

Transonic data memorandum

Example of the application of constrained multivariate optimisation techniques to the design of aerofoil sections

Design point	: Single
Geometry variation	: Leading and trailing edge flap deflections
Initial aerofoil	: Combat aircraft section
CFD	: BVGK (viscous flow)

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THE PREPARATION OF THIS DATA ITEM

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EXAMPLE OF THE APPLICATION OF CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES TO THE DESIGN OF AEROFOIL SECTIONS

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1. INTRODUCTION

This Data Item presents an example of the application of the constrained multivariate optimisation technique CODAS to minimisation of aerofoil drag coefficient, the particular example being one that was known, from previous experience, to demonstrate the effect of problems within the CFD method on the operation of the optimisation process.

The optimisation objective in this example is to minimise the C_D value at a single design point: $M = 0.79$, $C_L = 0.75$ with $R = 20 \times 10^6$ and transition fixed at 5% chord on both surfaces. The flow calculation method used is BVGK for viscous flow, set up to calculate C_D at the required C_L value. The basic section was designed originally to meet various design points applicable to a modern combat aircraft by deflection of plain leading and trailing edge flaps (Reference 1). The thickness distribution of the section (5.821% t/c at 42.39% chord, 0.2552% ρ_0/c , trailing edge thickness 0.7335% chord) was designed to ensure a good supersonic capability. The section is highly cambered to provide a very high manoeuvre performance at $M = 0.79$. Figure 1 shows the section shape and camber line. Figure 2 shows the pressure distribution, calculated by BVGK, at $M = 0.79$, $C_L = 0.75$.

Reduction of the drag of this aerofoil section at the selected flight condition is sought by the changes in effective camber available from deflection of leading and trailing edge plain flaps, hinged (in this case) at 15% and 75% chord, respectively. For convenience, deflection angles (in degrees) of the leading and trailing edge flaps are called LE and TE , respectively, and individual configurations are sometimes referred to in the form of Cartesian pairs (LE , TE).

Calculated drag coefficients are generally quoted here to 6 decimal places, not because they are believed to be correct to this level, but in order to indicate adequately the (sometimes small) differences between individual cases. The end results are not considered to be accurate to better than 4 decimal places.

The Data Item is one of a series illustrating the process of using such optimisation techniques, indicating their advantages and revealing the kinds of problems that can arise. To facilitate this, the Data Item is in narrative form.

An introduction to the application of constrained multivariate optimisation techniques to the design of aerofoil section shapes is given in Data Item No. 99019 (Reference 4). Data Item No. 99020 (Reference 5) gives examples of single design point optimisation that have no problems arising from the CFD method.

2. NOTATION

2.1 Algebraic

C_D	drag coefficient	
C_{DW}	wave drag coefficient	
C_L	lift coefficient	
C_p	pressure coefficient	
c	aerofoil chord	
$\bar{H}_{0.99}$	transformed boundary layer shape parameter at 99% chord on upper surface	
LE	deflection of leading edge flap, positive downwards	degree
M	free stream Mach number	
M_{local}	local Mach number	
R	Reynolds number, based on chord and freestream conditions	
TE	deflection of trailing edge flap, positive downwards	degree
t	maximum thickness of aerofoil, <i>i.e.</i> maximum value of $z_U(x) - z_L(x)$	
x	chordwise coordinate	
z	normal coordinate	
$z_L(x)$	normal coordinate of lower surface of aerofoil at x	
$z_U(x)$	normal coordinate of upper surface of aerofoil at x	
ρ_0	leading edge radius	

2.2 Computational

NFE	number of calls by RQPMIN to the CFD routine
NFEMAX	maximum number of calls by RQPMIN to the CFD routine (set in input data)
RES	maximum residual in inviscid-flow element
VRES	maximum residual in viscous-flow element

2.3 Definitions

CFD	“Computational Fluid Dynamics”, programs used to calculate the flow around aerofoil sections. In the example in this Data Item the chosen CFD method is BVGK (see Reference 2). The generic term “CFD” is used except where the behaviour under discussion is specific to BVGK.
CODAS	“Constrained Optimisation Design of Aerodynamic Surfaces”, a design method for aerofoils and wings that couples CFD and numerical optimisation methods (see Reference 4).
Optimiser	Software capable of solving constrained multivariate optimisation problems.
RQPMIN	The chosen optimiser in the example in this Data Item (see Reference 4).

