

Transonic data memorandum

Examples of the application of constrained multivariate optimisation techniques to the design of aerofoil sections

Design point	: Single
Geometry variation	: Leading and trailing edge flap deflections
Initial aerofoil	: Combat aircraft section
CFD	: BVGK (inviscid flow) and Euler code

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Mr G.R. Richards	— BAE SYSTEMS, Military Aircraft and Aerostructures, Farnborough
Mr D.R. Stanniland	— Aircraft Research Association
Mr P.W.C. Wong	— Aircraft Research Association.

The technical work in the assessment of the available information and the construction and subsequent development of the Data Item was carried out by

Mr A. Stanbrook — Consultant to ESDU.

The person with overall responsibility for the work in this subject area is Mr D.J. Mitchell.

EXAMPLES OF THE APPLICATION OF CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES TO THE DESIGN OF AEROFOIL SECTIONS

Design point : Single
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1. INTRODUCTION

This Data Item presents examples of the application of constrained multivariate optimisation techniques in inviscid flow. The constrained multivariate optimisation technique for the design of aerofoil sections, CODAS (Reference 6), has two inviscid CFD options – an Euler code (Reference 2), only available for inviscid flow, and BVGK (Reference 4) in its inviscid mode. The Euler code offers the capability of use for supersonic free stream conditions whilst BVGK (inviscid) can be useful in situations where BVGK (viscous) has difficulties. This Data Item compares the behaviour of BVGK (inviscid) with the Euler code in identical transonic CODAS studies.

The particular case being used in these examples is that of deflection of simple leading and trailing edge flaps on the combat aircraft aerofoil used in Data Item No. 00022 (Reference 8). The optimisation objective is to minimise the C_D value at a single design point, $M = 0.79$, $C_L = 0.75$. The basic section was designed originally to meet various design points applicable to a modern combat aircraft by deflection of plain leading and trailing edge flaps (Reference 1). The thickness distribution of the section (5.821% t/c at 42.39% chord, 0.2552% ρ_0/c , trailing edge thickness 0.7335% chord) was designed to ensure a good supersonic capability. The section is highly cambered to provide a very high manoeuvre performance at $M = 0.79$. The section shape and camber line may be seen in Reference 8.

Reduction of the drag of this aerofoil section at the selected flight condition is to be sought by the changes in effective camber available from deflection of leading and trailing edge plain flaps, hinged (in this case) at 15% and 75% chord, respectively. For convenience, deflection angles (in degrees) of the leading and trailing edge flaps will be called LE and TE , respectively, and individual configurations will sometimes be referred to in the form of Cartesian pairs (LE , TE).

Calculated drag coefficients are generally quoted here to 6 decimal places, not because they are believed to be correct to this level, but in order to indicate adequately the (sometimes small) differences between individual cases. The end results are not considered to be accurate to better than 4 decimal places.

The Data Item is one of a series illustrating the process of using such optimisation techniques, indicating their advantages and revealing the kinds of problems that can arise. To facilitate this, the Data Item is in narrative form.

An introduction to the application of constrained multivariate optimisation techniques to the design of aerofoil section shapes is given in Data Item No. 99019 (Reference 6). Data Item No. 99020 (Reference 7) gives examples of single design point optimisation in which no problems arise from the CFD method. Data Item No. 00022 (Reference 8) shows the effect of problems within the CFD method on the reliability of the multivariate optimisation technique.

2. NOTATION

2.1 Algebraic

C_D	drag coefficient	
C_{DW}	wave drag coefficient	
C_L	lift coefficient	
C_m	pitching moment coefficient (about quarter-chord point, positive nose-up)	
C_p	pressure coefficient	
c	aerofoil chord	
LE	deflection of leading edge flap, positive downwards	degree
M	free stream Mach number	
M_{local}	local Mach number	
TE	deflection of trailing edge flap, positive downwards	degree
t	maximum thickness of aerofoil, <i>i.e.</i> maximum value of $z_U(x) - z_L(x)$	
x	chordwise coordinate	
z	normal coordinate	
$z_L(x)$	normal coordinate of lower surface of aerofoil at x	
$z_U(x)$	normal coordinate of upper surface of aerofoil at x	
α	angle of incidence of reference line (<i>i.e.</i> for aerofoil section with leading and trailing edge flaps undeflected)	degree
ρ_0	leading edge radius	

