

Loading on a rigid aeroplane in steady lateral manoeuvres

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Prof. M.A. Woodhead	— University of Loughborough
Prof. J.R. Wright	— University of Manchester
Prof. R.J. Zwaan*	— Independent.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Mr C.J. Loughton — Head of Mechanical Motion and System Dynamics Group.

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LOADING ON A RIGID AEROPLANE IN STEADY LATERAL MANOEUVRES

CONTENTS

	Page
1. INTRODUCTION	1
2. NOTATION	2
3. BACKGROUND DESCRIPTION	5
3.1 Axis System and Motion	5
3.2 Overall Loads and Moments	6
3.3 Symmetric Loads	8
3.4 Antisymmetric Loads	13
4. STEADY HORIZONTAL TURN	17
5. STEADY STRAIGHT MOTION WITH AN ENGINE OUT	23
6. STEADY STRAIGHT SIDESLIPPING FLIGHT	27
7. AERODYNAMIC DATA AVAILABILITY AND LOADING DISTRIBUTIONS	33
7.1 Summary of Aerodynamic Data Required	33
7.2 ESDU Data	34
7.3 Loading Distributions	35
7.4 Bending Moment and Torque Distributions	36
8. REFERENCES	39
8.1 General	39
8.2 ESDU Derivative Data Availability	39
APPENDIX A DERIVATION OF SYMMETRIC AEROPLANE LOADS WHEN THE THRUST IS DIRECTLY DEPENDENT UPON ANGLE OF ATTACK	41
A1. Derivation	41
APPENDIX B DERIVATIONS OF AILERON ANGLE, FIN SIDEFORCE AND RUDDER ANGLE	42
B1. Aileron angle	42
B2. Fin sideforce	42
B3. Rudder angle	43
APPENDIX C COMPONENTS OF SIDEFORCE, ROLLING AND YAWING MOMENTS DUE TO STEADY SIDESLIP	44
C1. Definitions	44

LOADING ON A RIGID AEROPLANE IN STEADY LATERAL MANOEUVRES

1. INTRODUCTION

This Data Item describes the loading characteristics of a rigid aeroplane during steady lateral manoeuvres which include a steady horizontal turn, steady horizontal straight motion with an engine out and steady horizontal straight sideslipping flight.

The overall symmetric (longitudinal) and antisymmetric (lateral) aerodynamic and inertial loads and moments for an aeroplane in a steady turning manoeuvre involving sideslip and bank are identified. The longitudinal and lateral equations of motion are formed and the conditions for independent longitudinal and lateral motion are established.

The equilibrium equations to determine the longitudinal characteristics of wing-fuselage lift, tailplane lift, aeroplane angle of attack and elevator deflection are developed for cases when the longitudinal and lateral equations of motion are both independent and interdependent.

The three steady lateral motions are identified as special cases of the general steady equations and it is shown how to determine the three lateral unknowns of:

- aerodynamic sideforce on the fin,
- asymmetric rolling moment on the wing,
- angle of bank.

The horizontal turn is variously described as a steady sustained turn, a steady banked turn or a true banked turn, and is a motion in which the aeroplane is banked to generate a component of lift in the direction in which the turn is required and flies at constant pitch angle, θ , and at constant incidence, α , with no sideslip, no aeroplane speed change and no height loss or gain.

The ability to fly an aeroplane with unbalanced thrust due to engine failure is a safety requirement that needs to be met on all civil transport aeroplanes. The aeroplane must be able to be flown steadily in straight flight and so needs sufficient rudder power to counteract the yawing moment on the aeroplane due to the unbalanced engine thrust. An increase is also required in the elevator angle and the thrust of the operating engine(s) to prevent height and speed loss and use of aileron is required to prevent rolling motion. In general this steady flight condition can be achieved with a number of combinations of bank and sideslip angles, but in this Item the analysis is completed for an aeroplane at constant bank angle and at zero sideslip velocity.

Steady straight sideslipping flight is used almost exclusively for landing an aeroplane in a steady crosswind. To counteract the drift of the aeroplane induced by the crosswind, the aeroplane is flown in sideslip in order that the resultant direction of the motion of the aeroplane is along the direction of the runway. A combination of rudder and aileron and allowing the aeroplane to bank is used to fly the aeroplane in steady straight sideslip. An alternative way of flying an aeroplane in a landing approach in a crosswind is known as crabbing, in which the aeroplane is flown at an angle to the runway into the crosswind, such that the resultant velocity of the aeroplane relative to the ground is in the direction of the runway. Just before touchdown the aeroplane is yawed to bring the fuselage in line with the runway.

