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The acoustic attenuation of absorbent linings in cylindrical flow ducts

**Associated software: ESDUpac A0012
ESDUpac B0012**

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THE PREPARATION OF THIS DATA ITEM

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THE ACOUSTIC ATTENUATION OF ABSORBENT LININGS IN CYLINDRICAL FLOW DUCTS

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THE ACOUSTIC ATTENUATION OF ABSORBENT LININGS IN CYLINDRICAL FLOW DUCTS

1. NOTATION

		Units	
		SI	British
a	radius of duct (see Sketch 6.2)	m	ft
a_1, b_1, s_1	constants in Equation (4.5)		
c	speed of sound	m/s	ft/s
D	diameter of duct (see Sketch 6.2)	m	ft
f	frequency	Hz	Hz
f^*	non-dimensional frequency ($= fD/c$)		
i	subscript denoting i th duct section		
J_n	Bessel function of first kind of order n		
k	wave number ($= 2\pi f/c$)	m^{-1}	ft^{-1}
k_r^m	eigenvalue in radial direction	m^{-1}	ft^{-1}
$k_{r^*}^m$	non-dimensional eigenvalue in radial direction ($= ak_r^m$)		
k_z^{mn}	eigenvalue in axial direction	m^{-1}	ft^{-1}
$k_{z^*}^{mn}$	non-dimensional eigenvalue in axial direction ($= ak_z^{mn}$)		
L	duct length	m	ft
M_1	Mach number in boundary layer region		
M_w	Mach number at wall of duct		
M_0	Mach number in uniform flow region		
MM_n	total number of propagating modes for given n		
m	mode number in radial direction		
NN	highest circumferential mode number		

$N1$	specified circumferential mode number		
n	mode number in circumferential direction		
q	number of points across boundary layer		
r_0	radial distance from duct centreline to edge of boundary layer (see Sketch 6.1)	m	ft
r_1	radial distance to points in boundary layer (see Sketch 6.2)	m	ft
SR	slip ratio ($= M_w/M_0$)		
Z	specific acoustic impedance (see Section 6.1.4)	Ns/m ³	lbf s/ft ³
Z^*	normalized acoustic impedance ($= Z/\rho c$) (see Section 6.1.4)		
β_z^{mn}	$\text{Im}(k_z^{mn})$	m ⁻¹	ft ⁻¹
ρ	density of fluid	kg/m ³	lb/ft ³
Δ	attenuation of sound power level due to duct liner	dB	dB
δ	boundary layer thickness (see Sketch 6.2)	m	ft

2. INTRODUCTION

This Item provides a computerized means of estimating the sound power attenuation due to the introduction of acoustically absorbent linings in circular ducts from a knowledge of the normalized acoustic impedance of the linings at each frequency. The computed result may also be interpreted as the liner insertion loss, defined as the reduction in sound power following the introduction of the duct liner. The prediction method used is based on a theoretical model which allows a selection of typical velocity profiles across the duct. The program has the capability to consider up to ten different liner sections in series in a circular duct of constant cross-sectional area.

The Item addresses the total power reduction within the duct and does not consider subsequent radiation to the far-field.

3. APPLICATION AND ACCURACY

The prediction method given in this Item allows the user to estimate the sound power attenuation produced by acoustical liners in ducts, a common requirement if, for example, the noise from an aircraft engine or a ventilating system is to be predicted. The computer program that is provided with this Item gives solutions for straight cylindrical ducts of circular cross-section and is applicable to ducts with both positive and negative flows. If the flow is in the direction of sound propagation, it is considered positive. The user is given the option to estimate the attenuation for various modal energy distributions of the incident sound including a modal structure thought to be representative of rotor-stator interaction noise from a single stage fan. The several combinations of circumferential modes and radial modes are described in Section 6.1.1.

Results can be obtained for one-third octave band centre frequencies in the range of 50 Hz to 20 kHz, an array of specified frequencies or a frequency range with a constant step increment. Note, however, that in

the first case calculations are only carried out at the band centre frequencies. In circumstances where either the attenuation spectrum or the source spectrum or both are changing rapidly with frequency, this simplification will lead to results which should be treated with caution.

