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Supersonic pressure and Mach number distributions over conical, circular-arc and parabolic boat-tails

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SUPERSONIC PRESSURE AND MACH NUMBER DISTRIBUTIONS OVER CONICAL, CIRCULAR-ARC AND PARABOLIC BOAT-TAILS

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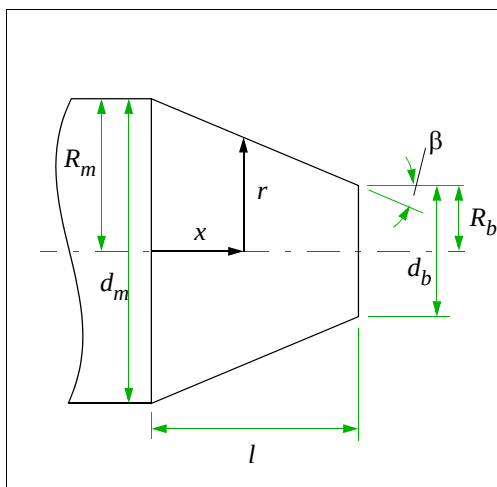
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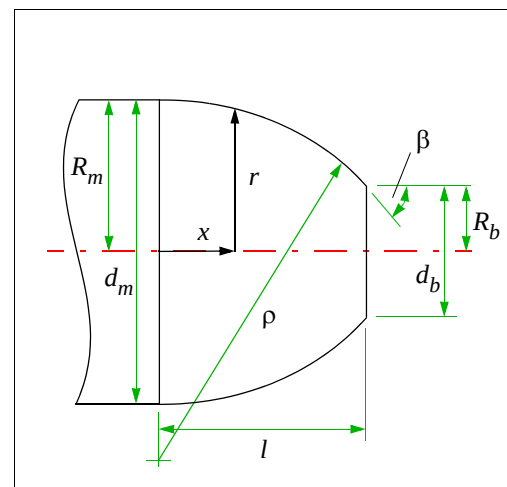
SUPERSONIC PRESSURE AND MACH NUMBER DISTRIBUTIONS OVER CONICAL, CIRCULAR-ARC AND PARABOLIC BOAT-TAILS

1. NOTATION AND UNITS

		SI	British
C_p	local pressure coefficient, $(p - p_\infty)/\frac{1}{2}\gamma p_\infty M_\infty^2$		
d_b	base diameter, $2R_b$	m	ft
d_m	maximum body diameter, $2R_m$	m	ft
l	boat-tail length, see Sketch 1.1	m	ft
M	local Mach number		
M_∞	freestream Mach number		
p	local static pressure	N/m ²	lbf/ft ²
p_∞	freestream static pressure	N/m ²	lbf/ft ²
R_b	base radius, see Sketch 1.1	m	ft
R_m	maximum body radius, see Sketch 1.1	m	ft
r	local radius, see Sketch 1.1	m	ft
x	axial distance along boat-tail aft of cylinder/boat-tail junction, see Sketch 1.1	m	ft
β	boat-tail angle, see Sketch 1.1	deg	deg
γ	ratio of specific heat capacities		
ρ	radius of curvature of circular arc, see Sketch 1.1	m	ft



Cone



Circular arc

Sketch 1.1 Boat-tail geometry

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2. INTRODUCTION

This Item presents data for the pressure coefficient and Mach number distributions along conical and circular-arc boat-tails at zero angle of attack over the Mach number range 1.5 to 4. The data are based on theoretical estimates obtained from the Method of Characteristics. It should be noted that data for boat-tail pressure and Mach number distributions are also available in Item No. Aero B.S. 05.03.01 (Reference 1), but the presentation therein is inappropriate for boat-tails where the ratio of length to maximum body diameter ratio is small. The present Item is restricted to a maximum value of l/R_m of 5 and the user is referred to Item No. B.S.05.03.01 when l/R_m is greater than 5. The boat-tail angles considered are limited to a maximum of 10 degrees for the case of conical boat-tails. For circular-arc boat-tails the data are given as a function the radius ratio of ρ/R_m , and these data may also be applied to parabolic boat-tails, see Section 3. A discussion of the use of the Item is given in Section 4, and the method is illustrated by a worked example in Section 6.

The method of this Item excludes situations where there are jet interference effects, see Section 4.

