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EIA STANDARD

EIA 61649

Weibull Analysis

EIA 61649
(IEC 61649:2008 Ed.2.0, IDT)

May 2017



Electronic Components Industry Association

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WEIBULL ANALYSIS

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International Standard IEC 61649 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition, published in 1997, and constitutes a technical revision.

The main changes with respect to the previous edition are as follows:

- the title has been shortened and simplified to read "Weibull analysis";
- provision of methods for both analytical and graphical solutions have been added.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/1269/FDIS	56/1281/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

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INTRODUCTION

The Weibull distribution is used to model data regardless of whether the failure rate is increasing, decreasing or constant. The Weibull distribution is flexible and adaptable to a wide range of data. The time to failure, cycles to failure, mileage to failure, mechanical stress or similar continuous parameters need to be recorded for all items. A life distribution can be modelled even if not all the items have failed.

Guidance is given on how to perform an analysis using a spreadsheet program. Guidance is also given on how to analyse different failure modes separately and identify a possible weak population. Using the three-parameter Weibull distribution can give information on time to first failure or minimum endurance in the sample.



WEIBULL ANALYSIS

1 Scope

This International Standard provides methods for analysing data from a Weibull distribution using continuous parameters such as time to failure, cycles to failure, mechanical stress, etc.

This standard is applicable whenever data on strength parameters, e.g. times to failure, cycles, stress, etc. are available for a random sample of items operating under test conditions or in-service, for the purpose of estimating measures of reliability performance of the population from which these items were drawn.

This standard is applicable when the data being analysed are independently, identically distributed. This should either be tested or assumed to be true (see IEC 60300-3-5).

In this standard, numerical methods and graphical methods are described to plot data, to make a goodness-of-fit test, to estimate the parameters of the two- or three-parameter Weibull distribution and to plot confidence limits. Guidance is given on how to interpret the plot in terms of risk as a function of time, failure modes and possible weak population and time to first failure or minimum endurance.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary – Part 191: Dependability and quality of service*

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 61810-2, *Electromechanical elementary relays – Part 2: Reliability*

ISO 2854:1976, *Statistical interpretation of data – Techniques of estimations and tests relating to means and variances*

ISO 3534-1:2006, *Statistics – Vocabulary and symbols – Part 1: General statistical terms and terms in probability*

3 Terms, definitions, abbreviations and symbols

For the purposes of this document, the definitions, abbreviations and symbols given in IEC 60050-191 and ISO 3534-1 apply, together with the following.

3.1 Terms and definitions

3.1.1

censoring

terminating a test after either a given duration or a given number of failures

NOTE A test terminated when there are still unfailed items may be called a “censored test”, and test time data from such tests may be referred to as “censored data”.

3.1.2

suspended item

item upon which testing has been curtailed without relevant failure

NOTE 1 The item may not have failed, or it may have failed in a mode other than that under investigation.

NOTE 2 An "early suspension" is one that was suspended before the first failure. A "late suspension" is suspended after the last failure.

3.1.3

life test

test conducted to estimate or verify the durability of a product

NOTE The end of the useful life will often be defined as the time when a certain percentage of the items have failed for non-repairable items and as the time when the failure intensity has increased to a specified level for repairable items.

3.1.4

non-repairable item

item that cannot, under given conditions, after a failure, be returned to a state in which it can perform as required

NOTE The given conditions may be technical, economic, ecological and/or others.

3.1.5

operating time

time interval for which the item is in an operating state

NOTE "Operating time" is generic, and should be expressed in units appropriate to the item concerned, e.g. calendar time, operating cycles, distance run, etc. and the units should always be clearly stated.

3.1.6

relevant failure

failure that should be included in interpreting test or operational results or in calculating the value of a reliability performance measure

NOTE The criteria for inclusion should be stated.

3.1.7

reliability test

experiment carried out in order to measure, quantify or classify a reliability measure or property of an item

NOTE 1 Reliability testing is different from environmental testing where the aim is to prove that the items under test can survive extreme conditions of storage, transportation and use.

NOTE 2 Reliability tests may include environmental testing.

3.1.8

repairable item

item that can, under given conditions, after a failure, be returned to a state in which it can perform as required

NOTE The given conditions may be technical, economic, ecological and/or others.

3.1.9

time to failure

operating time accumulated from the first use, or from restoration, until failure

NOTE In applications where the time in storage or on standby is significantly greater than "operating time", the time to failure may be based on the time in the specified service.

