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Sound power radiation from isotropic plates

Associated software: ESDUpac A0002
ESDUpac B0002

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SOUND POWER RADIATION FROM ISOTROPIC PLATES

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1. NOTATION

Averages are generally indicated by a bar, *e.g.* frequency averaged power $\overline{P}_0$ and modal averaged radiation efficiency $\overline{\sigma}$. However, spatial averages are indicated by brackets, *e.g.*, the mean square velocity, $\langle v^2 \rangle$. Vectors are indicated by bold letters, *e.g.* $\boldsymbol{r}_s$.

		Units	
		SI	*British*
a	length of plate in x-direction, $a \geq b$	m	in
b	width of the plate in y-direction, $b \leq a$	m	in
c_0	phase velocity of sound in fluid	m/s	in/s
$c_{B,x}$	phase velocity of bending waves in x-direction	m/s	in/s
$c_{B,y}$	phase velocity of bending waves in y-direction	m/s	in/s
c_B	phase velocity of bending waves ($= \sqrt[4]{D/\rho h}\ \sqrt{\omega}$)	m/s	in/s
D	bending stiffness of plate ($= E\ h^3 / [12(1-\nu^2)]$)	N m	lbf in
d	distance between two parallel plates	m	in
E	Young's modulus of plate	N/m^2	lbf/in^2
E_i	Young's modulus of plate 'i'	N/m^2	lbf/in^2
F	force	N	lbf
F_{line}	line force	N/m	lbf/in
$F(\boldsymbol{r}_s)$	force in position $\boldsymbol{r}_s$	N	lbf
f	frequency	Hz	Hz
f_c	coincidence frequency, see Equation (4.1)	Hz	Hz
$f_{c,i}$	coincidence frequency of plate 'i'	Hz	Hz
$f_{c,m}$	intermediate coincidence frequency, see Equation (8.4)	Hz	Hz
$f_{m,n}$	natural frequency of mode (m, n)	Hz	Hz
f_0	reference frequency	Hz	Hz
Δf	frequency bandwidth	Hz	Hz
H	function, see Figure 1		

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h	thickness of plate	m	in
h_i	thickness of plate 'i'	m	in
h_r	height of rib	m	in
i, j	integers		
k_0	wave number for fluid ($= \omega / c_0$)	m^{-1}	in^{-1}
L	length of one-dimensional acoustical system	m	in
l_i	length of side i	m	in
l_p	length of adjoining plate, see Sketch 6.6	m	in
m	mode number in x-direction		
N	number of modes, see Equation (3.6)		
n	mode number in y-direction		
n_i	modal density of subsystem 'i'	modes/Hz	modes/Hz
$n(f)$	modal density	modes/Hz	modes/Hz
P_0	radiated sound power	W	ft lbf/s
P_{ref}	reference radiated sound power, see Equation (8.3)	W	ft lbf/s
$p(\boldsymbol{r}_0)$	sound pressure in position $\boldsymbol{r}_0$	Pa	lbf/in^2
Q	correction, see Appendix A		
Q_e	correction for corner modes		
Q_{est}	estimated correction		
Q_ε	correction due to edge constraint, see Figure 2		
Q_φ	correction due to angle, φ, see Sketch 6.5		
$Q_{\varphi,\varepsilon}$	correction for combined influence of edge constraint and angle		
q	volume source	m^3/s	in^3/s
$q(\boldsymbol{r}_0)$	volume source in position $\boldsymbol{r}_0$	m^3/s	in^3/s
$\boldsymbol{r}_0$	vector to position in fluid	m	in
$\boldsymbol{r}_s$	vector to position on surface of plate	m	in
S	area of plate	m^2	in^2
U	circumference of plate ($= 2\ (a + b)$)	m	in

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V	volume of room	m^3	in^3
v	velocity	m/s	in/s
$v(\boldsymbol{r}_s)$	velocity in position $\boldsymbol{r}_s$	m/s	in/s
$\langle v^2 \rangle$	mean square velocity, see Equation (3.2)	m^2/s^2	in^2/s^2
w	displacement of plate	m	in
x	coordinate in x-direction, see Sketch 2.1	m	in
y	coordinate in y-direction, see Sketch 2.1	m	in
α, β	angles, see Equation (5.1)	rad	rad
γ	ratio of phase velocities, see Equation (5.4)		
ε	compliance of rotational spring along edges	N^{-1}	lbf^{-1}
ε_{avg}	average compliance of rotational spring	N^{-1}	lbf^{-1}
η	loss factor		
η_0	loss factor of fluid in cavity between two plates		
η_i	loss factor of plate 'i'		
η_{ij}	coupling loss factor from subsystem 'i' to subsystem 'j'		
θ	angle of propagation	rad	rad
λ_0	wave length of fluid (= c_0/f)	m	in
λ_B	wave length of bending wave (= c_B/f)	m	in
$\lambda_{B,x}$	wave length of bending wave in x-direction	m	in
$\lambda_{B,y}$	wave length of bending wave in y-direction	m	in
ν	Poisson's ratio of plate		
ν_i	Poisson's ratio of plate 'i'		
ξ	angle of propagation of bending waves	rad	rad
φ	angle of baffle, see Sketch 2.1	rad	rad
φ_{avg}	average angle of baffle, see Equation (6.16)	rad	rad
φ_i	angle of side i of baffle	rad	rad
ρ	density of plate	kg/m^3	lb/in^{3*}
ρ_0	density of fluid	kg/m^3	lb/in^{3*}

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ρ_i	density of plate '*i*'	kg/m^3	lb/in^{3*}
σ	radiation efficiency, see Equation (3.1)		
σ	modal averaged radiation efficiency		
$\overline{\sigma}_f$	modal averaged radiation efficiency for forced response		
$\overline{\sigma}_{ss}$	modal averaged radiation efficiency for simply supported plate		
σ_γ	radiation efficiency, see Appendix A		
χ, ψ	angles, see Equation (5.2)	rad	rad
ω	radian frequency ($= 2\pi f$)	rad/s	rad/s

* A density value expressed in British units as pounds per cubic inch has to be devided by 386.4 before it can be used in this Item.

