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Elasto-plastic buckling of flat isotropic stiffened panels and struts in compression

**Associated software:
ESDUpacs A0101
and B0101**

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Mr B. Popham	— Astrium Space Ltd
Prof. A. Rothwell	— Technische Universiteit Delft, The Netherlands
Mr J.H. van der Sloot	— Independent
Mr K. van Katwijk*	— European Space Agency, Noordwijk, The Netherlands.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken, under contract to ESDU, by

Mr J.H. van der Sloot.

The person with overall responsibility for the work in this subject area is Mr A.C. Quilter, Head of Strength Analysis Group.

ELASTO-PLASTIC BUCKLING OF FLAT ISOTROPIC STIFFENED PANELS AND STRUTS IN COMPRESSION

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ELASTO-PLASTIC BUCKLING OF FLAT ISOTROPIC STIFFENED PANELS AND STRUTS IN COMPRESSION

1. INTRODUCTION, THEORY AND APPLICATION

1.1 INTRODUCTION

This Data Item accompanies ESDUpac A0101 which provides a Fortran computer program that calculates initial compressive buckling loads for stiffened rectangular panels and struts made of isotropic materials. The purpose of the program is to extend the range of applicability of Data Item No. 98016 into the plastic range of the material stress-strain curve. Hence, both elastic and plastic material behaviour are treated. The method on which the program is based is the finite strip method (FSM), which is particularly well-suited to the analysis of prismatic structures that show periodic deformation patterns under loading. A brief outline of the finite strip method is presented in Section 1.3 of Data Item No. 98016*.

Table 1.1 summarizes the relevant ESDU Data Items, for which the program provides an alternative computational method for estimating the elasto-plastic buckling loads of panels and struts. These Items are mainly examples of applications of the general theory presented in Data Item Nos Struct 02.01.28† and 94005‡. Although the program is based on FEM-like discretization procedures, it possesses significant advantages in speed compared to a full-scale finite element approach.

Since the inclusion of the treatment of plastic material behaviour requires additional computations to be performed both at the strip element level and at the overall system matrix solution level, it may be of advantage to realize that it is likely to be more efficient to analyse panels or struts that are expected to buckle in the elastic range with the program associated with Data Item No. 98016, ESDUpac A9816 (though if those panels or struts are subsequently found to buckle in the plastic range then obviously ESDUpac A0101 should be used).

The treatment and formulation of the buckling problem with regard to the extension of material behaviour into the plastic range of the material curve is considered in Section 1.3. The modified stress-strain relations applicable to the buckling condition that are required are provided by the deformation theory of plasticity. These relations are computed for smooth stress-strain curves. The main reasons behind the choice of deformation theory may be found in Derivations 4 and 5.

Since ESDUpac A0101 is intended to be an extension of the program of ESDUpac A9816 to allow for plastic material behaviour, all options provided in the latter are also available in program A0101. Section 1.3.2 summarizes these options. For further details on the predefined skin-stringer sections, skin-stringer joint models and types of analysis the user is referred to Appendix A of this Data Item and Section 1.3 and Appendices B and C of Data Item No. 98016.

The computational capabilities of the program are demonstrated in Section 1.4. Firstly, in Section 1.4.2, direct comparisons of results produced by the program with those given by some of the Data Items listed in Table 1.1 are made. Secondly, Section 1.4.3 and subsequent sub-sections contain selected examples that illustrate the scope of the program and highlight its capabilities.

Typical features that are inherent in the finite strip formulation used by the program are outlined in Section 1.4.4 of Data Item No. 98016 and apply equally to results obtained with program A0101.

* Data Item No. 98016 "Elastic buckling of flat isotropic stiffened panels and struts in compression".

† Data Item No. Struct 02.01.28 "Information on the use of Item No. 73007".

‡ Data Item No. 94005 "Buckling of flat rectangular orthotropic plates".

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Section 2 of the Data Item may be considered as a Program User's Manual. It consists of descriptions of the input and output files, interwoven with appropriate examples to explain or elucidate detailed aspects of the finite strip input/output models. It is worth noting that, as far as the user is concerned, the extension of the capabilities of the program into the non-linear range of material behaviour manifests itself only as an increase in the amount of information required in the material data file compared to that required in the corresponding file used by program A9816 of Data Item No. 98016.

TABLE 1.1 Relevant Data Items on Buckling

<i>Data Item No.</i>	<i>Volume No.</i>	<i>Title</i>
Struct 01.01.08	11	Local instability of struts with flat sides.
Struct 01.01.10	11	Torsional instability of stringers and struts of angle section.
Struct 01.01.19	11	Local buckling and crippling of rectangular tube section struts.
70003	8	Local buckling of compression panels with unflanged integral stiffeners.
71014	8	Local buckling of compression panels with flanged stringers.
76023	11	Buckling of struts. Lipped and unlipped channel sections.
77030	11	Buckling of struts. Lipped and unlipped Z sections.
78020	11	Local buckling and crippling of I-, Z- and channel-section struts.
78021	11	Guide to Items on the strength and stability of struts.
89007	11	Flexural and torsional-flexural buckling of thin-walled open section struts.

1.1.1 General Limitations and Scope of the Method of ESDUpac A0101

ESDUpac A0101 has been written to compute the initial buckling loads of models of panels that are presumed to behave within the realm of classical thin plate theory. See Reference 14 for the pertinent assumptions that are basic to that theory and Paragraph 2 of Section 1.1.1 of Data Item No. 98016 for a broader perspective of its application in the context of the finite strip method.

Because of the inherent freedom, on account of the finite strip formulation, for the buckling modes to develop across the panel, the scope of the computable modes includes the combined torsional-flexural mode. No particular fixed points of rotation of the section, or parts of the section, are presumed to exist, other than those enforced by the user through the imposition of boundary conditions. A comprehensive account of local, flexural and torsional-flexural buckling may be found in Section 3 of Data Item No. 72012*.

Thus, the present version of ESDUpac A0101 is able to compute the elasto-plastic buckling stresses and – if relevant – the associated modes for local, flexural and torsional or torsional-flexural buckling. It is emphasized, however, that the method presented has, as yet, no capabilities to analyse the response of a

* Data Item No. 72012 "Information on the use of Data Items on the buckling of plates and compression panels manufactured from isotropic materials".

panel loaded under axial compression into the post-buckling range. See Data Item No. 78021* for more information on that subject.

The program provides the initial **elasto-plastic** buckling load for panels and struts directly. It is not necessary to apply a plasticity correction factor to reduce the elastic buckling stresses.

1.1.2 Limitations Inherited from Data Item No. 98016

ESDUpac A0101 is subject to the same restrictions and limitations as ESDUpac A9816. Details of those restrictions and limitations are given in Data Item No. 98016 and, because of their importance, a brief summary is repeated here.

- (i) Although the buckling modes that may be computed are general in form, it should be noted that inter-rivet buckling is not treated. Data for such cases are provided in Data Items Nos Struct 02.01.08[†] and Struct 02.01.09[‡].
- (ii) The plate strip element matrix is derived for flat elements. A buckling analysis for a model of curved panel or shell-type geometry, where, amongst other things, significant membrane action could be expected as a result of its overall curvature, could be carried out, by approximating the curved geometry with flat strip elements, but this is not recommended. The main reason for this is the fact that the present program only treats 'bifurcation' buckling behaviour, whereas curved panels may well buckle or even collapse in a 'limit point' buckling mode, the prediction of which is beyond the scope of the program.