This Data Item complements two previously issued Items, Nos. 94009^[3] and 94045^[4], associated with aircraft loading in symmetric steady manoeuvres. The former deals with "Symmetric steady manoeuvre loads on rigid aircraft of classical configuration at subsonic speeds" and the latter with "Shear force, bending moment and torque of rigid aircraft in symmetric steady manoeuvring flight".

2. NOTATION

Any coherent system of units may be used.

A	wing aspect ratio
BM	bending moment
b, b_T, b_{Fin}	wing span, tailplane span and fin height, respectively
C_{D0}	drag coefficient at zero lift ($= D_0 / (\frac{1}{2}\rho V_{trim}^2 S)$)
C_L	lift coefficient ($= L / (\frac{1}{2}\rho V_{trim}^2 S)$)
$C_{L\alpha}$	lift curve slope ($= dC_L/d\alpha$)
$(C_{L\alpha})_T, (C_{L\eta})_T$	lift curve slopes of tailplane due to incidence and elevator deflection, respectively ($= (dC_L/d\alpha)_T, (dC_L/d\eta)_T$)
$(C_{M0})_{WF}$	pitching moment coefficient at $\alpha = 0$ and $q = 0$ ($= (M_0)_{WF} / (\frac{1}{2}\rho V_{trim}^2 S \bar{c})$)
C_Q	dynamic pressure coefficient ($= (W/S) / (\frac{1}{2}\rho V_{trim}^2)$)
$\bar{c}$	geometric mean chord
D, D_0	aeroplane drag and aeroplane drag at zero lift, respectively
g	acceleration due to gravity
$H\{argument\}$	step function ($= 0$ when $argument < 0$; $= 1$ when $argument \geq 0$)
h_{Fin}, h_{Nac}	distances in z -direction from aeroplane mass centre to aerodynamic centre of fin and line of action of thrust, respectively
K	number of identical engine nacelles
k	lift dependent drag factor
L, M, N	overall moments about axes Ox, Oy and Oz , respectively
L	aeroplane lift
L_p, L_r, L_v	aero-normalised rolling moment derivatives due to p, r and v , respectively ($= \partial L / \partial p / (\frac{1}{2}\rho V_{trim} S b^2), \partial L / \partial r / (\frac{1}{2}\rho V_{trim} S b^2), \partial L / \partial v / (\frac{1}{2}\rho V_{trim} S b)$)
L_ξ	aero-normalised rolling moment derivative due to ξ ($= \partial L / \partial \xi / (\frac{1}{2}\rho V_{trim}^2 S b)$)
$l_{Fin}, l_{Nac}, l_{Nose}, l_T, l_W, l_{WF}, l_W)_F$	distances in x -direction from aeroplane mass centre to aerodynamic centres of fin, nacelle, fuselage nose, tailplane, wing, wing-fuselage and wing carry-over lift, respectively
l_{front}	distance in x -direction from aeroplane mass centre to extreme tip of aeroplane nose
ℓ	local lift

ℓ_0	local lift at zero angle of attack and zero pitch rate
$\ell_\alpha, \ell_\beta, \ell_\xi, \ell_\zeta$	local lifts arising from unit changes in α , β , ξ and ζ , respectively
ℓ_p, ℓ_q, ℓ_r	local lifts arising from unit changes in (pb / V_{trim}) , $(q\bar{c} / V_{trim})$ and (rb / V_{trim}) , respectively
$(M_q)_{WF}$	aero-normalised pitching moment derivative due to q ($= \partial \mathbf{M} / \partial q / (\frac{1}{2} \rho V_{trim} S \bar{c}^2)$)
$(\mathbf{M}_0)_{WF}$	pitching moment of wing-fuselage about aerodynamic centre at $\alpha = 0$ and $q = 0$
m	mass
N_p, N_r, N_v	aero-normalised yawing moment derivatives due to p , r and v , respectively ($= \partial \mathbf{N} / \partial p / (\frac{1}{2} \rho V_{trim} S b^2)$, $\partial \mathbf{N} / \partial r / (\frac{1}{2} \rho V_{trim} S b^2)$, $\partial \mathbf{N} / \partial v / (\frac{1}{2} \rho V_{trim} S b)$)
N_ξ	aero-normalised yawing moment derivative due to ξ ($= \partial \mathbf{N} / \partial \xi / (\frac{1}{2} \rho V_{trim}^2 S b)$)
O	origin of axis system for overall aeroplane
$O\{\text{argument}\}$	order of magnitude of <i>argument</i>
p, q, r	aeroplane angular velocities about x-, y- and z-directions, respectively
R	radius of circle in steady turn
S, S_T, S_{Fin}, S_F	wing area, tailplane area, fin area and fuselage side area, respectively
T	torque
Th	overall aeroplane thrust
V	velocity of aeroplane in x-direction when sideslipping
V_{trim}	trimmed velocity of aeroplane in flight direction
u, v, w	aeroplane velocities in x-, y- and z-directions, respectively
v_G	crosswind velocity
W	aeroplane weight
X_g, Y_g, Z_g	aeroplane inertia forces in the x-, y- and z-directions, respectively
$(X_g)_\alpha$	rate of change of X_g with α ($= dX_g / d\alpha$)
x, y, z	body axis directions for overall aeroplane with respect to origin O
x', y', z'	intermediate values of x, y and z, respectively, for purposes of evaluating bending moment and torque
Y	overall force in y-direction
Y_p, Y_r, Y_v	aero-normalised sideforce derivatives due to p , r and v , respectively ($= \partial Y / \partial p / (\frac{1}{2} \rho V_{trim} S b)$, $\partial Y / \partial r / (\frac{1}{2} \rho V_{trim} S b)$, $\delta Y / \delta v / (\frac{1}{2} \rho V_{trim} S)$)