2.2 Computational

NFE	number of calls by RQPMIN to CFD routine
NFEMAX	maximum number of calls by RQPMIN to CFD routine (set in input data)
RES	residual in inviscid-flow element, maximum value in BVGK, average value in Euler code
XTOLU	controls how accurately constraints are to be met (default value: 10^{-6})

2.3 Definitions

CFD	“Computational Fluid Dynamics”, programs used to calculate the flow around aerofoil sections. In the examples in this Data Item the chosen CFD methods are BVGK (Reference 4) in its inviscid mode and an Euler code (Reference 2), using default values for the control parameters. The generic term “CFD” is used except where the behaviour under discussion is specific to BVGK or the Euler code.
CODAS	“Constrained Optimisation Design of Aerodynamic Surfaces”, a design method for aerofoils and wings that couples CFD and numerical optimisation methods (see Reference 6).
Optimiser	Software capable of solving constrained multivariate optimisation problems.
RQPMIN	The chosen optimiser in CODAS (see Reference 5).

3. EXAMPLES

3.1 Characteristics of the Basic Aerofoil, as Calculated by the BVGK (Inviscid) and the Euler Codes

Figure 1 compares the variation of C_L with α as calculated by the two CFD methods. It is seen that there is close agreement for α from -3 to about 0.5 degree, which encompasses the value to be used in the subsequent optimisation study ($C_L = 0.75$). Figures 2 and 3 show similar comparisons for C_D and C_m , respectively, and Figure 4 compares pressure distributions at $C_L = 0.75$ obtained by the two methods. The differences are again slight, consisting of $0.05c$ variation in the position of a weak shock at about $c/3$ on the upper surface. For $C_L = 0.75$ the values of C_D obtained from the two methods are

0.004819 at $\alpha = -0.267$ degree from the BVGK calculations, and

0.005245 at $\alpha = -0.272$ degree from the Euler code calculations.

It should be noted that there are differences in the way in which C_D is calculated in the two CFD methods. The Euler code used in CODAS calculates C_D by simple integration of surface pressures, whereas BVGK makes use of the Machcont method (Reference 3) to take more detailed account of the wave drag element.

The standard of convergence within the two codes is illustrated in Figures 5a and 5b, which show the variation of RES^* with the number of iterations in the CFD calculations at $C_L = 0.75$, for BVGK and the Euler code respectively. A satisfactory level of convergence is taken to be $RES < 10^{-4}$ for BVGK and $RES < 10^{-3}$ for the Euler code. BVGK was set up for a maximum of 900 coarse and 900 fine grid iterations but the calculation terminates when a suitable degree of convergence is obtained. It will be seen that convergence was achieved in this case with only 229 coarse iterations and 20 fine iterations. In contrast, the Euler code continued for the total number of iterations set, which in this case was 500 coarse, 250 medium and 900 fine grid iterations.

It is of interest to analyse the relative times taken by the two methods (remembering that the actual times quoted are machine- and case-dependent). The overall calculation (for a single RQPMIN iteration) took 2.055 seconds for BVGK and 17.346 seconds for the Euler code. The coarse grid for BVGK was set at 80×15 and the fine grid at 160×30 ; hence the total number of grid point calculations was 370 800 (*i.e.* $229 \times 80 \times 15 + 20 \times 160 \times 30$). The time taken per grid point calculation for BVGK was therefore 5.5×10^{-6} seconds. Similarly, the total number of grid point calculations for the Euler code was 4 816 250 (*i.e.* $500 \times 49 \times 7 + 250 \times 93 \times 13 + 900 \times 193 \times 25$) and, hence, the time per grid point calculation for the Euler code was 3.6×10^{-6} seconds. Thus the individual grid point calculation for BVGK takes 50% longer than for the Euler code but the larger number of grid point calculations used by the Euler code results in a much greater overall time. The convergence history in Figure 5b shows that, *for this case*, adequate convergence levels might have been obtained by 350 coarse, 250 medium and 300 fine grid iterations. This would have reduced the run time to 6.7 seconds. However, there is no certainty that these lower iteration numbers would provide adequate convergence in all cases. Furthermore, one case where the lower number of Euler code iterations was used required more RQPMIN iterations – so the gain in overall run time was reduced.

* Note that RES is the maximum value of the residual for BVGK and the average value for the Euler code.