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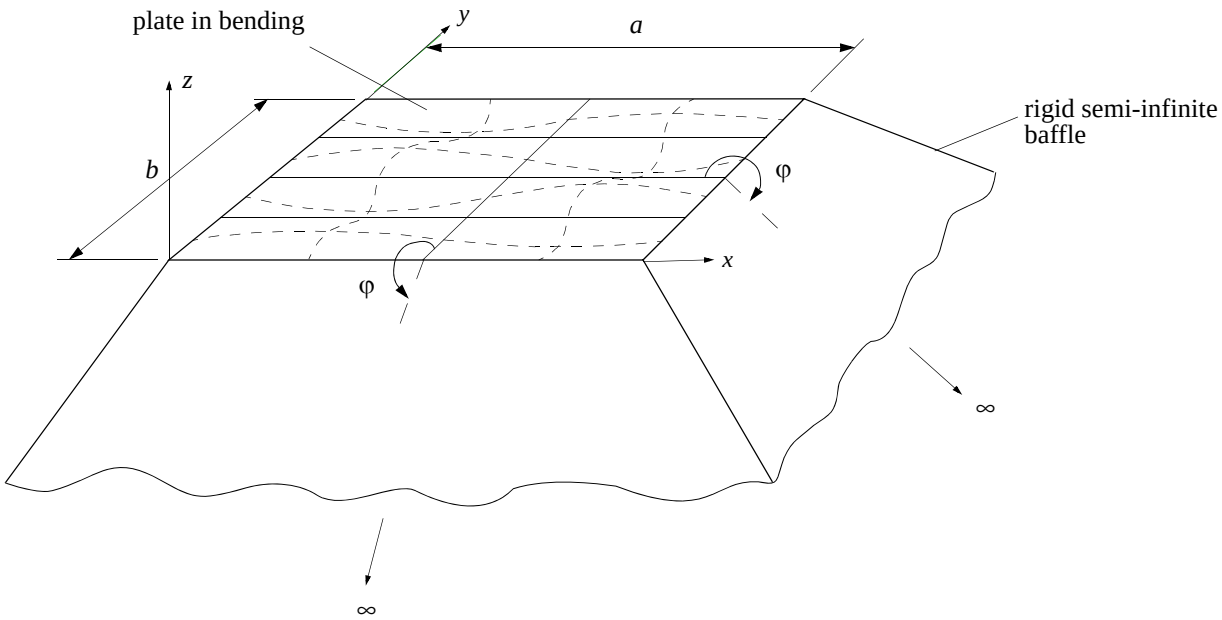
2. INTRODUCTION

Bending waves in plates can set the surrounding fluid into motion and by this action cause sound power to be radiated away from the plate into the fluid.

This Item describes how rectangular isotropic plates radiate sound power into a three-dimensional space. Two plate configurations are considered: a single plate and a double plate.

The ability of a single plate to radiate sound power is described by its radiation efficiency which depends on the properties of the plate and of the fluid, the boundary conditions and the adjacent structural elements such as ribs and adjoining plates. In the mathematical model for calculation of the radiation efficiencies, the nearby surrounding structure of the plate is modelled as a semi-infinite rigid baffle and the plate is assumed to radiate sound into a three-dimensional semi-infinite space. The system is shown in Sketch 2.1. Such a plate will have two radiation efficiencies, one for each side. This Item deals with radiation in the positive *z*-direction.



Sketch 2.1 Rectangular plate of length *a* and width *b* placed in a rigid semi-infinite baffle.

The assumption of semi-infinite baffles and spaces might seem restrictive for practical applications, but systems of finite dimensions can be reasonably well described by their equivalent semi-infinite systems if the wave length is sufficiently short in comparison to their dimensions. This is employed in Section 3.2 for a cavity and in Section 6.3.1 for adjoining plates and ribs. The angle of the baffle, φ, can in principle assume values between 0 and 2π. However, angles between 0 and $\pi/2$ can be problematic as the radiation efficiencies given herein might not be applicable. If the angle is less than $\pi/2$ the baffle may form a cavity above the plate that is so small that its average properties do not correspond to that of a semi-infinite space. However, a small angle can be permissible if the baffle refers to a set of structural elements such as ribs that do not form a cavity above the plate. If the angle of the baffle, φ, is $\pi/2$ then it is possible to make two physical interpretations of the system: a floor or wall radiating into a room or a plate radiating into a tube. The first case is covered by this Item as the radiation efficiency for this case is three-dimensional. However, the second case is not as the tube is a one-dimensional system (below its cut-off frequency) and therefore has a one-dimensional radiation efficiency. The space above a plate can be considered to be three-dimensional when the wave length, λ_0, is much smaller than the dimensions of the plate, a and b.

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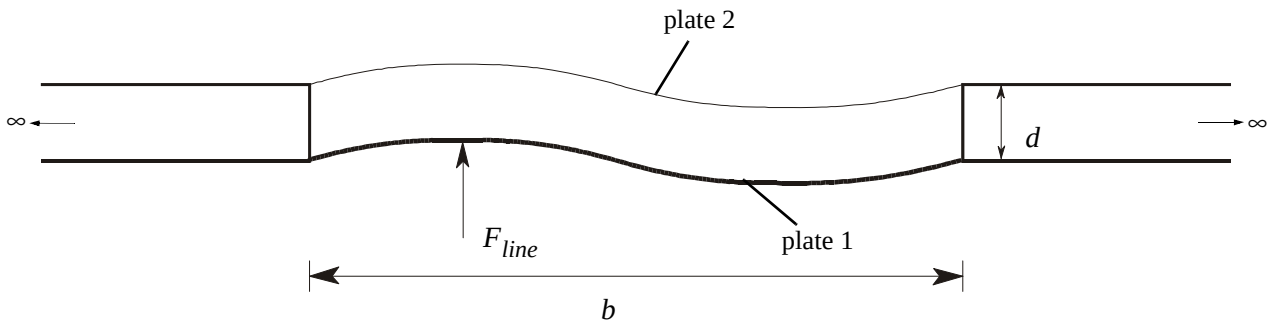
The second structural configuration considered in this Item consists of two parallel plate strips separated by a short distance as shown in Sketch 2.2. Such plates could form a simplified model of a double wall construction that, for example, can be found in aircraft.



