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Drag of a rectangular planform cavity in a flat plate with a turbulent boundary layer for Mach numbers up to 3. Part I: Closed flow



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DRAG OF A RECTANGULAR PLANFORM CAVITY IN A FLAT PLATE WITH A TURBULENT BOUNDARY LAYER FOR MACH NUMBERS UP TO 3. PART I: CLOSED FLOW

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DRAG OF A RECTANGULAR PLANFORM CAVITY IN A FLAT PLATE WITH A TURBULENT BOUNDARY LAYER FOR MACH NUMBERS UP TO 3. PART I: CLOSED FLOW

1. NOTATION AND UNITS

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A_{fw0}	coefficient for cavity front wall in Equation (4.5)
A_{rw0}	coefficient for cavity rear wall in Equation (4.5)
a	coefficient in Equation (4.1), see Equation (4.2)
a_0	value of a for low-speed flow, see Figure 3 or Equation (4.10)
a_{rw0}	contribution to a_0 from cavity rear wall, see Equation (4.20)
b	term in Equation (4.1), see Equation (4.3)
b_0	value of b for low-speed flow, see Figure 5 or Equation (4.12)
b_{rw0}	contribution to b_0 from cavity rear wall, see Equation (4.21)
C_D	increment in drag coefficient due to presence of cavity, based on planform (floor) area wl and q_{1m}
$(C_D/C_{fm})_{F_{sev}=0}$	value of C_D/C_{fm} in the absence of side-edge vortices, see Equation (4.17)
C_{Dp}	value of C_D based on integrated front and rear wall pressures
C_{Dref}	cavity drag coefficient based on wing reference area S_{ref} and q_∞ , Equations (5.3) to (5.5)
C_{Drw}	contribution to C_D from cavity rear wall in method for drag due to side-edge vortices, see Equation (4.22)
C_f	local skin friction coefficient on flat plate, in absence of cavity, based on q_1
C_{fm}	value of C_f at cavity mid-length, see Figure 13 or Equation (4.25)
$C_0, \dots, C_4$	coefficients in Equation (4.26), see Equations (4.26a)
F	cavity width factor in Equations (4.5), (4.17) and (4.23), see Figure 2 or Equation (4.6) or (4.8)

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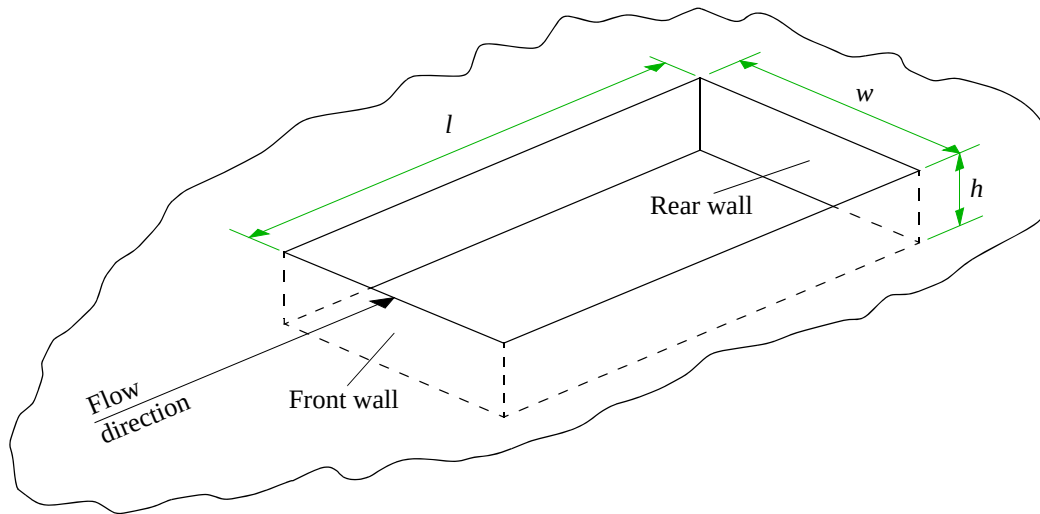
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F_{Ma}	Mach number factor in Equations (4.2) and (4.17), see Figure 6		
F_{Marw}	Mach number factor in Equations (4.19) and (4.23), see Figure 8		
F_{Mb}	Mach number factor in Equations (4.3) and (4.17), see Figure 7		
F_{Mbrw}	Mach number factor in Equations (4.19) and (4.23), see Figure 9		
F_{M1}	Mach number factor in Equation (4.25), see Equation (4.27) or Figure 12		
F_{M2}	Mach number factor in Equation (4.25), see Equation (4.28) or Figure 12		
F_{sev}	factor on C_{Drw}/C_{fm} in Equation (4.18) to account for side-edge vortices in supersonic flow, see Figure 10 or Equation (4.24)		
G	cavity width factor in Equations (4.12), (4.17) and (4.23), see Figure 4 or Equation (4.13) or (4.15)		
h	cavity depth, see Sketch 1.1	m	ft
l	length of rectangular cavity, see Sketch 1.1	m	ft
M_1	Mach number at edge of boundary layer at cavity mid-length, in absence of cavity		
M_∞	free-stream Mach number		
m_d	drag magnification factor in Equation (5.3)		
n	exponent in Equation (4.6) for F , see Equation (4.7)		
p	exponent in Equation (4.25) for C_{fm} , see Figure 11 or Equation (4.26)		
q_1	local kinetic pressure at edge of boundary layer, in absence of cavity, $\frac{1}{2} \rho_1 U_1^2$	N/m ²	lbf/ft ²
q_{1m}	value of q_1 at cavity mid-length	N/m ²	lbf/ft ²
q_∞	free-stream kinetic pressure	N/m ²	lbf/ft ²
R_{fl}	friction Reynolds number based on l , $u_\tau l/\nu_w$, see Equation (4.4)		
R_l	Reynolds number based on l and R_{1m} , $R_{1m}l$		
$R_{x'm}$	Reynolds number based on x'_m and R_{1m} , $R_{1m}x'_m$		

