed a good agreement, while there were systematic discrepancies, that could not be explained so far, between the values for the first two geometries. Such discrepancies are sometimes found in literatures, but they cannot be understood theoretically.

Then

$$\beta = \pi q \Omega_0, \quad q = L/E \quad (3)$$

where q is the luminous coefficient, E the illuminance at that point F , F and Ω_0 the solid angle of magnitude of 1 steradian. This relation is often used as the basis of the measurement of β . A difficulty exists in the large ratio of the luminous flux incident on the photometer in the measurement of E to the luminous flux in a small solid angle in the measurement of the reflected luminance L . There have been several methods recorded in the literatures to solve this problem.

Preston and Gordon-Smith⁶ used three apertures of different areas in order to make both the readings of the photocell in the measurements of incident and reflected luminous flux nearly equal. As shown in Figure 1, first the smallest aperture X is placed at a distance ' a ' from a lamp and the photometer reading R_1 for the total luminous flux passing through it is taken. Then the aperture plate is removed and replaced by the sample to be measured in the same plane as the aperture X , and the photocell is placed to view the surface at 45° through another larger aperture Y . The sensitive area of the cell is defined by the third aperture Z and the distance ℓ between these two apertures is fixed and known. Thus the reading R_2 for the reflected flux limited by these apertures is taken. The ratio R_2/R_1 is expressed as

$$\frac{R_2}{R_1} = \beta \frac{YZ}{\pi d^2 X} \quad (4)$$

where β is the luminance factor, X , Y and Z are the areas of the apertures, and d is the diameter of a sphere passing through the perimeters of the apertures Y and Z . From this formula β can be obtained without reference to the luminous intensity of the lamp and the distance 'a'. Corrections due to the nonuniformities in the sensitivity over the cell surface and in the illuminance distribution over the sample surface as well are applied.

A logical development of the method of Preston and Gordon-Smith is described by Clarke, Lambert, Sparrock and Garforth.^{6a} The sample and a black glass mirror are interchangeably mounted on a slide so that either may be brought into the parallel beam from a collimating mirror having the irradiating lamp at its focus. The black glass is used as an attenuator to help bridge the gap between the incident and diffusely reflected radiant fluxes, and as a mirror to permit minimum changes of lay-out in measuring these two fluxes. It is calibrated in the same way as any specular reflector, by evaluating the flux in a beam of radiation both before and after reflection from the glass.

The glass provides an attenuation of 25 times which, together with that of a sector in front of the irradiating lamp, provides a full reduction of the incident radiation, during its measurement, down to 1/2500 to give a detector reading close to that obtained when measuring the radiation reflected diffusely from the sample.

The reflected radiation from the black mirror is collected by a special collector unit of mirrors which then forms a reduced image of the lamp filament entirely within the entrance slit of a monochromator. With the sample in place the whole of the slit will be irradiated. Equal fractions of radiation passing the slit are then transmitted to the detector, resulting in readings R_1 and R_2 respectively. The spectral radiance factor for the 45/0-geometry is then given by

$$\beta_{45/0}(\lambda) = \frac{R_2(\lambda)}{R_1(\lambda)} \frac{\tau \rho(\lambda) \pi d^2}{S \cdot n^2 \cdot \cos 45^\circ} \quad (5)$$

where τ is the transmittance of the sector, $\rho(\lambda)$ the reflection factor of the black mirror, d the focal length of the concave mirror of the collector unit and S the area of the entrance slit of the monochromator, and n is the refractive index of the material composing the collector unit.

There are thus two photometric ratios τ and $\rho(\lambda)$, and two dimensional quantities d and S to be determined.

Morren⁷ used a photocell to measure illuminances over a wide range, that is, the illuminance E on the sample and the illuminance E' on a plane P located at a distance from an opal glass, which was illuminated by a lamp from the rear side as shown in Figure 2. The luminance of the opal glass is adjusted to be equal to L , which is the luminance of the sample in given direction, by means of a high precision luminance meter, and L can be calculated from the following equation

$$E' = L \frac{a_e a_r}{4R^2 S_r} \quad (6)$$

where R is the radius of a sphere to which the apertures S_e and S_r are inscribed, and a_e and a_r are the areas of caps of the sphere having these two apertures as the bases. The calibration of the cell to measure the large ratio E/E' is eventually done by the adoption of the inverse square law and, if necessary, by making use of additional neutral filters. By using Eq. (3), he obtained from E and L the luminance factors of opal glasses and pressed powder tablets.⁵⁹ The special feature of this method is that the luminance measurement is replaced by the illuminance measurement, which is more basic and easier to perform.

Kartashevskaya and Skachkova⁸ used (see Fig. 3), as the illuminator, a BaSO_4 plate which was contained in a hollow box of mat, white walls and illuminated uniformly by two projector lamps. In front of the illuminator an aperture of known diameter d_0 is placed. A photocell combined with apertures and lenses gives rise to photocurrent i_x and i_0 when it is illuminated by the sample (Fig. 3a) and by the illuminator (Fig. 3b) respectively, and the luminance factors $\beta_{0/45}$ and $\beta_{45/0}$ are calculated from the equation

$$\beta = \frac{i_x}{i_0} \frac{4\ell^2 \tau}{d_0^2 \cos \zeta} \quad (7)$$

where ℓ is the distance between the source and the sample, ξ the angle of incidence on the sample and τ the transmittance of a rotating sector which is used to make i_x/i_0 nearly unity. They measured MgO and BaSO₄ samples and gave a table for comparison among the results obtained in NRC, NPL and VNIIM. It was concluded that the lack of agreement among the results might be due to actual difference in the materials measured, in the technique involved in preparing plates, as well as errors in the measurement.

