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Frictional pressure gradient in adiabatic flows of gas-liquid mixtures in horizontal pipes: prediction using empirical correlations and database

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1. NOTATION AND UNITS

The SI system of units should be used throughout all the calculations to ensure compatibility with the dimensional quantities in the text.

		SI
A	cross-sectional area of pipe	m^2
B	parameter used in Chisholm equations	—
C	confidence level, or parameter used in Lockhart and Martinelli correlation (Appendix B)	—
D	pipe inside diameter	m
E	parameter in Friedel correlation (Appendix D)	—
$\bar{E}$	mean correction factor within selected batch of N data points given by $\bar{E} = 10^{\bar{e}}$	—
ΔE	uncertainty factor within selected batch of N data points given by $\Delta E = 10^{\Delta e}$	—
e	logarithmic error ratio defined by $\log_{10}$ (measured value of friction component of pressure gradient/calculated value of friction component)	—
$\bar{e}$	mean value of e within batch of N points, given by $\bar{e} = \frac{1}{N} \sum_{1}^N e$	—
Δe	tolerance on $\bar{e}$ appropriate to specified confidence and probability given by $\Delta e = \sigma_{be} k_C$	—
F	parameter in Friedel correlation (Appendix D), or modified Froude number defined by Equation (J1.2)	—
f	Fanning friction factor	—
f_H	value of Fanning friction factor used in homogeneous theory	—
Fr	Froude number, in Friedel correlation (Appendix D) and Olujic correlation (Appendix H)	—
g	acceleration due to gravity, = 9.81 m/s^2	m/s^2
H	parameter in Friedel correlation (Appendix D)	—
h_L	liquid height in idealised stratified flow	m
K	factor in Equation (8.2), or dimensionless parameter defined by Equation (J1.3)	—
K_2	Moussalli parameter (Appendix H2)	—
k_C	factor on standard deviation giving tolerance Δe	—

L	length of experimental test section	m
$\dot{M}_G$	mass flow rate of gas or vapour component of mixture	kg/s
$\dot{M}_L$	mass flow rate of liquid component of mixture	kg/s
$\dot{M}$	mass flow rate of whole mixture	kg/s
$\dot{m}$	mass flux of whole mixture, $= 4(\dot{M}_G + \dot{M}_L)/\pi D^2$	kg/s m ²
N	number of data points within selected batch	–
n	Blasius exponent defined in Appendix G	–
P	pipe perimeter	m
P	probability	–
p	absolute static pressure	N/m ² (Pa)
dp/dz	pressure gradient	N/m ³ (Pa/m)
dp_f/dz	component of pressure gradient due to friction	N/m ³ (Pa/m)
dp_g/dz	component of pressure gradient due to gravity	N/m ³ (Pa/m)
dp_a/dz	component of pressure gradient due to accelerational change	N/m ³ (Pa/m)
Re_G	Reynolds number defined by $Re_G = x\dot{m}D/\eta_G$	–
Re_{GO}	Reynolds number defined by $Re_{GO} = \dot{m}D/\eta_G$	–
Re_H	Reynolds number defined by $Re_H = \dot{m}D/\eta_H$	–
Re_L	Reynolds number defined by $Re_L = (1-x)\dot{m}D/\eta_L$	–
Re_{LO}	Reynolds number defined by $Re_{LO} = \dot{m}D/\eta_L$	–
Re_{TP}	Reynolds number defined by $Re_{TP} = \dot{m}D/\eta_{TP}$	–
s	standard deviation of e within selected batch of N points, given by	–
$s = \sqrt{\frac{1}{N} \sum_0^N (\bar{e} - e)^2}$		
s	parameter in Beggs and Brill correlation (Appendix E)	–
T	dimensionless parameter defined by Equation (J1.4)	–
U	superficial velocity (volume flow per unit cross-sectional area of pipe)	m/s
u_{mix}	flow velocity in Beggs and Brill correlation (Appendix E)	m/s
u_C	factor proportional to deviation of sample percentile from population percentile corresponding to confidence level C	–
u_P	number of standard deviations, σ_{be} , between percentile and estimated mean of population	–
We	parameter in Friedel correlation (Appendix D)	–
x	mass quality of gas component defined by $x = \dot{M}_G/(\dot{M}_G + \dot{M}_L)$	–
X	Martinelli parameter defined by $\sqrt{(dp_f/dz)_L/(dp_f/dz)_G}$	–
y	parameter in Beggs and Brill correlation (Appendix E)	–
z	pipe axial length, positive in direction of flow	m
α	angle of inclination of pipe to horizontal	degree