$(R_{x'm})_{min}$	lower limit of $R_{x'm}$ for fully-turbulent boundary layer, see Equation (4.29)		
R_1	local unit Reynolds number based on conditions at edge of boundary layer, in absence of cavity	m^{-1}	ft^{-1}
R_{1m}	value of R_1 at cavity mid-length	m^{-1}	ft^{-1}
$R_{1\infty}$	free-stream unit Reynolds number	m^{-1}	ft^{-1}
R_{δ_m}	Reynolds number based on δ_m and R_{1m} , $R_{1m}\delta_m$, see Figure 14 or Equation (5.2)		
r	exponent in Equation (4.13) for G , see Equation (4.14)		
S_{ref}	wing reference area	m^2	ft^2
U_1	local velocity at edge of boundary layer, in absence of cavity	m/s	ft/s
u_τ	friction velocity, $(\tau_w/\rho_w)^{1/2}$	m/s	ft/s
w	width of rectangular cavity, see Sketch 1.1	m	ft
x'_m	length of fully-turbulent flow from origin of turbulent boundary layer to cavity mid-length, in absence of cavity	m	ft
δ_m	boundary-layer thickness (at $0.99 U_1$) at cavity mid-length, in absence of cavity, obtained via Figure 14 or Equation (5.2)	m	ft
ν_w	kinematic viscosity at wall	m^2/s	ft^2/s
ρ_1	local density at edge of boundary layer, in absence of cavity	kg/m^3	$slug/ft^3$
ρ_w	density at wall	kg/m^3	$slug/ft^3$
τ_w	shear stress at wall	N/m^2	lbf/ft^2

Subscripts

$expt$	denotes experimental value
$pred$	denotes predicted value
w	denotes wall conditions (taken here as conditions on flat plate at cavity mid-length, in absence of cavity)
0	denotes value for $M_1 = 0$



Sketch 1.1 Cavity geometry

2. INTRODUCTION

The flow patterns associated with a rectangular planform cavity are traditionally divided into three categories, termed *closed*, *open* and *transitional*, as described in Section 3.1 and covered in more detail in ESDU 02008 (Derivation 17). This Data Item provides an empirical method for predicting the drag of rectangular planform cavities with *closed* flow for Mach numbers up to 3. The method applies only to cavities without doors. For the effect on cavity drag of a pair of doors open at 90° see ESDU 10016 (Reference 23). Closed flow occurs for low values of cavity depth to length ratio (h/l), typically less than about 0.1, the actual value being a function of cavity width (w) and flow Mach number (M_1), see Section 3.2. The drag prediction method, which is presented in Section 4, applies to cavities in a flat plate with a fully-turbulent boundary layer. Section 5 addresses the applicability and accuracy of the method and Section 6 lists the Derivation and References. Section 7 presents two worked examples, one relating to a light transport aircraft at low speed and the other to a supersonic combat aircraft. Appendix A details a cavity drag prediction method for closed flow, based only on the contributions from the front and rear walls. That method was used as the basis of the prediction method of Section 4. Appendix A also includes a justification for the use of the local skin friction coefficient at the mid-length of a flat plate in the determination of the skin friction drag of the plate, instead of the mean skin friction coefficients at the upstream and downstream ends of the plate. This simplification, which involves little loss of accuracy, was used in the development of the cavity drag prediction method of Section 4. Appendix B expands on the mathematical treatment of supersonic flow when side-edge vortices are present.

The drag prediction method in Section 4 was developed using wind-tunnel data for cavities in flat plates at zero angle of attack, *i.e.* in uniform flows. In order to apply the method to a practical situation in which a cavity is in a non-uniform flow, an artifice employed in the application of other excrescence drag data is used, see ESDU 90029 (Reference 19). In essence, that involves the use of local flow values for Mach number, unit Reynolds number and kinetic pressure at the edge of the boundary layer, at the cavity mid-length, but in the absence of cavity, and that is detailed in Section 5.1.