3. METHOD

Figure 1a gives the geometric relationship between the local boat-tail radius ratio, r/R_m , and the dimensionless axial station, x/R_m , as a function of boat-tail angle for conical afterbodies. The equation is

$$r/R_m = 1 - (x/R_m) \tan \beta. \quad (3.1)$$

Rearrangement of Equation (3.1) with the substitution of the base values, $r = R_b$ and $x = l$, gives the boat-tail angle for a given conical afterbody

$$\beta = \tan^{-1}[(1 - R_b/R_m)/(l/R_m)]. \quad (3.2)$$

Figure 1b gives the geometrical relationship between the radius ratio, r/R_m , and dimensionless axial station, x/R_m , as a function of the radius of curvature ratio, ρ/R_m , for circular-arc boat-tails. The equation is

$$r/R_m = [(\rho/R_m)^2 - (x/R_m)^2]^{1/2} - \rho/R_m + 1. \quad (3.3)$$

Rearrangement of Equation (3.3) and substitution of the base values, $r = R_b$ and $x = l$, gives the radius of curvature ratio for a given circular-arc boat-tail as

$$\rho/R_m = [(l/R_m)^2 + (1 - R_b/R_m)^2]/[2(1 - R_b/R_m)]. \quad (3.4)$$

The boat-tail angle, β , for a circular-arc boat-tail is given by

$$\beta = 2 \tan^{-1}[(1 - R_b/R_m)/(l/R_m)]. \quad (3.5)$$

Figures 2a, 2b to 5a, 5b give the local pressure coefficient and Mach number distributions for freestream Mach numbers of 1.5, 2, 3 and 4, respectively, for conical boat-tails with boat-tail angles of 2, 4, 6, 8 and 10 degrees.

Similarly, Figures 6a, 6b to 9a, 9b give the local pressure coefficient and Mach number distribution for freestream Mach numbers of 1.5, 2, 3 and 4, respectively, for circular-arc afterbodies. The curves are given for values of ρ/R_m up to 30, together with the limiting cases of a cylindrical body ($\rho/R_m \rightarrow \infty$).

Note that the relationship between local pressure coefficient and local Mach number is given by

$$C_p = \frac{1}{0.7 M_\infty^2} \left[\left(\frac{1 + 0.2 M_\infty^2}{1 + 0.2 M^2} \right)^{7/2} - 1 \right], \quad (3.6)$$

which assumes a value of γ of 1.4.

To the order of accuracy of the theory the data in Figures 6a, 6b to 9a, 9b may also be applied to boat-tails of parabolic profile, the equivalent radius of curvature being obtained from Equation (3.4).

Interpolation should be used to obtain estimates at intermediate values of β or ρ/R_m for each Mach number, and then against M_∞ .

4. DISCUSSION

The nature of the pressure distributions on boat-tails differs according to the type of boat-tail. For conical boat-tails, theory predicts a sudden expansion of the flow at the surface discontinuity at the cylinder/boat-tail junction. As there is no further change in surface slope the recompression that occurs results from the three-dimensional nature of the flow as the boat-tail reduces in diameter. The conical boat-tail pressure distribution is therefore characterised by sudden expansion of the flow followed by recompression.

For 'smooth' boat-tail profiles, such as circular-arc and parabolic, there is a continuous increase in surface slope aft of the cylinder/boat-tail junction and expansion of the flow along the boat-tail. This is countered and, provided that the boat-tail is sufficiently long, eventually overcome by the increasingly three-dimensional nature of the flow as the boat-tail reduces in diameter. The pressure distribution on a 'smooth' boat-tail is therefore characterised by expansion of the flow down to a minimum pressure followed by recompression.

As the method is derived from inviscid calculations, no account is taken of the presence of the boundary layer, the primary effect of which is flow displacement, the magnitude being dependent on Reynolds number and Mach number. Moreover, the growth of the boundary layer along the boat-tail itself will also be strongly dependent on the pressure gradient, which is determined by the severity of the boat-tail profile. In particular, the magnitude of the sudden expansion exhibited by theoretical data at the junction of the cylinder and a conical boat-tail is not realised as the boundary layer acts locally to smooth the discontinuity. The Item is expected to provide realistic results provided that the boundary layer is thin compared to the local body radius. Quantitative assessment of the effect of Reynolds number is beyond the scope of this Item.

Application of the method excludes circumstances where interference effects arise as a result of a propulsive jet issuing from the base of the body. However, such interference only occurs when the nozzle operating conditions and geometry result in a base pressure exceeding that on the end of the boat-tail. Under such circumstances the higher base pressure feeds onto the boat-tail via the subsonic portion of the boundary layer. For a highly under-expanded jet, producing a high base pressure and a large jet plume, a shock wave may form of sufficient strength to separate the flow on the boat-tail.

5. REFERENCE

- ESDU Pressure distribution on forebodies and afterbodies of revolution at zero incidence in inviscid flow.
ESDU International plc, Item No. Aero B.S.05.03.01, June 1954.