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used. Additional notation is defined where required.

A	cross-sectional area	m^2	in^2
B	coefficient in representation of stress-strain curve		
b	width	m	in
C	coefficient in representation of stress-strain curve, see Equation (1.2)		
c	width	m	in
c_1, c_2	0.1 and 0.2 per cent compressive proof stresses respectively (also designated as F_{cy} and $\sigma_{0.2}$)	N/m^2	lbf/in^2
D	diameter	m	in
$[D]$	matrix in stress-strain relation	N/m^2	lbf/in^2
d	width	m	in
E	Young's modulus	N/m^2	lbf/in^2

* Data Item No. 78021 "Guide to Items on the strength and stability of struts".

† Data Item No. Struct 02.01.08 "Buckling in compression of sheet between rivets".

‡ Data Item No. Struct 02.01.09 "Inter-rivet buckling curves for specific materials".

E_t	tangent modulus of stress-strain curve: $\frac{d\sigma}{d\varepsilon}$	N/m ²	lbf/in ²
E_s	secant modulus of stress-strain curve: $\frac{\sigma}{\varepsilon}$	N/m ²	lbf/in ²
e_1, e_2	edge widths, see Sketch A1.2	m	in
f	stress	N/m ²	lbf/in ²
f_n	stress at which $E_t = E/2$	N/m ²	lbf/in ²
G	shear modulus	N/m ²	lbf/in ²
h	height	m	in
I	second moment of area	m ⁴	in ⁴
ID_{type}	predefined skin-stringer section identifier		
I_{y0}	second moment of area of element about axis through its centroid and parallel to model y-axis (see Sketches A2.1 to A2.10 for location of y-axis)	m ⁴	in ⁴
I_{yy}	second moment of area of element about axis joining its node points	m ⁴	in ⁴
I_{zz}	second moment of area of element about axis through its centroid and perpendicular to axis joining its node points	m ⁴	in ⁴
i	counter or index		
k_1, k_2	boundary condition identifiers		
L	buckling half-wavelength	m	in
L_0	starting length, see Section 1.3.3 and Table 2.2	m	in
L_{ij}	element (i,j) of elasto-plastic stress-strain relation, see Equation (1.6)	N/m ²	lbf/in ²
L_{inc}	incremental length	m	in
L_p	panel length	m	in
LP_1	limit of proportionality between stress and strain	N/m ²	lbf/in ²
MC_i	material property code		
m	power law exponent in representation of stress-strain curve (sometimes called 'material characteristic')		
N_{st}	number of stringers		
OP	output specifier		
P	load	N	lbf
p	pitch	m	in

R	stringer corner inside bend radius	m	in
Runtype	run type specifier		
t	thickness	m	in
u, v, w	displacements in x -, y - and z -directions, respectively, in buckling mode, see Sketch 1.1	m	in
W_i	width	m	in
x, y, z	co-ordinates, see Sketch 1.1	m	in
z_{avg}	distance of centroid of element from y -axis	m	in
α	angle of inclination of web of machined top-hat section, see Sketch A2.9	rad	rad
γ	shear strain		
ε	strain		
θ	angle of orientation of element about x -axis, measured anticlockwise from y -axis ('Theta' in program output)	rad	rad
ν	Poisson's ratio ('Nu' in program output)		
σ	stress ('Sigma' in program output)	N/m ²	lbf/in ²
τ	shear stress	N/m ²	lbf/in ²
ϕ	rotational displacement about x -axis in buckling mode ('Phi' in program output), see Sketch 1.1	rad	rad

1.2.1 Suffices

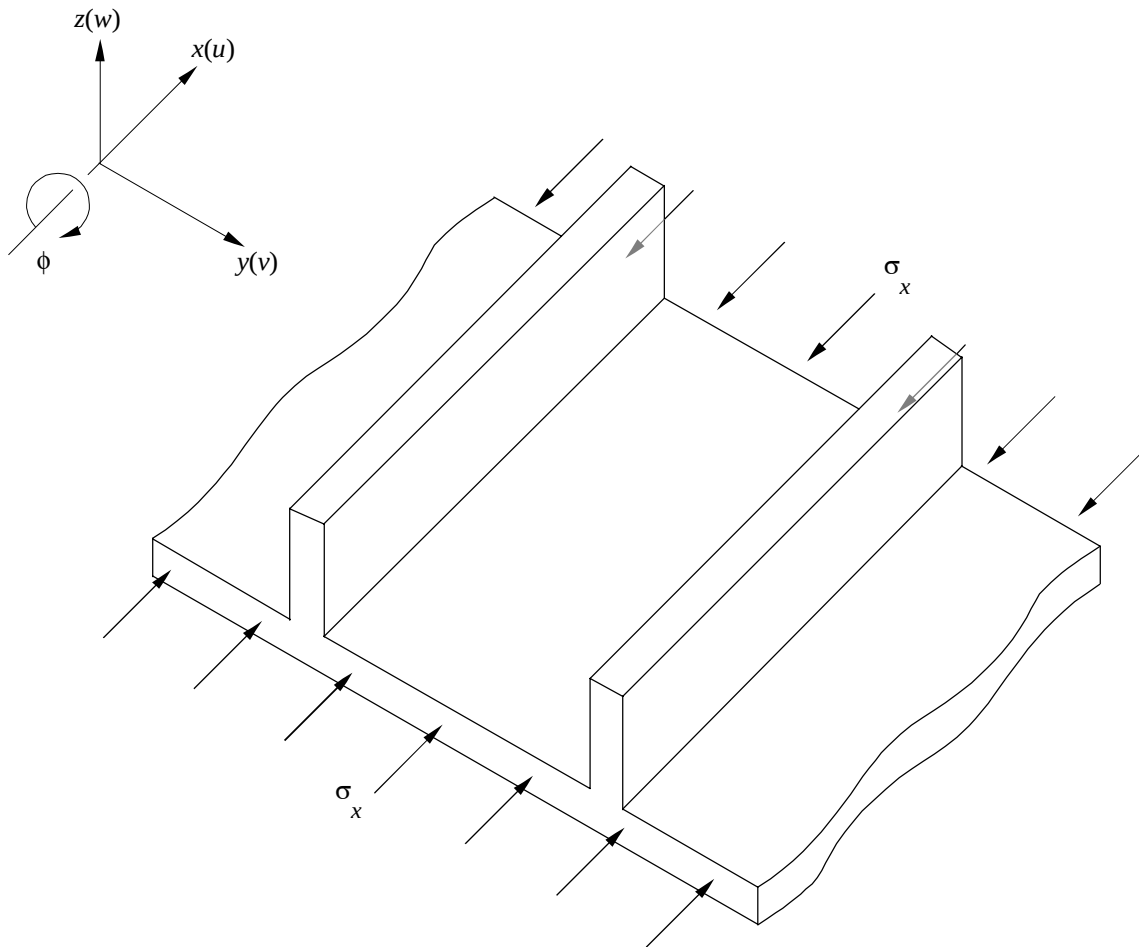
0	start value
1, 2, ...	indices
a, adh	adhesive
avg	average
$chord$	chord of sector of stringer corner
cr	critical (or buckling)
E	Euler
EL, El	element, elastic
f	flat (in this Data Item, a flat refers to any of the individual components that form part of a section)
i, j, k	indices
l, lb	local, local buckling
Nf	number of flats
Pl, pl	plastic
p	limit of proportionality, panel
r, riv	rivet
sk	skin
st	stringer
x, y	in x - and y -directions respectively

1.2.2 Matrix and Vector Notation

$[A]$	matrix
$\tilde{x}$	vector

1.2.3 Sign Convention

Tensile loads, stresses and strains are positive; compressive loads, stresses and strains are negative. An exception to this rule occurs in Section 1.3.1 of the Data Item where, for simplicity, the opposite convention is used.



