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Power Law Model - Goodness-of-Fit Tests and Estimation Methods

EIA 61710
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May 2017



Electronic Components Industry Association

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

POWER LAW MODEL – GOODNESS-OF-FIT TESTS AND ESTIMATION METHODS

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International Standard IEC 61710 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition, published in 2000, and constitutes a technical revision.

The main changes with respect to the previous edition are listed below:

- the inclusion of an additional Annex C on Bayesian estimation for the power law model.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/1500/FDIS	56/1508/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
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INTRODUCTION

This International Standard describes the power law model and gives step-by-step directions for its use. There are various models for describing the reliability of repairable items, the power law model being one of the most widely used. This standard provides procedures to estimate the parameters of the power law model and to test the goodness-of-fit of the power law model to data, to provide confidence intervals for the failure intensity and prediction intervals for the length of time to future failures. An input is required consisting of a data set of times at which relevant failures occurred, or were observed, for a repairable item or a set of copies of the same item, and the time at which observation of the item was terminated, if different from the time of final failure. All output results correspond to the item type under consideration.

Some of the procedures can require computer programs, but these are not unduly complex. This standard presents algorithms from which computer programs should be easy to construct.



POWER LAW MODEL – GOODNESS-OF-FIT TESTS AND ESTIMATION METHODS

1 Scope

This International Standard specifies procedures to estimate the parameters of the power law model, to provide confidence intervals for the failure intensity, to provide prediction intervals for the times to future failures, and to test the goodness-of-fit of the power law model to data from repairable items. It is assumed that the time to failure data have been collected from an item, or some identical items operating under the same conditions (e.g. environment and load).

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

3 Terms and definitions

For the purposes of this document, the terms and definitions of IEC 60050-191 apply.

4 Symbols and abbreviations

The following symbols and abbreviations apply:

β	shape parameter of the power law model
$\hat{\beta}$	estimated shape parameter of the power law model
β_{LB}, β_{UB}	lower, upper confidence limits for β
C^2	Cramer-von-Mises goodness-of-fit test statistic
$C_{1-\gamma}^2(M)$	critical value for the Cramer-von-Mises goodness-of-fit test statistic at γ level of significance
χ^2	Chi-square goodness-of-fit test statistic
$\chi_{\gamma}^2(\nu)$	γ th fractile of the χ^2 distribution with ν degrees of freedom
d	number of intervals for groups of failures
$E[N(t)]$	expected accumulated number of failures up to time t
$E[t_j]$	expected accumulated time to j th failure

$\hat{E}[N[t(i)]]$	estimated expected accumulated number of failures up to $t(i)$
$\hat{E}[t_j]$	estimated expected accumulated time to j th failure
$F_\gamma(\nu_1, \nu_2)$	γ th fractile for the F distribution with (ν_1, ν_2) degrees of freedom
i	general purpose indicator
j	general purpose indicator
k	number of items
L, U	multipliers used in calculation of confidence intervals for failure intensity
λ	scale parameter of the power law model
$\hat{\lambda}$	estimated scale parameter of the power law model
M	parameter for Cramer-von-Mises statistical test
N	number of relevant failures
N_j	number of failures for j th item
$N(t)$	accumulated number of failures up to time t
$N[t(i)]$	accumulated number of failures up to time $t(i)$
R	difference between the order number of future (predicted) failure and order number of last (observed) failure
T	accumulated relevant time
T^*	total accumulated relevant time for time terminated test
T_j	total accumulated relevant time for j th item
T_{RL}, T_{RU}	lower, upper prediction limits for the length of time to the R th future failure
$\hat{T}_{N+1}$	estimated median time to $(N+1)$ th failure
t_i	accumulated relevant time to the i th failure
t_{ij}	i th failure time for j th item
t_N	total accumulated relevant time for failure terminated test
t_{Nj}	total accumulated relevant time to N th failure of j th item
$t(i-1), t(i)$	endpoints of i th interval of time for grouped failures
$z(t)$	failure intensity at time t
$\hat{z}(t)$	estimated failure intensity at time t
z_{LB}, z_{UB}	lower, upper confidence limits for failure intensity

5 Power law model

The statistical procedures for the power law model use the relevant failure and time data from the test or field studies. The basic equations for the power law model are given in this clause. Background information on the model is given in Annex A and examples of its application are given in Annex B.