The drag of rectangular cavities with open or transitional flow is dealt with in ESDU 00007 (Reference 22).

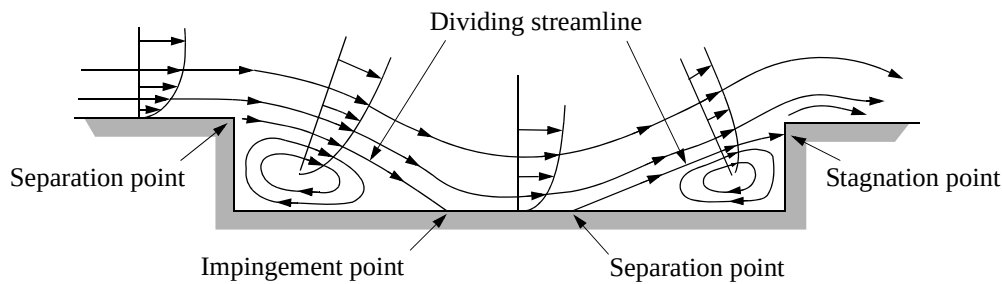
3. CLOSED, OPEN AND TRANSITIONAL CAVITY FLOWS

3.1 Flow Regimes

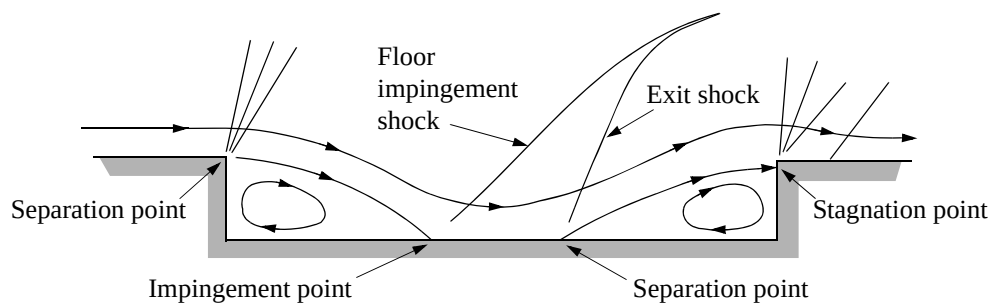
ESDU 02008 (Derivation 17) contains an extensive discussion of the various flow regimes occurring in rectangular cavities in both subsonic and supersonic flows. The effects of cavity geometry and free-stream flow properties, together with the effects of boundary-layer properties in the approach flow, are addressed. The material pertinent to the use of the present Data Item has been extracted for presentation and further analysis herein.

For subsonic speeds extensive wind-tunnel tests on pressure-tapped cavities (Derivation 13), together with earlier but much less extensive tests for supersonic speeds (Derivations 9 and 11), have enabled two main types of flow to be identified, with a transitional region between them. The names and associated definitions of the two flow types have become firmly established in the literature since their first use in or around 1961. It is important to recognise that the flow regimes were characterised on the basis of mean pressure distributions on the cavity centre line and so the flow patterns described are typical of that section. Away from the centre-line section, and particularly near the cavity side walls, the flow patterns can be more complex, with local three-dimensional effects. Ideally, in testing to determine the flow type, the mean floor pressure distribution should be taken in conjunction with the unsteady pressures, see Reference 21. This is because unsteady pressures are a very clear indicator of the presence of acoustic tones associated with open flow, see Derivation 17.

Closed flow occurs when the cavity length is large compared to the depth (*i.e.* small h/l) and the shear flow enters the cavity and attaches to the floor before separating to pass over the rear wall with a stagnation point near the top of the wall. Sketch 3.1(a), given in Derivation 13, depicts an idealised concept of closed flow for subsonic speeds. Sketch 3.1(b) illustrates the corresponding situation for supersonic speeds and shows the typical associated expansion and shock systems.