β	gas volume flow fraction given by Equation (5.6)	–
Γ	coefficient used in Chenoweth and Martin and Chisholm methods, and defined by Equation (5.9)	–
ε	equivalent sand roughness height	m
ε_G	void fraction (fraction of cross-section of pipe occupied by gas phase)	–
ε_L	void fraction (fraction of cross-section of pipe occupied by liquid phase)	–
$\varepsilon_1, \varepsilon_2, \varepsilon'$	parameters in Olujic correlation (Appendix H)	–
η_G	viscosity of gas or vapour phase	N s/m ²
η_L	viscosity of liquid phase	N s/m ²
η_H	mean viscosity in homogeneous model	N s/m ²
Λ	fluid property index, $(\rho_G/\rho_L)/(\eta_G/\eta_L)^{0.2}$	–
ρ_G	density of gas or vapour phase	kg/m ³
ρ_H	density defined by Equation (5.1) for use with homogeneous correlations	kg/m ³
ρ_L	density of liquid phase	kg/m ³
ρ_{TP}	two-phase density defined by Equation (3.2)	kg/m ³
σ_{be}	unbiased estimate of standard deviation of e , defined by $\sigma_{be} = Ks$, see Section 8.2	–
σ	surface tension of two-phase mixture	N/m
τ_o	wall shear stress	N/m ²
ϕ^2	two-phase multiplier defined in Section 3	–

Suffixes

<i>be</i>	refers to best estimate value
<i>G</i>	refers to gas or vapour flowing alone at same mass flow rate as gas or vapour component of two-phase mixture
<i>GO</i>	refers to gas or vapour flowing alone at same mass flow rate as complete two-phase mixture
<i>L</i>	refers to liquid phase flowing alone at same mass flow rate as liquid component of two-phase mixture
<i>LO</i>	refers to liquid phase flowing alone at same mass flow rate as complete two-phase mixture

2. INTRODUCTION

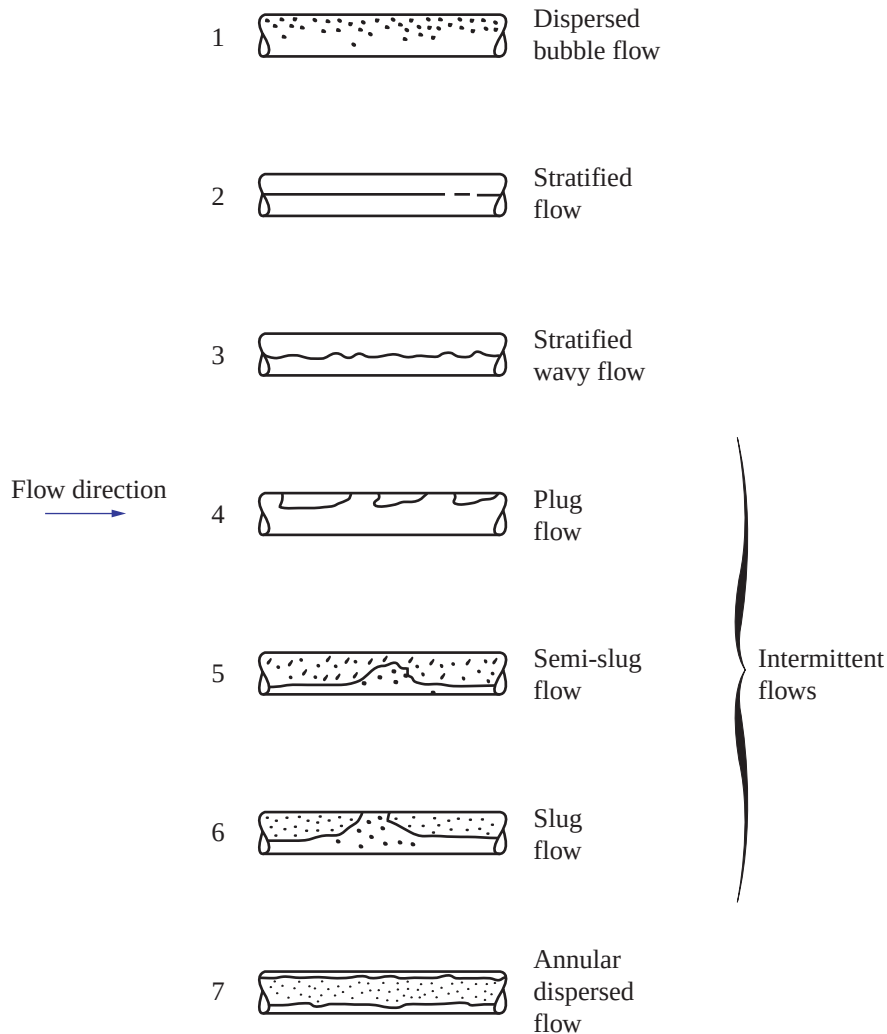
This Data Item is one of a group on the prediction of pressure gradient, and hence pressure drop, in two-phase gas-liquid mixture flows. It provides techniques for the assessment of the frictional pressure gradient in horizontal pipes and allows estimates to be made of the accuracy of the predictions.