The method used in this Item makes the following assumptions:

- (i) The liner is considered to be locally-reacting, with impedance that is independent of the angle of the incident sound.
- (ii) Only the modes that would propagate in an unlined duct will propagate in a lined duct.
- (iii) The flow is in the axial direction.
- (iv) There are no end effects, *i.e.* no waves are reflected either at the ends of the duct or at transition regions if more than one duct is being considered.
- (v) Viscosity and heat transfer effects are not significant.
- (vi) Velocity profiles are axisymmetric and constant along the entire duct length when boundary layer flow is considered.
- (vii) There is equal energy in all propagating modes. This assumption may be overridden by the user by adjusting the levels of individual modes. This weighting of modes is described in Section 6.1.5.

The data required for the program includes values for the normalized acoustic impedance for each lined section. If the duct Mach number is low, say < 0.1 , it is unlikely that there will be substantial error if no-flow impedance data which may be obtained from sources such as ESDU 74003 and 74004 are used. At higher Mach numbers, however, it would be usual to use data from the liner manufacturers since the determination of the impedance is not readily achievable using simple apparatus or by calculation.

Occasionally, inconsistencies in results may indicate that at certain frequencies some modes have been missed by the calculation procedure. These missing modes may sometimes be located by adjusting the value of the variable ERR in the main program for the particular frequency which has modes missing.

However, it is not always possible to locate missing modes by this technique. If progressive alteration of ERR fails to find the modes then it may be necessary to omit that frequency.

Occasionally the program may fail with a 'floating divide by zero' error at a particular frequency. Again it may be possible to recover the situation by altering the value of ERR for this frequency. If the program continues to fail, this frequency must be neglected.

In general, results show good agreement with measured data (References 2, 5 and 6). Over a range of $1 < ka < 200$, an rms[†] error of less than 2½ dB is evident at two different flow rates.

For values of ka significantly outside this range the results should be treated with caution.

[†] The rms error is an indicator of the accuracy of prediction. It has been calculated here from comparisons between measured data and predictions using the equation

$$\text{rms error} = \sqrt{\sum[(\text{measured value} - \text{predicted value})^2]/(\text{number of samples})}$$

The ms error can be used in a way analogous to the usage of standard deviation to obtain degrees of confidence in the prediction, *i.e.* twice rms error corresponds to 90% confidence and thrice rms error to 95% confidence.

3.1 Modelling Fan Noise

In order to assist the user in applying this program to fan noise, the following procedure is recommended. The broadband and the tonal noise content from the fan should be treated by separate applications of the program to derive the separate attenuation applicable to each noise component. Firstly the attenuation for the broadband noise should be calculated using Option 1 in Section 6.1.1 *i.e.* using the assumption of equal energy in all propagating modes. Then the attenuation for the rotor-stator interaction noise from the fan should be calculated using Option 4 in Section 6.1.1 employing the input values of fan speed and rotor and stator blades numbers. Calculations are performed for the rotor-stator interaction tone and the next three harmonics (Reference 1).

4. THEORETICAL BACKGROUND

The propagation of sound in a duct can be described by the wave equation which is conveniently expressed in cylindrical coordinates for a circular duct. From the eigenvalues which form the solutions of the wave equations, the attenuation of the duct can be derived. Detailed derivations for a circular duct can be found in References 2, 3, 4 and 6, but the main working equations on which the prediction method is based are given here. For a circular duct where the fluid flow is assumed to be in an axial direction and the lining assumed to be uniform through the duct, the sound power level attenuation at each frequency, assuming equal energy in all propagating modes in the incident field, is given by

$$\Delta = 10 \log_{10} \left\{ \frac{\sum_{n=0}^{NN} MM_n}{\sum_{n=0}^{NN} \sum_{m=1}^{MM_n} e^{2\beta_z^{mn} L}} \right\}, \quad (4.1)$$

where $\beta_z^{mn} = \text{Im}(k_z^{mn})$,
 L = lined length of duct section
 and MM_n = total number of propagating modes for a given n .

