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Aerofoil profile drag for Mach numbers below the drag-rise condition

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes Item Nos Aero W.02.04.02 and 03, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

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Mr J. Tweedie	— Short Brothers plc, Belfast
Mr S.F. Wood	— BAE SYSTEMS, Woodford

* Corresponding Member

The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr C.J. Batt — Engineer.

The person with overall responsibility for the work in this subject area is Mr R.W. Gilbey, Head of the Aircraft Aerodynamics Group.

AEROFOIL PROFILE DRAG FOR MACH NUMBERS BELOW THE DRAG-RISE CONDITION

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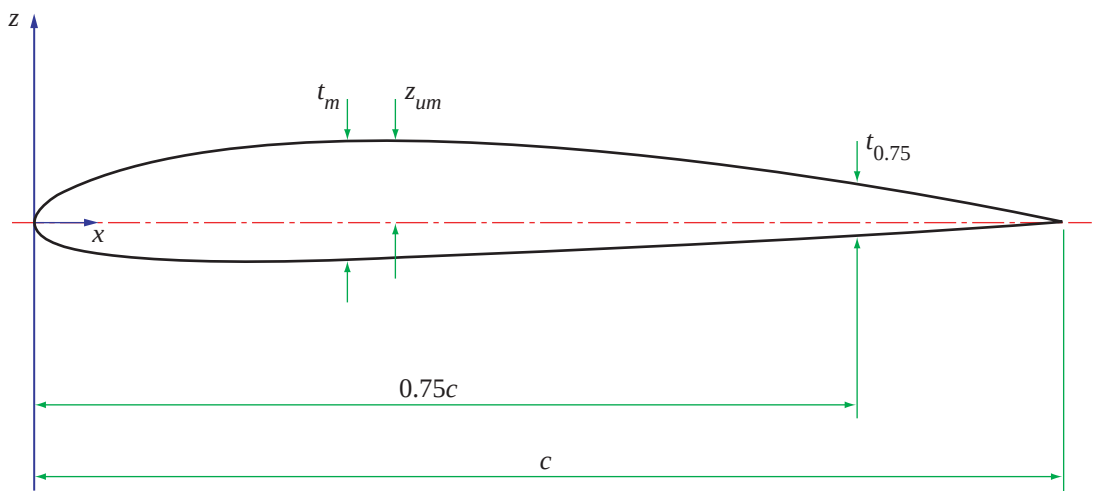
AEROFOIL PROFILE DRAG FOR MACH NUMBERS BELOW THE DRAG-RISE CONDITION