Sketch 1.1 Basic co-ordinate system

1.3 ASPECTS OF THE STRIP METHOD AND PLASTICITY

1.3.1 Introduction

The finite strip method may be used to advantage for structural analysis in cases where the analysis pertains to models of prismatic structures that exhibit periodic deformation behaviour. These conditions prevail when considering buckling phenomena that occur in stiffened panels under compressive loading. The uniformity of the pre-buckling state and the usually prismatic character of the panel allow the adoption of the assumption of periodicity for the buckling deformations. Since an extension of material behaviour into the plastic range of the material stress-strain relation does not modify the character of the governing differential equations, the assumption of the periodic character of the solutions still holds. This implies that the strip method – employed as a general formulation to solve the buckling problem for complicated strut and panel designs – can be utilized in the present application with similar advantages.

A more detailed account of the strip method is presented in Section 1.3 of Data Item No. 98016. Here it suffices to mention the fact that all formulations employed in ESDUpac A9816 for setting up the system matrices for the finite strip buckling problem have been utilized in ESDUpac A0101 without modification.

Two main aspects of the formulation of the buckling condition for application to the plastic regime require consideration; they are,

- (a) the relation between stress and strain that characterizes the panel material(s), and
- (b) the formulation of the generalized stress-strain relations or constitutive relation for the plane stress state as applicable to the buckling problem for struts and panels.

These subjects are briefly treated in the following sub-sections. There, no sign convention is used to discriminate between tensile and compressive stresses and strains.

1.3.1.1 The uniaxial stress-strain curve

The major change that is required to extend the analysis of panels and struts from purely elastic to plastic material behaviour occurs in the computation of the strip element material properties, which vary with the strain level. This notion is illuminated in Sketch 1.2, where it is seen that, at a given strain value, the stresses and moduli differ significantly between the elastic and plastic regions of the curves.

Generally, these curves will differ in form for different types of material. The analysis presented in this Data Item and the accompanying program is applicable to panels and struts fabricated from materials whose uniaxial stress-strain relations may be represented by smooth, continuous curves, such as that depicted in Sketch 1.2. For such curves, three parameters suffice to obtain an approximate description of the non-linear function relating the uniaxial stress to the uniaxial strain, and these parameters are usually obtained from uniaxial tension or compression tests. It should be noted that the properties derived from 'tensile' tests may not be directly applicable to 'compressive' buckling problems; see for example Derivation 6 and Reference 17.

Three models for describing these curves are provided in the program of the Data Item and available to the user. The first and second models are variants of the stress-strain curve originally proposed by Ramberg and Osgood in Derivation 1. Its form is:

$$\varepsilon = \frac{\sigma}{E} + B \left(\frac{\sigma}{E} \right)^m,$$

where ε and σ represent strain and stress respectively, E is Young's modulus and B can be related to the quantities E , c_2 and m via:

$$B = 0.002 \left(\frac{E}{c_2} \right)^m,$$

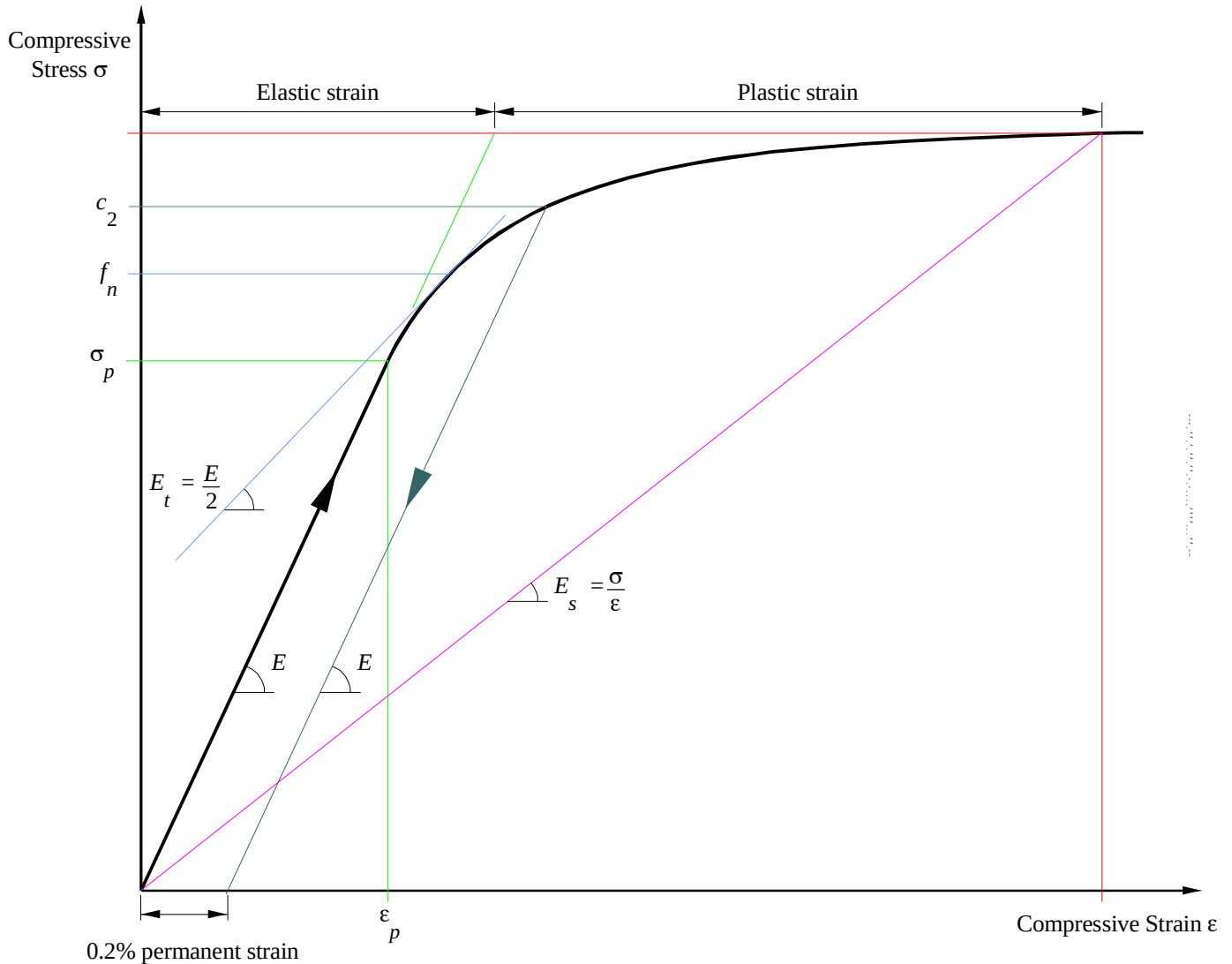
where c_2 is the stress at 0.2 per cent permanent strain, see Sketch 1.2.