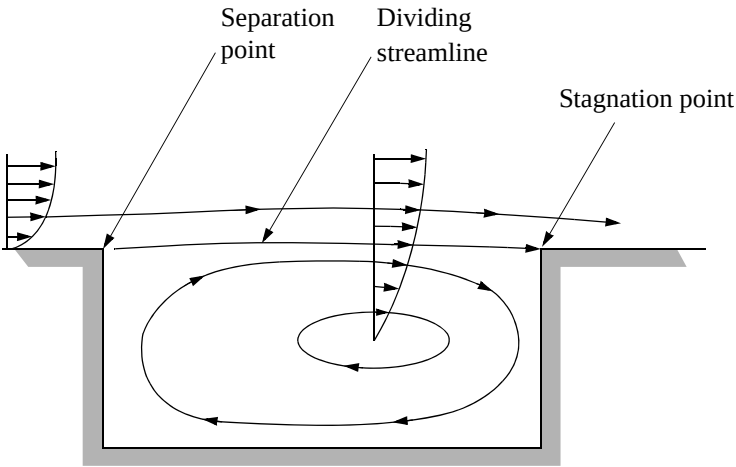
(a) Subsonic speeds



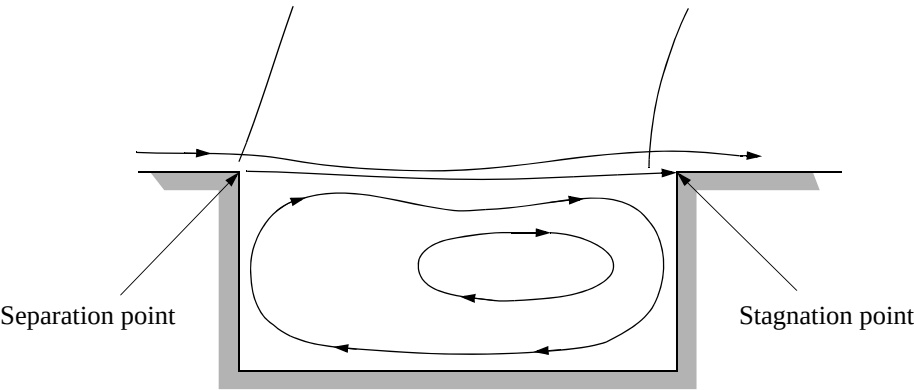
(b) Supersonic speeds

Sketch 3.1 Closed flow

At the other extreme, when the cavity length is small compared to the depth (*i.e.* large h/l), the shear flow bridges the cavity, with the dividing streamline ending in a stagnation point at, or near, the top of the rear wall. This type of flow is called *open flow*. Sketches 3.2(a) and 3.2(b) illustrate the main flow features for subsonic and supersonic speeds, respectively. It is worth noting that in practice the single trapped vortex shown is typical of h/l values between about 0.5 and 1. For lower h/l values a tandem pair of vortices can occur, while for higher h/l values vertically stacked double, or even triple, vortices can occur, see Derivation 17.



(a) Subsonic speeds



(b) Supersonic speeds

Sketch 3.2 Open flow

For a given cavity width and depth, at given flow conditions, as the length is progressively increased from zero there is a range of values, called the *transitional* range, over which, for subsonic speeds, the flow type gradually changes from open to closed, accompanied by associated changes in the characteristic shape of the floor pressure distribution. For supersonic speeds a similar range exists, but the change from open to closed flow is more complex, passing through two intermediate stages, termed *transitional-open* and *transitional-closed* occurring near the closed flow boundary. Furthermore, the change in flow type from transitional-closed to transitional-open is extremely abrupt, occurring over a very small h/l range. In fact, as noted in Derivation 13, the floor pressure distributions within the transitional range for subsonic and transonic speeds also pass through the transitional-open and transitional-closed shapes identified for supersonic flow. However, they are not normally referred to because, in contrast to the supersonic situation, no abrupt changes occur. These features are discussed in more detail in Derivation 17. The extent of the transitional flow region, in both subsonic and supersonic flows, is expressed in terms of h/l , but there is subsidiary dependence on both cavity width and Mach number.

Tests over a wide range of unit Reynolds number, for subsonic speeds at least (see Derivation 17), produced small effects on cavity pressures but no changes in flow type, even for cases close to the transitional boundaries. The tests involved quite thin boundary layers with h/δ values ranging from about 1 to nearly 4. Although it is possible (even probable) that variation of h/δ over a much larger range could affect the transitional region, no evidence of such effects has been found. It is therefore assumed that the conclusions of Section 3.2 also apply to thick boundary layers, with h/δ much less than 1 (down to 0.05, say).

3.2 Closed Flow Boundary

For the purposes of this Item a knowledge of the boundary for closed flow is required. In order to obtain an approximation to the upper limit in h/l for closed flow the data of Derivations 7, 9, 11 and 13 were analysed. The data of Derivation 13 for $M_1 = 0.2$ to 0.95 and $w/h = 1, 4, 8$ and 16 are comprehensive enough to allow a reasonable estimate to be made of the closed flow boundary for subsonic speeds. Even so, to provide a smooth variation with M_1 , for a given w/h , a local deviation at about $M_1 = 0.6$ for $w/h = 4$ had to be ignored. The available data for supersonic speeds in Derivations 7, 9 and 11 are restricted almost wholly to $w/h = 5$ and 5.3 for $M_1 = 1.5$ to 2.86. There is, however, some conflict between the results for the closed flow boundary and its variation with Mach number as deduced from the pressure data of Derivation 9, backed up by the balance data of Derivation 7 for $w/h = 5$, and those deduced from the combined pressure and balance drag data of Derivation 11 for $w/h = 5.3$. The latter Derivation suggests a boundary that is lower by about 0.01 in terms of h/l than that suggested by the data of Derivations 7 and 9. This situation highlights the problems involved in trying to define what in practice is a somewhat amorphous boundary. In the interests of conservatism the data of Derivation 11 were taken as "correct". The variation with w/h was deduced from *ad hoc* observations in Derivation 9 concerning the presence of closed flow at various values of w/h in the range $0.5 \leq w/h \leq 4$. In the overall analysis it was assumed that the closed flow boundary would not vary with w/h once that parameter was large enough for two-dimensional flow to be assumed. This was taken to be when $w/h \geq 7$, as in the method for the drag of grooves parallel to the flow in ESDU 75028 (Derivation 5), see also Section 4.1.