Y_{ξ}, Y_{ζ}	aero-normalised sideforce derivatives due to ξ and ζ , respectively (= $\partial Y / \partial \xi / (\frac{1}{2} \rho V_{trim}^2 S)$, $\partial Y / \partial \zeta / (\frac{1}{2} \rho V_{trim}^2 S)$)
y_{Nac}	spanwise location of nacelle
α	aeroplane angle of attack
$(\alpha_0)_W, (\alpha_0)_T$	wing setting angle and tailplane setting angle, respectively, relative to their zero lift lines
β	aeroplane sideslip angle (= v / V_{trim})
$\epsilon, \bar{\epsilon}$	downwash and average downwash at tailplane, respectively
ζ	rudder angle
η	elevator angle
θ	aeroplane pitch angle
μ	relative density parameter (= $(W/g) / (\frac{1}{2} \rho S b)$)
ξ	aileron angle
ρ	air density
ϕ	aeroplane bank angle
ψ	aeroplane yaw angle
Ω	aeroplane steady turn rate

Subscripts

E	denotes external
F	fuselage
Fin	fin
G	denotes steady crosswind
$Initial$	denotes a first approximation
Nac	nacelle
$Nose$	nose of aeroplane
S	denotes steady known sideslip
T	tailplane
W	wing
WF	wing-fuselage combination

3. BACKGROUND DESCRIPTION

The aim of this Data Item is to describe the procedure to obtain the loading distributions on the wing, tailplane and fin for a rigid aeroplane undergoing the steady lateral motions of:

- steady horizontal turn (also termed sustained turn, steady banked turn or true banked turn),
- steady horizontal motion with an engine out,
- steady horizontal sideslipping flight (applicable to crosswind landings).

In order to obtain the loading distributions in each of the steady manoeuvres, it is necessary to determine the overall loads and moments on the various elements of the aeroplane, together with the trim incidence angles of attack, α , and sideslip, β , the associated control angles of elevator, η , aileron, ξ , and rudder, ζ , and the attitude angles of pitch, θ , bank, ϕ , and yaw, ψ .

3.1 Axis System and Motion

The chosen aeroplane axis system, $Oxyz$, is a body-fixed system which is orthogonal and right-handed with the x - and z -axes taken in the plane of symmetry of the aeroplane, the x -axis pointing forward, the z -axis pointing downwards and the y -axis pointing to starboard. The origin, O , is located at the centre of mass of the aeroplane.

The axis system is shown in Sketch 3.1 and illustrates the aeroplane pitch angle, θ , defined as the angle between the Ox axis and the horizontal in the vertical plane through Ox , and the aeroplane bank angle, ϕ , defined as the angle between the Oy axis and the horizontal in the Oyz plane. Both these angles are constant for steady lateral manoeuvres.

The aeroplane motion about and along the axes, as shown in Sketch 3.1, is defined by

the components of linear velocity in the x -, y - and z - axis directions, respectively denoted by u , v and w , and the overall resultant velocity of the aeroplane, relative to air, which is denoted by V_{trim} ,

the aeroplane angular velocities about axes Ox , Oy and Oz , respectively denoted by p , q and r .