3.1.10**time between failures**

time duration between consecutive failures

NOTE 1 The time between failures includes the up time and the down time.

NOTE 2 In applications where the time in storage or on standby is significantly greater than operating time, the time to failure may be based on the time in the specified service.

3.1.11**B life****L percentiles**

age at which a given percentage of items have failed

NOTE "B₁₀" life is the age at which 10 % of items (e.g. bearings) have failed. Sometimes it is denoted by the L (life) value. B lives may be read directly from the Weibull plot or determined more accurately from the Weibull equation. The age at which 50 % of the items fail, the B₅₀ life, is the median time to failure.**3.2 Abbreviations**

ASIC	application specific integrated circuit
BGA	ball grid array
CDF	cumulative distribution function
PDF	probability density function
MLE	maximum likelihood estimation
MRR	median rank regression
MTTF	mean time to failure

3.3 Symbols

t	time – variable
η	Weibull characteristic life or scale parameter
β	Weibull shape parameter
t_0	starting point or origin of the distribution, failure free time
r^2	coefficient of determination
$f(t)$	probability density function
$F(t)$	cumulative distribution function
$h(t)$	hazard function
$\lambda(t)$	instantaneous failure rate
$H(t)$	cumulative hazard function
F_1	number of failures with failure mode 1
F_2	number of failures with failure mode 2
F_3	number of failures with failure mode 3

4 Application of the techniques

Table 1 shows the circumstances in which particular aspects of this standard are applicable. It shows the three main methods for estimating parameters from the Weibull distribution, namely graphical, computational and WeiBayes, and indicates the type of data requirements for each of these three methods.

Table 1 – Guidance for using IEC 61649

Method/ Kinds of data	Graphical methods	Computational methods	WeiBayes
Interval censored	√	NC	√
Multiple censored	√	NC	√
Singly censored	√	√	√
Zero failures	NC	NC	√
Small sample (≤20)	√	NC	√
Large sample	√	√	NC
Curved data	√	NC	NC
Complete data	√	√	√
NOTE NC means not covered in this standard.			

5 The Weibull distribution

5.1 The two-parameter Weibull distribution

The two-parameter Weibull distribution is by far the most widely used distribution for life data analysis. The Weibull probability density function (PDF) is shown in Equation (1):

$$f(t) = \beta \cdot \frac{t^{\beta-1}}{\eta^{\beta}} \cdot e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad (1)$$

where

t is the time, expressed as a variable;

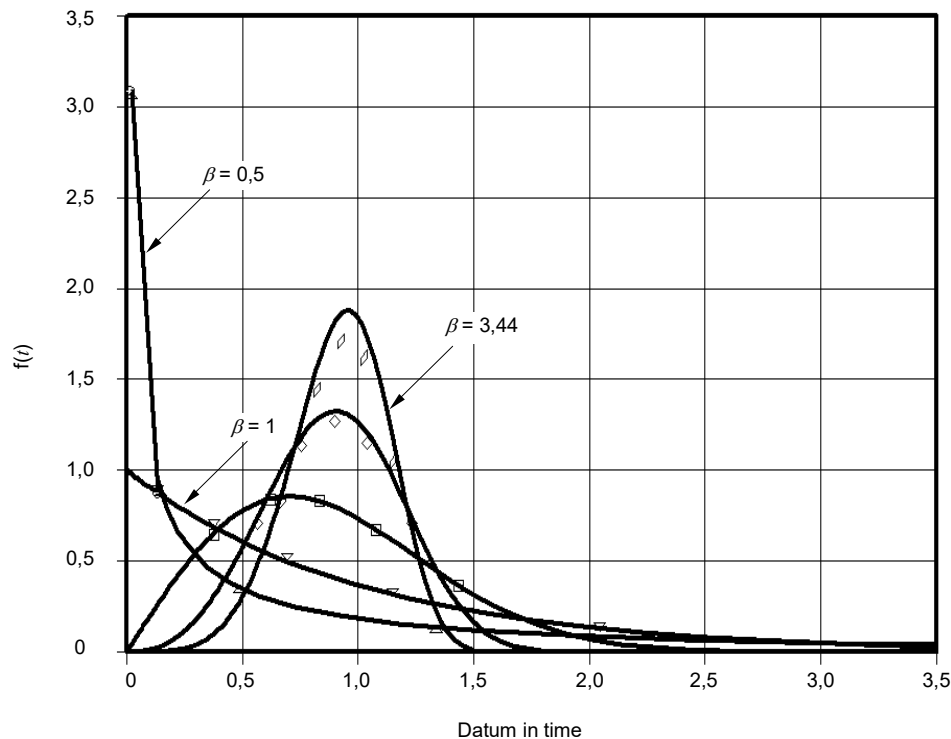
η is the characteristic life or scale parameter;

β is the shape parameter.

