



ESDU 00024

Issued December 2000

Re-issued March 2011

The acoustic attenuation of absorbent linings in rectangular flow ducts with application to annular flow ducts

Associated software: ESDUpac A0024
ESDUpac B0024



Endorsed by
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Duct Liners Working Party which has the following membership::

Dr P. Joseph	— Institute of Sound and Vibration Research, University of Southampton
Prof. Y.W. Lam	— University of Salford
Mr R.A. Pinker	— Defence Evaluation Research Agency, Pyestock
Mr D.C. Riordan	— Bombardier Aerospace Short Brothers plc
Mr P. Robinson	— Dowty Aerospace Propellers

and which was appointed by the Aircraft Noise Committee. The Aircraft Noise Committee, first met in 1971 and now has the following membership:

Chairman	
Mr R.A. Pinker	— Defence Evaluation Research Agency, Pyestock
Vice-Chairman	
Mr L.W. Illston	— BAE SYSTEMS
Members	
Mr W.D. Bryce*	— Bryce Research, Farnham
Dr M.J. Fisher	— Institute of Sound and Vibration Research, University of Southampton
Mr P.J. Hopkins	— Rolls-Royce plc, Derby
Mr W.J. Readman	— Civil Aviation Authority, Airworthiness Division
Mr D.C. Riordan	— Bombardier Aerospace Short Brothers plc, Belfast
Mr P. Robinson	— Dowty Aerospace Propellers, Gloucester
Mr D.P. Garber†	— NASA Langley Research Center, USA.

* Member Emeritus

† Corresponding Member.

The construction and subsequent development of the Data Item were undertaken by

Dr C.B. Chinoy	— Head of the Aircraft Noise and Structural Dynamics Group.
----------------	---

THE ACOUSTIC ATTENUATION OF ABSORBENT LININGS IN RECTANGULAR FLOW DUCTS WITH APPLICATION TO ANNULAR FLOW DUCTS

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	3
3. APPLICATION AND ACCURACY	3
4. THEORETICAL BACKGROUND	6
5. MULTI-SECTIONED DUCTS	7
6. COMPUTER PROGRAMS	8
6.1 Input Data and Output	8
6.1.1 Flow profiles	11
6.1.2 Definition of impedance	11
6.1.3 Weighting of modes	13
6.1.4 Common data files	13
7. REFERENCES	15
8. EXAMPLE	15
APPENDIX A – ESDU DATA FILE	28
A.1 NOTATION	28
A.2 DATA FILE STRUCTURE STANDARD	30
A.3 USING THE DATA FILE	30

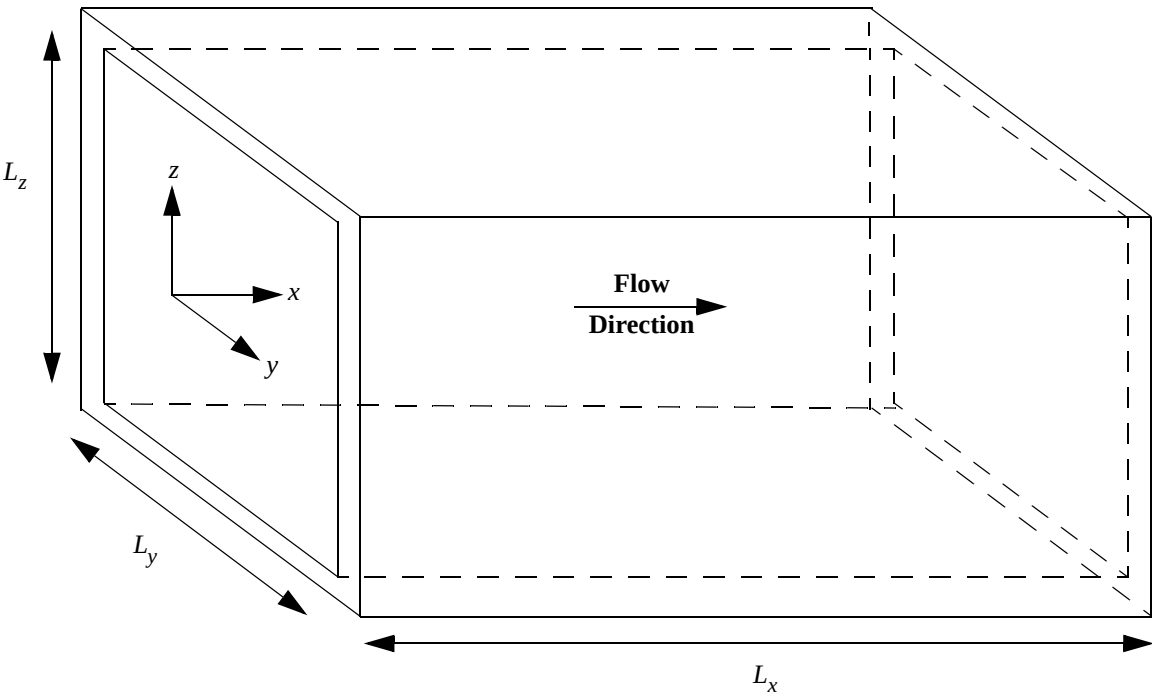
THE ACOUSTIC ATTENUATION OF ABSORBENT LININGS IN RECTANGULAR FLOW DUCTS WITH APPLICATION TO ANNULAR FLOW DUCTS