The velocities in the y - and z -directions, v and w , are respectively related to the angle of attack, α , and angle of sideslip, β , by

$$w = u \tan \alpha \quad (3.1)$$

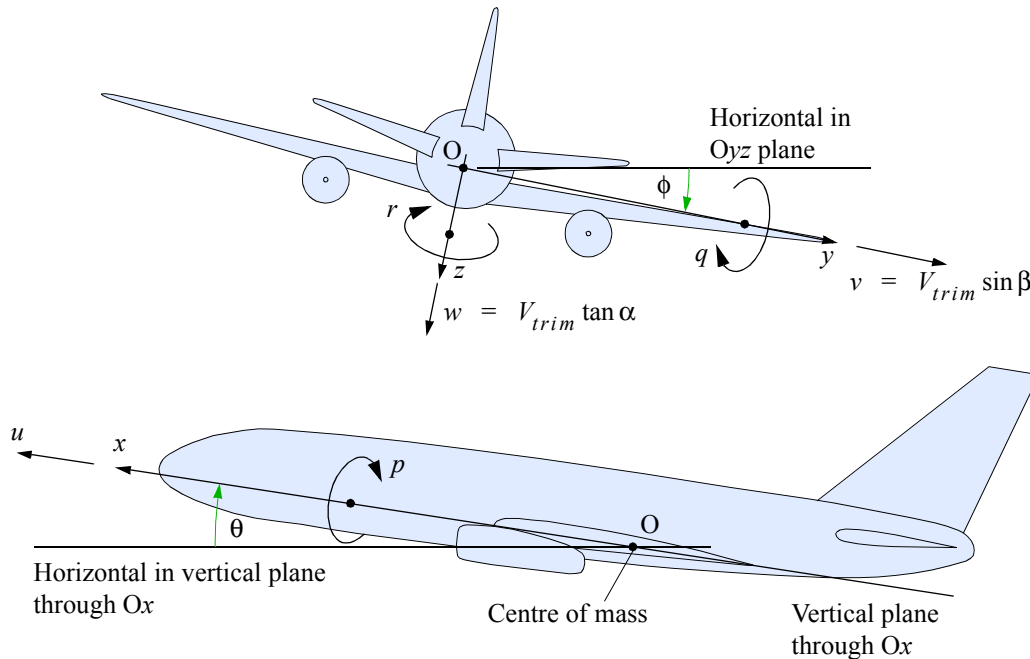
$$\text{and} \quad v = V_{trim} \sin \beta, \quad (3.2)$$

can be written $w \approx u \alpha$ and $v \approx V_{trim} \beta$, when α and β are small.

The resultant aeroplane velocity, V_{trim} , is

$$V_{trim} = \left(u^2 + v^2 + w^2 \right)^{1/2} \approx \left(u^2 + (V_{trim} \beta)^2 + (u \alpha)^2 \right)^{1/2}, \quad (3.3)$$

which reduces to $V_{trim} \approx u$ for small α and β .



Sketch 3.1 Aeroplane axis system and motion

In horizontal flight, V_{trim} is in a horizontal plane, so that the angles θ , α , β and ϕ are then related by the condition that the sum of the components of u , v and w normal to the horizontal plane are zero, that is

$$u \sin \theta = (w \cos \phi + v \sin \phi) \cos \theta,$$

which by reference to Equations (3.1) and (3.2) for small θ , α and β becomes

$$\theta \approx (\alpha \cos \phi + \beta \sin \phi). \quad (3.4)$$

So the pitch angle, θ , depends upon the angles of attack, α , sideslip, β , and roll, ϕ .