In this formulation, k_z^{mn} is the eigenvalue in the axial direction, the imaginary part of which represents the rate of decay in the axial direction. $\text{Im}(k_z^{mn}) < 0$ for decaying wave propagation.

Values of k_z^{mn} for uniform flow are obtained by solving the eigenvalue equation expressed in non-dimensional form below.

$$\frac{i\pi f^*}{Z^*} \left(1 - \frac{k_z^{mn}}{k} M_0 \right)^2 = k_{r^*}^m \frac{J_{n+1}(k_{r^*}^m)}{J_n(k_{r^*}^m)} - n, \quad (4.2)$$

where $k_{r^*}^m = k_r^m a$,
 $f^* = f \frac{D}{c}$
 and $Z^* = Z/\rho c$.

The eigenvalues k_z^{mn} and k_r^m are related by the equation

$$k_z^{mn} = \frac{-kM_0 + [k^2 - (1 - M_0^2)(k_r^m)^2]^{1/2}}{1 - M_0^2}. \quad (4.3)$$

This equation also provides a means of establishing the number of propagating radial modes.

Making the assumption that only the modes that would propagate in an unlined duct will propagate in a lined duct, the number of radial modes in a hard-walled duct is calculated as follows.

For propagation in a hard-walled duct, k_z^{mn} has to be real. Therefore, from Equation (4.3)

$$k^2 \geq (1 - M_0^2) \cdot (k_r^m)^2$$

or

$$k_r^m \leq \frac{ka}{\sqrt{1 - M_0^2}}.$$

For a hard-walled duct, k_{r*}^m is obtained by solving for increasing values of m

$$\frac{dJ_n(k_{r*}^m)}{dk_{r*}^m} = 0. \quad (4.4)$$

When the value of k_{r*}^m found as m increases is greater than the frequency cut-off parameter, $ka/(1 - M_0^2)^{1/2}$, the number of propagating radial modes, MM_n , is taken as m .

The equations with the eigenvalues k_z^{mn} and k_r^m are cycled using the Newton-Raphson iteration procedure and if there is no convergence, k_r^m is set to zero.

For boundary layer flow, the eigenvalue equation that has to be solved is, in non-dimensional form,

$$\begin{aligned} (b_1 - 1)J_n(k_{r*}^m) - \left\{ (1 + b_1)s_1 + (a_1 - 2) \right\} \cdot \left\{ J_n \left[k_{r*}^m - \frac{1}{q} \left(\frac{a}{r_0} - 1 \right) k_{r*}^m \right] \right. \\ \left. + \frac{2}{q} \left(\frac{a}{r_0} - 1 \right) k_{r*}^m \cdot \left[\left(\frac{n}{k_{r*}^m} \right) J_n(k_{r*}^m) - J_{n+1}(k_{r*}^m) \right] \right\} = 0, \end{aligned} \quad (4.5)$$

where a_1 , b_1 and s_1 are constants defined in Reference 2.

After the eigenvalues have been found the same calculation process as described for uniform flow is used to determine the sound power level attenuation.

5. MULTI-SECTIONED DUCTS

The computer program provides the user with the option to consider up to ten lined lengths in series. However, the duct must be of constant radius and cross-sectional area but can have different linings over different lengths. It is assumed that there are no scattering effects at transition points.

Then, to obtain the total attenuation for a number of ducts in series, $L_1, L_2 \dots L_i$, Equation (4.1) is modified as follows.

$$\Delta = 10 \log_{10} \left\{ \frac{\sum_{n=0}^{NN} MM_n}{\sum_{n=0}^{NN} \sum_{m=1}^{MM_n} e^{2\beta_1^{mn} L_1 + 2\beta_2^{mn} L_2 + \dots + 2\beta_i^{mn} L_i}} \right\}. \quad (5.1)$$

Thus, for example, inspection of Equation (5.1) will show that two ducts in series with identical liners and of length L each will, theoretically, give the same attenuation for each mode as a single duct of length $2L$.