3.2 Contour Plots

3.2.1 BVGK

In the present case, as in the examples in Data Item Nos 99020 (Reference 7) and 00022 (Reference 8), the limitation of the number of independent variables to two enables the design surface to be displayed as a contour map of C_D versus LE and TE . However, the contour map for BVGK* shown in Figure 6 exhibits some peculiarities which need explanation.

In its inviscid mode, BVGK cannot be set to calculate the flow field at a given value of C_L . Hence, the spot data on which the contour map is based were produced by CODAS runs set up with $C_L = 0.75$ as a constraint and α as a variable. In a well behaved case, as at $LE = 0$, $TE = 0$ (Figure 7), C_L quickly converges to its target value of 0.75. However, at some rogue points, such as $LE = -3$, $TE = 1$ (Figure 8), C_L fails to converge on the target value at all. This behaviour does not depend on the starting value for α ; starting values above or below the target would all fail to converge on the target C_L . Further investigation reveals a discontinuity in the calculated C_L , α curve (see Figure 9); between $\alpha = -0.92682$ and -0.92683 there is a jump of about 0.02 in C_L . The variation of C_D with C_L (see Figure 10) also shows a discontinuity. The correct value for C_D is probably somewhere between level B and C (extrapolated from the two separate parts of the curve) but the value recorded by CODAS would be either level A or D, depending purely on which occurs when the iterations (NFE) reach the set value of NFEMAX. It is this variability that results in the waviness of some of the contours in Figure 6. The cause of this phenomenon is indicated in the pressure distributions and shock positions (see Figure 11). Between $\alpha = -0.92682$ and -0.92683 there is a small incremental change in shock position, probably due to a jump between grid lines. It must be emphasised that there is no suggestion that this would actually occur in practice; this is simply a phenomenon resulting from the calculation process.

Of the 121 points used to produce the contour map of Figure 6, the 28 that are affected by this phenomenon (to varying extent) are indicated in Figure 12.

A cross section of the contour map at $TE = -1.20$ was produced by spot point calculations, (a) to indicate the precise nature of the design surface near the optimum, and (b) to investigate the degree of convergence of the CFD method. The results are shown in Figure 13 and it will be seen that the degree of convergence was virtually constant across the cross section. Of the 30 points shown, only two suffered from the effect discussed above, *i.e.* $LE = -1.1$ and -2.0 .

The contour map in Figure 14 was produced by (a) including appropriate C_D values from the $TE = -1.2$ cross section, and (b) adjusting C_D values (in cases where C_L had not converged) to those equivalent to point B in Figure 10. The result† is thought to be a better representation of the design surface although there are some discrepancies between the contours and the cross section in Figure 13 (the global optimum is the region of zero drag and is probably a little to the right of the region shown by the contours). The contour map shows that the optimum lies in a valley and also suggests the existence of more than one optimum‡.

In operation, CODAS has to use unadjusted values of C_D . There is no way in which a failure to converge on the target C_L at any stage in the search procedure can be detected and the corresponding value of C_D corrected. The contours as shown in Figures 6 and 14 do not represent the true state of affairs as seen by RQPMIN (which calculates C_D afresh at each point as required). Isolated points have these convergence errors, neighbouring points may have other errors (greater or smaller, of the same or opposite sign) or may be correct. This essentially erratic behaviour will have consequential effects on the calculation of gradients and, thereby, affect the efficiency of the search procedure.

* The contours are based on spot calculations by BVGK at the nodes of a square grid where LE and TE take the values $-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2$ degree.

† Here, the contours are based on spot calculations by BVGK at the nodes of a grid where LE takes the values $-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2$ degree and TE takes the values $-3, -2.5, -2, -1.5, -1.2, -1, -0.5, 0, 0.5, 1, 1.5, 2$ degree.

‡ It must be remembered that the contour maps are only an approximation. Being produced by interpolations from a grid of points, the locations of some characteristics, such as the global and local optima, may be displaced from their true locations.

3.2.2 Euler code

The output from the Euler code calculations suffered none of the convergence problems discussed in Section 3.2.1. All runs converged quickly to $C_L = 0.75$. The resulting contour map, based on the same grid of points as in Figure 14, is shown in Figure 15. Again, the optimum is seen to lie at the end of a valley (more prominent in this case) and the existence of more than one optimum point is suggested*. Additional calculations locate the global optimum at $(-0.145, -1.047)$, where $C_D = -0.000329$.