3. EXAMPLE

3.1 Contour Plots

In the present case, as in Example 1 in Data Item 99020 (Reference 5), the limitation of the number of independent variables to two enables the design surface to be displayed as a contour map of C_D versus LE and TE . The contour map for this example* (Figure 3) appears to be somewhat simpler than that in Reference 5 with only one optimum point, which additional spot point calculations show to be at $(-1.015, -0.860)$ where $C_D = 0.007569$. Figure 4 shows a similar contour map for $\bar{H}_{0.99}$.

3.2 Optimisation Searches

Several optimisation search runs were made, starting from different points. The resulting tracks across the LE, TE plane are shown in Figure 5a, with the end points shown separately in Figure 5b for clarity. All terminated with a “Converged B” message†. Note that the search starting from point 3 (2, 0) made very little progress from its starting point before terminating. The search was restarted, starting from point 3a (1.5, -0.25), nearer to the global optimum. This search made more progress but still reached its “Converged B” status well away from the global optimum. From examination of the contour map, it appears that the path followed is along a valley leading to the global optimum but, presumably, the gradient along this path is sufficiently low for the optimiser to consider that it has reached the optimum. The contour map for $\bar{H}_{0.99}$ (Figure 4) is included to check whether $\bar{H}_{0.99}$ is particularly high in this region of the LE, TE plane as this might have had a bearing on the early conclusion of these runs. In fact, as is seen from Figure 4, $\bar{H}_{0.99}$ is highest at $(-3, 0)$, where the corresponding run is well behaved. No further searches starting from points near points 3 and 3a were made.

The remaining six runs all finished in the vicinity of the global optimum. The values of C_D obtained are plotted as a bar chart, in size order, in Figure 6, from which it is seen that four (1, 5, 7, and 6) are very close (the largest being 0.39% greater than the global optimum and the smallest only 0.08% greater).

* The contours are based on spot calculations by BVGK at the nodes of a grid formed by the values $LE = -3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2$ degrees and $TE = -3, -2, -1, -0.75, -0.5, -0.25, 0$ degrees. Further calculations have shown that C_D values tend to be erratic in the vicinity of the minimum (see, for instance, Figures 8 and 18). The global optimum corresponds to zero wave drag.

† Indicating that RQPMIN had located a point which satisfied the constraints to the tolerance specified and that a value of the objective function sufficiently smaller than the current value cannot be found (Reference 3).

Convergence of the CFD calculations may be judged from the behaviour of the residuals from the inviscid and viscous phases (RES and VRES respectively). Ideally, these show a steady decrease as the number of iterations increases; the lowest level that can be hoped for is dependent on the accuracy limits of the calculations. The actual behaviour for each of these six runs^{*} is displayed in Figures 7a to 7f, compared with arbitrarily chosen standards of acceptable convergence[†]. The nature of the convergence for each run is as follows.

- From Starting point 1, the last call has good convergence in the coarse grid calculations. The fine grid calculations start well but at about 500 iterations RES suddenly increases to an unacceptable level and stays there. At about 700 iterations VRES also increases but then recovers its previous rate of improvement.
- From Starting point 2, the last call has poor convergence, never reaching the acceptable levels for RES in either coarse or fine grid nor for VRES in fine grid calculations. Both RES and VRES exhibit an oscillatory behaviour with the fine grid.
- From Starting point 4, the last call has good convergence throughout although, for the fine grid, both RES and VRES exhibit a slight upward drift after good levels have been reached.
- From Starting point 5, the last call has poor convergence for the inviscid phase, staying at a constant value after about 450 fine grid iterations. The viscous phase has good convergence although after VRES has reached a good level there is a slight upward drift (as with starting point 4).
- From Starting point 6, the last call makes reasonable progress initially, reaching acceptable levels for RES and VRES at about 600 fine grid iterations, after which there is a slight upward drift. At about 700 fine grid iterations both increase suddenly, RES going back above the acceptable level and staying there whilst VRES stays just below the acceptable level.
- From Starting point 7, the last call reaches the acceptable levels of convergence for both RES and VRES. Note that the RES history contains, from about 450 to 610 fine grid iterations, a stretch of constant value only slightly lower than those seen for starting points 1 and 5.