The first model is the stress-strain curve defined in Data Item No. 76016*. It has the form, in the notation of that Data Item:

$$\frac{\varepsilon E}{f_n} = \frac{f}{f_n} + \frac{1}{m} \left(\frac{f}{f_n} \right)^m, \quad (1.1)$$

* Data Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials".

where f represents stress, f_n is the stress at which the tangent modulus, E_t , equals $E/2$ and m is the power law exponent. Thus the three parameters are E , f_n and m . Data Item No. 89052* may be consulted to obtain these quantities from other data.



Sketch 1.2 Elasto-plastic material curve characterization

The second model is the form that utilizes the stress corresponding to 0.2 per cent permanent strain, c_2 , instead of f_n and reads:

$$\frac{\epsilon E}{c_2} = \frac{\sigma}{c_2} + C \left(\frac{\sigma}{c_2} \right)^m, \quad (1.2)$$

* Data Item No. 89052 "Construction of inelastic stress-strain curves from minimal materials data (computer program)".

where $C = 0.002E/c_2$. Here the three parameters are E , c_2 and m . In addition it may be mentioned that Equations (1.1) and (1.2) are related to each other via the expression:

$$f_n = c_2 \left(\frac{1}{mC} \right)^{\frac{1}{m-1}} = \left(\frac{1}{0.002m \frac{E}{c_2}} \right)^{\frac{1}{m-1}}.$$

The third model is a different representation of the relation between uniaxial tensile stress and strain, which is often found in the literature, see for instance Derivation 4, and is of advantage when a strictly linear relationship is desired below a specific stress-strain level. The curve is represented by the two branches:

$$\text{for } \sigma \leq \sigma_p, \quad \frac{\varepsilon}{\varepsilon_p} = \frac{\sigma}{\sigma_p} \quad (1.3a)$$

$$\text{and for } \sigma \geq \sigma_p, \quad \frac{\varepsilon}{\varepsilon_p} = \frac{1}{m} \left(\frac{\sigma}{\sigma_p} \right)^m + 1 - \frac{1}{m}, \quad (1.3b)$$

where σ_p is the limit of proportionality. Since σ_p/E equals ε_p , E , σ_p and m are the three parameters that characterize the non-linear region of the stress-strain curve.

The respective tangent and secant moduli are easily obtained from Equations (1.1), (1.2) or (1.3a) and (1.3b) by calculating either $\frac{df}{d\varepsilon}$ or $\frac{d\sigma}{d\varepsilon}$ for E_t and $\frac{f}{\varepsilon}$ or $\frac{\sigma}{\varepsilon}$ for E_s .

The means of specifying which relations are used for each of the materials from which the panel or strut to be analysed is constructed are described in Section 2.3.3 of Part 2.

Note that the value of Poisson's ratio in the elastic range is required in order to compute the stress-strain relation at all strain levels. Typical values for the required material properties may be found in Data Item No. 76016, Derivation 6 or Reference 17.

1.3.1.2 The generalized stress-strain law or constitutive relation

In the elastic range the stress-strain relation, or Hooke's law, for plane stress formulations appropriate to the models analysed in this Data Item, reads:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0 \\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix}, \quad (1.4)$$

$$\text{or} \quad \tilde{\sigma} = [\mathbf{D}_{EL}] \tilde{\varepsilon}. \quad (1.5)$$

The tilde indicates a vector and the boldface letter indicates a matrix.

In the case of plates compressed into the plastic range of the material stress-strain curve, Equations (1.4) and (1.5) are replaced by:

$$\begin{bmatrix} d\sigma_x \\ d\sigma_y \\ d\tau_{xy} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & 0 \\ L_{21} & L_{22} & 0 \\ 0 & 0 & L_{33} \end{bmatrix} \begin{bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\gamma_{xy} \end{bmatrix} \quad (1.6)$$

$$\text{and} \quad d\tilde{\sigma} = [\mathbf{D}_{PL}] d\tilde{\varepsilon}. \quad (1.7)$$

The differential form of Equation (1.6) focuses attention on the dependency on ε of the matrix elements. The expressions for L_{ij} are provided by the plasticity theory. The theory adopted in this Data Item to formulate the plastic stress-strain relation for application to buckling problems, deformation theory, is extensively described in Derivations 3, 4 and 5; the first provides the detailed expressions for the elements L_{ij} in Equation (1.6). For an exposition on how the system of resulting equations is solved for the minimum critical strain the user is referred to Derivation 7.

1.3.2 Stringer Section Models

This section describes the various models that may be utilized to solve buckling problems for a variety of geometries. As mentioned in the introduction, all these features are the same as those available in the program of Data Item No. 98016. Therefore, the following sections are condensed reproductions of what was presented in that Data Item.

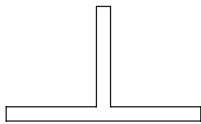
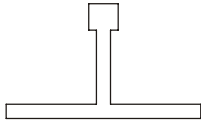
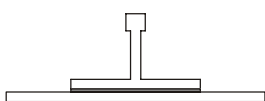
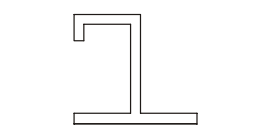
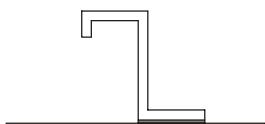
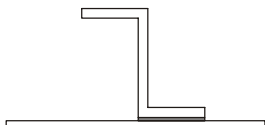
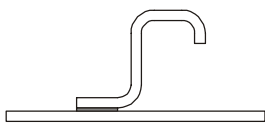
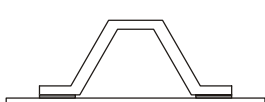
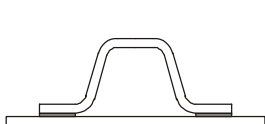
1.3.2.1 Predefined skin-stringer sections

One of the features of the program is a pre-processor routine which simplifies the input of data for a number of commonly used stringer sections by generating the strip model data file for a panel consisting of a skin and a number of stringers. This option frees the user from the need to generate a usually large strip model data file. The stringer types for which this feature is available are illustrated in Table 1.2. Detailed

geometrical and model information for these skin-stringer sections is presented in Appendix A and input details are presented in Tables 2.2 and 2.3 of Section 2.3.2.

In most cases, use of the element meshes that are given for the predefined sections is expected to give reasonably accurate results; if, however, a very local buckling mode arises, it is suggested that the strip density in the region of the buckle is increased. See Section 1.4.2 of Data Item No. 98016 for more detailed comments on this aspect of the strip models.