The expected accumulated number of failures up to test time t is given by:

$$E[N(t)] = \lambda t^\beta \text{ with } \lambda > 0, \beta > 0, t > 0$$

where

λ is the scale parameter;

β is the shape parameter ($0 < \beta < 1$ corresponds to a decreasing failure intensity; $\beta = 1$ corresponds to a constant failure intensity; $\beta > 1$ corresponds to an increasing failure intensity).

The failure intensity at time t is given by:

$$z(t) = \frac{d}{dt} E[N(t)] = \lambda \beta t^{\beta-1} \text{ with } t > 0$$

Thus the parameters λ and β both affect the failure intensity in a given time.

Methods are given in 7.2 for maximum likelihood estimation of the parameters of λ and β . Subclause 7.3 gives goodness-of-fit tests for the model and 7.4 and 7.5 give confidence interval procedures. Subclause 7.6 gives prediction interval procedures and 7.7 gives tests for the equality of the shape parameters. The model is simple to evaluate. However when $\beta < 1$, theoretically $z(0) = \infty$ (i.e. $z(t)$ tends to infinity as t tends to zero) and $z(\infty) = 0$ (i.e. $z(t)$ tends to zero as t tends to infinity); but this theoretical limitation does not generally affect its practical use.

6 Data requirements

6.1 General

6.1.1 Case 1 – Time data for every relevant failure for one or more copies from the same population

The normal evaluation methods assume the observed times to be exact times of failure of a single repairable item or a set of copies of the same repairable item. The figures below illustrate how the failure times are calculated for three general cases.

6.1.2 Case 1a) – One repairable item

For one repairable item observed from time 0 to time T , the relevant failure time, t_i , is the elapsed operating time (that is, excluding repair and other down times) until the occurrence of the i -th failure as shown in Figure 1.

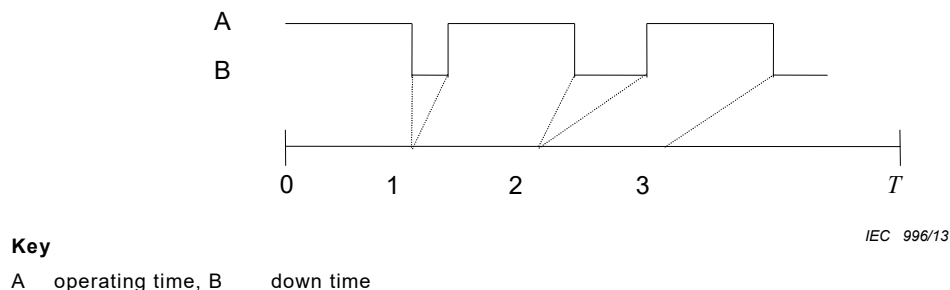
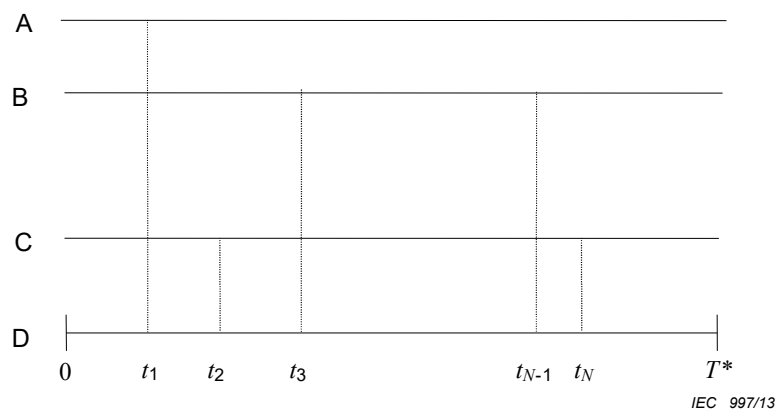


Figure 1 – One repairable item

Time terminated data are observed to T^* , which is not a failure time, and failure terminated data are observed to t_N , which is the time of the N th failure. Time terminated and failure terminated data use slightly different formulae.

6.1.3 Case 1b) – Multiple items of the same kind of repairable item observed for the same length of time

It is assumed there are k items, which all represent the same population. That is, they are nominally identical items operating under the same conditions (e.g. environment and load). When all items are observed to time T^* , which is not a failure time (i.e. time terminated data), then the failure time data are combined by superimposing failure times $(t_i, i = 1, 2, \dots, N)$ for all k items on the same time line as shown in Figure 2.