1. NOTATION AND UNITS

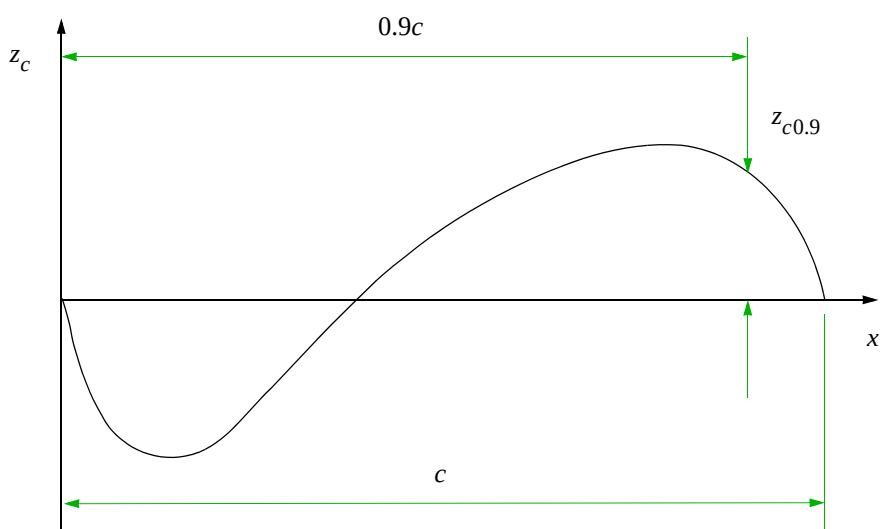
		SI	British
C_D	aerofoil profile drag coefficient, based on c		
C_{Dmin}	value of C_D at C_{Lmin} in plot of C_D against C_L for fixed x_{tr}/c , R_c and M , see Sketches 4.1 and 4.2		
$(C_{Dmin})_{th}$	value of C_{Dmin} for basic aerofoil thickness distribution in incompressible flow, see Section 3		
C_L	aerofoil lift coefficient, based on c		
C_{Lmin}	value of C_L corresponding to minimum value of C_D with $x_{tr}/c = 0.01$, see Sketches 4.1 and 4.2		
c	aerofoil chord	m	ft
H	stagnation pressure in undisturbed stream	N/m ²	lbf/ft ²
k_c	camber factor, dependent on M , see Section 3		
k_M	compressibility factor applied to $(C_{Dmin})_{th}$, see Section 3		
M	freestream Mach number		
M_{crit}	critical Mach number, see Section 4.4		
M_D	drag-rise Mach number, see Section 4.4		
R_c	Reynolds number based on c and freestream flow conditions		
t_m	maximum thickness of aerofoil section, see Sketch 1.1	m	ft
$t_{0.75}$	aerofoil thickness at $x = 0.75c$, measured perpendicular to chord line, see Sketch 1.1	m	ft
x	chordwise distance aft of aerofoil leading edge, see Sketch 1.1	m	ft
x_{tr}	mean of chordwise distances to upper and lower surface boundary-layer transition positions, measured aft of leading edge	m	ft
z	ordinate, see Sketch 1.1	m	ft

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z_c	aerofoil camber-line ordinate, measured perpendicular to chord line, see Sketch 1.2	m	ft
$z_{c0.9}$	value of z_c at $x = 0.9c$, see Sketch 1.2	m	ft
z_{um}	maximum upper surface ordinate, see Sketch 1.1	m	ft



Sketch 1.1 Aerofoil geometric parameters



Sketch 1.2 Camber-line geometry

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2. INTRODUCTION

This Data Item provides a means of predicting the profile drag coefficient of a wide range of symmetrical and cambered aerofoil sections with attached flow at low lift coefficient. The predicted value, C_{Dmin} , for a given mean boundary-layer transition position and at fixed Reynolds number and Mach number, is taken to be that which occurs at the lift coefficient C_{Lmin} , where C_{Lmin} is the lift coefficient for minimum aerofoil drag coefficient when the boundary-layer transition position is fixed at one per cent from the aerofoil leading edge on both upper and lower surfaces. When the mean boundary-layer transition position is further aft than $0.01c$, the value of C_{Dmin} , although not a true minimum, closely approximates the minimum aerofoil drag coefficient. The prediction of the value of C_{Lmin} is outside the scope of the Item but it can be expected to be near to the value of C_L at zero incidence.

The method was created from an analysis of a database built by using the VGK program* (Derivations 1 and 2) to obtain C_D versus C_L plots, and hence C_{Dmin} values, for a varied selection of aerofoils, see Table 5.2. A systematic study was conducted to investigate the effects of camber.

Section 3 describes the method for predicting C_{Dmin} , in which two factors that allow for compressibility and camber are applied to the profile drag coefficient associated with the basic aerofoil thickness distribution. Knowledge of the Reynolds number and the mean chordwise position of boundary-layer transition is required, and one camber and two thickness parameters are used to characterise the aerofoil geometry, see Sketches 1.1 and 1.2.

Section 4.1 discusses the important differences between the C_D versus C_L plots for an aerofoil when transition is free and when it is fixed and includes sketches that illustrate the difference in two particular cases. Section 4.2 elaborates on the use of C_{Dmin} as a measure of profile drag coefficient. Aerofoil profile drag is governed by the momentum thicknesses of the upper and lower surface boundary layers at the trailing edge, which in turn are influenced by the aerofoil pressure distribution, Reynolds number, Mach number and boundary-layer transition positions. The effects of thickness and camber at fixed mean transition position, Reynolds number and Mach number are discussed in Section 4.3, and the effect of transition position, Reynolds number and Mach number for a given geometry are discussed in Section 4.4.