3.3 Poor Convergence of CFD Code

3.3.1 Cross section of design surface at $TE = -0.86$

Because of the variable standard of the CFD convergence in the optimisation search runs, a detailed examination was made of a cross section of the design surface taken along the $TE = -0.860$ line, *i.e.* through the global optimum at $(-1.015, -0.860)$. Values of C_D and $(C_D - C_{DW})$ for values of LE from -3 to 2 along this line are shown in Figure 8, together with corresponding final values of RES and VRES. From the C_D plot it can be seen that values taken only at 0.5 degree intervals would suggest a smooth curve, with a possible drop to a zero wave drag zone in the vicinity of $LE = -1$. Figure 9 (an enlarged view of the cross section from $LE = -1.7$ to -0.7) shows that the situation is far more complex. Points with poor CFD convergence[†] are identified by use of different symbols and it is seen that the region from $LE \approx -1.24$ to -1.02 appears to consist entirely of poorly converged results with erratic C_D values. There are further regions of poor RES convergence near $LE = -1.5$ and -1.0 . The CFD code failed in the region from $LE = -1.491$ to -1.487 (tested at 0.001 intervals), resulting in the small gap in the curves in Figure 9.

^{*} CODAS provides full details of the last call to BVGK in any run, from which RES and VRES convergence histories may be obtained.

[†] "Acceptable convergence" is defined here as either $RES < 4 \times 10^{-6}$ or $VRES < 3.6 \times 10^{-4}$ (or both). These levels allow all those results in References 5 and 6 for which RQPMIN reached either "Converged B" status or NFEMAX to be taken as having achieved acceptable CFD convergence.

It only requires extremely small changes in flap deflection to cause the change in the calculated regime. For instance, increasing LE by 0.001, from -1.240 to -1.239 , produces an increase in C_D together with a severe degradation in the standard of convergence within the CFD calculation. The effect on CFD convergence may be seen in Figure 10 which shows RES and VRES histories for both values of flap deflection. The curves are nearly identical until about 260 fine grid iterations are completed, after which the calculation for $LE = -1.240$ continues to converge while that for $LE = -1.239$ switches to some form of cycle, repeated roughly every 180 iterations. The corresponding calculated pressure distributions and the shock wave positions (deduced by BVGK in the regions of steep pressure gradients) may be seen in Figure 11 (for the complete section). Figure 12 gives a larger scale view in the vicinity of the shock waves. Four curves are shown for $LE = -1.239$, at successively greater numbers of iterations – chosen to be located at similar points in the convergence cycle (see Figure 10). It is seen, particularly from Figure 12, that identical points in the cycle have similar pressure distributions and shock positions, although the top end of the shock wave is slightly further downstream for 677 iterations than for 872. However, it is unlikely that any of the flow patterns for $LE = -1.239$ represents the true one (which is probably close to that at $LE = -1.240$).

At $LE = -1.140$, in the middle of the band of poorly converged results in Figure 9, the convergence history (Figure 13) is similar in nature to that for $LE = -1.239$ (Figure 10). An oscillatory cycle is again established, with a period of about 180 iterations. Figure 14 shows the calculated pressure distributions and deduced shock positions at various points in the cycle. These are shown in greater detail in Figure 15. As before, there is similarity at corresponding points in adjacent cycles, although in this case there are somewhat greater differences in the shock positions, away from the aerofoil surface. Again, it is unlikely that any of these flow patterns represents the true one.

An increase of 0.001 in LE , from -1.017 to -1.016 , produces a decrease in C_D , accompanied by a pronounced decrease in RES and VRES (see Figure 9). Convergence histories (Figure 16) show that RES and VRES follow near identical patterns until about 650 iterations for the fine grid, after which the $LE = -1.016$ calculation continues to converge but the $LE = -1.017$ calculation diverges. Figure 17 shows that BVGK deduced that there is a shock wave in the calculation for $LE = -1.017$ but not for -1.016 (despite the fact that surface pressure gradients (Figure 17) in the vicinity appear to be slightly smaller for $LE = -1.017$). There are only small differences between the calculated pressure distributions for the two flap deflections. Yet again, it is unlikely that the calculated flow field at $LE = -1.017$ represents the true one.

3.3.2 Cross section of design surface at $TE = -0.90$

Values of C_D , RES and VRES along a neighbouring cross section of the design surface (at $TE = -0.90$, shown in Figure 18 and, to an enlarged scale, Figure 19) show that there is little consistency between the two cross sections. At $TE = -0.90$, the region of poor convergence is of greater extent but is interspersed with converged results. This may, of course, be merely a matter of the particular values at which the spot calculations were made. It does, however, emphasise the variability of the pattern.

3.3.3 Points visited during optimisation runs

In addition to the two cross sections, a large number of results had been obtained at the various points visited during the optimisation searches of Section 3.2. Each step involves CFD calculations at a series of points in order to determine the gradients. Any one of these points may happen to be at a point of poor convergence with a possibility of high or low values for C_D , and consequent effects on the calculated gradients.