1. NOTATION

		Units	
		SI	British
c	speed of sound	m/s	ft/s
f	frequency	Hz	Hz
i	subscript denoting i th duct section		
K	constant in Equation (4.3)		
k	wave number ($= 2\pi f/c$)	m^{-1}	ft^{-1}
k_x^{mn}	wave number in axial direction	m^{-1}	ft^{-1}
k_y^m	wave number in y -direction	m^{-1}	ft^{-1}
$k_{y^*}^m$	eigenvalue in y -direction ($= k_y^m y_0$)		
k_z^n	wave number in z -direction	m^{-1}	ft^{-1}
$k_{z^*}^n$	eigenvalue in z -direction ($= k_z^n z_0$)		
L_x	duct length (see Sketch 1.1)	m	ft
L_y	distance between two lined walls normal to the y -axis (see Sketch 1.1)	m	ft
L_z	distance between two lined walls normal to the z -axis (see Sketch 1.1)	m	ft
M_1	Mach number in boundary layer at distance y_1 from duct centre-plane (see Sketch 6.1)		
M_w	Mach number close to wall of duct		
M_0	Mach number in uniform flow region		
MM	total number of propagating modes		
m	mode number in y -direction		
NN_m	total number of propagating modes for given m		

Issued December 2000 – 30 pages
Re-issued March 2011 – 30 pages

n	mode number in z-direction		
$Q_t(x)$	total energy flow at location x	W	ft lbf/s
x, y, z	coordinates (see Sketch 1.1)		
SR	Slip Ratio ($= M_w/M_0$)		
y_0	distance from duct centre-plane to edge of boundary layer in the y-direction (see Sketch 6.1)	m	ft
y_1	distance from duct centre-plane to points in boundary layer defining velocity profile (see Sketch 6.1)	m	ft
Z	specific acoustic impedance (see Section 6.1.2)	Ns/m ³	lbf s/ft ³
Z^*	normalized acoustic impedance ($= Z/\rho c$) (see Section 6.1.2)		
z_0	distance from duct centre-plane to edge of boundary layer in the z-direction	m	ft
β_x^{mn}	$\text{Im}(k_x^{mn})$	m ⁻¹	ft ⁻¹
ρ	density of fluid medium	kg/m ³	lb/ft ³
Δ	attenuation of sound power level due to duct liner	dB	dB
δ	boundary layer thickness (see Sketch 6.1)	m	ft



Sketch 1.1 Geometry of Duct

2. INTRODUCTION

This item provides a computerized means of estimating the sound power attenuation due to the introduction of acoustically absorbent linings in rectangular ducts. It may also be used for annular ducts under special circumstances discussed below. For circular ducts, ESDU 00012 should be used. The computed result may also be interpreted as the liner insertion loss, defined as the reduction in sound power following the introduction of the duct liner. The prediction method used requires knowledge of the normalized acoustic impedance at each frequency and is based on a theoretical model which allows a selection of typical velocity profiles across the duct cross-section. The program has the capability to consider up to ten different liner sections in series in a duct of constant cross-sectional dimensions.