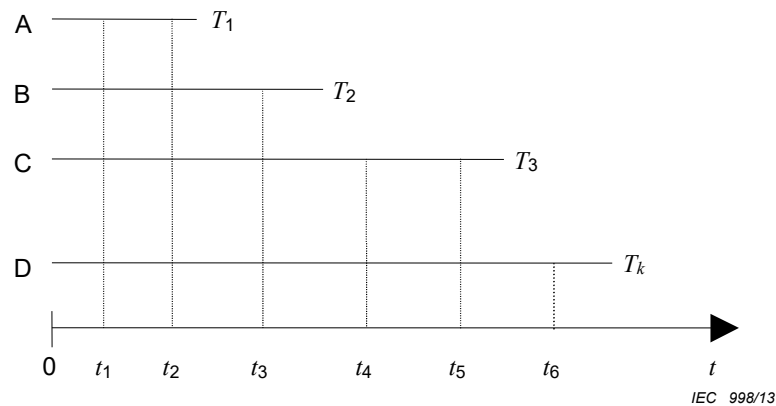
Key

- A item 1
- B item 2
- C item k
- D superimposed process

Figure 2 – Multiple items of the same kind of repairable item observed for same length of time

6.1.4 Case 1c) – Multiple repairable items of the same kind observed for different lengths of time

When all items do not operate for the same period of time, then the time at which observation of the j th item is terminated $T_j (j = 1, 2, \dots, k)$, where $T_1 < T_2 < \dots < T_k$, is noted. The failure data are combined by superimposing all the failure times for all k items on the same time line as shown in Figure 3. The times to failure are $t_i, i = 1, 2, \dots, N$, where N = the total number of failures observed accumulated over the k items.



Key

- A item 1
- B item 2
- C item 3
- D item k
- t time

Figure 3 – Multiple repairable items of the same kind observed for different lengths of time

If each item is a software system then the repair action should be done to the other systems which did not fail at that time.

6.2 Case 2 – Time data for groups of relevant failures for one or more repairable items from the same population

This alternative method is used when there is at least one copy of an item and the data consist of known time intervals, each containing a known number of failures.

The observation period is over the interval $(0, T)$ and is partitioned into d intervals at times $0 < t(1) < t(2) < \dots < t(d)$. The i th interval is the time period between $t(i-1)$ and $t(i)$, where $i = 1, 2, \dots, d$, $t(0) = 0$ and $t(d) = T$. It is important to note that the interval lengths and the number of failures per interval need not be the same.

6.3 Case 3 – Time data for every relevant failure for more than one repairable item from different populations

It is assumed there are k items which do not represent the same population and are to be compared. It should be noted that if each item is to be considered individually then it is appropriate to use case 1a) in 6.1.2.

If direct comparisons of the items are to be made then as an extension of 6.1 the following notation is used:

- t_{ij} denotes the i th failure time for the process corresponding to the j th item;
- N_j denotes the number of failures observed for the j th item;
- t_{N_j} is the time of the N th failure for the j th item;

where $i = 0, 1, 2, \dots, N_j$ and $j = 1, 2, \dots, k$.

7 Statistical estimation and test procedures

7.1 Overview

In case 1 – time data for every relevant failure – the formulae given for failure terminated data assume one repairable item, that is $k = 1$. All output results correspond to that item. The formulae given for time terminated data assume k copies of the item observed for the same length of time. If there is only one repairable item then $k = 1$. The point estimation procedures for all the aforementioned cases are given in 7.2.1. The appropriate procedures for the case when all copies are observed for different lengths of time are given in 7.2.2. Procedures for the case of time data for groups of relevant failures are given in 7.2.3.

An appropriate goodness-of-fit test, as described in 7.3 shall be performed after the parameter estimation procedures of 7.2. Note that these tests, and the procedures given in 7.4 to 7.7 for constructing interval estimates and carrying out statistical tests, distinguish only between the cases of time data for every relevant failure (i.e. all instances of case 1 data – 1a), 1b) and 1c)) and time data for groups of relevant failures (i.e. case 2)).

The inference procedures that follow provide approximate estimates in some circumstances and so caution is required if they are to be applied if the number of observed failures is less than 10.

7.2 Point estimation

7.2.1 Case 1a) and 1b) – Time data for every relevant failure

This method applies only when the time of failure has been logged for every failure as described in 6.1.2 and 6.1.3.