Two-phase gas-liquid flow is of increasing importance in a wide range of applications including chemical and power plant, refrigeration systems, offshore petroleum pipelines, *etc.* Such flows are complex in nature and it is notoriously difficult to obtain accurate predictions of pressure drop. There are two main methods.

- (1) The use of more or less arbitrary empirical correlations, which usually attempt to relate the pressure drop in the two-phase situation to that which would occur in the single-phase flow of one of the phases flowing alone in the pipe. Development of empirical correlations was the main focus of work on two-phase flow in the 1950s to 1970s. Perhaps the best illustration of the problems of this approach to prediction is to compare the results obtained from the published correlations; these predictions are often widely different from each other, let alone from the available data. The correlations are generally based on limited data and systematic comparisons within the data were difficult before the widespread availability of computers. Nevertheless, these correlations are still widely used in the design of new systems, and in predicting the performance of existing systems.
- (2) In recent years, there has been increasing use of *phenomenological models* which recognise the differences between the respective *flow patterns*, where a flow pattern is defined as being a characteristic type of distribution of the interface between the gas and the liquid phases. Typical flow patterns for horizontal flow are shown in Sketch 2.1. These range from dispersed bubble flow, through stratified flow, intermittent flow (plug, semi-slug and slug) to annular dispersed flow. As will be obvious, frictional pressure losses may follow very different laws depending on the type of flow pattern encountered. Thus, the phenomenological approach attempts to identify the flow pattern and then to develop specific relationships for each pattern in the hope of obtaining more general results.

Though, from a scientific point of view, the phenomenological approach is clearly preferable, it should be recognised that there is still a large measure of empiricism in this approach. First of all, the means for assessing the flow pattern are not entirely unequivocal and, secondly, the phenomenological models for each flow pattern depend in themselves on empirical relationships for such quantities as wall friction, interfacial friction, bubble size, slug frequency, slug velocity, *etc.* It does not follow, therefore, that these models always offer significant increases in accuracy compared to the empirical models. Recognising that the empirical models are widely used in industry, it has been felt that, for the purposes of this present Data Item, an assessment of their accuracy with recommendations on the ranges of parameters over which they are the best suited serves a useful purpose.

Though the phenomenological approach has not been pursued in the work carried out for this Data Item, it is nevertheless quite likely that specific empirical correlations may be more suitable for a given flow pattern than for other flow patterns. Thus, where flow pattern data were available, comparisons between the empirical models and the pressure drop data were also classified in terms of flow pattern and the results included in the selection tables. These flow-pattern-based tables may therefore be used if the flow pattern is known in a particular application. If, as is usually the case, the flow pattern is not known, then it can be predicted from a model or correlation. As was mentioned above, such models and correlations lack generality; however, for reference, one of the most widely used models for horizontal flows (that of Taitel and Dukler³⁷, 1976) is given as Appendix J of this Data Item. This Appendix enumerates some of the uncertainties of this method.



Sketch 2.1 Flow patterns in horizontal flow

3. CALCULATION METHODS

Pressure gradient in steady-state two-phase gas-liquid flow is often represented in terms of the combined momentum equation (see for instance: Hewitt and Hall-Taylor⁶⁵, 1970, Hewitt⁶⁶, 1982, Lahey and Moody⁷⁴, 1997, Whalley⁸⁷, 1987 and Collier and Thome⁵⁴, 1994):

$$-\frac{dp}{dz} = \frac{\tau_o P}{A} + \frac{d}{dz} \left[\frac{(1-x)^2}{\rho_L (1-\epsilon_G)} + \frac{x^2}{\rho_G \epsilon_G} \right] \dot{m}^2 + g \rho_{TP} \sin \alpha \quad (3.1)$$

where dp/dz is the pressure gradient (Pa/m), τ_o the wall shear stress (N/m²), P the pipe perimeter (m), A the cross-sectional area of the pipe (m²), $\dot{m}$ the total mass flux of the fluid (kg/m² s), x the quality (fraction of the mass flow which is in the form of gas or vapour), ϵ_G the void fraction (fraction of the cross-section of the pipe occupied by the gas phase), ρ_L and ρ_G the liquid and gas densities (kg/m³), g the acceleration due to gravity (m/s²) and α the angle of inclination of the pipe from the horizontal