TABLE 1.2 Predefined Skin-stringer Sections

Type No.	Section	Relevant section of Appendix A
1		A2.1
2		A2.2
3		A2.3
4		A2.4
5		A2.5
6		A2.6
7		A2.7
8		A2.8
9		A2.9
10		A2.10

1.3.2.2 General panel model

It is emphasized that the user may still generate the strip model data file for a panel or strut of completely arbitrary cross-section, without utilization of the predefined sections shown in Table 1.2. The input file definitions required in that case are provided in Section 2.4.2. When analysing struts it will often be necessary to generate the strip model data file in this way.

In many cases, however, modification of a strip model data file generated for one of the standard predefined sections may satisfy the user's needs. A detailed example of adapting such a file to specific requirements is presented in Appendix C of Data Item No. 98016, where the strip model data file for an I-section stiffened panel is modified to create space in the cross-section of the panel to accommodate hydraulic system parts.

1.3.2.3 Special single-stringer section

An added option for use with the predefined skin-stringer sections is the possibility of representing a wide panel having many stringers by a special single skin-stringer model with predefined boundary conditions.

The special single-stringer model consists of a piece of skin of width equal to the stringer pitch with the web of the stringer located centrally (except for the top-hat sections which are simply located centrally on the skin).

The predefined boundary conditions have been selected in such a way as to give complete freedom in the z-direction whilst not allowing lateral in-plane displacements and rotations in the buckling mode of the outer edges of the skin section, see Sketch 1.2 of Data Item No. 98016. This option allows quick exploration of the variation of buckling loads for ranges of geometrical and physical parameter values that may occur in a new design.

Though extremely useful for practical purposes, especially in cases where symmetric stringer sections are analysed, care should be exercised in cases where asymmetry of the section, or other irregularities, may give rise to buckling deformations for the full panel cross-section that do not fit the predefined boundary conditions for this special single-stringer case. This is shown more explicitly in the example problem in Section 1.4.4.2 of Data Item No. 98016. It is recommended that the results of special single-stringer analyses of panels with asymmetric stiffeners always be verified by a multi-stringer analysis.

A model for a single stringer with user-specified boundary conditions can obviously be analysed as well. The typical differences in the definitions for the two single-stringer models are discussed in Section A1.1 of Appendix A.

1.3.2.4 Rolled-sheet and machined sections

In the definitions of the various predefined skin-stringer section types shown in Table 1.2 and described in Table 2.3, a distinction is made between 'rolled-sheet' sections and other types, for example, 'machined' sections. The 'rolled-sheet' type sections have a different finite strip model for the stringer corners which takes the value of the bend radius into account. This aspect is elaborated in Section A1.2 of Appendix A where information on how to model a corner when not using the predefined skin-stringer sections is provided. The effect of different bend radii on the critical stress is illustrated by the example in Section 1.4.4.1 of Data Item No. 98016. For the non-rolled-sheet cases, the effect of a corner radius is ignored.

1.3.2.5 Skin-stringer joint models

The use of the finite strip method allows the scope of the computational models to be extended beyond that of the Data Items listed in Table 1.1 with little additional effort. One way in which this has been done is through the development of adhesive layer and rivet line elements. These elements function as joining elements between the stringer flange and the skin and allow the creation of a more realistic panel model.

For the adhesive layer, simplified strain-displacement relationships are adopted on the basis that the stress distribution is assumed to be constant through the thickness of the adhesive layer. The strain energy can then be formulated in terms of the degrees of freedom of the two plate strips that constitute the joint. Also, in order to accommodate a wide range of adhesive tension/compression and shear stress-strain behaviour, the Young's modulus, E_{adh} , and shear modulus, G_{adh} , are uncoupled in the stress-strain relationship for the adhesive layer. A more detailed account of the specific features of the adhesive layer element, and of the mechanical joint model to be described next, may be found in Appendix B of Data Item No. 98016. It should be remembered that the description of plastic material behaviour is restricted to the plate strip elements in the present Data Item. The adhesive material behaviour is presumed to be elastic in all analyses.

To make some allowance for a rivet line connection between the skin and stringer, a rivet line element is derived for the model depicted in Sketch B2.2 of Appendix B of Data Item No. 98016. The two-dimensional model of the panel cross-section allows the connection of adjacent skin and stringer nodes via a simple beam element. This element is presumed to function strictly as a tension-compression-bar/bending-in-two-directions beam element.

In practical cases, the rivet line is usually located about halfway across the skin flange of the stringer. Unless preventive measures are taken (see below), the finite strip approximation for the actual panel model may show a displacement pattern in the buckling mode in which the node point at the stringer heel or the node point at the flange tip displaces beyond the adjacent skin nodal point. In the real structure, that is not possible and the buckled shape is less extreme because the skin and flange constrain each other.

This freedom of the model stringer to rotate and for the edges to penetrate the skin will result in a conservative estimate of the buckling load, but will be physically unrealistic. Some alleviation of the phenomenon can be introduced by placing additional elements between the skin and the stringer, and over the otherwise unsupported nodes, to represent the contact stiffness or the stiffness of an adhesive or sealant layer when this is present.

Note that neither of these connection elements contributes to the load-carrying cross-sectional area of the panel. Nor has any attempt – as yet – been made to incorporate non-linear material effects in the adhesive layer or the rivet line elements.

It should be emphasized again here that no attempt has been made to equip the present version of the program with the means to analyse inter-rivet buckling phenomena.

1.3.3 Analysis Options

For all the models described in the previous sections, the following three options for obtaining buckling loads are provided.

- (1) Calculation of the buckling load for the buckling half-wavelength equal to the specified panel length, that is, $L = L_p$. (Note that lower buckling loads associated with buckling half-wavelengths less than L_p will be ignored.)

- (2) Calculation of the complete $P-L$ curve with L ranging from a 'small' value up to the actual panel length.
- (3) A more precise search, starting from a user-specified value, L_0 , for a minimum in the load-length characteristic of the panel.

The three options are explained fully in Section 2.4.1.

The second option is available because the buckling phenomenon may be rather complicated on account of the various deformation patterns that may occur at different load levels, such as overall panel buckling, torsional, flexural, or a combination of the last two, and/or local inter-stringer buckling, *et cetera*.

It is envisaged that, in cases where the first option does not provide sufficiently detailed results, the second option will be used to identify areas of the $P-L$ curve that are of interest for a particular panel; the last option will then be used to perform more detailed analyses of these areas.

A summary of the various models and analysis options is given in Table 1.3.