The result of the analysis is shown in Figure 1, in which the upper limit in h/l for closed flow is plotted against w/l for a family of Mach number values from 0 to 3. For further information, lines of constant w/h are also given. The lines of constant w/h are given for two reasons. Firstly, they are a reminder that the boundaries were originally constructed in terms of w/h , and in particular they show that when $w/h > 7$ the upper limit in h/l is assumed invariant with w/l for a given Mach number. Secondly, if it is more convenient to use w/h , rather than w/l , then the combination $w/h, M_1$ can be used as an alternative means of determining the upper limit in h/l , although the use of $w/l, M_1$ is to be preferred since interpolation on Figure 1 in w/l is easier than it is in w/h .

In the use of Figure 1 it should be recognised that the h/l values are not definitive. It is suggested that for low speeds an accuracy of about ± 0.005 on h/l might be appropriate, rising to about ± 0.01 at high subsonic and supersonic speeds. The lines for supersonic speeds are shown broken in Figure 1 to indicate the uncertainty due to the lack of comprehensive data. Note the change in direction of the trend with h/l as M_1 rises above about 2.

4. DRAG PREDICTION METHOD FOR CLOSED FLOW

The method developed here is for cavities without doors. For the effect on cavity drag of a pair of doors open at 90° see ESDU 10016. A simple method in Appendix A for the drag of a rectangular planform cavity with closed flow, based on the contributions from the front and rear walls only, leads to an equation (Equation (A2.6)) of the form

$$C_D / C_{fm} = a \log_{10}(R_{fl}) - b, \quad (4.1)$$

where

$$a = F_{Ma} a_0 \quad (4.2)$$

and

$$b = F_{Mb} b_0. \quad (4.3)$$

In Equations (4.2) and (4.3) a_0 and b_0 are constant for given cavity geometry at low speeds and are discussed in Sections 4.1 and 4.2, respectively. The factors F_{Ma} and F_{Mb} are Mach number corrections operating on a_0 and b_0 , respectively, see Section 4.3. For low-speed flow, $F_{Ma} = F_{Mb} = 1$.

For supersonic flow, side-edge vortices can occur for certain cavity planforms resulting in an additional drag contribution, see Section 4.4.

From Appendix A it is seen that Equation (4.1) was developed from the analysis of the excrescence drag data for two-dimensional steps given in ESDU 75031 (Derivation 6). Those data were for steps of small height compared to the boundary-layer thickness of the approach flow. In common with such analyses, the drag coefficient increment due to the excrescence is normalised in terms of the flat plate local skin friction coefficient at the excrescence location, in the absence of the excrescence. For rectangular planform cavities it is appropriate to take this as C_{fm} , the value at the cavity mid-length, see Section 4.5. Also in common with such analyses, a Reynolds number that characterises the shear flow at the wall is used. In the present case the friction Reynolds number, R_{fl} , based on cavity length, l , is employed, given to a good approximation by

$$R_{fl} = (C_{fm}/2)^{1/2} R_l (1 + 0.178 M_1^2)^{-1.3}, \quad (4.4)$$

in which R_l is a Reynolds number based on cavity length and edge of boundary-layer conditions at the cavity mid-length, and M_1 is the Mach number at the edge of the boundary layer at the cavity mid-length, all in the absence of the cavity.

In this Data Item, the drag predicted from Equation (4.1) is the drag due to the **presence** of the cavity, *i.e.* the gross drag of the cavity installed in a flat plate minus the drag due to that area of plate (wl) occupied by the cavity. Accordingly, the definition of C_D includes a reduction of C_{fm} . Thus, when $C_D = 0$ the drag of the installed cavity is equal to that of the area of flat plate it replaced, see Section A3 of Appendix A for a justification of the use of C_{fm} for the calculation of flat plate drag.

With the form of Equation (4.1), it is possible for negative values of C_D/C_{fm} to be calculated for values of R_{fl} below a certain value, dependent on the values of a and b . Such values are considered to be unrealistic and should be taken as zero.