Step 1: Calculate the summation:

$$S_1 = \sum_{j=1}^N \ln \left(\frac{T^*}{t_j} \right) \quad (\text{time terminated})$$

$$S_2 = \sum_{j=1}^N \ln \left(\frac{t_N}{t_j} \right) \quad (\text{failure terminated})$$

Step 2: Calculate the (unbiased) estimate of the shape parameter β from the formula:

$$\hat{\beta} = \frac{N-1}{S_1} \quad (\text{time terminated})$$

$$\hat{\beta} = \frac{N-2}{S_2} \quad (\text{failure terminated})$$

Step 3: Calculate the estimate of the scale parameter λ from the formula:

$$\hat{\lambda} = \frac{N}{k \left(T^* \right)^{\hat{\beta}}} \quad (\text{time terminated})$$

$$\hat{\lambda} = \frac{N}{k(t_N)^{\hat{\beta}}} \quad (\text{failure terminated})$$

Step 4: Calculate the estimate of the failure intensity $z(t)$, for any time $t > 0$, from the formula:

$$\hat{z}(t) = \hat{\lambda} \hat{\beta} t^{\hat{\beta}-1}$$

$\hat{z}(t)$ estimates the current failure intensity for t over the range represented by the data. "Extrapolated" estimates for a future t may be obtained similarly, but should be used with the usual caution associated with extrapolation.

Step 5: Given N observed failures the last of which occurred at t_N , the median time to the $(N+1)$ th failure can be estimated from the formula:

$$\hat{T}_{N+1} = t_N \exp \left[\frac{0.5 \frac{-1}{N+1} - 1}{N\hat{\beta} / (N-1)} \right] \quad (\text{time terminated})$$

$$\hat{T}_{N+1} = t_N \exp \left[\frac{0.5 \frac{-1}{N+1} - 1}{N\hat{\beta} / (N-2)} \right] \quad (\text{failure terminated})$$

7.2.2 Case 1c) – Time data for every relevant failure

This method applies only when the time of failure has been logged for every failure as described in 6.1.4.

Step 1: Assemble the data into the times to failure, t_i , $i=1,2,...,N$, where N is the total number of failures over the k copies and T_j , $j=1,2,...,k$, is the end of the observation period for the j th copy.

Step 2: The maximum likelihood estimate of the shape parameter β is the value of $\hat{\beta}$ which satisfies the formula:

$$\frac{N}{\hat{\beta}} + \sum_{i=1}^N \ln t_i - \frac{N \sum_{j=1}^k T_j^{\hat{\beta}} \ln T_j}{\sum_{j=1}^k T_j^{\hat{\beta}}} = 0$$

An iterative method shall be used to solve the formula for $\hat{\beta}$.

Step 3: Calculate the estimate of the scale parameter λ from the formula:

$$\hat{\lambda} = \frac{N}{\sum_{j=1}^k T_j^{\hat{\beta}}}$$

Step 4: Calculate the estimate of the failure intensity $z(t)$, for any time $t > 0$, from the formula:

$$\hat{z}(t) = \hat{\lambda} \hat{\beta} t^{\hat{\beta}-1}$$

$\hat{z}(t)$ estimates the current failure intensity for t over the range represented by the data. "Extrapolated" estimates for a future t may be obtained similarly, but should be used with the usual caution associated with extrapolation.

7.2.3 Case 2 – Time data for groups of relevant failures

This method applies when the data set consists of known time intervals, each containing a known number of failures as described in 6.2.

Step 1: Assemble into a data set the number of relevant failures N_i recorded in the i th interval $[t(i-1), t(i)]$, $i = 1, 2, \dots, d$. The total number of relevant failures is

$$N = \sum_{i=1}^d N_i$$

Step 2: The maximum likelihood estimate of the shape parameter β is the value of $\hat{\beta}$ which satisfies the formula:

$$\sum_{i=1}^d N_i \left[\frac{[t(i)]^{\hat{\beta}} \ln t(i) - [t(i-1)]^{\hat{\beta}} \ln t(i-1)}{[t(i)]^{\hat{\beta}} - [t(i-1)]^{\hat{\beta}}} - \ln t(d) \right] = 0$$

Note that $[t(0)]^{\hat{\beta}} = 0$ and $[t(0)]^{\hat{\beta}} \ln t(0) = 0$. All $t(\cdot)$ terms may be normalized with respect to $t(d)$ and then the final term $\ln[t(d)]$ disappears. An iterative method shall be used to solve the formula for $\hat{\beta}$.