(degrees). The two-phase density, ρ_{TP} , is defined as:

$$\rho_{TP} = \rho_L(1 - \epsilon_G) + \rho_G \epsilon_G. \quad (3.2)$$

The three terms of the right hand side of Equation (3.1) are designated as being the *frictional*, *accelerational* and *gravitational* pressure gradients respectively, written as:

$$-\frac{dp}{dz} = -\left(\frac{dp_f}{dz} + \frac{dp_a}{dz} + \frac{dp_g}{dz}\right). \quad (3.3)$$

In a horizontal pipe (the case considered in this Data Item) the gravitational term (dp_g/dz) is zero since α is zero. The accelerational pressure gradient (dp_a/dz) is often very small; in the data considered here, the changes in quality, x , (due to flashing, say), gas density, ρ_G , (due to expansion under the pressure gradient) and void fraction, ϵ_G , are small. However the magnitude of this term should be checked, especially in cases where the change of pressure along the pipe is a significant portion of the absolute pressure (greater than 10%, say). Calculations on the database described in Section 4 indicated that the accelerational pressure gradient was typically less than 1% of the total pressure gradient for the adiabatic flow considered here. Of course, if significant changes in quality are induced as a result of evaporation or condensation, then significant accelerational pressure gradients can occur. However, in dealing with the database discussed here, this term has also been ignored, leaving only the frictional term (dp_f/dz) to be considered. For a round horizontal pipe it follows that:

$$-\frac{dp_f}{dz} = \frac{\tau_o P}{A} = \frac{\tau_o(\pi D)}{(\pi D^2/4)} = \frac{4\tau_o}{D}. \quad (3.4)$$

Generally, there are two commonly used empirical approaches for calculating gas-liquid phase pressure drops.

- (1) To treat the flow as a *homogeneous mixture* to which is assigned a characteristic density and viscosity. The pressure gradient is then calculated using standard single-phase relationships or, alternatively, by the assignment of special friction factors for the two-phase flow.
- (2) The use of *two-phase multipliers* which relate the pressure gradient in the two-phase flow to that for a hypothetical single-phase flow of either of the phases flowing alone in the pipe or for the total flow in the pipe of a fluid with the properties of the gas or liquid. The following relationships are commonly used in this context:

$$-\frac{dp_f}{dz} = -\phi_G^2 \left(\frac{dp_f}{dz}\right)_G = -\frac{\phi_G^2 2x^2 \dot{m}^2}{\rho_G D} f_G, \quad (3.5)$$

$$-\frac{dp_f}{dz} = -\phi_{GO}^2 \left(\frac{dp_f}{dz}\right)_{GO} = -\frac{\phi_{GO}^2 2\dot{m}^2}{\rho_G D} f_{GO}, \quad (3.6)$$

$$-\frac{dp_f}{dz} = -\phi_L^2 \left(\frac{dp_f}{dz}\right)_L = -\frac{\phi_L^2 2\dot{m}^2 (1-x)^2}{\rho_L D} f_L, \quad (3.7)$$

$$-\frac{dp_f}{dz} = -\phi_{LO}^2 \left(\frac{dp_f}{dz}\right)_{LO} = -\frac{\phi_{LO}^2 2\dot{m}^2}{\rho_L D} f_{LO}. \quad (3.8)$$

In Equations (3.5) to (3.8), the multipliers ϕ_G^2 and ϕ_L^2 multiply the respective pressure gradients for the gas and liquid phases flowing alone in the pipe. The multipliers ϕ_{GO}^2 and ϕ_{LO}^2 multiply the pressure gradients for a flow at the *total* flow rate having the gas and liquid properties respectively, and f_G , f_{GO} , f_L and f_{LO} are the respective friction factors. The single-phase equivalent friction factors can be calculated if the geometries and physical properties are specified; normally, friction factor correlations include the effect of pipe roughness. Most of the data considered here are for relatively smooth pipes. The effect of roughness in two-phase flow appears to be much less than in single-phase flow. It is recommended, therefore, that the roughness be taken as zero in carrying out two-phase flow calculations though this would obviously not be appropriate as the quality closely approaches zero or unity. The friction factor relationships used in the present work are stated in Appendix I.

The first stage of the work on which this Data Item is based was to construct a new data bank. This work is described in Section 4. Section 5 discusses the choice of correlations and Section 6 discusses the bases of the comparisons of the data bank with predictions using the selected correlations. Section 7 presents the results of the analysis of the data bank and Section 8 outlines how these results may be used. Section 9 gives some examples of the methodology. Appendices A to H describe how to use the respective methods.

4. DEVELOPMENT OF DATABASE

In a previous study by ESDU (ESDU Data Item 76018⁸⁹, 1976), comparisons of frictional pressure gradient were made with a database of 2210 measurements. In correlating frictional pressure gradient, no distinction was made in that Data Item between horizontal, vertically upwards and vertically downwards flows. The database consisted of 57% horizontal flow cases, 42% vertically upwards flow cases and 1% of vertically downwards flow cases. In view of the large variations in flow patterns between vertical and horizontal flows, the approach in the present Data Item was to develop a data bank only for horizontal flows. The data collected were obtained from the following key sources.