6. EXAMPLE

Calculate the local Mach number and pressure coefficient at a station located at $x = 1.5R_m$ on a circular-arc boat-tail at a freestream Mach number of 2.5. The geometry of the boat-tail is defined by

$$l = 2R_m \text{ and } R_b = 0.7R_m.$$

From Equation (3.4), with $l/R_m = 2$ and $R_b/R_m = 0.7$,

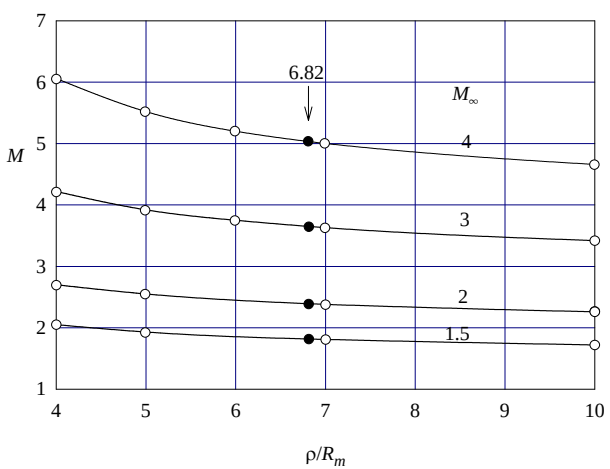
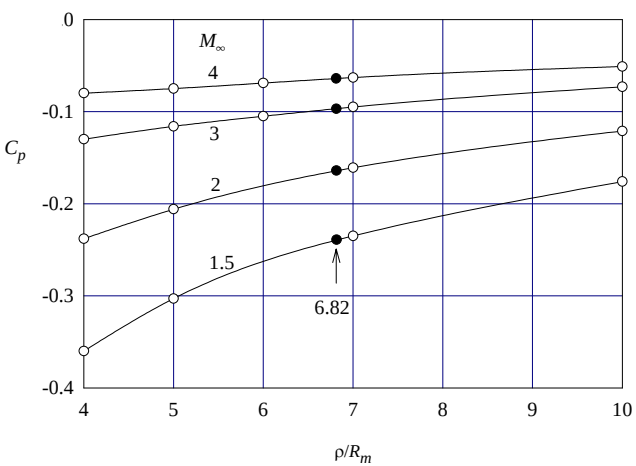
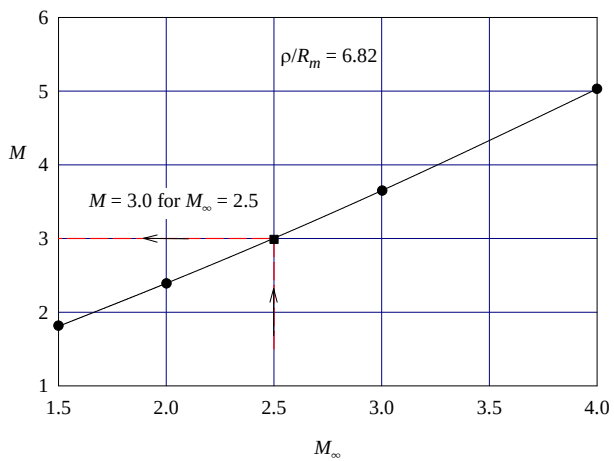
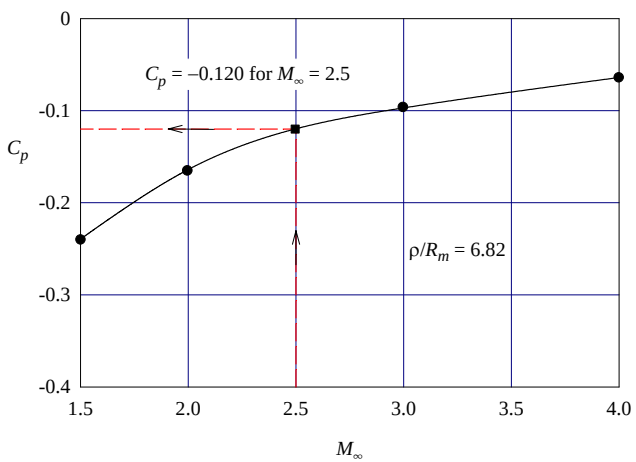
$$\begin{aligned} \rho/R_m &= [(l/R_m)^2 + (1 - R_b/R_m)^2]/[2(1 - R_b/R_m)] \\ &= [2^2 + (1 - 0.7)^2]/[2(1 - 0.7)] \\ &= 6.82. \end{aligned}$$

Data for both C_p and M are obtained for sufficient values of ρ/R_m at each of the freestream Mach numbers represented by Figures 6a, 6b to 9a, 9b, as given in Table 6.1. Sketch 6.1 summarises the procedure of cross-plotting both C_p and M against ρ/R_m to interpolate values at $\rho/R_m = 6.82$, and then cross-plotting them against M_∞ to interpolate at $M_\infty = 2.5$.