After the accuracy and applicability have been discussed in Section 5, and the Derivation and References listed in Section 6, there are worked examples showing the application of the method in Section 7, including a comparison with experiment.

3. METHOD

The value of C_{Dmin} in shock-free compressible flow is calculated from the relationship

$$C_{Dmin} = k_c k_M (C_{Dmin})_{th}, \quad (3.1)$$

where $(C_{Dmin})_{th}$ is the value associated with the basic thickness distribution. Figures 1 to 4 provide a method of obtaining $(C_{Dmin})_{th}$, for values of the rear geometry parameter, $t_{0.75}/t_m$, of 0.35 and 0.8, and

* The VGK (Viscous Garabedian and Korn) method is a computational method for determining two-dimensional attached flow past an aerofoil immersed in a subsonic freestream, with allowance for viscous effects. The method couples finite difference solutions of inviscid flow about the aerofoil with solutions for the displacement effects of the boundary layers. External to the boundary layer the inviscid flow element is governed by the full potential equation. The boundary-layer development is predicted by forward integration of three simultaneous ordinary differential equations (momentum integral equation, entrainment equation and an equation derived from the turbulent energy equation). An iterative approach allows boundary-layer displacement and momentum thickness distributions to be calculated, with allowance for the effects of the pressure field obtained from the inviscid flow element. The turbulent boundary layer is assumed to start at a selected transition point and the laminar calculations at this point form the initial conditions. The method of coupling the viscous and inviscid flow elements permits converged solutions to be obtained for flows with attached boundary layers.

mean boundary-layer transition positions, x_{tr}/c , of 0 and 0.5. Linear interpolation is required for other values of $t_{0.75}/t_m$ and x_{tr}/c . The factor k_M , introduced in Section 3.1.2 and given in Figures 5 and 6, accounts for the effect of compressibility on the profile drag of the basic aerofoil thickness distribution. The factor k_c , introduced in Section 3.2 and given in Figure 7, accounts for the effect of aerofoil camber, and includes an additional significant influence of compressibility.

3.1 Basic Aerofoil Thickness Distribution

3.1.1 Incompressible flow, $M = 0$

The parametric equation used to analyse the VGK aerofoil database and plot Figures 1 to 4 is,

$$(C_{Dmin})_{th} = k_1 + k_2 \frac{t_m}{c} + k_3 \frac{t_{0.75}}{t_m} + k_4 \left(\frac{t_m}{c} \right) \left(\frac{t_{0.75}}{t_m} \right). \quad (3.2)$$

In Equation (3.2), the factors k_1 , k_2 , k_3 and k_4 model the linear variation of $(C_{Dmin})_{th}$ with aerofoil thickness to chord ratio t_m/c and rear-geometry parameter $t_{0.75}/t_m$, for any particular condition of Reynolds number R_c and mean boundary-layer transition position x_{tr}/c , and are given by

$$k_1 \times 10^3 = 4.0 + 0.84(8.5 - \log_{10} R_c)^2 - 3.40 \frac{x_{tr}}{c} - 0.25 \frac{x_{tr}}{c} (8.5 - \log_{10} R_c)^2, \quad (3.3)$$

$$k_2 \times 10^3 = 8.46 + 1.15(8.5 - \log_{10} R_c)^2 - 27.40 \frac{x_{tr}}{c} - 3.97 \frac{x_{tr}}{c} (8.5 - \log_{10} R_c)^2, \quad (3.4)$$

$$k_3 \times 10^3 = -0.29 - 0.12(8.5 - \log_{10} R_c)^2 + 0.31 \frac{x_{tr}}{c} - 0.12 \frac{x_{tr}}{c} (8.5 - \log_{10} R_c)^2 \quad (3.5)$$

$$\text{and} \quad k_4 \times 10^3 = 6.97 + 3.10(8.5 - \log_{10} R_c)^2 + 16.60 \frac{x_{tr}}{c} + 2.10 \frac{x_{tr}}{c} (8.5 - \log_{10} R_c)^2. \quad (3.6)$$