- (1) A database set up at the Harwell Laboratory of the United Kingdom Atomic Energy Authority (UKAEA) in the 1970s and used by the Heat Transfer and Fluid Flow Service (HTFS) at Harwell as a basis for their proprietary correlations for two-phase flow.
- (2) A database compiled at Stanford University* (Aziz and Petalas⁴⁶, 1994). This database is valuable in that it contains a wealth of industrial data (particularly for vertical flows), in addition to some of the older data incorporated in the earlier databases (for instance, that of Dukler⁵⁶, 1969).

It was decided to use the Harwell database as the prime source. However, many of the points are common to those in the Stanford database. Since both databases were available, this presented a unique opportunity to cross check between the two. In accumulating data, there is always a possibility of mis-typing of individual numbers *etc.* and when the two databases were compared for nominally the same data sources, minor discrepancies of this kind were identified. This amounted to only a few per cent of the whole but the simple procedure was adopted of eliminating all data points where there was this discrepancy, leaving only data which were confirmed between the two databases. For horizontal flows, this left 3127 data points. These were adopted as a principal component of the present database. All of the data sources used are tabulated in Table 13.1 and those which are common to the Harwell and Stanford databases are given the designatory letters AC and IC.

Additional data contained only in the Harwell database were incorporated. These additional data contained material on natural gas-oil, air-oil, nitrogen-mercury, air-glycol-water and steam-water flows which

* A copy of this data bank was obtained (with Stanford University's permission) through Prof. H. Müller-Steinhagen, formerly of the University of Surrey and now at the University of Stuttgart.

broadened the coverage considerably.

In addition to the above data points arising from the Harwell database (cross checked wherever possible with the Stanford database), ESDU has accumulated further sources of data as follows:

- (a) Data from Spedding and Van Nguyen (1975)³⁶ for air-water flow in a 45.5 mm tube.
- (b) Data from Ali (1992)² for air-water flow in a 203 mm diameter tube.
- (c) Data from Andrews (1966)³ for air-water flow in a 52 mm diameter tube.
- (d) Data from Aynsley (1968)⁴ for air-water flow in a 28.6 mm diameter pipe.
- (e) Data from Chen (1979)¹⁰ for air-water flow in a 45.5 mm diameter pipe.

These new data sources accounted for 1141 data points; the characteristics of these additional data are also listed in Table 13.1. In all, a total of 6453 data points were available in the new database, making it comparable in size (for the specific case of horizontal flows) with any previous database.

5. CHOICE OF CORRELATIONS

The correlations tested are as follows.

- A. **Homogenous Models.** In this class of models, the fluid is treated as a single-phase flow with suitably averaged properties. All the correlations use identical average density, ρ_H , given by:

$$\rho_H = \left[\frac{x}{\rho_G} + \left(\frac{1-x}{\rho_L} \right) \right]^{-1} \quad (5.1)$$

where x is the quality and ρ_G and ρ_L are the gas and liquid densities respectively. The four correlations considered here differed in their definition of mixture viscosity. Details of the homogeneous-type correlations are given in Appendix A. The correlations selected had the following characteristics.

- A1: **Dukler *et al.* (1964)¹⁶**. In this correlation, mean viscosity was calculated according to the expression:

$$\eta_H = \eta_G x \frac{\rho_H}{\rho_G} + \eta_L (1-x) \frac{\rho_H}{\rho_L}. \quad (5.2)$$

- A2: **Isbin (1958)²³** used a reciprocal mean viscosity defined as follows:

$$\eta_H = \left[\frac{x}{\eta_G} + \left(\frac{1-x}{\eta_L} \right) \right]^{-1}. \quad (5.3)$$

- A3: **Cicchitti *et al.*¹⁵ (1960)**. Here, a linear averaged viscosity was taken as follows:

$$\eta_H = x\eta_G + (1-x)\eta_L. \quad (5.4)$$

- A4: **Beattie and Whalley⁶ (1982)**. Here, a more complex form of relationship for viscosity was employed as follows:

$$\eta_H = \eta_L (1 - \beta)(1 + 2.5\beta) + \eta_G \beta, \quad (5.5)$$

where β is the gas volume flow fraction (the void fraction in homogeneous flow) given by:

$$\beta = \frac{\rho_L x}{\rho_L x + \rho_G (1-x)}. \quad (5.6)$$

- B. **Lockhart and Martinelli (1949)²⁸**. Though obtained in the first half of the twentieth century, this correlation is still very widely used; it relates the pressure drop multipliers ϕ_L and ϕ_G to the Martinelli parameter X defined as follows:

$$X = \sqrt{\left(\frac{dp}{dz} \right)_L / \left(\frac{dp}{dz} \right)_G} \quad (5.7)$$

where $(dp_f/dz)_L$ and $(dp_f/dz)_G$ are the single-phase frictional pressure gradients for the liquid and gas phase flowing alone in the pipe respectively. In the original paper, the relationships between the pressure drop multipliers and X were presented in graphical form. However, Chisholm⁵¹ (1967) provided convenient analytical expressions (see Appendix B).