TABLE 6.1 Data for cross-plots, $x/R_m = 1.5$

M_∞	ρ/R_m	4	5	6	7	10
1.5 (Figures 6a and 6b)	C_p	-0.360	-0.303	—	-0.235	-0.176
	M	2.05	1.93	—	1.81	1.72
2.0 (Figures 7a and 7b)	C_p	-0.238	-0.206	—	-0.161	-0.121
	M	2.70	2.55	—	2.38	2.26
3.0 (Figures 8a and 8b)	C_p	-0.130	-0.116	-0.105	-0.095	-0.073
	M	4.22	3.92	3.75	3.63	3.42
4.0 (Figures 9a and 9b)	C_p	-0.080	-0.075	-0.069	-0.063	-0.051
	M	6.06	5.52	5.20	5.00	4.66

The resulting values for C_p and M are -0.120 and 3.0, respectively.



Sketch 6.1 Cross plots for example

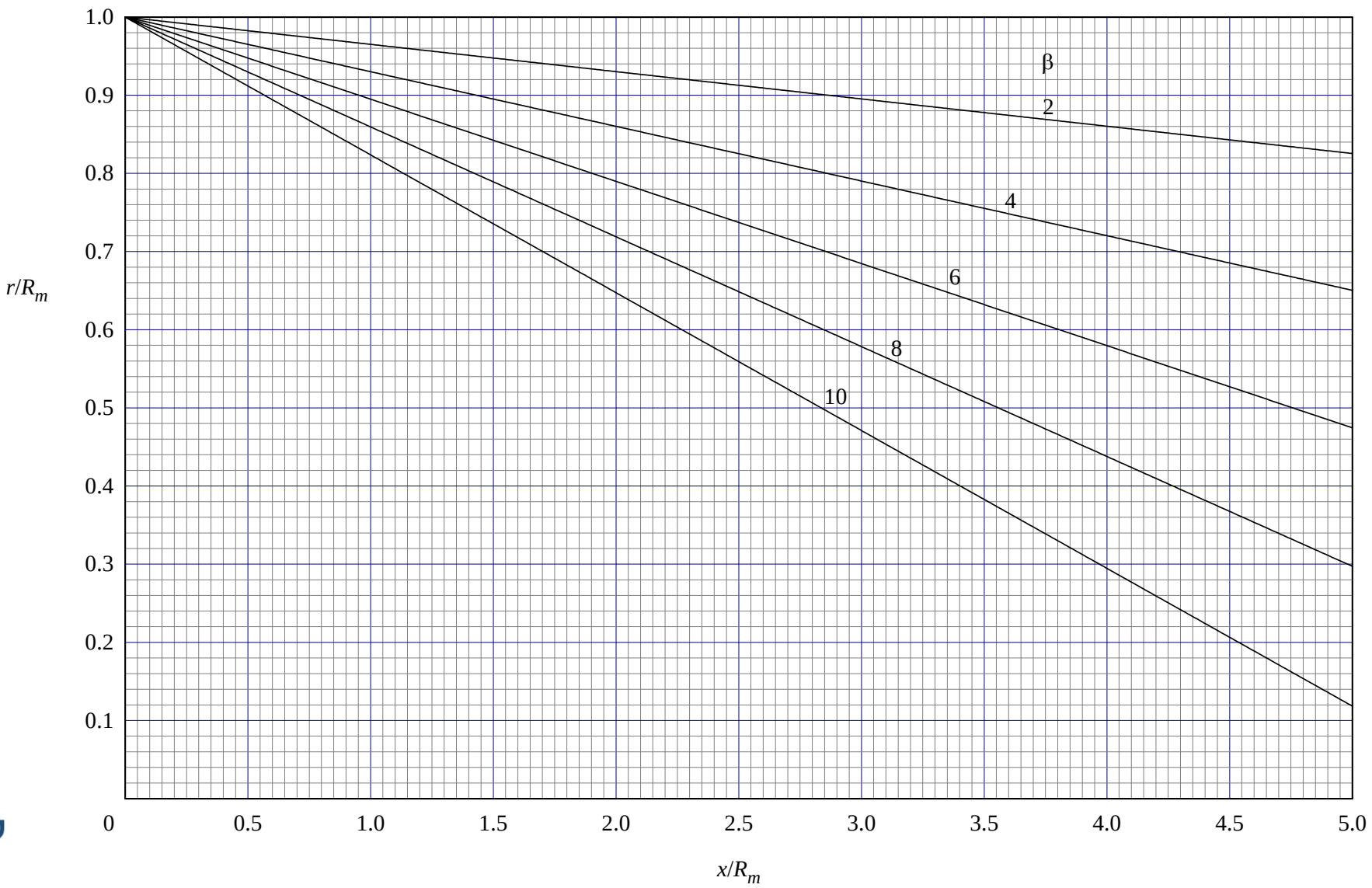


FIGURE 1a r/R_m versus β and x/R_m for conical boat-tails