3.1.2 Compressible flow

The factor k_M , introduced in Equation (3.1), accounts for the effect of compressibility on the profile drag due to the basic aerofoil thickness distribution. This factor is the ratio of C_{Dmin} in compressible flow to its value in incompressible flow for the aerofoil without any camber. Thus $k_M = 1$ at $M = 0$ and varies as a function of M , t_m/c and $t_{0.75}/t_m$. This is shown in Figures 5 and 6, which present carpets of k_M versus M and t_m/c for values of $t_{0.75}/t_m = 0.35$ and 0.8, respectively. For fixed values of M and t_m/c , the variation of k_M with $t_{0.75}/t_m$ is linear and so linear interpolation may be used for intermediate values of $t_{0.75}/t_m$. An approximate limit to the applicability (to remain below the drag-rise condition) is shown dashed on the Figures, see Section 5.

3.2 Camber Correction

The factor k_c , presented in Figure 7, accounts for the significant effect of aerofoil rear camber. It is the ratio of C_{Dmin} to its value for the basic thickness distribution at the Mach number of interest. Thus $k_c = 1$ for symmetrical aerofoils and increases as a function of both rear camber parameter, $z_{c0.9}/c$, and Mach number. The significant non-linear effect of rear camber is demonstrated by the use of $(z_{c0.9}/c)^2$ in the plot. The upper limit of the camber parameter is set at $(z_{c0.9}/c)^2 = 4 \times 10^{-4}$. An approximate limit to the applicability (for the flow to remain fully attached) is shown dashed on the Figure, see Section 5.

4. EFFECTS OF GEOMETRY AND FLOW CONDITIONS ON AEROFOIL PROFILE DRAG COEFFICIENT

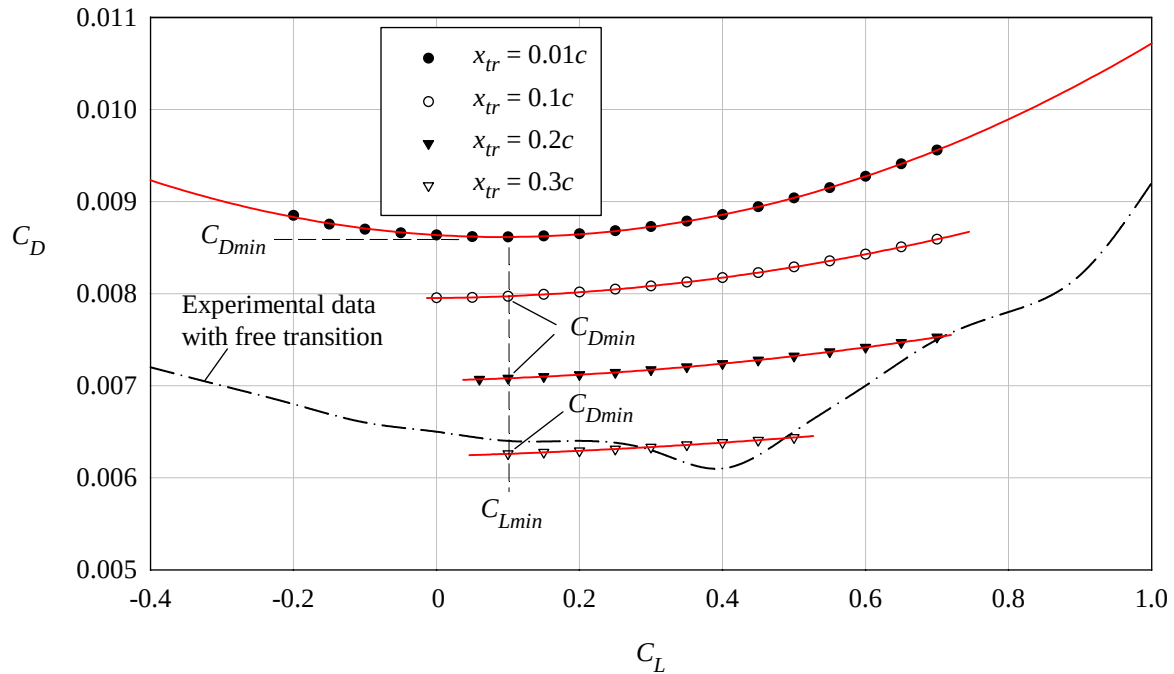
Profile drag, which derives from the presence of the boundary layer, is sometimes divided into two components. These are boundary-layer normal pressure drag and skin friction drag. Boundary-layer normal pressure drag (or form drag) results from the departure from the inviscid pressure distribution due to the effective modification of the aerofoil shape caused by the presence of the boundary layer. Skin friction drag is the resultant force in the drag direction due to shear stresses at the surface of the aerofoil. The method of this Item does not attempt to isolate the two components. However, aerofoil skin friction drag coefficient may be predicted separately using Item No. 06001 (Reference 9).