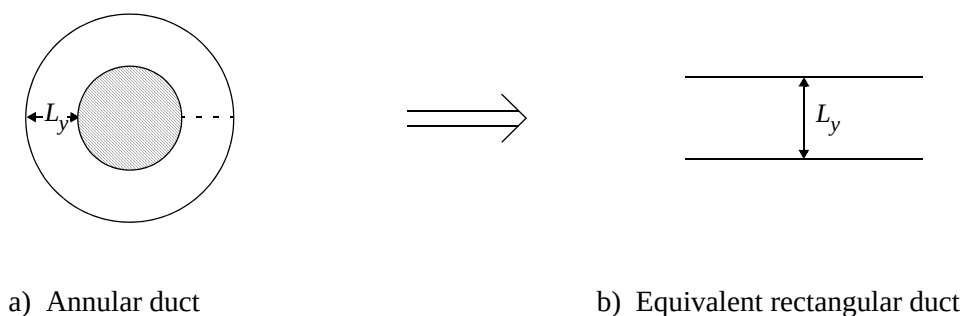
The Item addresses the total power reduction within the duct and does not consider subsequent radiation to the far-field.

3. APPLICATION AND ACCURACY

The prediction method given in this Item allows the user to estimate the sound power attenuation produced by acoustic liners in ducts, a common requirement if, for example, the noise from an aircraft engine, an auxiliary power unit or a ventilating system needs to be reduced. The computer program that is provided with this Item gives solutions for straight ducts of rectangular cross-section in any of the three following configurations.

- (i) An infinitely wide duct with one or both walls lined with absorbent material.
- (ii) A duct with two opposite walls lined with absorbent material.
- (iii) All four walls of the duct lined.

For the purposes of this program, an annular duct can be approximated by a rectangular duct with two parallel walls lined and the other two walls infinitely far apart, *i.e.* the first of the three configurations mentioned above (Reference 1). The reasoning behind this method of simulation is clarified if a cut through the cross-section of the duct along the dashed line shown in Sketch 3.1a is imagined and the duct 'unwrapped' flat to form the equivalent rectangular duct section shown in Sketch 3.1b. Although in theory this approximation has limitations because it does not consider spinning modes, in practice this approach has been used with acceptable accuracy with full size jet engines and auxiliary power units. However, it should be noted that as the ratio of the inner radius to the outer radius of the annular duct decreases to less than 0.5, this approximation tends to become less valid.



Sketch 3.1 Cross-section of duct

The program makes provision for results to be obtained for one-third octave band centre frequencies in the range from 50 Hz to 20 kHz, an array of specified frequencies or a frequency range with a constant step

increment. Note, however, that in the first case calculations are only carried out at the band centre frequencies. Note also that in any given application, the range will be dependent on the geometry of the duct. In circumstances where either the attenuation spectrum or the source spectrum or both are changing rapidly with frequency, this simplification will lead to results which should be treated with caution.

The maximum mode number calculated by the programme in either the y or z direction is 50. This restriction imposes a maximum frequency below which all the modes are included in the calculation. Above this frequency, the programme uses only a subset of the total number of propagating modes, which may produce error. By way of an example, for unheated flow in a duct in which the maximum cross sectional dimension is 1 m, this maximum frequency is approximately 8.5 kHz. More generally, the frequency can be found from the formula $kL = 50\pi$, where L is the greater of L_y and L_z .

The method used in this Item makes the following assumptions.