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4.1 Determination of a_0

Unfortunately, there are no comprehensive drag data in which R_{fl} was varied that would enable the dependence of a_0 on cavity geometry to be established for low-speed closed flow. In the absence of such data the analysis of Appendix A was relied on, with Equation (A2.13) giving

$$a_0 = F(h/l)(A_{fw0} + A_{rw0}). \quad (4.5)$$

In Equation (4.5), F is the cavity width factor given in Figure 3 of ESDU 75028 as a function of w/h . In the derivation of F in that Item it was found that the front and rear walls were effectively two-dimensional for $w/h > 7$. For computational purposes, F is well represented, for $w/h \leq 7$, by

$$F = \{ \sin[90(w/h)/7] \}^n, \quad (\text{with } [\] \text{ in degrees}) \quad (4.6)$$

where

$$n = -0.56 \log_{10}[177(w/h)] + 1.422(w/h)^{0.1046}, \quad (4.7)$$

and for $w/h > 7$, by

$$F = 1. \quad (4.8)$$

Figure 2 gives F against w/h as obtained from Equations (4.6) to (4.8).

Section A2.1 of Appendix A to ESDU 75031 (Derivation 6) gives $A_{fw0} = 16$ and $A_{rw0} = 60$, so that

$$A_{fw0} + A_{rw0} = 76, \quad (4.9)$$

and Equation (4.5) becomes

$$a_0 = 76F(h/l), \quad (4.10)$$

which is a function of w/h (or w/l) and h/l , as shown in Figure 3. For values of $w/h > 7$, $F = 1$ and so,

$$a_0 = 76(h/l). \quad (4.11)$$

The boundary for low-speed closed flow from Figure 1 is also given in Figure 3, which shows that the values of a_0 applicable to closed flow are never greater than about 8.

4.2 Determination of b_0

The analysis in Appendix A, leading to Equation (A2.20), gave unsatisfactory values of b_0 . Such values, when used in Equation (4.1) together with a_0 from Equation (4.10), gave consistent overpredictions when compared with test data for cavity drag. So the limited experimental data of Derivations 1 to 3 and 14 for low-speed closed flow, together with values of a_0 taken from Equation (4.10) and values of R_{fl} taken from Equation (4.4), were used in Equation (4.1) to derive empirical values of b_0 . The results, when plotted against h/l , showed trends with both w/l and h/l very similar to those for a_0 in Figure 3. Consequently, a curve fit similar to Equation (4.10) was sought, resulting in

$$b_0 = 230G(h/l), \quad (4.12)$$

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in which the cavity wall width factor G is given, for $w/h \leq 7$, by

$$G = \{ \sin [90(w/h)/7] \}^r, \quad (\text{with } [] \text{ in degrees}) \quad (4.13)$$

where

$$r = -0.386 \log_{10} [1.29(w/h)] + 0.415, \quad (4.14)$$

and, for $w/h > 7$, by

$$G = 1. \quad (4.15)$$

Figure 4 gives G against w/h as obtained from Equations (4.13) to (4.15).

Like a_0 , the parameter b_0 from Equation (4.12) is a function of w/h (or w/l) and h/l , as shown in Figure 5. Similarly, when $w/h > 7$

$$b_0 = 230(h/l). \quad (4.16)$$

Again, as for a_0 , the boundary for low-speed closed flow is given in Figure 5, which shows that the values of b_0 for closed flow are never greater than about 23.

4.3 Mach Number Corrections to a_0 and b_0

In order to allow the values of a_0 and b_0 from Sections 4.1 and 4.2 to be used for compressible flow, the Mach number factors F_{Ma} and F_{Mb} are applied, as shown in Equations (4.2) and (4.3).

The Mach number factors are determined from Equation (A2.14) in Appendix A for F_{Ma} and Equation (A2.21) for F_{Mb} , with the required values of the data for the front and rear walls being obtained from Section A2.1 and Tables A2.1 and A2.2 of Appendix A in ESDU 75031. The results are shown in Figure 6 for F_{Ma} and Figure 7 for F_{Mb} for values of M_1 up to 3. It is seen that while F_{Ma} is a function of M_1 only, F_{Mb} has additional dependence on h/l , and $F_{Mb} \rightarrow F_{Ma}$ when $h/l \rightarrow 0$.

So Equation (4.1) becomes, using Equation (4.2) with (4.10) for a and Equation (4.3) with (4.12) for b ,

$$C_D/C_{fm} = F_{Ma} 76 F(h/l) \log_{10}(R_{fl}) - F_{Mb} 230 G(h/l), \quad (4.17)$$

with R_{fl} given by Equation (4.4).

4.4 Supersonic Flow with Side-edge Vortices

For cavities typical of weapons bays ($w/l \leq 0.5$, say) Derivations 7 and 9 show that, for supersonic closed flow, very significant three-dimensional effects can occur. These effects relate to vortices and their associated shock wave systems that are generated as the shear flow spills over the side edges of the cavity. The twin vortices pass downstream to impinge on the cavity rear wall, thereby having a significant influence on the drag. The scope of the available test data is not wide enough to be able to determine the ranges of geometry and flow variables, including those for the shear flow, over which side-edge vortices occur. Therefore, any method developed using the data will be tentative if used outside the ranges of variables applying to them. However, rear wall pressure data for subsonic speeds up to $M_1 = 0.95$ (Derivation 10) show no sign of side-edge vortices, so it is probably safe to assume that they only occur for supersonic speeds.