TABLE 1.3 Finite Strip Models for Panels and Struts

Parameter/Feature	Model		Remarks
	Predefined Skin-stringer Section	Panel of Arbitrary Geometry	
Number of Stringers	- Special single-stringer as 'wide panel' model, or N_{st} stringers with $N_{st} = 1$ to 10-18 (depending on stringer type, see Section 2.5.2)	User-defined	The maximum value of N_{st} depends on the program parameter constants (see Section 2.5.2)
Boundary Conditions	Program-defined for the buckling displacements: - special single-stringer with 'wide panel' boundary conditions, or - multi-stringer models, with panel edges - free, - simply-supported, or - clamped	User-defined	The user may create other types of support conditions by adding or removing nodal degrees of freedom
Skin-stringer Joint Models	Adhesive layer and/or rivet line/sealant layer		The user can modify a joint model generated for the predefined sections
Analysis Options	1. Single buckling load calculation 2. Generation of complete $P-L$ curve 3. Search for particular minimum in $P-L$ curve		

1.4 RESULTS

1.4.1 Introduction

This section presents a number of example cases. The purpose of these examples is to demonstrate how the scope of the methods employed in Data Item No. 98016 has been extended in this Item with respect to aspects of plastic material behaviour. Some cases selected from the examples in Data Items listed in Table 1.1 are presented in Section 1.4.2. Section 1.4.3 presents results for problems treated in the literature. Section 1.4 of Data Item No. 98016 presents results for special cases to emphasize differences that may result from various models that might appear to be equally applicable to a particular problem. It is important that the user be aware of these observations and realize that they also hold for problems treated in this Data Item.

1.4.2 Comparison Cases from the Data Items listed in Table 1.1

This section presents values for the buckling stresses of panels and struts. All the cases were taken from selected Data Items mentioned in Table 1.1. Assumptions that were basic to the derivation of the methods employed in those Data Items have been incorporated, for the purposes of comparison, in the strip models for these cases. These assumptions entail, among other things, omitting a detailed geometry for the corner radii of stringer sections and ignoring the offset of the skin flange of the stringer relative to the skin. It was also assumed that there was no special mechanical joint connection between skin and stringer elements.

Some of the cases that appear in the Data Items listed in Table 1.1 do not result in stresses in the plastic range of the specified material curve and therefore not all are revisited below. In the descriptions of the problem geometries, the notation of the reference Data Item is usually employed.

1.4.2.1 Example problem from Data Item No. 70003*

The example problem treats the computation of the local buckling stress of the blade-stiffened panel shown in Sketch 1.3. The material properties, as given in the Data Item, are:

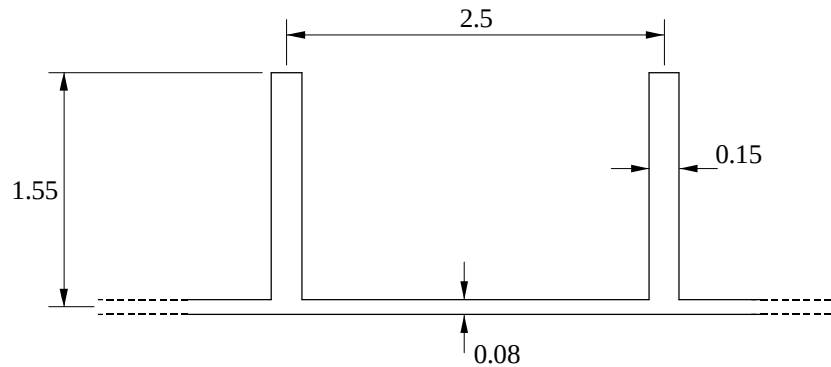
Young's modulus, E = 10.6×10^6 lbf/in²,

the reference stress, f_n = 46 000 lbf/in², and

the power law exponent, m = 15.0.

Poisson's ratio, ν , was taken as 0.30.

* Data Item No. 70003 "Local buckling of compression panels with unflanged integral stiffeners".



Dimensions in inches

Sketch 1.3 Blade-stiffened panel of Data Item No. 70003

A direct computation with program A0101 is made using the predefined skin-stringer section with $ID_{type} = 1$, see Table 1.2, with the special single-stringer option. The results are compared with the values given in Data Item No. 70003 in the following table.

TABLE 1.4 Comparison of Local Buckling Stresses for the Example of Data Item No. 70003

	Data Item No. 70003		A0101	
	Critical Stress lbf/in ²	Associated Half-wavelength in	Critical Stress lbf/in ²	Associated Half-wavelength in
Elastic local buckling	– 49 000	5.0	– 48 300	4.8
Plastic local buckling	– 44 900		– 45 000	4.0

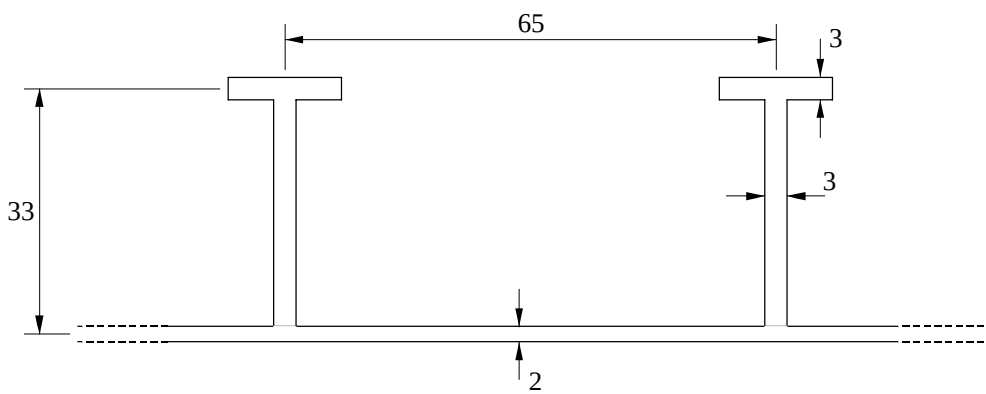
Reasonable agreement is seen to exist for this simple problem. It is of interest to note the distinct difference between the elastic and plastic buckling half-wavelengths. The elastic case buckles at $L = 4.8$ in, close to the value read from Data Item No. 70003, which is 5.0 in, whereas the plastic case shows a half-wavelength, L , of 4.0 in. All the input and output files for this particular problem – obtained under various run conditions – are presented in Section A1.6.1.

1.4.2.2 Example problem from Data Item No. 71014*

The model analysed in the example given in Data Item No. 71014 is that of an integrally stiffened T-section panel. The local buckling stress is sought. The panel is shown in Sketch 1.4. The material properties are:

* Data Item No. 71014 “Local buckling of compression panels with flanged stringers”.

Young’s modulus, E = 73 600 N/mm²,
the reference stress, f_n = 317 N/mm²,
the power law exponent, m = 15.0, and
Poisson’s ratio, ν = 0.34.



Dimensions in millimetres

Sketch 1.4 T-section stiffened panel of Data Item No. 71014

For this problem, the preparation of the data files is most conveniently accomplished through the predefined skin-stringer section option, employing stringer section type 4 suitably adapted for the non-existence of the stringer skin flange. The results are presented in Table 1.5.