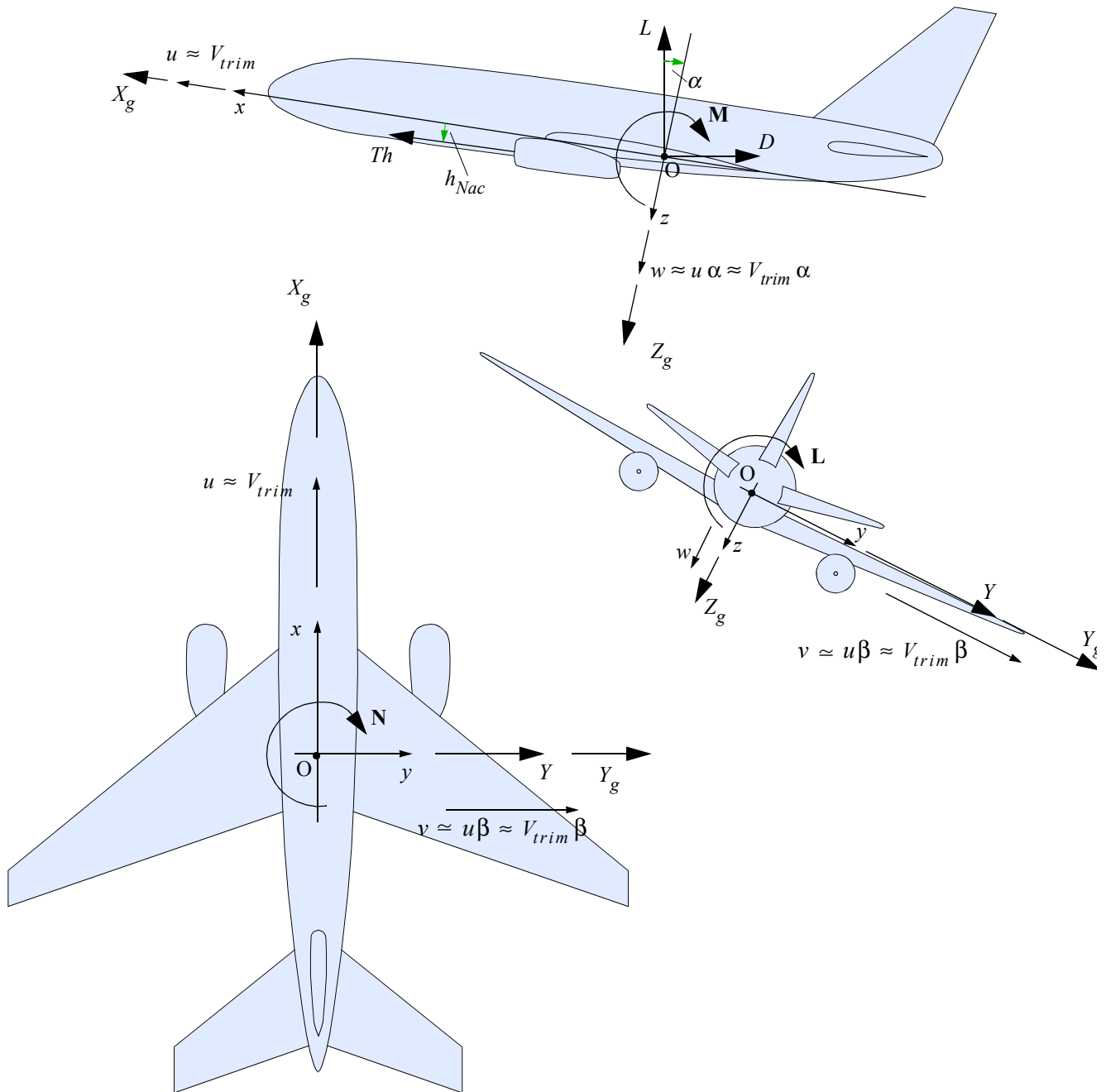
3.2 Overall Loads and Moments

The overall aerodynamic loads on the aeroplane consist of lift, L , and drag, D , that act in the xz plane, the sideforce, Y , that acts in the y -direction, together with the overall engine thrust, Th , which is taken to act in the x -direction.

The overall aerodynamic rolling, pitching and yawing moments about the x -, y - and z -axes are respectively denoted by $\mathbf{L}$, $\mathbf{M}$ and $\mathbf{N}$.

The resultant inertia forces in the x -, y - and z -directions that result from the linear and angular accelerations on the aeroplane are a combination of the weight and d'Alembert's reaction forces and are respectively denoted by X_g , Y_g and Z_g .

All of these forces and moments, apart from the thrust, act through or about the centre of mass and the positive direction and sense of these forces and moments are shown in Sketch 3.2.



Sketch 3.2 Overall aeroplane forces and moments

The overall symmetric (longitudinal) force equations, in the x - and z -directions, and moment equation, about the Oy axis, are respectively,

$$L \sin \alpha - D \cos \alpha + Th + X_g = 0, \quad (3.5)$$

$$-L \cos \alpha - D \sin \alpha + Z_g = 0, \quad (3.6)$$

$$\text{and} \quad M + Th h_{Nac} = 0, \quad (3.7)$$

in which h_{Nac} is the z -distance from the centre of mass defining the line of action of the thrust, Th .

The overall antisymmetric (lateral) force equation, in the y -direction, and moment equations, about the Ox and Oz axes, are respectively,

$$Y + Y_g = 0, \quad (3.8)$$

$$L = 0 \quad (3.9)$$

and $N = 0.$ (3.10)

It should be pointed out that even when the angle of attack, α , and angle of sideslip, β , are small the symmetric and antisymmetric equations are linked because the inertia terms, X_g , Y_g and Z_g , can depend on the bank angle, ϕ .