The Weibull cumulative distribution function (CDF) has an explicit equation as shown in Equation (2):

$$F(t) = 1 - e^{-(t/\eta)^{\beta}} \quad (2)$$

The two parameters are η , the characteristic life, and β , the shape parameter. The shape parameter indicates the rate of change of the instantaneous failure rate with time. Examples include: infant mortality, random or wear-out. It determines which member of the Weibull family of distributions is most appropriate. Different members have widely different shaped PDFs (see Figure 1). The Weibull distribution fits a broad range of life data compared with other distributions. The variable t is generic and can have various measures such as time, distance, number of cycles or mechanical stress applications.



IEC 1321/08

Figure 1 – The PDF shapes of the Weibull family for $\eta = 1,0$

From Figure 1, the PDF shape for $\beta = 3,44$ (indicated) looks like the normal distribution: it is a fair approximation, except for the tails of the distribution.

The instantaneous failure rate $\lambda(t)$ (or $h(t)$, the hazard function) of the two-parameter Weibull distribution is shown in Equation (3):

$$\lambda(t) = h(t) = \beta \cdot \frac{t^{\beta-1}}{\eta^\beta} \quad (3)$$

Three ranges of values of the shape parameter, β , are salient:

- for $\beta = 1,0$ the Weibull distribution is identical to the exponential distribution and the instantaneous failure rate, $\lambda(t)$, then becomes a constant equal to the reciprocal of the scale parameter, η ;
- $\beta > 1,0$ is the case of increasing instantaneous failure rate; and
- $\beta < 1,0$, is the case of decreasing instantaneous failure rate.

Characteristic life, η , is the time at which 63,2 % of the items are expected to fail. This is true for all Weibull distributions, regardless of the shape parameter, β . If there is replacement of items, then 63,2 % of the times to failure are expected to be lower or equal to the characteristic life, η . Further discussion of the issues concerning repair and non-repairable items can be found in IEC 60300-3-5. The 63,2 % comes from setting $t = \eta$ in Equation (2) which results in Equation (4):

$$F(\eta) = 1 - e^{-(\eta/\eta)^\beta} = 1 - e^{-(1)^\beta} = 1 - (1/e) = 0,632 \quad (4)$$

5.2 The three-parameter Weibull distribution

Equation (5) shows the CDF of the three-parameter Weibull distribution:

$$F(t) = 1 - e^{-\left(\frac{t-t_0}{\eta}\right)^\beta} \quad (5)$$

The parameter t_0 is called the failure-free time, location parameter or minimum life.

The effect of location parameter is typically not understood well until a poor fit is observed with a 2-parameter Weibull plot. When a lack of fit is observed, engineers attempt to use other distributions that may provide them with a better fit. However, the lack of fit can be reconciled when the data is plotted with a 3-parameter Weibull distribution (see 8.5). Using the location parameter, it becomes evident that the product failures are offset by a fixed period of time, called the threshold. The effect of location parameter is normally observed when a product sees “shelf-life” after which the first failure occurs. A good indicator of the effect of a location parameter is the convex shape of a plot.

6 Data considerations

6.1 Data types

Life data are related to items that “age” to failure. Weibull failure data are usually life data but may also describe material data where the “aging” may be stress, force or temperature. “Age” may be operating time, starts and stops, landings, takeoffs, low-cycle fatigue cycles, mileage, shelf or storage time, cycles or time at high stress or high temperature, or many other continuous parameters. In this standard the “age” parameter will be called time. When required, “time” can be substituted by any of the “age” parameters listed above.