6. COMPUTER PROGRAMS

There are two programs associated with this Item, programs A0012 and B0012.

Program A0012 permits the user to estimate the sound power level attenuation due to acoustic duct liners in cylindrical ducts of circular cross-section. This program when run requires the user to enter the name of the file containing the input data. The structure of this input file is shown in Section 6.1. Program B0012 facilitates the generation of input files.

Typically, the program to estimate sound attenuation can be run either by typing in the program name, A0012V30, or by clicking on the icon associated with the executable version of the program. The user is then prompted to enter the name of the input file.

6.1 Input Data and Output

The structure of the input file is shown in Table 6.1. Although most of the entries in this table require no further explanation, some do and these are discussed in Sections 6.1.1 to 6.1.6. An example is given in Section 8.

Three lines of text may be entered to title a run. The output file name must be specified in the input file.

TABLE 6.1

First line of run title (maximum of 75 characters)		
Second line of run title (maximum of 75 characters)		
Third line of run title (maximum of 75 characters)		
Output file name		
Common Data Input File name (enter 'NONE' if there is none)		
Common Data Output File name (enter 'NONE' if there is none)		
Select calculations for		
One-third octave bands [†]		Enter 0
Array of discrete frequencies (maximum of 200)		Enter 1
Constant step increment		Enter 2
If one-third octave bands selected, enter Lower one-third octave band centre frequency Upper one-third octave band centre frequency (The maximum frequency range is 50 Hz to 20 kHz) If array of frequencies selected, enter Number of frequencies Enter each frequency Press the return key after each frequency If constant step increment selected, enter Lower frequency limit Upper frequency limit Frequency step size		
Select form of output		
Total attenuation at each frequency		Enter 0
Total attenuation and attenuation for each circumferential mode		Enter 1
Total attenuation and attenuation for all modes		Enter 2
Select modes at which calculations are to be performed		
All modes		Enter 1
Circumferential mode $n = 0$		Enter 2
Circumferential mode $n = 0$ and all from $n = 1$ to NN		Enter 3
Rotor-stator interaction modes		Enter 4
Circumferential modes $n = 0$ and $n = N1$ only		Enter 5
Circumferential mode $n = N1$ only		Enter 6
If the previous entry has been 1 or 2, go to the next entry If the previous entry has been 3, enter highest circumferential mode number NN If the previous entry has been 4, enter Fan speed, rpm Number of rotor blades Number of stator blades If the previous entry has been 5 or 6, enter specific mode number $N1$		
Diameter of duct [‡]		
Speed of sound, distance per second [‡]		

TABLE 6.1 (Continued)

Flow Mach number, M_0 (positive, if flow is in direction of sound propagation; negative otherwise)
If the flow is uniform, enter 0. If not, enter Boundary layer thickness, $\delta = D/2 - r_0^{\ddagger}$ If slip is to be considered enter the slip ratio, otherwise enter 0 Select boundary layer flow profile Linear Enter 0 1/7 power curve Enter 1 Number of calculation points across boundary layer (a number around 10 is normally adequate for the power curve)
Number of lined segments of duct
Enter length of each lined section of duct. Press Return key after each entry. [‡]
Enter normalized acoustic impedance (complex) for first duct section at first frequency Enter normalized acoustic impedance for first duct section at second frequency Enter normalized acoustic impedance for first duct section at last frequency
Enter normalized acoustic impedance for second duct section at first frequency Enter normalized acoustic impedance for second duct section at second frequency Enter normalized acoustic impedance for second duct section at last frequency
.
Enter normalized acoustic impedance for last duct section at first frequency Enter normalized acoustic impedance for last duct section at last frequency
Enter the number of modes (maximum of 50) that are to be weighted If no weighting is required, enter 0 and the input is complete
For each mode, enter <i>n m weighting</i> (dB) with spaces as separators

[‡] The same units of length must be used for these parameters. The speed of sound must be in distance units per second.