Korte and Schmidt⁹ measured the luminous intensity I of the sample at various angles and that of the source I_0 illuminating the sample. The luminance factor β is obtained as $\beta = \pi \Omega_0 L/E$, where L is the sample luminance calculated from the luminous intensity and the projected area of the sample, and E is the illuminance on the sample calculated from the luminous intensity of the source and the distance between the source and the sample. They determined the luminance factor $\beta_{0/45}$ and the reflectance $\rho_{0/d}$ by integration of β . The relation between the luminous intensity of the sample (≈ 0.05 cd) and that of the source (~ 200 cd) is mediated through a stable source (≈ 1 cd). The results obtained were 0.3 to 0.4 percent higher than those obtained by them using a hemispherical irradiator (sphere) mentioned in Chap. 4.

For the measurement of the radiance factor in 45/0 geometry, Erb¹⁰ used a method similar to that of Kartashevskya and Skachkova mentioned above. A small integrating sphere¹¹ is used as the irradiator, in which a lamp and a reflector are mounted, and its radiance is uniform to better than ± 0.1 percent over the radiating area. Radiances of the irradiator and the sample irradiated by

it can then be measured with the same instrument. As the ratio of two radiances amounts to about 10^4 , a second irradiator which is similar to the above irradiator and has a radiance of about 10^2 times as large as that of the sample is used to avoid linearity errors of the photoelectric detector used. The irradiance E on the sample is calculated from the measured radiance and the area of the irradiator, the distance between the irradiator and the sample, and the angle (45°) between the sample normal and the direction of incidence. The radiance factor $\beta_{45/0}$ is obtained from the relation $\beta = \pi \Omega_0 L/E$ between the radiance L and the irradiance E of the sample. Some results on freshly pressed BaSO_4 are reported, and compared with the results obtained in the measurements of the radiance factor in $d/0$ geometry.⁹

Brockes and Helm¹² evaluated the influence of the aperture angle of observation on the luminance factor measured in $45/0$ geometry from the results of goniophotometric measurements with a very narrow aperture angle, and found that the change in the luminance factor was much less than 0.005 for the aperture angles between 0° and 10° for most of 25 samples.

In the goniophotometric method, the geometry is in general asymmetrical, so that the effect of polarization of radiation on the luminance factor must be taken into consideration. It has recently been made clear that even mat surfaces used as the diffuse reflectance standards are not perfect depolarizers and that reflected radiation from such surfaces is more or less polarized even if incident radiation is unpolarized, so that one must take care to avoid the polarization effect by the use of a

photometer insensitive to the direction of polarization. The polarization characteristics are studied by Billmeyer et al.¹³ and Kaase¹⁴ on BaSO₄ samples, and by Carmer and Bair,¹⁵ Robertson,¹⁶ and Clarke et al.¹⁷ on MgO, BaSO₄ and other reflecting diffusers.

4. Methods Using Hemispherical Irradiators

The geometry in these methods is that the source subtends the maximum solid angle of 2π at the sample and has a luminance uniformly distributed over its whole extent, so that the sample is illuminated under completely diffusing conditions, and viewed at 0° or other angles through a small opening in the source. The illuminance E on the sample is calculated by integrating the luminance L_0 of the source multiplied by $\cos\theta \, d\omega$, where θ is the angle with the sample normal and $d\omega$ the elementary solid angle with its apex at the sample, over the solid angle 2π . Then

$$E = \pi\Omega_0 L_0 \quad (\Omega_0 = 1 \text{ sr}), \text{ that is, } E/(\pi\Omega_0) = L_0.$$

If the perfect diffuser is put in place of the sample, its luminance is equal to that of the source, because the luminance of the perfect diffuser is equal to $E/\pi\Omega_0$. Then the luminance factor $\beta_{d/0}$ or $\beta_{d/\theta}$ of the sample is obtained by

$$\beta = \frac{L}{L_0}, \quad (8)$$

where L is the luminance of the sample in the direction concerned. In the measurement of white samples this ratio is so close to unity that the requirement for linearity of the photometer is

lightened, but a difficulty exists in realization of a hemispherical source having a very uniform luminance. Again, the effect of the opening from which no luminous flux is incident on the sample must be taken into account.

McNicholas¹⁸ measured the luminance factor of a sample at several fixed angles of observation under diffuse illumination. As shown in Figure 4, an opal glass hemisphere was illuminated by a large number of lamps situated between the outer surface of this sphere and a concentric hemispherical reflector smoked with MgO. The sample is located at the center of the hemisphere, and small holes are provided in both the hemispheres at selected intervals on a meridian so that the luminance $L(\theta)$ may be measured with a specially designed photometer from the outside of the larger hemisphere. The inner wall of the source is viewed through a hole in the base of the hemisphere, and the ratio $L/L_0 = \beta$ is determined by means of a variable sector disk photometer. The reflectance $\rho_{0/d}$ was calculated from $\beta_{d/0}$ by use of the reciprocity law.

Tingwaldt¹⁹ constructed an instrument which followed in principle the method of McNicholas. In his instrument, an integrating sphere is used and the sample is placed vertically at its center and rotated about the vertical diameter so that it may be viewed at any angle desired from the outside of the sphere through an aperture in the wall. The source is located over the sample inside the sphere and gives a uniform luminance to the sphere walls which irradiate the sample from 2π space.