Sketch 2.2 Two non-identical plate strips placed between two semi-infinite baffles

In addition, this Items allows the user to calculate coupling loss factors used in SEA (Statistical Energy Analysis) models that describe the coupling between fluids and plates.

Section 4 introduces the important parameter, the coincidence frequency, f_c, and considers sound radiation from infinite plates. Section 5 explains how individual plate modes of a finite plate radiate sound power, described by the so-called radiation efficiency, σ. However, for practical purposes knowledge of the radiation efficiency for a single mode is not always very useful as several modes normally will be excited strongly simultaneously.

A much more useful quantity is the modal averaged radiation efficiency, $\bar{\sigma}$. Knowledge of the modal averaged radiation efficiency, $\bar{\sigma}$, and the mean square velocity of the plate, $\langle v^2 \rangle$, permits the calculation of the radiated sound power, $\bar{P}_0$. Section 6 deals with the calculation of the modal averaged radiation efficiency, $\bar{\sigma}$, for the plate configuration shown in Sketch 2.1.

The modal averaged radiation efficiency, $\bar{\sigma}$, can be calculated at 1/3 octave centre frequencies from analytical expressions given in Table 6.1. These values can then be corrected for the influence of the angle between the plate and the baffle, φ, and the specific boundary condition described by a rotational spring compliance, ε. A number of figures are supplied that permit the interpolation of correction factors from knowledge of ε and φ. The main difficulty with this approach is to estimate ε. The auxiliary program ESDUpac A0002 permits estimation of the rotational compliance, ε, from knowledge of one or several measured resonance frequencies. Finally, the radiated sound power can be calculated from Equation (3.1). This, however, requires knowledge of the vibration level of the plate given as its mean square velocity, $\langle v^2 \rangle$. The mean square velocity, $\langle v^2 \rangle$, could for example be estimated from a Statistical Energy Analysis (SEA) model.

The analytical expressions for modal averaged radiation efficiencies, $\bar{\sigma}$, in Section 6, have been derived under the assumption that the plate radiates sound power into a semi-infinite space like a large room. The modal averaged radiation efficiencies cannot therefore be used to describe the power exchange between two plates that are placed close together, *e.g.* a double wall construction. The program ESDUpac B0002, described in Section 8 is based on other assumptions. It permits estimation of the sound power radiated by a double plate construction, that is, two infinite plate strips that are placed close together.

Section 9 gives analytical expressions that can be used to calculate SEA coupling loss factors that describe the coupling between plates and fluids.

Finally, the application of this Item is illustrated in Section 12 with two examples, one for each of the two plate configurations illustrated in Sketches 2.1 and 2.2.

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3. NOTES

3.1 Definitions

The radiation efficiency, σ, gives the important relation between the sound power, P_0, radiated away from a structure and into a fluid and the mean square velocity of the structure, $\langle v^2 \rangle$. The radiation efficiency is defined as

$$\sigma = \frac{P_0}{\rho_0 c_0 S \langle v^2 \rangle} , \tag{3.1}$$

where ρ_0 and c_0 are the density and phase velocity of the fluid and S is the surface area of the structure. The term 'radiation efficiency' can be somewhat misleading as it can assume values above unity. However, for most practical cases the radiation efficiency assumes values either below unity or close to unity.

For the general case, the mean square velocity, $\langle v^2 \rangle$, is defined as

$$\langle v^2 \rangle = \frac{1}{S\,\overline{\rho h}} \int_S \rho(x, y) h(x, y) |v(x, y)|^2 \, \mathrm{d}x\mathrm{d}y , \tag{3.2}$$

where the product $\rho(x,y)\ h(x,y)$ corresponds to the mass per unit area and $|v(x,y)|$ is the modulus of the velocity in the direction normal to the surface of the structure. The term $\overline{\rho h}$ is the average mass per unit area. For homogeneous structures of uniform thickness, Equation (3.2) reduces to

$$\langle v^2 \rangle = \frac{1}{S} \int_S |v(x, y)|^2 \, \mathrm{d}x\mathrm{d}y . \tag{3.3}$$

In practice, the mean square velocity, $\langle v^2 \rangle$, can be calculated as the average of the velocity squared measured at a number of discrete points. Obviously the points should be chosen randomly and the number of points should be sufficiently high to ensure a low standard deviation.

3.2 General Assumptions

The equations and programs on which this Item is based have been derived under the following two major assumptions: the first is that there is low fluid loading and the second that the structure is radiating into a free field, that is, a semi-infinite space.

The first assumption of low fluid loading means that the velocity distribution of the plate will be the same whether it is placed in vacuo or in the fluid. This assumption is fulfilled when

$$\frac{\rho_0 c_0}{\omega \rho h} \ll 1 , \tag{3.4}$$

where ρ_0 is the density of the fluid, c_0 is the phase velocity of the fluid, ω is the angular frequency and ρh is the mass density per unit area of the plate. Plates that radiate sound into a fluid with properties similar to that of air will experience low fluid loading, while plates submerged in fluid with properties like that of water will experience high fluid loading. The approach used in this Item cannot be applied to cases of high fluid loading.

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The second assumption, that the plate radiates sound into a free field simplifies the calculations enormously. This assumption might seem very restrictive. However, many spaces, such as large rooms, will on average have properties similar to a free space when a significant number of modes are excited and can therefore in this context be considered to be a free space. The cavity between two parallel plates can also be considered to be a free space when

$$k_0 d \gg 1 \ , \tag{3.5}$$

where k_0 is the acoustic wave number and d is the distance between the parallel plates.

3.2.1 Additional assumptions for radiation efficiencies

In the derivation of the modal averaged radiation efficiencies, σ, given in Section 6, two additional assumptions have been made: that the plates are lightly damped and that there is equipartition of modal energy.