3.3 Symmetric Loads

This section describes how expressions for the lift on the wing-fuselage combination, L_{WF} , and the lift on the tailplane, L_T , together with the trim angle of attack, α , and elevator angle, η , are determined.

The overall aeroplane lift, L , and thrust, Th , from Equations (3.5) and (3.6), are

$$L = Z_g \cos \alpha - Th \sin \alpha - X_g \sin \alpha$$

and
$$Th = \frac{D}{\cos \alpha} - Z_g \tan \alpha - X_g,$$

which for small α , noting that Th/L and X_g/L are small compared with Z_g/L , give

$$L \approx Z_g \quad (3.11)$$

and
$$Th = D - Z_g \alpha - X_g \equiv D - (Z_g + (X_g)_\alpha) \alpha, \quad (3.12)$$

in which $(X_g)_\alpha$ is the rate of change of X_g with α , and is independent of α (as seen later, X_g is a linear function of α and Z_g). Equation (3.11) shows that the overall lift essentially depends only on Z_g , *i.e.* on the manoeuvre.

Assuming that the drag, D , is independent of any lateral motion, then the thrust can be written as

$$Th = D \left\{ \frac{1}{2} \rho V_{trim}^2, Z_g^2, C_{D0}, A \right\} - (Z_g + (X_g)_\alpha) \alpha, \quad (3.13)$$

where the use of curly brackets $\{...\}$ indicates 'a function of', C_{D0} is the drag coefficient at zero lift and A is the aspect ratio of the wing.

If, for example, the drag can be expressed as the sum of the drag coefficient at zero lift, C_{D0} , and a lift dependent drag, then

$$D = \frac{1}{2} \rho V_{trim}^2 S C_{D0} + \frac{k}{\pi A} \frac{L^2}{\frac{1}{2} \rho V_{trim}^2 S}$$

$$\approx \frac{1}{2} \rho V_{trim}^2 S C_{D0} + \frac{k}{\pi A} \frac{Z_g^2}{\frac{1}{2} \rho V_{trim}^2 S}.$$

Now the overall lift, L , can be expressed in terms of wing-fuselage and tailplane components, as

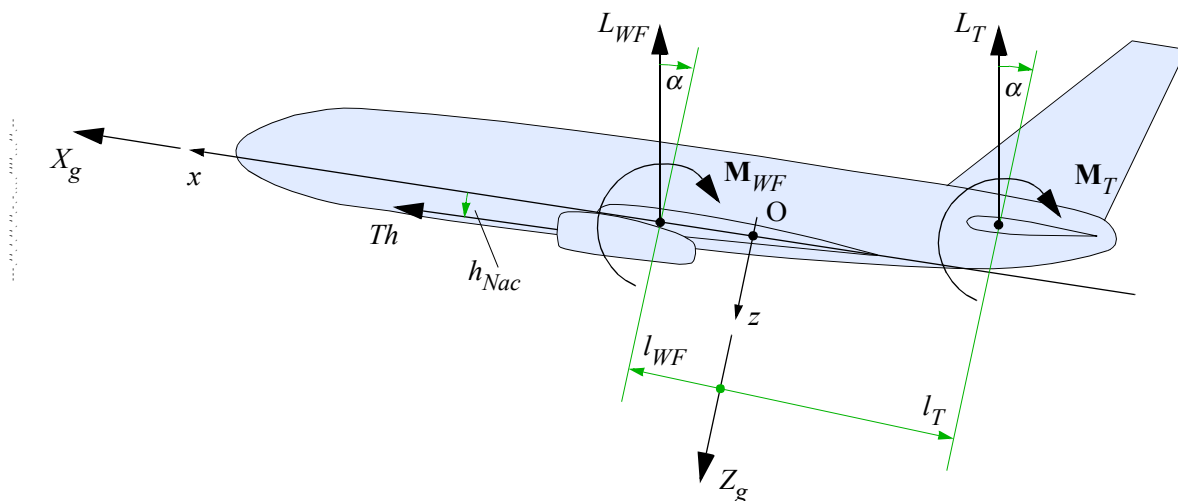
$$L = L_{WF} + L_T, \quad (3.14)$$

where L_{WF} is the lift on the wing-fuselage combination, *i.e.* the complete aeroplane configuration without the tailplane,

and L_T is the lift on the tailplane.