- (i) The liner is considered to be locally-reacting, with impedance that is independent of the angle of the incident sound.
- (ii) Only the modes that would propagate in an unlined duct are considered.
- (iii) The wall velocity is zero when shear flow is considered.
- (iv) There are no end effects, *i.e.* no waves are reflected either at the ends of the duct or at transition regions if more than one liner section is being considered.
- (v) Viscosity and heat transfer effects are not significant.
- (vi) Velocity profiles are constant along the entire duct length when boundary layer flow is considered.
- (vii) The lining impedance is the same on opposite walls. However, it is possible to consider the case of one wall lined and the opposite wall hard by using a duct length that is half of its actual length. Thus, ducts with different impedances on opposite walls can be included.

The user can choose between assuming equal energy per mode or assuming equal amplitude per mode (see Section 4). These modal power distributions may be altered by the user by adjusting the levels of individual modes as described in Section 6.1.3.

The data required for the program include values for the normalized acoustic impedance for each lined section. If the duct Mach number is of the order of 0.1 or less, it is unlikely that there will be substantial error if no-flow impedance data are used. These may be obtained from sources such as ESDU 74003 and 74004. At higher Mach numbers, however, it would be usual to use data from the liner manufacturers since the determination of the impedance is not readily achievable using simple apparatus or by calculation.

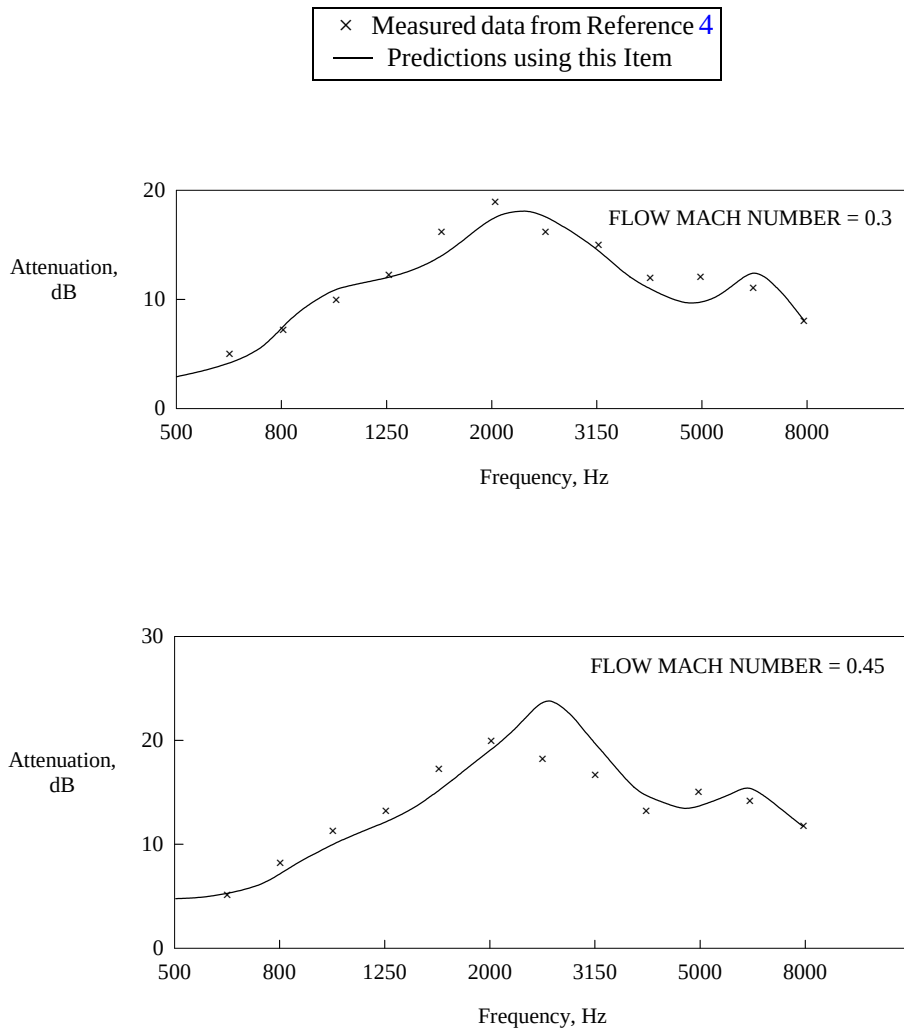
In general, results show good agreement with measured data (References 4 and 5). With ducts where the distance between lined walls ranged from 0.18 m to 0.75 m, over a frequency range of 400 Hz to 10 kHz, an rms[†] error of less than 2 dB is evident at several different flow rates.