- C. **Martinelli and Nelson** (1948)³⁰. This correlation was specifically for steam-water flows and corrected known discrepancies between the data for such flows (particularly at high pressure) and the predictions from the Lockhart-Martinelli²⁸ (1949) correlation. However, this correlation has been generalised in terms of the fluid property index Λ :

$$\Lambda = \frac{\rho_G/\rho_L}{(\eta_G/\eta_L)^{0.2}} \quad (5.8)$$

The original correlation was in graphical form.

- D. **Friedel** (1979)¹⁸. The widely used Friedel correlation is written in terms of the multiplier ϕ_{LO} (see Equation (3.8)), where ϕ_{LO} is related to a number of dimensionless parameters in a complex correlation involving seven fitted constants. Details are given in Appendix D.
- E. **Beggs and Brill** (1973)⁸. This correlation is used extensively in the oil industry and is, strictly speaking, a homogenous type of correlation. However, instead of relying on single-phase friction factor relationships involving Reynolds numbers with redefined densities and viscosities (as with the homogeneous models characterised under correlation A above), a specific correlation for friction factor is derived. This is in terms of phase holdup and so a correlation for phase holdup is also given as part of the package. Details are given in Appendix E.
- F. **Chenoweth and Martin** (1955)¹¹. This correlation is given in terms of ϕ_{LO} as a function of $(1-\beta)$ with a coefficient Γ defined as follows:

$$\Gamma^2 = \left(\frac{dp_f}{dz} \right)_{GO} / \left(\frac{dp_f}{dz} \right)_{LO} \quad (5.9)$$

$$(5.10)$$

Details are given in Appendix F.

- G. **Chisholm** (1973)^{13, 14}. This correlation relates ϕ_{LO} to Γ and quality by the expression:

$$\phi_{LO}^2 = 1 + (\Gamma^2 - 1) \{ Bx^{(2-n)/2} (1-x)^{(2-n)/2} + x^{2-n} \} \quad (5.11)$$

where n is the exponent on the Blasius-type equation for friction factor and B is a parameter which is correlated in terms of Γ and mass flux, $\dot{m}$. Details are given in Appendix G.

- H. **Olujic** (1985)³⁴. This correlation (described in Appendix H) attempts to take some account of the nature of the flow in the pipe, characterised by the flow pattern (see Sketch 2.1). Essentially two correlations are used, one applying to low quality flow (bubbly flow and plug flow) and the other to flows in which the phases are interacting more strongly (wavy, slug and annular-dispersed flow). The correlation for the low quality regime is based on that of Mousalli and Chawla⁸⁰ which is itself based on the variable-density model of Bankoff⁴⁹ (1959). A new correlation is developed for the higher quality regimes, based on complex empirical relationships involving physical properties and flow quality. Full details are given in Appendix H.

6. BASES OF PERFORMANCE COMPARISONS AGAINST DATABASE

There are considerable errors in estimating pressure drop from all empirical correlations. Here, the objective is to provide the designer with a realistic tool for estimating the errors involved in each case. The following method is used.

For each point in the database, a quantity e is defined as follows:

$$e = \log_{10} \left[\left(\frac{dp}{dz} \right)_{f, exp} / \left(\frac{dp}{dz} \right)_{f, calc} \right] \quad (6.1)$$

where $(dp / dz)_{f, exp}$ is the experimental value of the pressure gradient, and $(dp / dz)_{f, calc}$ is the value calculated from the particular correlation. A mean value $\bar{e}$ for the logarithmic error ratio is defined as follows:

$$\bar{e} = \frac{1}{N} \sum_{1}^N e \quad (6.2)$$

where N is the number of data points in the batch being considered. A mean correction factor, $\bar{E}$, for the batch of N data points to which Equation (6.2) has been applied is given by:

$$\bar{E} = 10^{\bar{e}}. \quad (6.3)$$

A standard deviation of e within the selected batch of N data points is defined as follows:

$$s = \sqrt{\frac{1}{N} \sum_{0}^N (\bar{e} - e)^2}. \quad (6.4)$$

For a perfect fit, $\bar{E} = 1$ and $s = 0$. In what follows, the data are categorised in terms of N , $\bar{E}$ and s . The use of these parameters in estimating the uncertainty of a prediction of pressure drop from any of the correlations is described in detail in Section 8.