The assumption of lightly damped plates allows the radiation efficiencies to be derived without taking damping into consideration. For more details on how damping influences the radiated sound power, see Section 6.5.1.

The radiation efficiencies for individual modes given in Section 5 are not always very useful for practical purposes as excitation at even a single frequency will excite several modes significantly. It is therefore desirable to calculate an average radiation efficiency that takes into account the fact that the sound power radiated from a plate will be the sum of the contributions of many different modes. However, to be able to calculate an average radiation efficiency, σ, it is necessary to make an assumption about the velocity distribution of the plate modes. The assumption about the velocity distribution is that there is equipartition of modal energy. This assumption leads to the so-called modal averaged radiation efficiency, σ. For a simply supported rectangular plate, equipartition of modal energy will mean that the velocity amplitudes of all the modes are the same.

3.3 Modal Average

The modal averaged radiation efficiencies, σ, and the related correction factors, Q_ε, Q_φ and $Q_{\varphi,\varepsilon}$, have been derived under the assumption that a high number of modes are present in the frequency band that is being considered. It is important to realise what this means in practice. The approximate number of modes, N, in a frequency band, Δf, can be found from

$$N = n(f)\ \Delta f \ , \tag{3.6}$$

where $n(f)$ is the modal density. For a plate with bending waves, the modal density, $n(f)$, can be written as

$$n(f) = S\sqrt{\frac{\rho\ h}{4\ D}} \tag{3.7}$$

and it is seen to be independent of frequency, f.

For example, for a 1 × 0.5 × 0.0025 m aluminium alloy plate the modal density is 0.064 modes/Hz. The electronic filters that are frequently used for noise and vibration analyses are of 1/3 octave bandwidth which corresponds to 23% constant percentage bandwidth. In the 1/3 octave band centred at 1 kHz there can be expected approximately 15 modes. As there is a high number of modes present, the modal average radiation efficiency, σ, can be expected to give a good representation of the average value of the radiation efficiencies,

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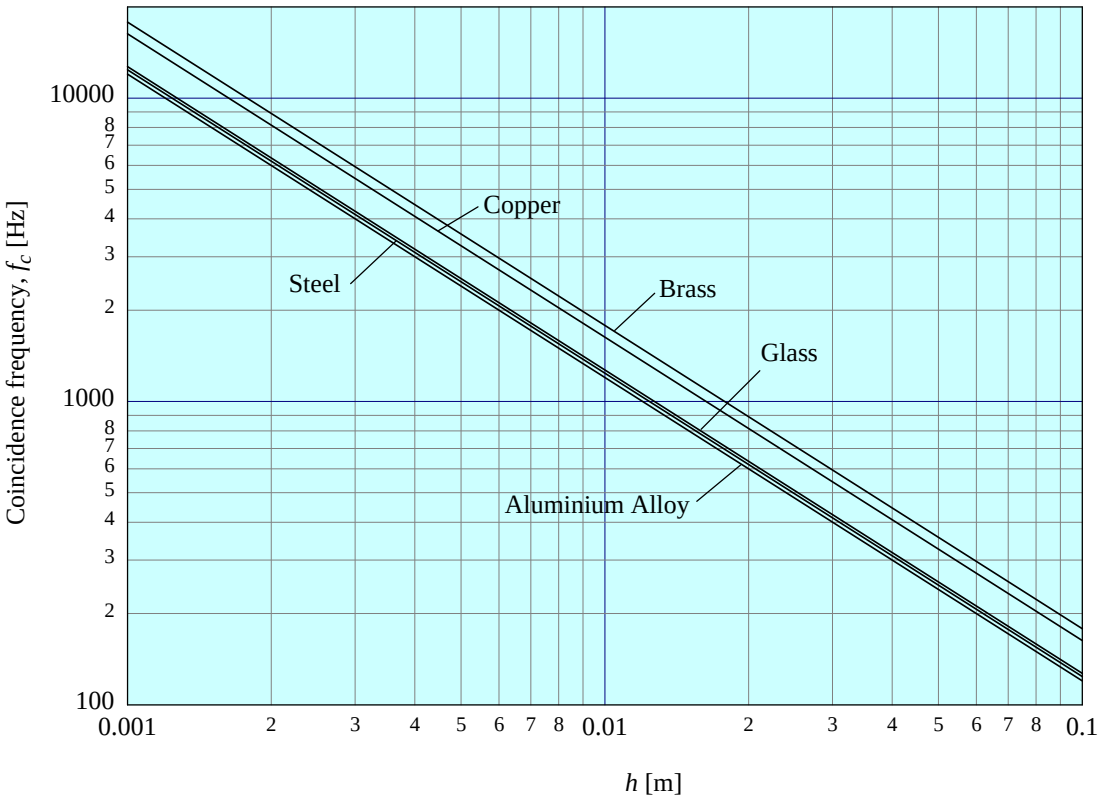
σ, for the individual modes (an example of this is shown later in Sketch 6.2). However, in the 1/3 octave band centred on 100 Hz the bandwidth is only 23 Hz and only 1.5 modes can be expected to be present on average. Thus in the 100 Hz band only 1 or 2 modes might be present or maybe none at all. The conclusion is that the agreement between the modal averaged radiation efficiency, σ, and the actual average of individual modes can be expected to be good at high frequencies where many modes are present and poor or even misleading at low frequencies where only a few or none modes might be present.

4. COINCIDENCE FREQUENCY

The coincidence* frequency, f_c, is the frequency at which the phase velocity of the bending waves in the plate, c_B, equals the phase velocity of sound in the fluid, c_0, and is given by

$$f_c = \frac{c_0^2}{2\pi}\sqrt{\frac{\rho h}{D}}, \tag{4.1}$$

where ρh is the mass of the plate per unit area of the plate and D is the bending stiffness of the plate. Sketch 4.1 shows the coincidence frequency, f_c, as a function of the thickness of the plate, h, for various materials.



Sketch 4.1 Coincidence frequency, f_c, for radiation into air from plates of various materials as a function of plate thickness, h

* The coincidence frequency is sometimes called the critical frequency as it is the lowest frequency where the coincidence phenomenon occurs.