Values of C_D from those points (239 altogether) for which TE is between -0.93 and -0.81 (*i.e.* they lie close to the cross sections shown in Figures 8 and 18) are shown in Figure 20, plotted against LE . No values of RES or VRES are available for comparison since CODAS only outputs detailed information for the final call to the CFD code, *i.e.* for the end point of the search. The full line is a least squares cubic curve fitted

to the points, the dotted lines cover a band ± 0.0001 wide around the cubic curve, and the broken line shows $C_D - C_{DW}$, *i.e.* zero wave drag (again a smoothed curve).

Of those 239 points, 18 have C_D values above the band and 65 are below (mostly at or close to the zero wave drag line). These are of exactly the same nature as the poorly converged results discussed in Sections 3.3.1 and 3.3.2 and it is clear that the searches encountered the associated problems with gradients occasionally being unreliable.

In the midst of this pattern of poorly converged (and, hence, unreliable) CFD results, the optimisation search appears to have found the global optimum. In a two variable case, such as this one, it is possible to confirm that the solution offered is indeed the global optimum. In a more general case this is not practicable and it is necessary to rely on searches from many different starting points. The irregular nature of the CFD results introduces the possibility of a large number of local pseudo-optima. These are surrounded by a region of largely unreliable CFD results and logical progress of a gradient-based search cannot be certain.

3.4 Failures Within the CFD Code

During spot calculations of a flow field an error can occur within the CFD code (usually appearing in the form of a “divide by zero”) which results in failure of the CFD code. Although this is disastrous to the case being calculated it is usually possible to make small adjustments to the case being studied or to the parameters controlling the CFD code which enable the case to be completed. However, since the remedial action depends on the individual flow condition, this is not possible in the case of CFD calculations within an optimisation search. It is, therefore, essential that the optimiser is able to identify that a failure has occurred and to take alternative action.

No such failures occurred at the nodes^{*} used to calculate the contour map. However, it is noted in Section 3.3.1 that failures in the CFD code occurred[†] when investigating the cross sections in the region from $LE = -1.491$ to -1.487 with $TE = -0.86$. Other CFD failures occurred during the optimisation searches, see Table 3.1.

TABLE 3.1 CFD code failures during optimisation searches

Starting point	Number of optimiser iterations	Number of calls to CFD code	Number of CFD failures
1	23	116	0
2	21	95	0
3	32	322	68
3a	26	406	0
4	18	135	3
5	29	159	0
6	26	228	0
7	24	188	12

^{*} The contours are based on spot calculations by BVGK at the nodes of a grid formed by the values $LE = -3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2$ and $TE = -3, -2, -1, -0.75, -0.5, -0.25, 0$. Further calculations have shown that C_D values tend to be erratic in the vicinity of the minimum (see, for instance, Figures 8 and 18). The global optimum corresponds to zero wave drag..

[†] No remedial action was taken here since it was important that the cross section should correspond to the contour map as closely as possible.

In total, failure occurred in 90 of the 1874 calls to CFD code, *i.e.* about 5% overall*. The actual distribution probably varies from area to area. The high frequency of failure (21%) in the area covered by the search that started from point 3 may well have contributed to that search stopping well away from the optimum. Nevertheless, it is clear that in most cases these failures have not affected the capability of CODAS to locate an optimum. In general, CODAS can survive CFD failure because RQPMIN is able to identify the situation and to take appropriate action† to circumvent the problem. However, if CFD failure occurs in the initial call to the CFD code there is no such possibility and the only option available is to alter the problem, either by using a different starting point or by adjusting the parameters of the CFD calculation.

4. CONCLUDING REMARKS

Two particular problems can occur in CFD calculations during optimisation searches,

- poor convergence, and
- failure within the code.

Poor convergence of the CFD calculations (which may be associated with the presence in the flow field of an incipient weak shock wave) and the associated unreliability of the calculated C_D results can lead to uncertainty in the reported solution. It is essential to review the convergence standard of the results and, as always, to make several runs starting from widely different points. However, despite poor convergence, the method may still produce the global optimum.

Failure of the CFD code during the progress of an optimisation run should not prevent a satisfactory conclusion of the optimisation process, provided that the failure is detected by the optimiser and an alternative strategy for gradient calculation is available. However, if failure happens at the starting point then it is impossible to continue and the only option available is to try another starting point.

5. REFERENCES

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* This compares with a total of 47 failures in 17308 calls to CFD, *i.e.* about 0.3%, in the optimisation runs in References 5 and 6. This suggests that the present example is more prone to CFD failures.

† For instance, if the gradient was being calculated by forward differences, RQPMIN would try using backward differences. This process was not available in earlier versions of CODAS, causing the optimisation run to fail.