7. RESULTS OF PERFORMANCE COMPARISONS AGAINST DATABASE

In the analysis of the performance of the correlations, the objective was to prepare Performance Charts in which values of N , $\bar{E}$ and s would be determined for specific ranges of parameters. These Performance Charts are described in Section 7.1 and presented in Section 12. In Section 7.2, a selection of graphical representations of the results is presented. Section 7.3 describes the Method Selection Charts which are presented in Section 11, again based on ranges of parameters. Finally, in Section 7.4, trends are discussed in the performance of the best correlations with mass flux and quality.

7.1 Performance Charts

Two categories of Performance Charts (stating values of N , $\bar{E}$ and s) are presented.

- (i) Performance of the correlations classified according to the ranges of quality (x , the fraction by mass of gas or vapour in the flow) and mass flux ($\dot{m}$, in $\text{kg/m}^2 \text{ s}$).
- (ii) The performance of each correlation is classified according to the flow regime (annular, dispersed bubble, intermittent, stratified or stratified-wavy). Only data where the flow pattern was stated in the database were used in preparing these Performance Charts. Of the 6453 data points in the database, only for 3799 was the flow pattern specified.

The charts are designated according to the above categories and according to the designatory letter of the correlation as given in Section 5. Thus, for example, Performance Chart II-D refers to comparisons with the Friedel correlation (described in Appendix D) on the basis of flow pattern and quality. A total of 22 charts are presented. The following points should be noted.

- (1) Where there is a stated limitation in the range of the correlation, data points may exist within the data bank within particular ranges of parameter that are outside those limitations. Within Performance Charts, such points are indicated by the letter "R" and, of course, the correlation should not be used within those ranges. Limitations in the range can also lead to a gross reduction in the number of data points captured for particular ranges of conditions and, again, great care should be taken in application of the correlation in these cases. Such cases are easily identified by examination of Performance Charts and comparing the numbers of points for the various correlations.
- (2) In some cases, under some headings, there are no points within the range. For example, there may be no points in the particular flow regime for particular range of quality (see Performance Chart II). Such points are designated as "W" (*i.e.* Without) in the respective charts.
- (3) In some cases, there is only one data point in the range for the given correlation. In these cases, the standard deviation cannot, of course, be determined and the values of s in these cases are designated as "I" (*i.e.* Impossible to determine).

The use of the data for N , $\bar{E}$ and s is discussed in Section 8. The actual performance data are given in Section 12.

7.2 Graphical Representation of Results

In assessing the comparisons, very extensive graphical representations of the results were prepared. Only two of these are given for illustration here. The graphs were of two types.

- (1) Plots against specific quantities of the ratio of measured values to those calculated, for instance mass flux and quality.
- (2) Plots of the distribution of the number of points lying within certain ranges of error.

Examples of these two types of plot are shown in Figure 1 and 2. Figure 1 shows comparisons between the “best” correlation and the data in terms of quality, x , and mass flux, $\dot{m}$. Here “best” refers to the correlation performing best in a particular range of mass fluxes and qualities as given in the method selection charts (see Section 7.3). It should be remembered that the performance of any selected correlation will differ considerably over the ranges of quality and mass flux.

Figure 2 shows a comparison of the second type, in this case the distribution of error between predicted and measured values for the “best” correlation. In looking at Figure 2, it should be noted that the ranges change between 90-100% to 100-200%, hence the apparent peaks in the outer zones. This has been done to produce a compact presentation.

The first point to note in considering the graphical presentations is the very large differences which exist between predictions and experiments, even for the best correlations. Figure 1 is plotted in logarithmic form and errors up to around a factor of 100 may occur, though the preponderance of data is predicted with much less error. To some extent, the errors represent experimental errors; two-phase pressure gradient is quite difficult to measure accurately and there are number of experimental difficulties (in particular keeping the lines to the pressure tapings full of only one phase). However, experience would indicate that these errors would be of the order of 5-25% provided due care was taken. Thus, the gross errors seen in the plots are more likely to represent a failure of the prediction methods.

The designer of equipment for multiphase flows has, in reality, to live with large uncertainties in predictions and should take account of these in design.

7.3 Method Selection Charts

Method Selection Charts I and II are presented in Section 11 for the data classified in terms of quality and mass flux, or quality and flow pattern respectively. In these charts, for a particular range of quality and for a range of mass flux or for a designated flow regime, the methods are arranged in order of decreasing performance in terms of standard deviation, s . Thus, for a particular range of parameters, the best available method can be selected. The order of performance of the methods varies greatly from case to case and range to range. This reflects the large variability of the predictions referred to above. Judgement should also be exercised in selecting the best method for a particular range in relation to the number of points which were available within that range (in other words, the Performance Charts should also be examined in finalising a selection).