The wing-fuselage lift is taken to act at the aerodynamic centre of the wing-fuselage combination, taken to be at an x -distance of l_{WF} forward from the aeroplane mass centre (the origin of the aeroplane axis system). It introduces a pitching moment, M_{WF} , independent of aeroplane incidence, α , that acts about the aerodynamic centre in the xz plane.

Similarly, the tailplane lift is taken to act at the aerodynamic centre of the tailplane, taken to be at an x -distance of l_T aft of the aeroplane mass centre, that introduces a pitching moment, M_T , independent of tailplane incidence, α_T , that acts about the aerodynamic centre of the tailplane. These component forces and moment are illustrated in Sketch 3.3.



Sketch 3.3 Aeroplane symmetric loads and moments in horizontal flight

Combining Equations (3.11) and (3.14), the sum of the wing-fuselage and tailplane component lifts for small α , is given by

$$L_{WF} + L_T \approx Z_g. \quad (3.15)$$

Moment equilibrium in the xz plane is given by Equation (3.7), which in component terms becomes

$$\mathbf{M}_{WF} + L_{WF} l_{WF} - L_T l_T + Th h_{Nac} \equiv 0, \quad (3.16)$$

in which the pitching moment of the tailplane, $\mathbf{M}_T$, has been neglected as small compared with the other terms.

The pitching moment about the wing-fuselage combination, $\mathbf{M}_{WF}$, can be expressed in terms of the sum of the moment coefficient at zero pitch rate, $(C_{M0})_{WF}$, and the aero-normalised wing-fuselage moment coefficient due to pitch rate, $(M_q)_{WF}$, as

$$\begin{aligned} \mathbf{M}_{WF} &\equiv \frac{1}{2} \rho V_{trim}^2 S \bar{c} \left((C_{M0})_{WF} + (M_q)_{WF} \frac{q \bar{c}}{V_{trim}} \right) \\ &\approx \frac{W \bar{c}}{C_Q} \left((C_{M0})_{WF} + (M_q)_{WF} \frac{q \bar{c}}{V_{trim}} \right), \end{aligned} \quad (3.17)$$

where C_Q is the dynamic pressure coefficient defined by

$$C_Q = \frac{W}{\frac{1}{2} \rho V_{trim}^2 S}. \quad (3.18)$$

The pitch rate, q , is that resulting from any turn rate that the lateral manoeuvre may include.

Combining Equations (3.15), (3.16) and (3.17), expressions for the lift on the wing-fuselage combination and the lift on the tailplane can be found as

$$L_{WF} \approx \left(\frac{Z_g l_T - Th h_{Nac} - \frac{W \bar{c}}{C_Q} \left((C_{M0})_{WF} + (M_q)_{WF} \frac{q \bar{c}}{V_{trim}} \right)}{l_{WF} + l_T} \right) \quad (3.19)$$

and

$$L_T \approx \left(\frac{Z_g l_{WF} + Th h_{Nac} + \frac{W \bar{c}}{C_Q} \left((C_{M0})_{WF} + (M_q)_{WF} \frac{q \bar{c}}{V_{trim}} \right)}{l_{WF} + l_T} \right). \quad (3.20)$$

The terms Z_g and q in these last two equations depend, in general, upon the values of bank angle, ϕ , turn rate, Ω , and sideslip angle, β , and so depend upon the lateral manoeuvre being executed. Furthermore the thrust, Th , depends upon Z_g and the aeroplane angle of attack, α , in the steady manoeuvre.

However, when the factor $(Z_g + (X_g)_{\alpha})$ is zero in steady lateral manoeuvres, the thrust, Th , from Equation (3.13) is independent of α and only dependent upon Z_g and the flight condition, $\frac{1}{2} \rho V_{trim}^2$. Initial estimates for ϕ , Ω , and β then give an initial estimate for Z_g which in turn gives an estimate for Th and so allows the lift on the wing-fuselage combination, L_{WF} , and the lift on the tailplane, L_T , to be directly evaluated from Equations (3.19) and (3.20).

Expressions for L_{WF} and L_T are derived in Appendix A when the factor $(Z_g + (X_g)_\alpha)$ is not zero, *i.e.* when the thrust is directly dependent upon the aeroplane angle of attack.

The lift on the wing-fuselage can be separated into components as follows

$$L_{WF} = L_W + L_F, \quad (3.21)$$

where
$$L_F = (L_W)_F + L_{Nose}, \quad (3.22)$$

in which it has been assumed that the effect of underwing nacelles and pylons leaves unaltered the overall wing lift but changes the aerodynamic centre position of the wing-nacelle combination.