6.2 Time to first failure

The Weibull “time” variable is usually considered to be a measure of life consumption. The following interpretations can be used:

- time to first failure of a repairable item;
- time to failure of a non-repairable item;
- time from new to each failure of a repairable system if a non-repairable item in the system fails more than once during the period of observation. It has to be assumed that the repair (change of the item) does not introduce a new failure, so that the system after the repair can, with an approximation, be regarded as having the same reliability as immediately before the failure (commonly referred as the “bad as old” assumption);
- time to first failure of a non-repairable item, following scheduled maintenance, with the assumption that the failure is related to the previous maintenance.

6.3 Material characteristics and the Weibull distribution

Material characteristics such as creep, stress rupture or breakage and fatigue are often plotted on Weibull probability paper. Here the horizontal scale may be stress, cycles, load, number of load repetitions or temperature.

6.4 Sample size

Uncertainty with regard to the Weibull parameter estimation is related to the sample size and the number of relevant failures. Weibull parameters can be estimated using as few as two failures; however, the uncertainty of such an estimate would be excessive and could not

confirm the applicability of the Weibull model. Whatever the sample size, confidence limits should be calculated and plotted in order to assess the uncertainty of the estimations.

As with all statistical analysis, the more data that is available, the better the estimation but if the data set is limited, then refer to the advice given in 11.3.

6.5 Censored and suspended data

When analysing life data it is necessary to include data on those items in the sample that have not failed, or have not failed by a failure mode analysis. This data is referred to as censored or suspended data (see IEC 60300-3-5). When the times to failure of all items are observed, the data are said to be complete.

An item on test that has not failed by the failure mode in question is a suspension or censored item. It may have failed by a different failure mode or not failed at all. An "early suspension" is one that was suspended before the first failure time. A "late suspension" is suspended after the last failure. Suspensions between failures are called random or progressive suspensions.

If items remain unfailed, then the corresponding data are said to be censored. If a test is terminated at a specified time, T , before all items have failed, then the data are said to be time censored. If a test is terminated after a specified number of failures have occurred, then the data are said to be failure censored.

Further discussion of censoring is covered in IEC 60300-3-5.

7 Graphical methods and goodness-of-fit

7.1 Overview

Graphical analysis consists of plotting the data on Weibull probability paper, fitting a line through the data, interpreting the plot and estimating the parameters using special probability paper derived by transforming the Weibull equation into a linear form. This is illustrated in Annex I.

Data is plotted after first organizing it from earliest to latest, a process called ranking. The time to failure data are plotted as the X coordinate on the Weibull probability paper.

The Y coordinate is the median rank as specified in 7.2.1. For sample sizes above 30 the median rank is, in practice, the same as the per cent of failures. If the plotted data follow a linear trend, a regression line may be drawn.

The parameters may then be read off the plot. The characteristic life, η , is the time to 63,2 % of the items failing, called the "B63,2 Life". The shape parameter, β , is estimated as the slope on Weibull paper.

Median rank regression (MRR) is a method for estimating the parameters of the distribution using linear regression techniques with the variables being the median rank and lifetime or stress, etc.

Another graphical method that is used for estimating parameters of a Weibull distribution is called hazard plotting. This is described in 7.3.

7.2 How to make the probability plot

In order to make a probability plot, a sequence of steps needs to be carried out. These steps are described in detail below.

7.2.1 Ranking

To make the Weibull plot, rank the data from the lowest to the highest times to failure. This ranking will set up the plotting positions for the time, t , axis and the ordinate, $F(t)$, in percentage values. These will provide information for the construction of the Weibull line shown in Equation (6).

Median ranks are given in Annex C. Enter as an example the tables for 50 % median rank, for a sample size of five, and find the median ranks shown in Table 2 for five failure times shown in the middle column. The median rank plotting positions in Annex C are used with all types of probability paper, i.e. Weibull, log-normal, normal, and extreme value.

NOTE 1 If two data points have the same time, they are plotted at different median rank values.

Table 2 – Ranked flare failure rivet data

Order number I	Failure time t min (X)	Median rank % (Y)
1	30	12,94
2	49	31,38
3	82	50,00
4	90	68,62
5	96	87,06

The median estimate is preferred to the mean or average value for non-symmetrical distributions. Most life data distributions are skewed and, therefore, the median plays an important role.

If a table of median ranks and a means to calculate median ranks using the Beta distribution is not available, then Benard's approximation, Equation (6), may be used:

$$F_i = \frac{(i-0,3)}{(N+0,4)} \% \quad (6)$$

where N is the sample size and i is the ranked position of the data item of interest.