[†] The rms error is an indicator of the accuracy of prediction. It has been calculated here from comparisons between measured data and predictions using the equation

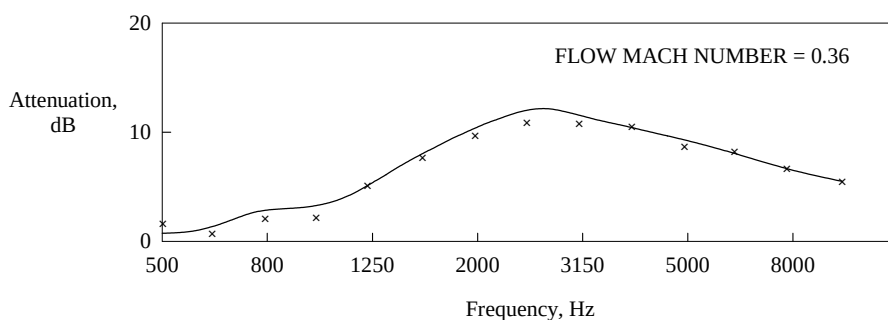
$$\text{rms error} = \sqrt{\sum[(\text{measured value} - \text{predicted value})^2]/(\text{number of samples})}.$$

The rms error can be used in a way analogous to the usage of standard deviation to obtain degrees of confidence in the prediction, *i.e.* $2 \times \text{rms error}$ corresponds to 90% confidence and $3 \times \text{rms error}$ to 95% confidence.

Sketches 3.2 and 3.3 are illustrations of comparisons between measured and predicted data for a rectangular duct with four lined walls and an annular duct respectively.



Sketch 3.2 Comparison of predicted with measured attenuation spectra for two flow Mach numbers in a duct with four lined walls



Sketch 3.3 Comparison of predicted with measured attenuation spectra for an annular duct

4. THEORETICAL BACKGROUND

The propagation of sound in a duct can be described by the wave equation and the boundary conditions. The attenuation of the duct modes can be derived from the eigenvalues which form the solutions of the equation. Detailed derivations for a rectangular duct can be found in References 1 to 4 but a summary of the prediction method is given in this Section.

The program gives the user the option to estimate the sound power level attenuation at each frequency either with the common assumption for the incident field of equal energy per mode or equal rms pressure amplitude per mode. This latter assumption is described in greater detail in Reference 2.

The sound power level attenuation, assuming equal energy per mode, is given by

$$\Delta = 10 \log_{10} \left\{ \frac{\sum_{m=1}^{MM} NN_m}{\sum_{m=1}^{MM} \sum_{n=1}^{NN_m} e^{2\beta_x^{mn} L_x}} \right\} \text{ dB}, \quad (4.1)$$

where MM = total number of propagating modes,
 NN_m = total number of propagating modes for a given m ,
 $\beta_x^{mn} = \text{Im}(k_x^{mn})$
 and L_x = lined length of duct section.

The sound power level attenuation, assuming equal pressure amplitude per mode, is given by

$$\Delta = 10 \log_{10} \left\{ \frac{Q_t(0)}{Q_t(L_x)} \right\} \text{ dB}, \quad (4.2)$$

where $Q_t(0)$ and $Q_t(L_x)$ are the total energy flow at $x = 0$ and $x = L_x$ respectively and defined by

$$Q_t(x) = K \sum_m \sum_n \left\{ M_0 \left[\left| \frac{k_y^m}{k} \right|^2 + \left| \frac{k_z^n}{k} \right|^2 + (M_0 - 1) \cdot \left| \frac{k_x^{mn}}{k} \right|^2 + 1 \right] + 2 \text{Re} \left(\frac{k_x^{mn}}{k} \right) \cdot \left(1 - M_0^2 \right) \right\} e^{2\beta_x^{mn} x}, \quad (4.3)$$

where K is constant for a given frequency.