Clearly, the method leading to Equation (4.17) only considers three-dimensionality in terms of simple end effects on the front and rear walls. Therefore, some additional means is required to allow for the drag due to side-edge vortices, which have their greatest effect on the rear wall. In simplistic terms it can be assumed that the effect of the side-edge vortices on the drag of the rear wall is proportional to the ratio of the impingement area to the rear wall area, *i.e.* the impingement area ratio. Furthermore, the application of side-edge vortex geometry in relation to a cavity of given dimensions in Derivation 7 suggests that the impingement area ratio is reduced as the aspect ratio (w/l) of the cavity is increased. If that is so, then it follows that the drag due to side-edge vortices should also reduce as w/l is increased. Indeed, for large values of w/l it is likely that the effect of side-edge vortices will be negligible. However, although the range of available test data is not large enough to determine an upper limit, it is felt that side-edge vortices are unlikely to occur anyway for w/l greater than about 1, say.

An appropriate way of predicting the drag increment due to side-edge vortices is via a factor, F_{sev} , on the drag contribution from the rear wall in the absence of side-edge vortices. Therefore,

$$\frac{C_D}{C_{fm}} = \left(\frac{C_D}{C_{fm}} \right)_{F_{sev} = 0} + F_{sev} \frac{C_{Drw}}{C_{fm}}, \quad (4.18)$$

where $(C_D/C_{fm})_{F_{sev} = 0}$ is given by Equation (4.17).

Equations (B2.1), (B2.2) and (B2.5) in Appendix B give

$$\frac{C_{Drw}}{C_{fm}} = F_{Marw} a_{rw0} \log_{10}(R_{fl}) - F_{Mbrw} b_{rw0}, \quad (4.19)$$

in which, from Equations (B2.3) and (B2.8),

$$a_{rw0} = 60F(h/l), \quad (4.20)$$

and, from Equation (B2.19),

$$b_{rw0} = 197G(h/l). \quad (4.21)$$

Therefore, Equation (4.19) becomes

$$\frac{C_{Drw}}{C_{fm}} = F_{Marw} 60F(h/l) \log_{10}(R_{fl}) - F_{Mbrw} 197G(h/l), \quad (4.22)$$

which is Equation (B2.20). Equation (4.22) gives good compatibility with the rear wall contribution to the overall cavity drag in Equation (4.17), as noted in Section B2.1 in Appendix B.

Equation (4.18) now becomes

$$\frac{C_D}{C_{fm}} = \left(\frac{C_D}{C_{fm}} \right)_{F_{sev} = 0} + F_{sev} \left[F_{Marw} 60F(h/l) \log_{10}(R_{fl}) - F_{Mbrw} 197G(h/l) \right]. \quad (4.23)$$

The cavity width factors F and G are given by Figures 2 and 4, respectively. The Mach number factors are determined from Equation (B2.4) for F_{Marw} and from Equations (B2.6) and (B2.8) for F_{Mbrw} . The

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variations with M_1 of F_{MArw} and F_{MBrw} in those Equations are given in Table A2.1 of Appendix A in ESDU 75031. The results are given in Figure 8 for F_{MArw} and Figure 9 for F_{MBrw} . Note that as the concern here is for supersonic speeds, the data in those Figures start at $M_1 = 1$. The data also only extend up to $M_1 = 1.5$, beyond which they remain constant up to $M_1 = 3$. In addition, the data in Figure 9 only cover the range of the experimental data, $0.04 \leq h/l \leq 0.07$, over which the factor 197 in the second term of Equation (4.22) is a good approximation, as shown by Sketch B2.1 in Appendix B.

The factor F_{sev} was deduced using the balance drag data of Derivations 7 and 11 via Equation (4.23) with $(C_D/C_{fm})_{F_{sev}=0}$ from Equation (4.17). The resulting variation of F_{sev} with M_1 is given by

$$F_{sev} = 0.1782(M_1 - 1)^2 + 0.0206(M_1 - 1)^4, \quad (4.24)$$

and is presented in Figure 10. In view of the limited range of test data used in developing Figure 10 (see Table 5.2 later) it is recommended that its use be restricted to cavities within the range $0.2 \leq w/l \leq 0.5$, say.