NOTE 2 This equation is mostly used for $N \leq 30$; for $N > 30$ the correction of the cumulative frequency can be neglected: $F_i = (i/N) \times 100 \%$.

7.2.2 The Weibull probability plot

After transforming the data, the plot can be constructed using three different methods:

- Weibull probability paper – Annex F shows Weibull probability paper;
- a computer spreadsheet program – Annex E gives a spreadsheet example;
- commercial off-the-shelf software.

7.2.3 Dealing with suspensions or censored data

Non-failed items or items that fail by a different failure mode are "censored" or "suspended" items, respectively. These data cannot be ignored. The times on suspended items have to be included in the analysis.

The formula below gives the adjusted ranks without the need for calculating rank increments. It is used for every failure and requires an additional column for reverse ranks. The procedure

is to rank the data with the suspensions and to use Equation (7) to determine the ranks, adjusted for the presence of the suspensions.

$$\text{Adjusted rank} = \frac{(\text{Reverse rank}) \times (\text{Previous adjusted rank}) + (N + 1)}{(\text{Reverse rank}) + 1} \quad (7)$$

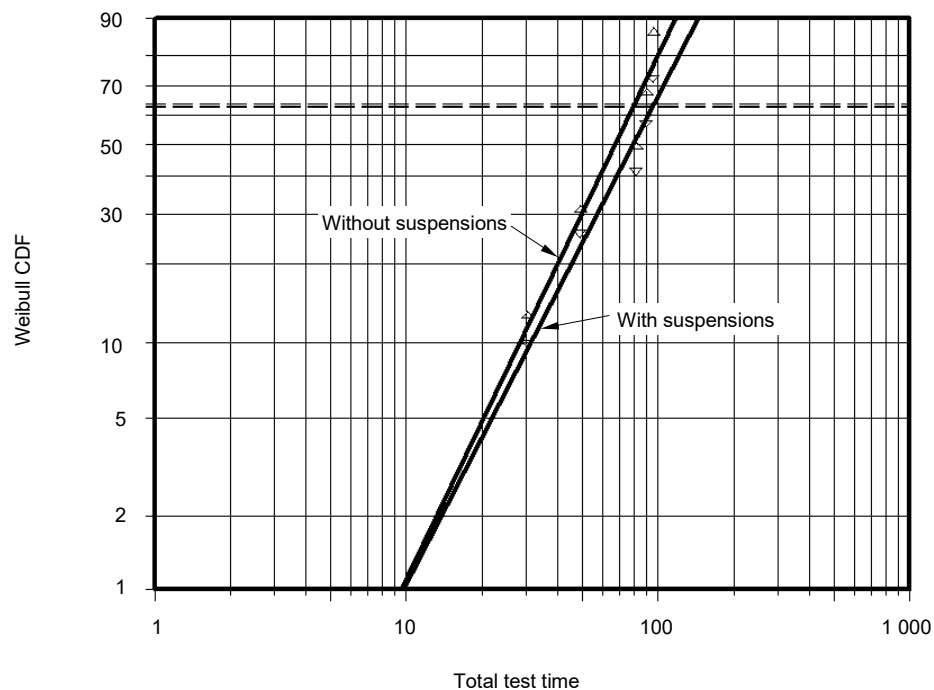
The rank order numbers are adjusted for the effect of the three suspended items in Table 3.

Table 3 – Adjusted ranks for suspended or censored data

Rank	Time	Status	Reverse rank	Adjusted rank	Median rank %
1	10	Suspension	8	Suspended...	
2	30	Failure	7	$[7 \times 0 + (8+1)] / (7+1) = 1,125$	9,8
3	45	Suspension	6	Suspended...	
4	49	Failure	5	$[5 \times 1,125 + (8+1)] / (5+1) = 2,438$	25,5
5	82	Failure	4	$[4 \times 2,438 + (8+1)] / (4+1) = 3,750$	41,1
6	90	Failure	3	$[3 \times 3,750 + (8+1)] / (3+1) = 5,063$	56,7
7	96	Failure	2	$[2 \times 5,063 + (8+1)] / (2+1) = 6,375$	72,3
8	100	Suspension	1	Suspended...	

In this example, the adjusted ranks use Benard's approximation to calculate the median ranks as it is easier than interpolating in the table. The results in Table 3 are plotted in Figure 2.