4.1 Effect of Aerofoil Lift Variation and Boundary-layer Transition Position

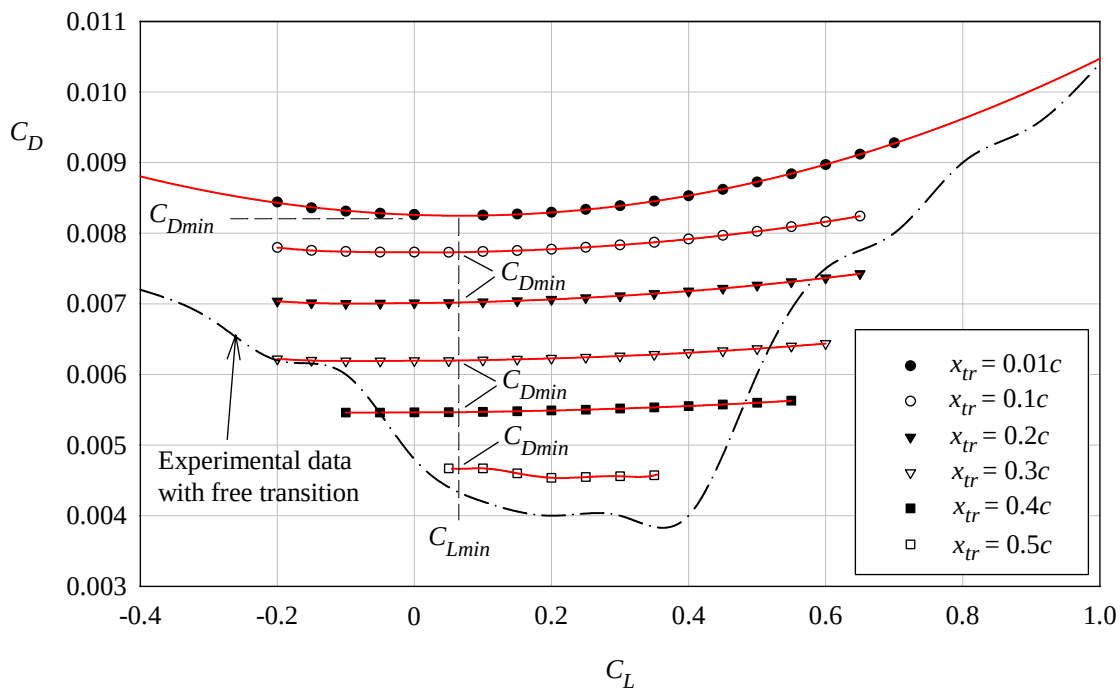
The extent of laminar flow on each aerofoil surface directly influences profile drag. If the boundary-layer transition positions on the upper and lower surfaces are not constrained they move as aerofoil incidence changes, causing the profile drag coefficient, C_D , to vary greatly with the lift coefficient, C_L . However, when boundary-layer transition is fixed the aerofoil profile drag coefficient varies relatively slowly with the lift coefficient about some minimum value, describing a shallow parabola.