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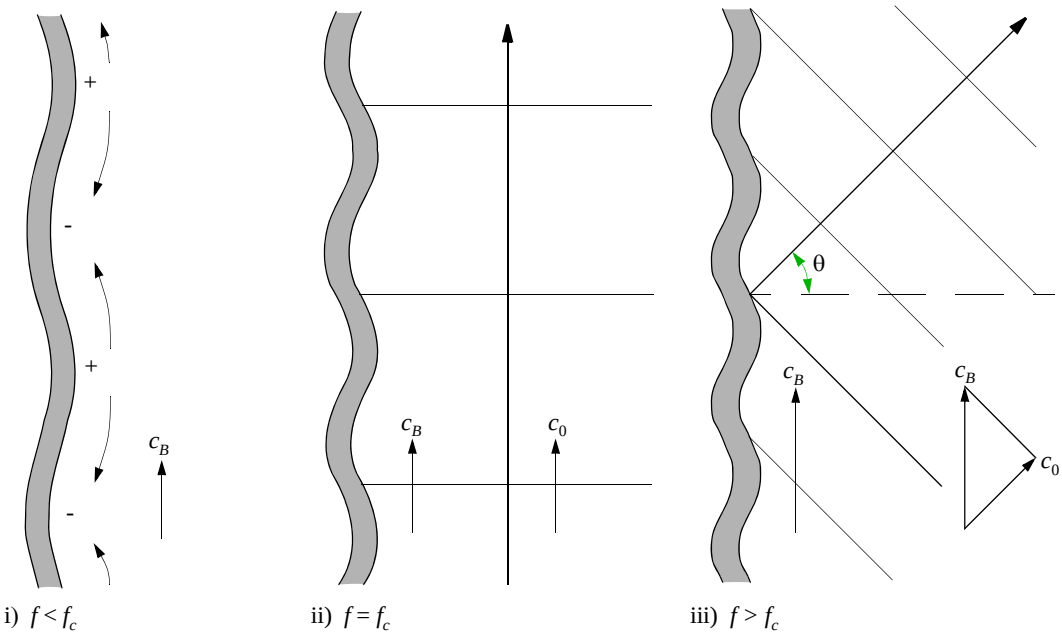
The ability of a plate to radiate sound power depends decisively on whether the frequency, f, is below, equal to or above the coincidence frequency, f_c. It is instructive to consider these three frequency ranges for an infinite plate coupled with a fluid. The three situations are illustrated in Sketch 4.2.



Sketch 4.2 Sound radiation from an infinite plate below, at and above the coincidence frequency, f_c

For an infinite plate the behaviour of the sound radiation may be described as follows:

(i) Below the coincidence frequency the two wave types are unable to couple as the phase velocity of the bending wave in the plate, c_B, is smaller than the phase velocity of the fluid, c_0. The bending wave can only cause local disturbances of the fluid. The phases of these disturbances are indicated by '+' and '–'. However, these disturbances counterbalance each other and no sound power is radiated from an infinite plate. This phenomenon is referred to as 'hydrodynamic short-circuiting'. As the short-circuiting is complete for an infinite plate, the radiation efficiency, σ, for an infinite plate is zero below the coincidence frequency.

(ii) At the coincidence frequency, the two wave types match perfectly as $c_B = c_0$. This is a very special case. The bending wave in the plate will cause a compression wave in the fluid to propagate parallel to the plane of the plate. At the coincidence frequency, the infinite plate is a very efficient radiator of sound power and the radiation efficiency, σ, will be large (theoretical infinite).

(iii) Above the coincidence frequency the phase velocity of the bending wave is greater than the phase velocity of the compression wave in the fluid. However, the two wave types can still couple if the compression wave in the fluid assumes an angle, θ, with the plane of the plate. The phase velocity of the two wave types will then be related by $c_B \sin\theta = c_0$. Well above the coincidence frequency, the radiation efficiency, σ, for the infinite plate will be close to unity.

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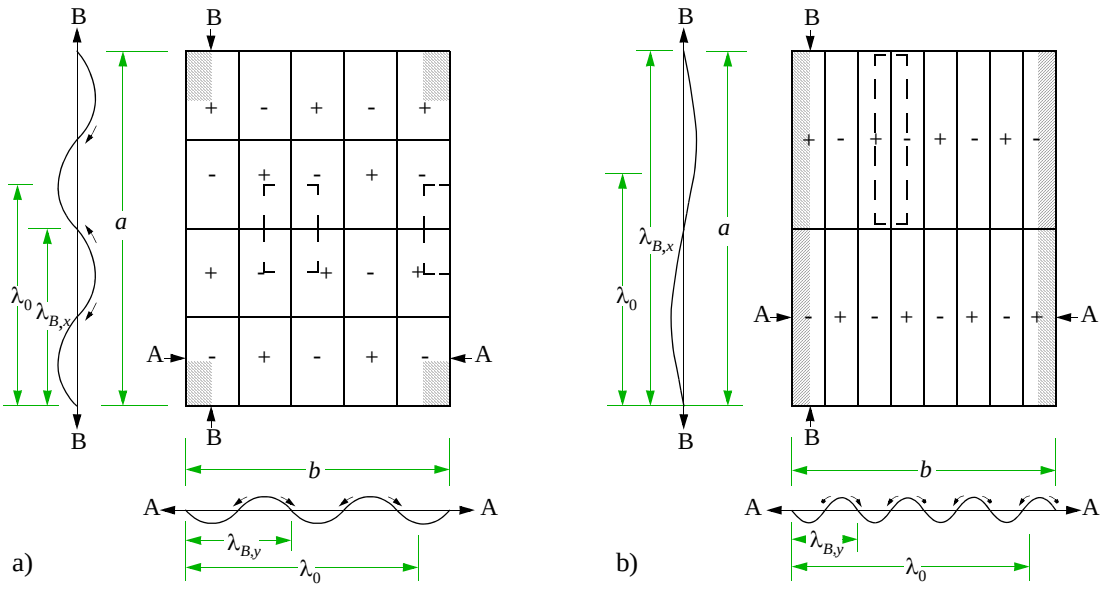
5. RADIATION EFFICIENCIES OF INDIVIDUAL MODES

In order to understand how plates of finite size radiate sound power it is instructive to consider the sound radiation from individual modes of a rectangular plate.