Step 3: Calculate the estimate of the scale parameter λ from the formula:

$$\hat{\lambda} = \frac{N}{t(d)^{\hat{\beta}}}$$

Step 4: Calculate the estimate of the failure intensity $z(t)$, for any test time $t > 0$, from the formula:

$$\hat{z}(t) = \hat{\lambda} \hat{\beta} t^{\hat{\beta}-1}$$

$\hat{z}(t)$ estimates the current failure intensity for t over the range represented by the data. "Extrapolated" estimates for a future t may be obtained similarly, but should be used with the usual caution associated with extrapolation.

7.3 Goodness-of-fit tests

7.3.1 Case 1 – Time data for every relevant failure

7.3.1.1 Cramer-von-Mises test

Step 1: Calculate $\hat{\beta}$ from step 2 in 7.2.1 or step 2 in 7.2.2.

Step 2: Calculate the Cramer-von-Mises goodness-of-fit test statistic given by the formula:

$$C^2 = \frac{1}{12M} + \sum_{j=1}^M \left[\left(\frac{t_j}{T} \right)^{\hat{\beta}} - \left(\frac{2j-1}{2M} \right) \right]^2$$

where

$$\begin{aligned} M = N \text{ and } T = T^* & \quad (\text{time terminated}) \\ M = N - 1 \text{ and } T = t_N & \quad (\text{failure terminated}) \end{aligned}$$

Step 3: Select the critical value $C_{0.90}^2(M)$ for the Cramer-von-Mises test corresponding to M from Table 1, which gives critical values at a 10 % significance level.

Step 4: If:

$$C^2 > C_{0.90}^2(M)$$

then the hypothesis that the power law model fits the data cannot be accepted. Otherwise, on the basis of the data analysed, the power law model can be used as a working hypothesis.

7.3.1.2 Graphical procedure

When the failure times are known, the graphical procedure described below may be used to obtain additional information about the correspondence between the model and the data. This involves plotting the expected time to the j th failure, $E(t_j)$, against the observed time to the j th failure. Further details about the approach are given in Annexes A and B.

Step 1: Calculate $\hat{\beta}$ from step 2 in 7.2.1 and $\hat{\lambda}$ from step 3 in 7.2.1.

Step 2: Calculate the estimate of the expected time to the j th failure, $j=1,2,\dots,N$, from the formula:

$$\hat{E}(t_j) = \left(\frac{j}{\hat{\lambda}} \right)^{\frac{1}{\hat{\beta}}}$$

Step 3: Plot $\hat{E}(t_j)$ against t_j on identical linear scales. The visual agreement of these points with a line of 45 ° through the origin is a subjective measure of the applicability of the model.

7.3.2 Case 2 – Time data for groups of relevant failures

7.3.2.1 Chi-square test

Step 1: Calculate $\hat{\beta}$ from step 2 in 7.2.3 and $\hat{\lambda}$ from step 3 in 7.2.3.

Step 2: Calculate the expected number of failures in the time interval $[t(i-1), t(i)]$ which is approximated by:

$$e_i = \hat{\lambda} \left\{ [t(i)]^{\hat{\beta}} - [t(i-1)]^{\hat{\beta}} \right\}$$

Step 3: For each interval, e_i shall not be less than 5, and if necessary, adjacent intervals should be combined before the test. For d intervals (after combination if necessary) and with N_i the same as in 7.2.3, calculate the statistic:

$$\chi^2 = \sum_{i=1}^d \frac{(N_i - e_i)^2}{e_i}$$

Step 4: Select the critical value from a χ^2 distribution with $(d-2)$ degrees of freedom and a 10 % significance level from Table 2, i.e. $\chi_{0,90}^2(d-2)$.

Step 5: If the test statistic χ^2 exceeds the critical value $\chi_{0,90}^2(d-2)$ then the hypothesis that the power law model fits the data cannot be accepted. Otherwise, on the basis of the data analysed, the power law model can be used as a working hypothesis.

The Chi-square test is a large sample test and so will need large data sets to detect deviations from the power law model that are practically important.

7.3.2.2 Graphical procedure

When the data set consists of known time intervals, each containing a known number of failures, the graphical procedure described below may be used to obtain additional information about the correspondence between the model and the data. This involves plotting the expected number of failures against those observed at each endpoint. Further details of the approach are given in B.5.

Step 1: For each endpoint $t(i)$, calculate the observed number of failures from 0 to $t(i)$ from the formula:

$$N[t(i)] = \sum_{j=1}^i N_j$$