TABLE 1.5 Comparison of Local Buckling Stresses for the Example of Data Item No. 71014

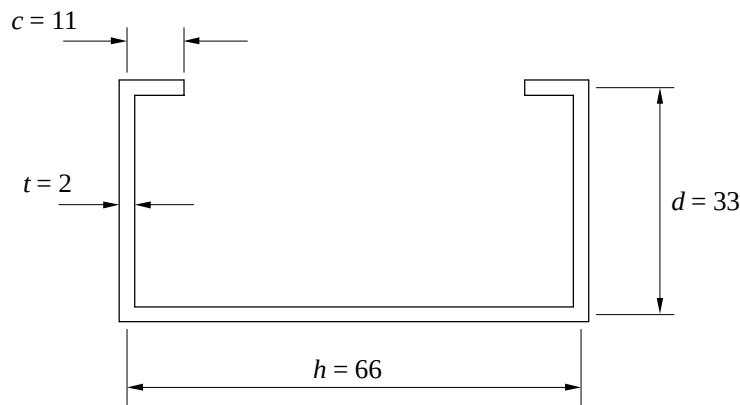
	<i>Data Item No. 71014</i> Critical Stress N/mm ²	<i>A0101</i> Critical Stress N/mm ²
Elastic local buckling	–382	–382
Plastic local buckling	–318	–320

1.4.2.3 Example problem from Data Item No. 76023*

The example problem estimates the local buckling stress of an inward-lipped channel-section strut for two lengths, 440 mm and 587 mm. The strut is shown in Sketch 1.5 and the material properties are as follows:

* Data Item No. 76023 “Buckling of struts. Lipped and unlipped channel sections”.

Young's modulus, E = 72 400 N/mm²,
the reference stress, f_n = 260 N/mm²,
the power law exponent, m = 14.0, and
Poisson's ratio, ν = 0.30.



Dimensions in millimetres

Sketch 1.5 Channel-section strut of Data Item No. 76023

The Data Item solutions are obtained from linear interpolation between values read from the figures. The results are presented in Table 1.6.

TABLE 1.6 Comparison of Local Buckling Stresses for the Example of Data Item No. 76023

Column Length mm	Data Item No. 76023 Critical Stress N/mm ²	A0101 Critical Stress N/mm ²
440	-250	-247
587	-211	-207

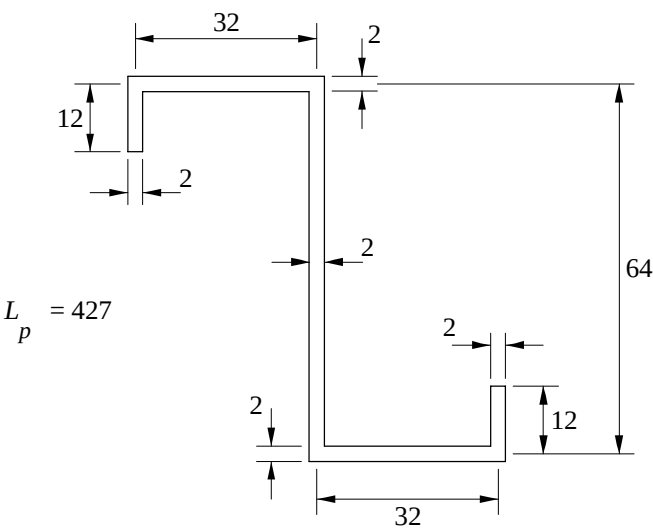
These values suggest good correspondence between the results obtained by the two methods.

1.4.2.4 Example problem from Data Item No. 77030*

The example problem from Data Item No. 77030 estimates the local buckling stress for the lipped Z-section strut shown in Sketch 1.6. The material properties are as follows:

* Data Item No. 77030 "Buckling of struts. Lipped and unlipped Z sections".

Young's modulus, E = 72 400 N/mm²,
the reference stress, f_n = 260 N/mm²,
the power law exponent, m = 14.0, and
Poisson's ratio, ν = 0.30.



Dimensions in millimetres

Sketch 1.6 Lipped Z-section strut of Data Item No. 77030

First, the elastic local buckling stress is checked with program B9816V12, which gives a value of -329 N/mm². This should be compared to the Data Item value of -319 N/mm²; a difference of 3 per cent, which is attributed to the interpolation procedure employed to read the value from Figure 8 in the Data Item. The opportunity is taken here to look briefly at the effect of model mesh density on the local buckling stress calculated by the program. The results are presented in Table 1.7. Convergence to the value of -329 N/mm² is seen to occur.

TABLE 1.7 Elastic Local Buckling Stress of a Lipped Z-section Strut. Example of Data Item No. 77030. (Results Obtained with ESDUpac B9816)

No. of Elements	Local Buckling Stress N/mm ²	Half-wavelength mm
8	-330.56	256
10	-330.12	256
14	-329.51	256
18	-329.22	256

When the model associated with the last entry in the table is analysed using program A0101, an identical 'elastic' local buckling stress, σ_{lb} , of -329.22 N/mm^2 at the same half-wavelength is obtained. An 'elastic' value is obtained from program A0101 by specifying a value of f_n equal to 1060 N/mm^2 in the related material data file (that is to say, the elastic limit of the material is artificially increased so that the stresses remain in the elastic region of the material stress-strain curve).

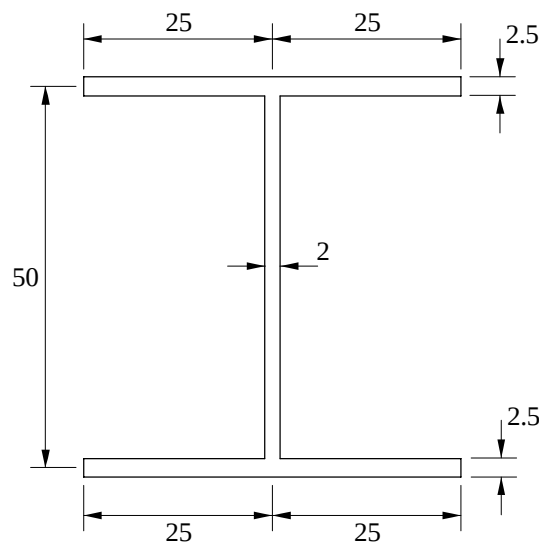
With f_n equal to its true value of 260 N/mm^2 , the corresponding plastic local buckling stress is found to be $\sigma_{lb,pl} = -259 \text{ N/mm}^2$. The value given in the Data Item is -263 N/mm^2 , obtained via the plasticity reduction factor of 0.825, estimated from Figure 10 in Data Item No. 77030 and based on the elastic local buckling value of -319 N/mm^2 . The results presented here yield a reduction factor of:

$$\eta = \frac{\sigma_{lb,pl}}{\sigma_{lb,el}} = \frac{-259}{-329} = 0.790.$$

1.4.2.5 Example problem from Data Item No. 78020*

The example problem in Data Item No. 78020 computes the elastic local buckling stress for the I-section strut shown in Sketch 1.7. It then applies a plasticity reduction factor to obtain the plastic local buckling stress and finally estimates the crippling stress of the strut. Here the local buckling stresses are compared. The material properties are:

Young's modulus, E	= $73\,800 \text{ N/mm}^2$,
the reference stress, f_n	= 264 N/mm^2 ,
the power law exponent, m	= 11.5, and
Poisson's ratio, ν	= 0.30.



Dimensions in millimetres

Sketch 1.7 I-section strut of Data Item No. 78020

* Data Item No. 78020 "Local buckling and crippling of I-, Z- and channel-section struts".