For frequencies at or above the coincidence frequency, f_c, the finite plate will behave like an infinite plate and be an efficient radiator of sound power. The whole surface will radiate sound and the radiation efficiency will be large at the coincidence frequency and close to unity above the coincidence frequency. However, in contrast to an infinite plate, a finite plate will radiate some sound power at frequencies below the coincidence frequency . The reason is that the aforementioned short-circuit process will not be complete. The sound radiation below the coincidence frequency can be ascribed to so-called corner and edge modes. These are illustrated in Sketch 5.1.



Sketch 5.1 (a) Sound radiation from a corner mode and (b) Sound radiation from an edge mode

Sketch 5.1(a) shows the mode $(m, n) = (4,5)$ that in this case is a corner mode. For a corner mode the projections of the phase velocity of the bending wave in both the x- and y-directions, $c_{B,x}$ and $c_{B,y}$, are smaller than the phase velocity of the fluid, c_0. Consequently, the wavelengths $\lambda_{B,x}$ and $\lambda_{B,y}$ are both shorter than the wavelength of the fluid, λ_0. The phases of the local disturbances are again indicated by '+' and '–'. The dashed lines indicate the short-circuiting process between the different parts of the plate. As can be seen, the short-circuiting process is not complete. The contributions from corner areas, indicated by hatching, are not annihilated and consequently they radiate a small amount of sound power. Sketch 5.1(b) illustrates an edge mode $(m, n) = (2, 8)$. For this mode $c_{B,y} < c_0$ while $c_{B,x}$ is greater than the phase velocity of the fluid, c_0. In terms of wavelengths this means that $\lambda_{B,y} < \lambda_0$ while $\lambda_{B,x} > \lambda_0$. The short-circuiting will therefore only occur in the y-direction for which $\lambda_{B,y} < \lambda_0$. The edge mode will radiate more sound power than the corner mode as the hatched area is greater for the edge mode than for the corner mode.

Various methods can be used to calculate the radiation efficiency of a single mode. In Derivation 3 the following expression is given for the radiation efficiency, σ, of mode (m, n) of a simply supported plate with the natural frequency, $f_{m,n}$, and angle of baffle $\varphi = \pi$.

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$$\sigma = \frac{256 f_{m,n}^2 ab}{c_0^2 \pi^4 m^2 n^2} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \left[\frac{\begin{matrix}\cos \\ \sin\end{matrix}\left(\frac{\chi}{2}\right) \begin{matrix}\cos \\ \sin\end{matrix}\left(\frac{\psi}{2}\right)}{\left(\left(\frac{\chi}{m\pi}\right)^2 - 1\right)\left(\left(\frac{\psi}{n\pi}\right)^2 - 1\right)} \right]^2 \sin\alpha \, d\alpha \, d\beta , \tag{5.1}$$

where

$$\chi = k_0 a \sin\alpha \cos\beta , \quad \psi = k_0 b \sin\alpha \sin\beta . \tag{5.2}$$

In Equation (5.1) $\cos(\chi / 2)$ is used when m is odd and $\sin(\chi / 2)$ is used when m is even. Similarly, $\cos(\psi / 2)$ is used when n is odd and $\sin(\psi / 2)$ is used when n is even. Equation (5.1) can only be evaluated using a numerical method, see Reference 19. The natural frequency, $f_{m,n}$, for a simply supported plate is given by

$$f_{m,n} = \frac{1}{2\pi} \left\{ \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right\} \sqrt{\frac{D}{\rho h}} . \tag{5.3}$$

Examples of the radiation efficiency, σ, for various modes (m, n) of a square and a rectangular plate are shown in Sketches 5.2 to 5.5. The parameter γ is given by

$$\gamma = c_B / c_0 = \frac{\sqrt{\omega h}}{c_0} \cdot \sqrt[4]{\frac{E}{12(1 - \nu^2)\rho}} . \tag{5.4}$$

The following characteristics may be noted:

$\gamma > 1$: Above coincidence, the radiation efficiencies approach unity for all modes, $\sigma \to 1$ as $\gamma \to \infty$.

$\gamma = 1$: At coincidence, the radiation efficiencies assume values greater than unity.

$\gamma < 1$: Below coincidence, the radiation efficiencies assume values less than unity, $\sigma \to 0$ as $\gamma \to 0$. The radiation efficiency depends on whether the modes are odd or even. The odd-odd modes are the most efficient radiators, followed by the odd-even modes while the even-even modes are the least efficient radiators.

If a plate is excited at a single frequency, several modes will respond simultaneously with different amplitudes and phases. The analysis of the radiated sound power for this situation becomes very complex and a simple analytical expression cannot be derived. A procedure that leads to simple analytical expressions is to calculate an average radiation efficiency for the modes within a frequency band. To calculate such an average it is necessary to make an assumption about the velocity distribution of the modes. The assumption that is used is that all the modes are equally energetic. These averages are referred to as modal average radiation efficiencies, σ, and are described in Section 6.