As with BVGK, a cross section of the design surface at $TE = -1.20$ was produced by spot point calculations, (a) to indicate the precise nature of the design surface near the optimum, and (b) to investigate the degree of convergence of the CFD method. The results are shown in Figure 16 and, again, it will be seen that the degree of convergence was virtually constant, at an acceptable level, across the cross section.

The negative values for C_D produced by the Euler code result from the fact that C_D is calculated by integrating surface pressures.

3.3 Optimisation Searches

For inviscid cases, CODAS runs have to be set up with $C_L = 0.75$ as a constraint and α as one of the variables. The starting value for α has to be determined by a preliminary run in which the only variable is α . If this is not done then the search is starting from outside the feasible region† and the first action of RQPMIN is to get within it by the most direct route, which may alter the effective starting point for the search (see Reference 7).

3.3.1 BVGK

Several optimisation search runs were made, starting from different points; the resulting tracks across the LE, TE plane are shown in Figure 17.

Initially, searches were made with the scaling factor‡ for incidence set at 2. The results are shown in Figure 17a and comparison with the contour map shows that the search paths tend to follow steepest descent paths. It will be seen that the three searches starting with $TE = 2$ all failed to make much progress across the plane. The search starting from point 1, (0, 0), ends at the local optimum*, those starting from points 2, (2, -3), and 6, (-1, -3), finish close (but not close enough) to the global optimum*, and that starting at point 5, (-3, -3), also ends in the valley leading to the global optimum – but even further from it. The endpoints, corresponding values of C_D and C_L , and solution status for the searches are listed in Table 3.1.

TABLE 3.1 Search solutions with BVGK/Incidence Scaling Factor = 2

Starting point number	End point values					Status (see Reference 7)
	LE	TE	C_D	C_L	α	
1	0.453	-1.077	0.000188	0.74999	0.371	No further progress possible
2	-0.717	-1.709	0.000420	0.75007	0.783	No further progress possible
3	1.852	1.231	0.012335	0.75000	-1.032	Converged B
4	-3.000	1.264	0.012031	0.75002	-1.119	No further progress possible
5	-2.257	-1.643	0.001905	0.75038	0.728	No further progress possible
6	-0.805	-1.504	0.000278	0.75060	0.658	No further progress possible
7	-1.102	1.856	0.015343	0.74822	-1.391	No further progress possible

* It must be remembered that the contour maps are only an approximation. Being produced by interpolations from a grid of points, the locations of some characteristics, such as the global and local optima, may be displaced from their true locations.

† See Reference 6 for an explanation of the feasible region.

‡ See Reference 6 for an explanation of scaling factors.

The message “No further progress possible” indicates that “RQPMIN can make no further progress in finding a point that satisfies the currently active constraints” (Reference 5). However, the only constraint is $C_L = 0.75$ and it is clear that the values of C_L achieved from starting points 1, 2 and 4 are quite acceptable. A higher value for the constraint tolerance (XTOLU) would probably result in “Converged B” status for these runs. The values of C_L achieved from starting points 5 and 6 are only marginally high; a slightly greater relaxation of the constraint tolerance would be necessary for these to be judged “Converged”.

The lowest drag solution is from starting point 1 with $C_D = 0.000188$ but Figure 14 makes it clear that this is at or near the local optimum. None of these searches located the global optimum.

The scaling factor for α was then increased to 4, resulting in the search paths shown in Figure 17b. Some of those paths double back on themselves, making the end points difficult to discern from the figure. Comparison with the contour map again shows that the search paths tend to follow steepest descent paths. It will be seen that each of these runs reached a solution lying close to a straight line joining (2, -0.7) to (-2.3, -1.3). The searches starting from points 1, (0, 0), and 2, (2, -3), appear to end between the global and local optima. However, the cross section at $TE = -1.20$ (Figure 16) suggests that the global optimum may occur at a slightly higher value of LE than the contour map indicates*. If this is the case, then the search starting at point 1 will be seen to end closer to the global than the local optimum.

The endpoints, corresponding values of C_D and C_L , and solution status are shown in Table 3.2.