In Equations (3.21) and (3.22) the component terms are defined as follows:

- L_W the aerodynamic lift on both halves of the net wing, that includes fuselage interference effects and lift on the nacelles, L_{Nac} , is taken to act at the aerodynamic centre of the net wing, an x -distance of l_W forward from the aeroplane mass centre,
 - L_F the fuselage lift,
 - $(L_W)_F$ the carry-over lift due to the wing on the fuselage in the region of that part of the fuselage common to the fuselage and gross wing, is taken to act at an x -distance of $(l_W)_F$ forward from the aeroplane mass centre
- and L_{Nose} the lift on the fuselage nose, is taken to act at an x -distance of l_{Nose} forward of the aeroplane mass centre.

The wing lift, L_W , can be expressed as

$$L_W \approx \frac{W}{C_Q} (C_{L\alpha})_W (\alpha - (\alpha_0)_W), \quad (3.23)$$

from which the aeroplane incidence, α , is then found from

$$\alpha = \frac{C_Q}{W} \frac{L_W}{(C_{L\alpha})_W} + (\alpha_0)_W, \quad (3.24)$$

where $(C_{L\alpha})_W$ is the lift-curve slope of the wing and $(\alpha_0)_W$ is the angle of attack for zero wing lift, which is usually a negative quantity. Once the aeroplane incidence, α , and wing setting angle, $(\alpha_0)_W$, are known, the wing loading distribution, which is also dependent upon the wing camber distribution and the wing twist distribution, can be determined (see Section 7.3).

The average downwash at the tailplane due to the wing is evaluated from

$$\bar{\epsilon} = \left(\frac{d\bar{\epsilon}}{d\alpha} \right) (\alpha - (\alpha_0)_W) \quad (3.25)$$

and the tailplane lift, L_T , can then be expressed as

$$L_T = \frac{W}{C_Q} \frac{S_T}{S} \left((C_{L\alpha})_T (\alpha + (\alpha_0)_T - \bar{\epsilon}) + (C_{L\eta})_T \eta \right) \quad (3.26)$$

and, assuming α and $(\alpha_0)_W$ are known, Equation (3.26) can be rearranged to find the elevator angle, η , from

$$\eta = \left(\frac{\frac{C_Q}{W} \frac{S}{S_T} L_T - (C_{L\alpha})_T (\alpha + (\alpha_0)_T - \bar{\epsilon})}{(C_{L\eta})_T} \right), \quad (3.27)$$

where $(C_{L\alpha})_T$ and $(C_{L\eta})_T$ are the lift-curve slopes of the tailplane due to the incidence and elevator deflection and $(\alpha_0)_T$ is the tailplane setting angle relative to the zero lift line of the tailplane. The tailplane setting angle, $(\alpha_0)_T$, may be zero, positive or negative. Once the elevator angle, η , and the downwash, $\bar{\epsilon}$, are known, the loading distribution on the tailplane can be determined (see Section 7.3).

The symmetric loads on the wing, tailplane, wing carry-over fuselage section and fuselage nose can be expressed as follows:

$$\begin{aligned} &L_W, \mathbf{M}_W, (Z_g)_W, \\ &L_T, \mathbf{M}_T, (Z_g)_T, \\ &(L_W)_F, (\mathbf{M}_W)_F, ((Z_g)_W)_F, \\ &L_{Nose}, \mathbf{M}_{Nose}, (Z_g)_{Nose}, \end{aligned}$$

where the lifts on the various elements due to angle of attack and pitch rate act at their respective aerodynamic centres, the moments on the various elements are in the xz plane, act about their respective aerodynamic centres and are independent of both incidence, α , and pitch rate, q , and the inertial loads on the wing, tailplane and fuselage, that result from a combination of their weight and their d'Alembert reaction force, together act at their mass centres. The tailplane lift due to elevator angle acts at approximately the half-chord point of the tailplane.

It is noted that engine net thrust will contribute to wing torque. The total symmetric loading on a nacelle is the determined from

$$L_{Nac} \quad \text{and} \quad (Z_g)_{Nac}.$$

There are symmetric 'in-plane' loadings on the wing, tailplane and fin, resulting from their component drags, and engine thrust, Th , as opposed to 'transverse' loads such as lift and pitching moment.

There is also an 'in-plane' centripetal force distribution due to pitch rate, q , given by an expression of the form,

$$\sum m(x, y) g x q^2,$$

where g is the acceleration due to gravity and x is the distance of an incremental element of mass, $m(x, y)$, from the aeroplane mass centre. This force is predominantly on the tailplane but is negligible.