It can be shown theoretically that, at a given frequency, there will be a maximum number of modes that will propagate in an unlined rectangular duct. All other modes above this value are said to be 'cut-off' and are not used in the calculation.

The eigenvalue equations for uniform flow for the case of walls lined in the y -direction are

$$\frac{iky_0}{Z^*} \left(1 - \frac{k_x^{mn}}{k} M_0 \right)^2 = k_{y^*}^m \tan k_{y^*}^m \quad (4.4)$$

for the symmetric modes, $m = 1, 3, 5 \dots$

$$\text{and} \quad \frac{iky_0}{Z^*} \left(1 - \frac{k_x^{mn}}{k} M_0 \right)^2 = (-k_{y^*}^m) \cot k_{y^*}^m \quad (4.5)$$

for the asymmetrical modes, $m = 2, 4, 6 \dots$

The eigenvalue equations for the case of walls lined in the z -direction are identical but with y_0 replaced by z_0 and $k_{y^*}^m$ by $k_{z^*}^n$.

If the two walls in the z -direction are unlined, the hard-wall values of $k_{z^*}^n$ are real and defined by

$$k_{z^*}^n = (n-1) \frac{\pi}{2}, \quad n = 1, 2, 3, \dots \quad (4.6)$$

Solutions for $k_{y^*}^m$ and, if required, $k_{z^*}^n$ are found using the Newton-Raphson iteration procedure from which k_x^{mn} is calculated using the equation

$$k_x^{mn} = \frac{-kM_0 + \sqrt{\{k^2 - (1 - M_0^2) \cdot [(k_y^m)^2 + (k_z^n)^2]\}}}{1 - M_0^2} \quad (4.7)$$

The calculation is made initially for the case with no flow. The Mach number is then incremented by a small amount and the entire calculation repeated until the specified Mach number is reached.

5. MULTI-SECTIONED DUCTS

The computer program provides the user with the option to consider up to ten lined lengths in series. The duct sections must have identical dimensions in cross-section but each section can have a different lining. It is assumed that there are no scattering effects at transition points.

To obtain the total attenuation for a number of ducts in series, $L_1, L_2 \dots L_i$, Equation (4.1), which assumes equal energy per mode, is modified as follows.

$$\Delta = 10 \log_{10} \left\{ \frac{\sum_{m=1}^{MM} NN_m}{\sum_{m=1}^{MM} \sum_{n=1}^{NN_m} e^{2\beta_{x_1}^{mn} L_1 + 2\beta_{x_2}^{mn} L_2 + \dots + 2\beta_{x_i}^{mn} L_i}} \right\} \text{ dB.} \quad (5.1)$$

Thus, for example, inspection of Equation (5.1) will show that two ducts in series with identical liners and of length L each will, theoretically, give the same attenuation for each mode as a single duct of length $2L$.

6. COMPUTER PROGRAMS

There are two programs associated with this Item, programs A0024 and B0024.

Program A0024 permits the user to estimate the sound power level attenuation due to the presence of acoustic duct liners in ducts of rectangular cross-section with application to annular ducts. This program requires the user to enter the name of the file containing the input data. The structure of this input file is shown in Table 6.1. Program B0024 facilitates the generation of the input files.

Both versions of these programs are provided. Typically, the program to estimate sound attenuation can be run either by typing in the program name, A0024V10, or by clicking on the icon associated with the executable version of the program. The user is then prompted to enter the name of the input file.

6.1 Input Data and Output

The structure of the input file is shown in Table 6.1. An example is given in Section 8.

Items which need further explanation are discussed in Sections 6.1.1 to 6.1.3.