4.5 Flat Plate Local Skin Friction Coefficient

The value of C_{fm} used in Equations (4.1) and (4.4) is obtainable from Figure 2 of ESDU 68020 (Derivation 4) as a function of Reynolds number $R_{x'm}$ at the cavity mid-length, in the absence of the cavity, for various values of M_1 . An excellent fit to the data is given by the modified form of the Prandtl-Schlichting equation*

$$F_{M1} C_{fm} = [2 \log_{10}(F_{M2} R_{x'm}) - 0.65]^p, \quad (4.25)$$

where p (taken as -2.3 by Schlichting for incompressible flow) is given by

$$p = C_4 [\log_{10}(F_{M2} R_{x'm})]^4 + C_3 [\log_{10}(F_{M2} R_{x'm})]^3 + C_2 [\log_{10}(F_{M2} R_{x'm})]^2 + C_1 \log_{10}(F_{M2} R_{x'm}) + C_0 \quad (4.26)$$

with

$$\left. \begin{aligned} C_4 &= -5.399 \times 10^{-5} \\ C_3 &= 2.031 \times 10^{-3} \\ C_2 &= -2.888 \times 10^{-2} \\ C_1 &= 1.8523 \times 10^{-1} \\ C_0 &= -2.7643. \end{aligned} \right\} \quad (4.26a)$$

The variation of p with $\log_{10}(F_{M2} R_{x'm})$ is shown in Figure 11.

The Mach number factors F_{M1} and F_{M2} in Equations (4.25) and (4.26) are given by

$$F_{M1} = \frac{0.178 M_1^2}{[\tan^{-1}(0.4219 M_1)]^2}, \quad \text{with } [] \text{ in radians}, \quad (4.27)$$

* See pages 323 and 324 of *The Mechanics of Fluids*, by W. J. Duncan, A. S. Thom and A. D. Young, published by Edward Arnold, 1960.

and

$$F_{M2} = \frac{[1 + 0.178M_1^2]^{-0.702}}{F_{M1}}. \quad (4.28)$$

These factors are presented in Figure 12 for M_1 up to 3.

Equation (4.25) fits the data of ESDU 68020 to within $\pm 0.02\%$ for $(R_{x'm})_{min} \leq R_{x'm} \leq 10^9$, and $(R_{x'm})_{min}$ is the approximate lower limit for a fully-turbulent boundary layer (see ESDU 68020), given by the equation

$$(R_{x'm})_{min} = (1 + 0.036M_1 + 0.17M_1^2 + 0.045M_1^3) \times 10^5. \quad (4.29)$$

Figure 13 presents C_{fm} against $R_{x'm}$ over the range $10^5 \leq R_{x'm} \leq 10^9$ for $M_1 = 0$ to 3 in increments of 0.5, obtained from Equations (4.25) to (4.28). Also shown is the boundary for $(R_{x'm})_{min}$ from Equation (4.29).

5. APPLICABILITY AND ACCURACY

5.1 Applicability

The method of Section 4 for cavity drag prediction applies to rectangular planform cavities having closed flow (see Sketches 3.1(a) and 3.1(b)) for Mach numbers up to 3. The cavities, which have no doors, lie in a flat plate with a fully-turbulent boundary layer at the cavity location. For the effect on cavity drag of a pair of doors open at 90° see ESDU 10016.

The form of Equation (4.1) and the parameters involved in it were derived from data where the cavity dimensions were small compared to the boundary-layer thickness of the approach flow. There are no force balance tests that could be used to demonstrate any effects of boundary-layer thickness on cavity drag for closed flow; but for transitional and open flows refer to Section 5.1 of ESDU 00007. Analysis of drag data derived from integrated pressures on cavity front and rear walls (Derivation 10) has shown that while, for closed flow, there is a significant effect of boundary-layer thickness on C_{Dp} , the effect on C_{Dp}/C_{fm} is considerably reduced, giving variations to within about ± 1 . It is therefore assumed that the use of C_D/C_{fm} largely accounts for likely variations of boundary-layer thickness. Experience with various excrescence types has shown that, in practice, methods based on the assumption of a large boundary-layer thickness compared to excrescence height or length appear to apply well even when the boundary-layer thickness is comparable to the height. This was true for circular planform cavities in ESDU 74036 (Reference 18) and it is true here, see Section 5.2. However, it would be unwise to assume that the method applied without loss of accuracy to very thin boundary layers, with h/δ_m greater than 2 to 2.5, say. In this context it is helpful to be able to estimate the boundary-layer thickness parameter, h/δ_m , where δ_m is the boundary-layer thickness at the cavity mid-length, in the absence of the cavity. This can be accomplished through the equation

$$h/\delta_m = (h/l)(l/\delta_m) = (h/l)R_l/R_{\delta m}. \quad (5.1)$$

Figure 14, derived from Figure 9 of ESDU 68020, presents $R_{\delta m}$ against $R_{x'm}$. The curve in Figure 14, which is independent of M_1 , is given by

$$R_{\delta m} = \frac{0.0641 R_{x'm} [\log_{10}(R_{x'm})]^{-1.06}}{1 - 3.03 [\log_{10}(R_{x'm})]^{-1.06}}. \quad (5.2)$$

Table 5.1 summarises the parameter ranges covered by the experimental data of Derivations 1 to 3 and 14 to 16 used to correlate b_0 in Equation (4.12).