NOTE If two items fail at the same age, they are assigned sequential rank order numbers. In case of later suspension, the procedure is to be repeated for the rest of the failures.



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Figure 2 – Total test time (in minutes)

Benard's approximation for the median rank is sufficiently accurate when using suspension for plotting Weibull distributions and estimating the parameters. Here " i " is the adjusted rank and " N " is the sum of failures and suspensions. The median ranks are converted to percentages for plotting on Weibull paper. For example, for the first failure in Table 3 with an adjusted rank of 1,125:

$$\text{Median Rank (\%)} = \frac{(1,125 - 0,3)}{(8 + 0,4)} \times 100 = 9,82\% \quad (8)$$

Figure 2 shows the correct Weibull plot as it includes the suspensions.

The following are the steps to plot data sets with suspensions:

- rank the times, both failures and suspensions, from earliest to latest;
- calculate the adjusted ranks for the failures (suspensions are not plotted);
- use Benard's approximation to calculate the median ranks;
- plot the failure times (x) versus the median ranks (y) on Weibull paper;
- estimate η by reading the B63,2 life from the plot;
- estimate β with a ruler or use special beta scales usually given on the Weibull probability paper;
- interpret the plot.

7.2.4 Probability plotting

Plotting the data on Weibull paper by hand or on a computer may be sufficient for checking the goodness-of-fit. Subjectively fitting a straight line by eye can give an indication of goodness-of-fit.

7.2.5 Checking the fit

If the data cluster around a straight line on a probability plot, it is evidence that the data is represented by the subject distribution. However, small samples make it difficult to gauge the goodness-of-fit. There are statistical measures of goodness-of-fit such as Chi-squared, Kolmogorov-Smirnoff and Nancy Mann's tests. This standard uses the correlation coefficient squared called the coefficient of determination.

This can be calculated using Equation (9):

$$r^2 = \frac{\left(\sum_{i=1}^N x_i y_i - \frac{\sum_{i=1}^N x_i \sum_{i=1}^N y_i}{N} \right)^2}{\left(\sum_{i=1}^N x_i^2 - N(\bar{x})^2 \right) \left(\sum_{i=1}^N y_i^2 - N(\bar{y})^2 \right)} \quad (9)$$

where x_i and y_i are the median rank and the failure time, respectively, $\bar{x}$ and $\bar{y}$ are averages of x_i and y_i and N is the sample size.

r^2 is the proportion of variation in the data that can be explained by the Weibull hypothesis. The closer this is to 1, the better the data are fitted to a Weibull distribution; the closer to 0 indicates a poor fit. The correlation coefficient, " r ," is intended to measure the strength of the fit.

linear relationship between two variables. "r" is a number between -1 and +1, depending on the slope. Alternatively, if the Weibull plot is constructed using a spreadsheet, then when using linear regression techniques the correlation coefficient is often given as part of the output when data are fitted with a straight line (usually a selective option). Similarly, commercial software packages provide the coefficient of determination.

This should be used with care and only if visual inspection concurs with observation.

7.3 Hazard plotting

Weibull probability plotting techniques first estimate the cumulative proportion failed, $F(t)$, using median ranks, and then plot times to failure against respective estimated cumulative probabilities on the Weibull probability paper.

The hazard plotting technique begins with the estimation of the instantaneous failure rate or the hazard function,

$$\lambda(t) = h(t) = \frac{f(t)}{[1 - F(t)]} , \quad (10)$$

using an estimate of the cumulative hazard function.

The cumulative hazard function,

$$H(t) = \int_0^t h(t)dt = -\ln[1 - F(t)] , \quad (11)$$

is estimated by the cumulative sum of the estimated hazard functions.

For the Weibull distribution,

$$H(t) = \left(\frac{t}{\eta}\right)^\beta , \quad (12)$$

taking natural logarithms of both sides, yields the following linear relationship for $\ln H(t)$ against $\ln t$:

$$\ln(H(t)) = \beta \ln(t) - \beta \ln(\eta) \quad (13)$$

The Weibull hazard paper is therefore ln-ln paper. The nominal slope of the fitted line to the data is β , and $t = \eta$ when $H(t) = 1$.

NOTE Either natural logs or base 10 logs can be used for Weibull hazard paper.

Though the Weibull hazard paper is available for some countries, this technique can be used with the usual Weibull probability paper applying the transformation:

$$F(t) = 1 - e^{-H(t)} \quad (14)$$

This can be very easily implemented using a spreadsheet program.