A straightforward computation with A0101 provides the following comparison.

TABLE 1.8 Comparison of Local Buckling Stresses for the Example of Data Item No. 78020

	<i>Data Item No. 78020 Critical Stress N/mm²</i>	<i>A0101 Critical Stress N/mm²</i>
Elastic local buckling	–381	–379
Plastic local buckling	–284	–287

Again, good correspondence between the results can be seen.

1.4.3 General Example Problems

In the following sections, results for example problems taken from the literature are presented.

1.4.3.1 Flat plate buckling problems

In Reference 16 flat plates with various boundary conditions are analysed for buckling in the plastic range with stress-strain relations that hold for deformation theory. In this section the reference values are compared with the critical stresses obtained via program A0101.

The first model is of a square simply-supported plate under uniaxial compression. The material curve selected is the following Ramberg-Osgood representation:

$$\varepsilon = \frac{\sigma}{10\,700} + 0.002 \left(\frac{\sigma}{61.4} \right)^{20},$$

which is Equation (4.12) from Reference 16 and is in the form of Equation (1.2) used here multiplied throughout by c_2/E . The values of E and c_2 are 10 700 and 61.4 ksi respectively and Poisson's ratio, ν , is 0.32. Therefore, using the form of Equation (1.2),

$$e = s + Cs^m,$$

where $e = \varepsilon \frac{E}{c_2}$, $s = \frac{\sigma}{c_2}$ and $C = 0.002 \frac{E}{c_2} = 0.3485$.

The following table summarizes the results for the square plate problem.

TABLE 1.9 Buckling Stresses for Square Simply-supported Plates of Reference 16

b/t	A0101 Critical Stress ksi	Ref. 16, Fig 2 Deformation Theory Critical Stress ksi
22	-60.2	-60.5
23	-59.0	-58.5
24	-57.6	-57.6
25	-55.9	-56.0
26	-54.0	-54.0
27	-51.8	-52.4
28	-49.2	-49.2

The length, b , of the plate was chosen arbitrarily as 10.0 in and the value of the thickness, t , was varied to give the values of b/t for which results were quoted in the reference.

A second case considered in the reference estimates the plastic local buckling stresses for a long flange. That is a plate with one unloaded edge simply supported and the other unloaded edge free. The loaded edges are both simply supported and thus the model satisfies the requirements for comparison with the results computed using program A0101. In the table below, b is the flange width and t is its thickness.

TABLE 1.10 Comparison of Plastic Local Buckling Stresses for Hinged-free Plate of Reference 16

b/t	A0101 Critical Stress ksi	Ref. 16 Deformation Theory Critical Stress ksi
4	-68.0	-
5	-65.8	~-66
6	-63.6	-63.8
7	-61.1	-61.2
8	-57.3	-57.5
9	-49.9	-49.9

1.4.3.2 Local buckling of square tubes

1.4.3.2.1 Introduction

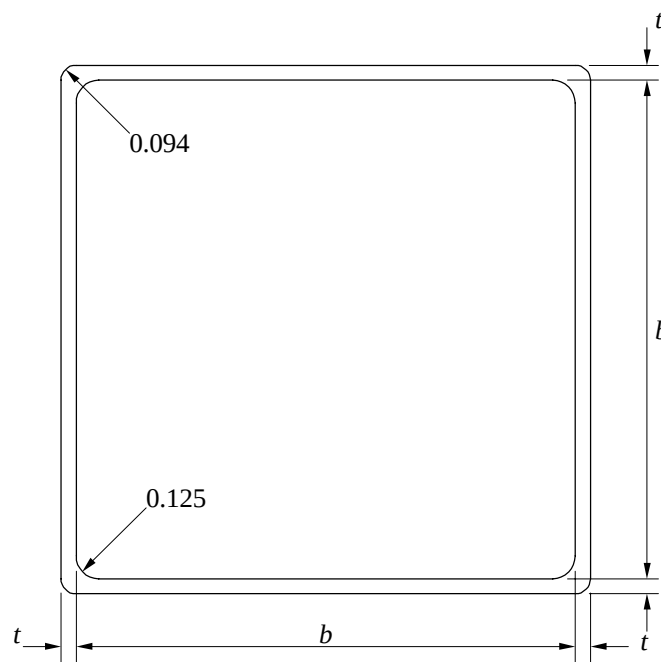
A comparison of local buckling stresses of square tubes of Al-14S-T6 material obtained from compression tests as reported in NACA TN No. 1817 (Reference 10) with values computed using program A0101 is presented in Table 1.11.

In contrast to the suggestion in the wording of the title of Reference 10, test results obtained from specimens of square tube cross-section are reported. The geometry is depicted in Sketch 1.8.

The report compiles detailed geometric data and material properties for square tubes of six distinguishable widths, ranging from 3.2 to 6.7 in, with nominal wall thickness of 0.16 in. Material properties are presented in the form of six stress-strain curves. In addition, values for the compressive proof stress σ_2 are listed. Values for Young's modulus, E , are not explicitly given, nor are values for Poisson's ratio.

1.4.3.2.2 Results

The test values and the computed critical stresses for local buckling are compared in Table 1.11. It is seen that good agreement in trends is obtained and that program A0101 produces slightly conservative values.



Dimensions in inches

Sketch 1.8 Square tube cross-section from Figure 1 of Reference 10

TABLE 1.11 Local Buckling Stresses of Square Tubes, $L/b > 4$. Comparison of Computed Values with Test Values

Identification used in NACA TN 1817 (Reference 10)	σ_{cr}		Ratio of Minimum Test Value to Computed Value
	Test Values from Table 1 of Reference 10 Minimum Maximum lbf/in ²	Result Computed by A0101 lbf/in ²	
A	21 200 22 000	20 200	1.05
B	22 500 22 600	21 200	1.06
C	42 400 42 900	40 400	1.05
D	53 300 53 900	52 900	1.01
E	58 100 59 000	56 700	1.03
F	60 900 61 600	61 200	1.00

1.4.3.3 Local buckling of stiffened panels

This section presents a comparison between computed local buckling stress values and those obtained from actual compression tests on stiffened panels.

In Section 1.4.3.3.1 the results of tests on blade-stiffened panels are compared to computed estimates. Section 1.4.3.3.2 presents comparison values for top-hat-section-stiffened panels.

1.4.3.3.1 Blade-stiffened Panels from Shorts Reports SD-39 (Reference 13) and SD-65 (Reference 15)

References 13 and 15 present the results of tests on machined compression panels which were performed in order to ascertain the influence of different values of corner radius. Since the value of the corner radius is not taken into account for the predefined blade-stiffened skin-stringer section ($ID_{type} = 1$), only the panels with the smaller bend radii and for which sufficient test results were available were examined.

Young's modulus, E , and the stress at the limit of proportionality, LP_1 , for both tension and compression control test specimens are given for DTD646 in the first report and for DTD5020 in the second. For those panels that were expected to buckle in the plastic range, five proof stresses were read from the control test registrations and listed in the reports. Poisson's ratio, ν , was not quoted and therefore a value of 0.3 was used in the computations. The scatter in the values of E and LP_1 can be seen in Table 1.12.