TABLE 3.2 Search solutions with BVGK/Incidence Scaling Factor = 4

Starting point number	End point values					Status
	LE	TE	C_D	C_L	α	
1	-0.049	-1.153	0.000027	0.74999	0.429	No further progress possible
2	0.004	-1.145	0.000035	0.74997	0.432	No further progress possible
3	1.985	-0.728	0.000335	0.74998	0.142	No further progress possible
4	-1.288	-1.324	0.000447	0.75006	0.519	No further progress possible
5	-2.335	-1.326	0.002442	0.75000	0.516	Converged B
6	-0.924	-1.347	0.000197	0.75000	0.547	Converged B
7	-0.650	-1.263	0.000022	0.75000	0.508	No further progress possible

The message “No further progress possible” occurs against the results from starting points 1, 2, 3, 4 and 7. However, the values of C_L achieved in each of these is within 0.0001 of the target, *i.e.* quite acceptable. A higher value for the constraint tolerance (XTOLU) would probably result in “Converged B” status for these runs. Further runs were made with $XTOLU = 10^{-4}$ instead of 10^{-6} to check this and to attempt to locate the optimum solution (*i.e.* within the zero wave drag region) by the optimisation search process. These runs were started from (-0.2, 0), (-0.4, 0), (-0.6, 0), (-0.8, 0) and (0, -3), labelled 1a to 1d and 6a, respectively. The results are shown in Figure 17c and Table 3.3.

TABLE 3.3 Additional search solutions with BVGK/Incidence Scaling Factor = 4

Starting point number	End point values					Status
	LE	TE	C_D	C_L	α	
1a	-0.238	-1.192	0.0	0.75014	0.459	Converged B
1b	-0.267	-1.203	0.0	0.75007	0.466	Converged B
1c	-0.382	-1.231	0.000007	0.74998	0.477	Converged B
1d	-0.638	-1.264	0.000022	0.75004	0.510	Converged B
6a	-0.463	-1.291	0.000185	0.75009	0.527	Converged B

* It must be remembered that the contour maps are only an approximation. Being produced by interpolations from a grid of points, the locations of some characteristics, such as the global and local optima, may be displaced from their true locations.

It is seen that runs 1a and 1b both located the global optimum ($C_D = 0.0$) and that all of the additional runs converged satisfactorily.

The individual values of C_D are compared in the bar charts* in Figure 18a.

3.3.2 Euler code

The same seven initial starting points (with incidence scaling factor = 4) were used for a series of optimisation search runs using the Euler code. Their tracks across the LE, TE plane are shown in Figure 19a; the endpoints, final C_D and C_L values, and solution status are shown in Table 3.4. The C_D values are compared in the bar charts* in Figure 18b.

TABLE 3.4 Search solutions with the Euler code

Starting point number	End point values					Status
	LE	TE	C_D	C_L	α	
1	0.399	-0.978	-0.000253	0.75000	0.302	Converged B
2	-0.254	-1.114	-0.000139	0.75000	0.405	No further progress possible
3	2.544	-0.583	-0.000110	0.75000	0.007	Converged B
4	-0.991	-1.274	-0.000131	0.75008	0.485	No further progress possible
5	-2.538	-1.220	+0.000589	0.75000	0.418	No further progress possible
6	-0.979	-1.300	-0.000170	0.75000	0.502	No further progress possible
7	-0.733	-1.190	-0.000121	0.75000	0.436	Converged B

None of these searches located the global optimum. As with BVGK, the cases which are marked with the message "No further progress possible" have quite acceptable values for C_L and a higher value for the constraint tolerance (XTOLU) would probably result in "Converged B" status for all of these runs. Further searches were made, starting from the same additional points as with BVGK and with the same higher constraint tolerance (XTOLU = 10^{-4} instead of 10^{-6}). The search paths are shown in Figure 19b and the results are summarised in Table 3.5.

TABLE 3.5 Additional search solutions with the Euler code

Starting point number	End point values					Status
	LE	TE	C_D	C_L	α	
1a	-0.338	-1.285	-0.000195	0.74995	0.507	Converged B
1b	-0.145	-1.047	-0.000329	0.75007	0.355	Converged B
1c	-0.365	-1.082	-0.000325	0.74997	0.378	Converged B
1d	-0.489	-1.136	-0.000311	0.74999	0.411	Converged B
6a	-0.397	-1.105	-0.000299	0.74997	0.395	Converged B

Four of the results lie on or close to the straight line joining $(-0.489, -1.136)$ to $(-0.145, -1.047)$, with values of C_D within 0.00003 of the global optimum. The lowest drag solution (1b) is the global optimum at $(-0.145, -1.047)$ with $C_D = -0.000329$.

* Note the change in C_D scale between Figures 18a and 18b.