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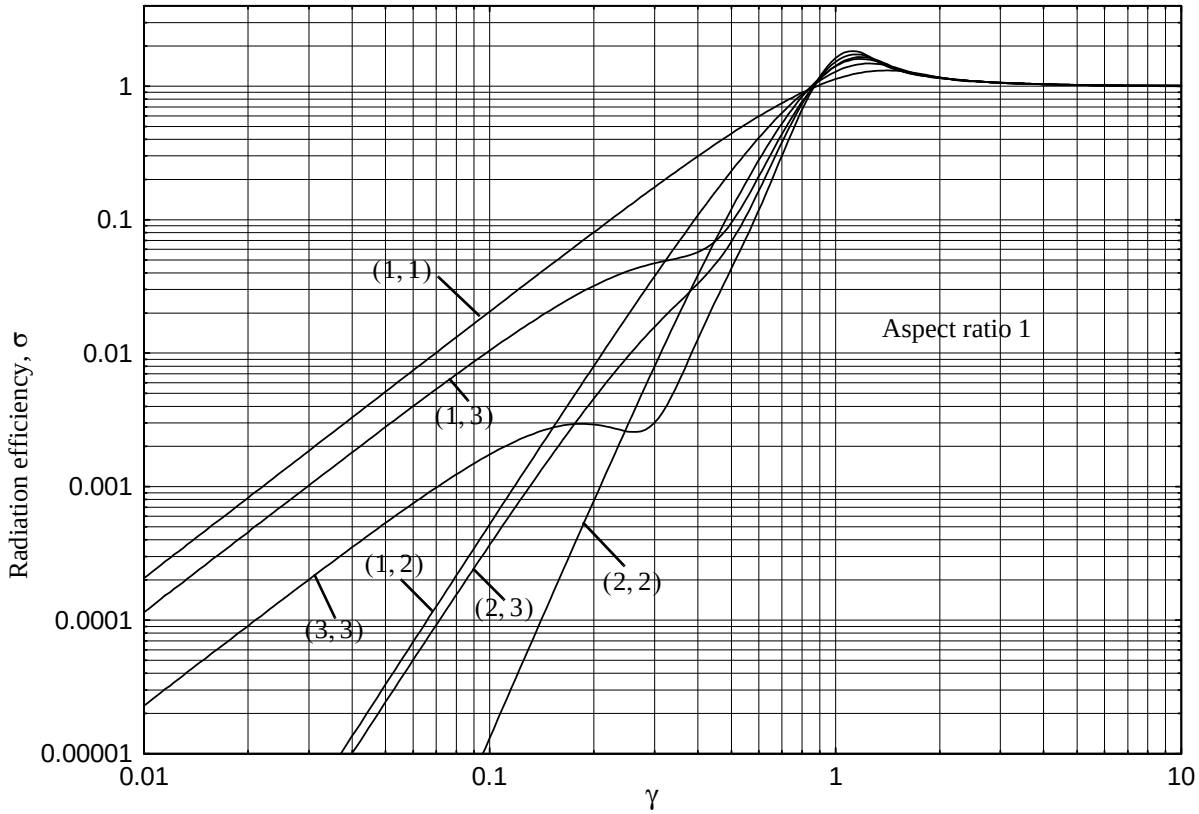
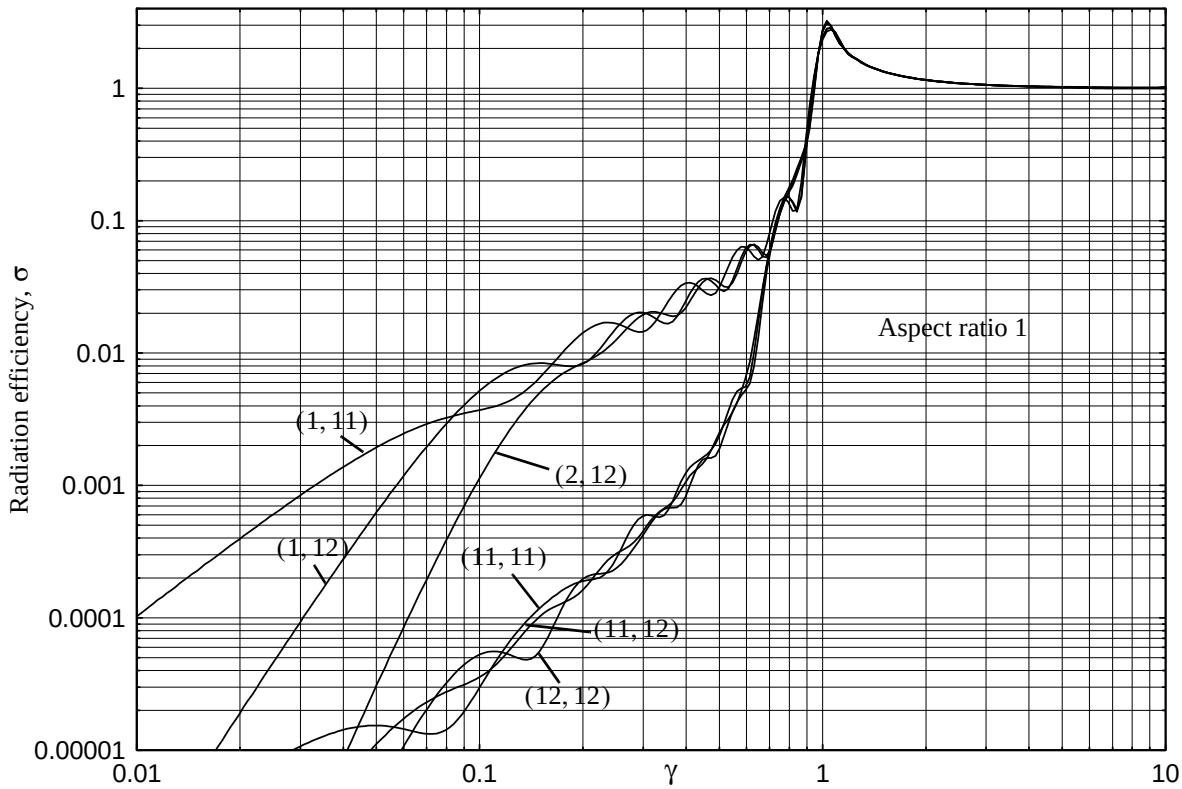


Sketch 5.2 Radiation efficiency, σ, for various low-numbered modes (*m*, *n*) of a square plate simply supported at all edges



Sketch 5.3 Radiation efficiency, σ, for various high-numbered modes (*m*, *n*) of a square plate simply supported at all edges