Sketches 4.1 and 4.2 show plots of C_D against C_L obtained by using the VGK program with boundary-layer transition positions specified at various chordwise stations (the same on both upper and lower surfaces) for two 12%-thick aerofoils. The curves in Sketches 4.1 and 4.2 cover ever narrower ranges of C_L as transition is specified further aft. This is because, outside the plotted C_L ranges, natural transition occurs upstream of the specified position. Low-drag aerofoils like the NACA 65-212 to which Sketch 4.2 applies have the ability to maintain transition positions further aft and over a wider C_L range than conventional aerofoils such as the NACA 2412 used for Sketch 4.1. Thinner aerofoils have a smaller variation of C_D with C_L .

Also shown in Sketches 4.1 and 4.2 as dashed lines are experimental data obtained from free-transition tests at the same Reynolds and Mach numbers as the VGK calculations. The much greater and irregular dependence of C_D on C_L is clear. Only the plots of C_D versus C_L associated with fixed transition show a minimum value of C_D that represents the profile drag coefficient over a modest range of C_L . Realistic C_D versus C_L plots would reflect independent movement of transition.



Sketch 4.1 NACA 2412 drag polars ($M = 0.05$, $R_c = 5.7 \times 10^6$)



Sketch 4.2 NACA 65-212 drag polars ($M = 0.05$, $R_c = 6 \times 10^6$)

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4.2 Minimum Drag Coefficient

The drag coefficient, C_{Dmin} , predicted by the method is the value of C_D at C_{Lmin} for fixed mean transition position, Reynolds number and Mach number, see Sketches 4.1 and 4.2. It is important to note that the lift coefficient C_{Lmin} is always that value corresponding to the minimum value of C_D on a plot of C_D versus C_L , when boundary-layer transition is fixed at $x_{tr}/c = 0.01$. For transition positions further aft, the drag minimum may occur at lift coefficients slightly away from C_{Lmin} but small changes of C_L near C_{Lmin} do not greatly affect the value of C_D , and C_{Dmin} provides a close approximation of the minimum drag coefficient. For all transition positions the value of C_{Dmin} has the advantage of being relatively unaffected by aerofoil camber (Reference 4) provided that the amount of aft camber is small (Section 4.3). The value of C_{Lmin} is largely independent of Reynolds number and Mach number.

Aerofoil pressure distribution, Reynolds number, Mach number and boundary-layer transition position all influence the momentum thicknesses of the boundary layers at the trailing edge and therefore C_{Dmin} . Sections 4.3 and 4.4 discuss these effects.

4.3 Effect of Geometry at Fixed Values of x_{tr}/c , R_c and M

Profile drag is influenced by pressure distribution which itself is influenced by the aerofoil geometry.

(i) Thickness effects

Pressure drag rises as aerofoil thickness increases, causing an increase in profile drag. The maximum thickness-to-chord ratio, t_m/c (Sketch 1.1) is used in the method to model the primary effects of thickness.

Rear aerofoil geometry can be significant due to its effect on pressure distribution in the trailing-edge region, particularly for thicker aerofoils. Reference 3 showed that aerofoils with cusped trailing edges produce lower values of C_D than those with wedge-shaped trailing edges. The effects of rear section shape are characterised in the method by the parameter $t_{0.75}/t_m$, where $t_{0.75}$ is the section thickness at $x = 0.75c$ (Sketch 1.1). The variation with t_m/c is the dominant effect while the secondary dependence on $t_{0.75}/t_m$ increases in importance as t_m/c becomes large.

Below the drag rise (see Section 4.4 (iii)) blunt trailing edges generally increase profile drag; for example, changing a sharp trailing edge to one with a trailing-edge thickness of $0.01c$ may increase C_D by as much as two drag counts (0.0002).