The hazard plotting procedure is as follows:

- sort the times, both failures and suspensions simultaneously, from earliest to latest;
- for each failure, calculate the instantaneous failure rate, given by $1/(\text{number of items remaining after the previous failure or censoring})$;
- for each failure, calculate the cumulative sum of instantaneous failure rates, as an estimate of the hazard function;
- plot the estimated cumulative hazards against the time of failure on either log-log paper or ln-ln paper;
- fit a straight line to the plots;
- estimate the parameters.

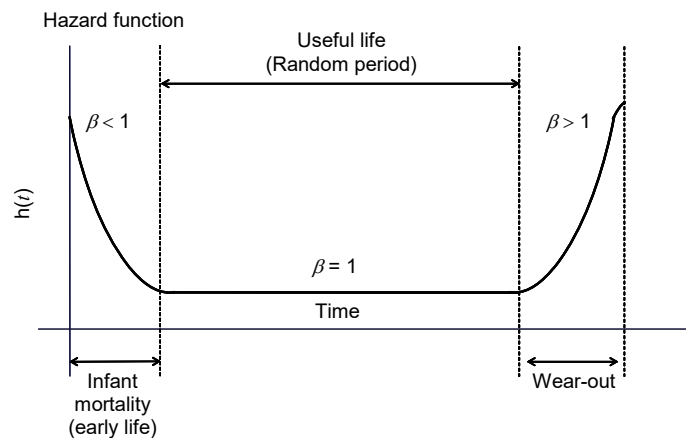
Annex E gives worked examples. IEC 61810-2 also provides examples of using hazard plotting to estimate the parameters of a Weibull distribution.

8 Interpreting the Weibull probability plot

8.1 The bathtub curve

8.1.1 General

The often-used bathtub curve (see Figure 3) shows the relationship between the Weibull shape parameter, β , and the hazard function throughout the life of an item. Not all items, however, display all elements of the bathtub curve during their lifetimes.



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Figure 3 – Typical bathtub curve for an item

8.1.2 $\beta < 1$ – Implies early failures

Both electronic and mechanical systems may initially have high failure rates. Manufacturers conduct production process control, production acceptance tests, "burn-in," or reliability stress screening (RSS), to prevent early failures before delivery to the customer. Therefore, shape parameters of less than one indicate the following:

- lack of adequate process control;
- inadequate burn-in or stress screening;
- production problems, mis-assembly, poor quality control;

- overhaul problems;
- mixture of populations;
- run-in or wear-in.

Many electronic components during their useful life show a decreasing instantaneous failure rate, thus featuring shape parameters less than 1. Preventive maintenance on such a component is not appropriate, as old parts are better than new.

8.1.3 $\beta = 1$ – Implies constant instantaneous failure rate

This is often called the random failure period as the failures occur randomly in time. These failure modes are considered independent of time. In this case, preventive maintenance would not improve the system.

Therefore, any of the following might be suspected:

- random maintenance errors, human errors;
- random overload;
- failures due to nature, foreign object damage, lightning strikes;
- mixtures of data from three or more failure modes (assuming they have different values of β) where no failure mechanism dominates the failure behaviour.

Here again, preventive maintenance is not appropriate. The Weibull distribution with $\beta = 1$ is identical to the exponential distribution. Of those that survive to time, t , a constant percentage fails in the next period of time. This is known as a constant instantaneous failure rate.

8.1.4 $\beta > 1$ – Implies wear-out

Some typical examples of these cases are as follows:

- wear;
- corrosion;
- crack propagation;
- fatigue;
- moisture absorption;
- diffusion;
- evaporation (weight loss);
- damage accumulation.

Design measures have to ensure that those phenomena do not significantly contribute to the probability of product failure during the expected operational life.

Three-parameter Weibull distribution estimates minimum time to first failure, which is highly advantageous in cases where the shape parameter is greater than 1 (see 5.2 or 8.5).

8.2 Unknown Weibull modes may be "masked"

The "masked" phenomenon might be encountered when there are two or more competing failure modes with high values of shape parameters and highly different scale parameters. This means that for a small sample size, most or all of the samples would fail in the failure mode with lower scale parameter. The other failure modes may not be identified until the first failure mode is eliminated. An example of two competing failure modes is shown in Figure 4.