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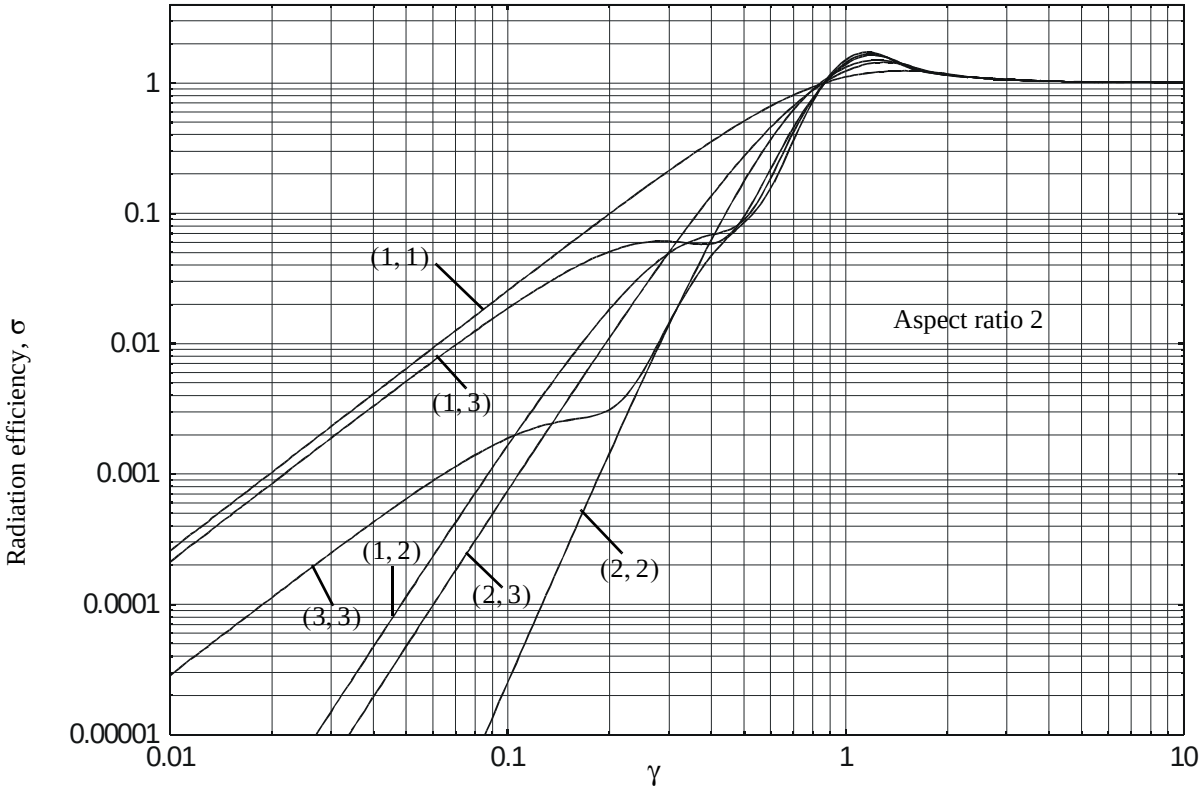
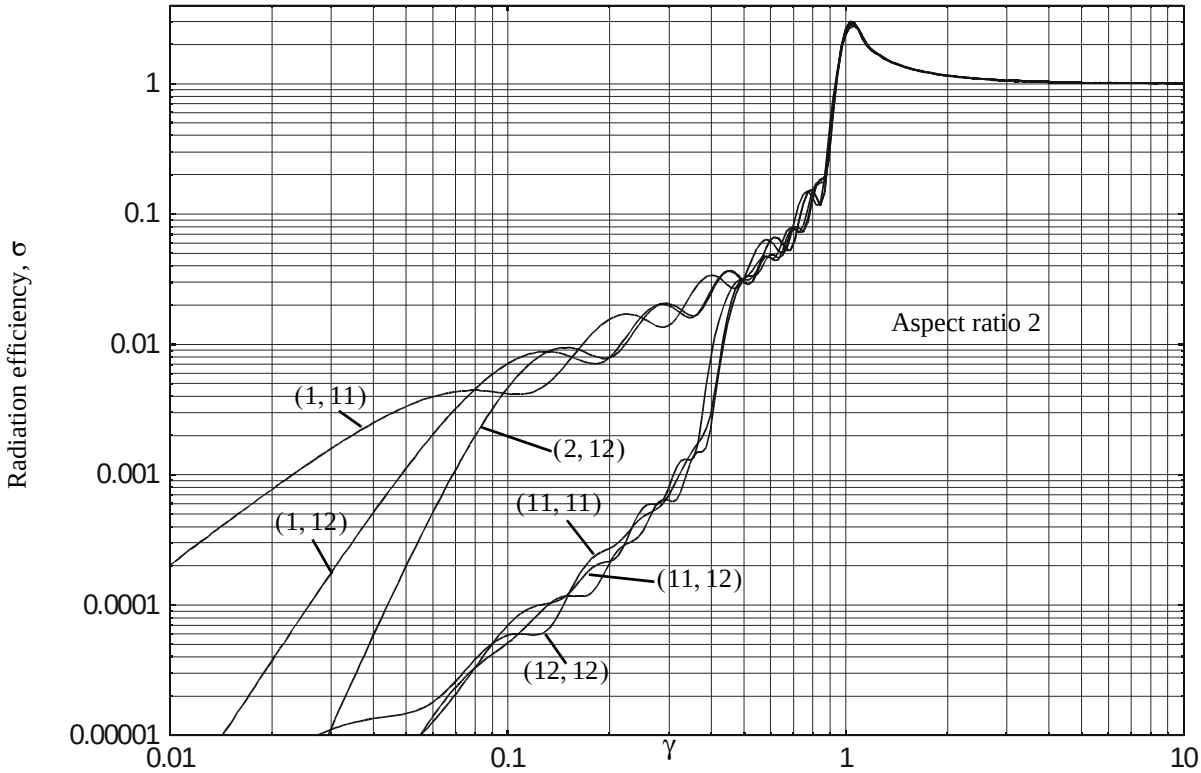


Sketch 5.4 Radiation efficiency, σ, for various low-numbered modes (m, n) of a rectangular plate with aspect ratio a/b = 2, simply supported at all edges



Sketch 5.5 Radiation efficiency, σ, for various high-numbered modes (m, n) of a rectangular plate with aspect ratio a/b = 2, simply supported at all edges

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