(ii) Camber effects

Aerofoil camber lines can vary considerably in shape and in some cases exhibit negative camber near the leading edge. A systematic study of the effect of camber-line variation showed that the camber-line shape forward of the trailing-edge region (see Section 4.4 (iii)) does not greatly affect profile drag. In contrast, an increase in the amount of rear camber produces a larger adverse pressure gradient near the upper-surface trailing edge, which significantly increases profile drag. For most NACA and GUNACA 23-series sections (References 4 and 8) the amount of rear camber exhibited is not large enough to affect profile drag greatly. However, there are many aerofoils with large rear loading for which the camber effect becomes important.

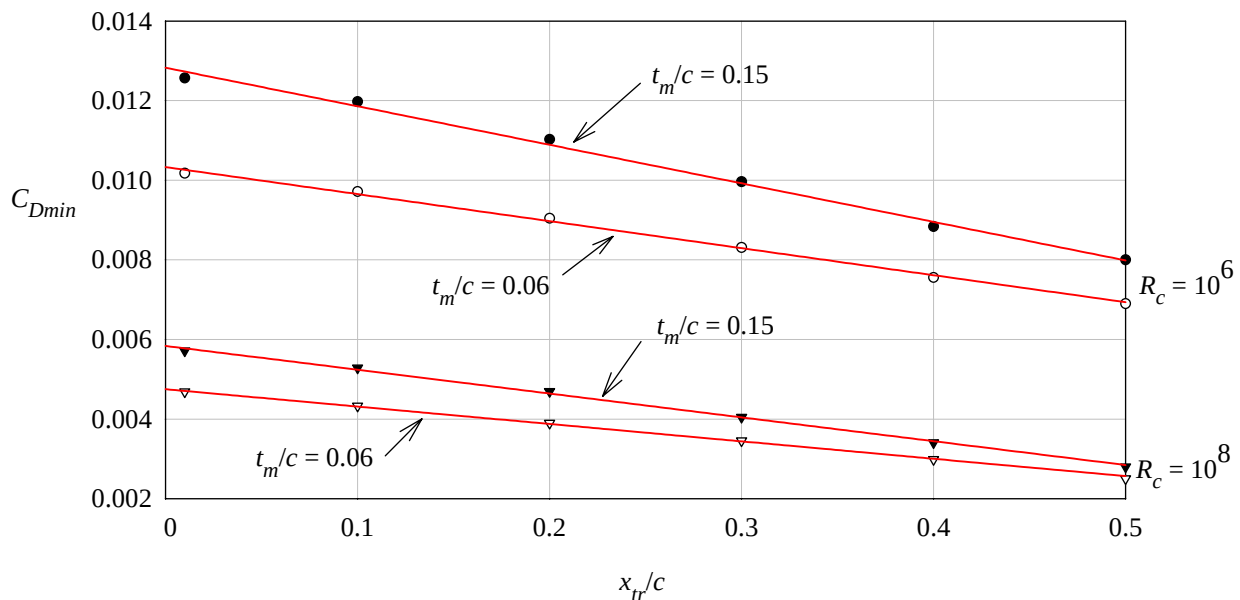
In the method, aerofoil rear camber is characterised by the parameter $z_{c0.9}/c$, where $z_{c0.9}$ is the camber line ordinate at $x = 0.9c$ (Sketch 1.2). The effects of compressibility become more marked for aerofoils with large amounts of rear camber, as demonstrated in Figure 7; see also discussion in Section 4.4 (iii).

4.4 Effect of x_{tr}/c , R_c and M for a Given Aerofoil

Profile drag is greatly influenced by boundary-layer transition position, Reynolds number and Mach number, each of which affects the development of boundary layers on both upper and lower surfaces.

(i) Effect of x_{tr}/c for fixed R_c

The variation of C_{Dmin} with the mean of the upper and lower boundary-layer transition positions, x_{tr}/c , is shown in Sketch 4.3 for $M = 0.05$, for aerofoil thickness ratios of 0.06 and 0.15 and two Reynolds numbers, 10^6 and 10^8 . The value of C_{Dmin} varies almost linearly with transition position. As transition moves aft, the increasing extent of laminar boundary layer causes a reduction in C_{Dmin} .



Sketch 4.3 The effect of boundary-layer transition variation ($M = 0.05$, $t_{0.75}/t_m = 0.5$)