To use Method Selection Chart I (Table 11.1), it is necessary to know the mass flux and quality. Since these are the prime independent variables, this would, of course, present no difficulty. However, to use Method Selection Chart II, the *flow pattern* must be known and this, in itself, is a *dependent* variable. There are no totally reliable methods of predicting flow pattern. Many empirical correlations (“flow pattern maps”) have been developed based on experimental data. A typical earlier correlation is that of Baker⁵ (1954) and a more recent empirical treatment of flow patterns in horizontal flows is that of Mandhane *et al.*⁷⁶ (1974). However, a *semi-theoretical* treatment that is in rather wide use is that of Taitel and Dukler³⁷ (1976). For reference purposes, this method is presented in Appendix J. It should be stressed strongly, however, that even this semi-theoretical approach is subject to large uncertainties.

7.4 Trends With Mass Flux and Quality

As will be seen from Method Selection Chart I (Table 11.1), the choice of “best” method varies in a somewhat random way with the ranges of quality and mass flux. The same applies when the data are classified in terms of flow regime (Method Selection Chart II, Table 11.2). Some efforts have, therefore, been made to see if a more consistent pattern of recommended correlations could be developed in terms of mass flux and quality. Though it is difficult to make very general statements on the basis of results shown in this chart, the following points may be made.

- (1) For low mass flux and high quality, the Lockhart and Martinelli correlation (B) shows consistently good performance.
- (2) The homogeneous models perform well, particularly at lower qualities.
- (3) The more complex separated flow models (such as that of Chisholm) begin to figure significantly in the chart at high mass fluxes and high qualities.

These are rather tentative general observations but may form some basis for setting up a generic design methodology.

8. METHODOLOGY FOR APPLICATION OF RESULTS

This section gives procedures for aligning the predicted values of dp_f/dz with the experimental data and for calculating a margin for error (assuming a normal distribution of error) about this best estimate value in accordance with any value of confidence level and probability that the user may nominate. Details of the statistical methods used here are given in ESDU FMI2⁹¹ (1997).

8.1 Best Estimate Pressure Gradient

In general, the best estimate value of pressure gradient $(dp_f/dz)_{be}$, is obtained from the product of the value of dp_f/dz calculated from the method in use and the mean correction factor, $\bar{E}$, obtained from the appropriate location in the Performance Chart corresponding to that method. However, a lower limit of $|(dp_f/dz)_{LO}|$ is set on $|(dp_f/dz)_{be}|$.

Thus,

$$\left(\frac{dp_f}{dz}\right)_{be} = \bar{E} \left(\frac{dp_f}{dz}\right) \quad (8.1)$$

or, if $(dp_f/dz)_{LO} > (dp_f/dz)$,

$$\left(\frac{dp_f}{dz}\right)_{be} = \bar{E} \left(\frac{dp_f}{dz}\right)_{LO}.$$

The reciprocal argument that $|(dp_f/dz)_{GO}|$ should be the upper limit of $|(dp_f/dz)_{be}|$ is not valid because, at high qualities, the magnitude of pressure gradient can exceed $|(dp_f/dz)_{GO}|$. Nevertheless, the user should exercise discretion over the validity of Equation (8.1) if $|(dp_f/dz)_{be}|$ greatly exceeds $|(dp_f/dz)_{GO}|$.

8.2 Uncertainty Factor

The uncertainty factor, ΔE , determines the upper and lower limits of pressure gradient between which the actual pressure gradient can be expected to lie if the confidence level C and probability P are specified. Thus the actual pressure gradient can be expected to lie in the range

$$\left(\frac{dp_f}{dz}\right)_{be} \Delta E \quad \text{and} \quad \left(\frac{dp_f}{dz}\right)_{be} / \Delta E.$$

The procedure for estimating ΔE is as follows.

- (1) Obtain K from Figure 3 from the specified value of N .
- (2) Calculate the best estimate standard deviation σ_{be} from

$$\sigma_{be} = Ks \quad (8.2)$$

where s , the standard deviation for the batch of data, is obtained from the Performance Charts.

- (3) Nominate a confidence C and a probability P and obtain u_C and u_P respectively from Figure 4. Values of $C = 90\%$ (giving $u_C = 1.25$) and $P = 99\%$ (giving $u_P = 2.3$) are suggested.
- (4) Evaluate k_C from

$$k_C = \frac{u_P + u_C \left\{ \frac{1}{N} \left[1 - \frac{u_C^2}{2(N-1)} \right] + \frac{u_P^2}{2(N-1)} \right\}^{0.5}}{1 - \frac{u_C^2}{2(N-1)}} \quad (8.3)$$

where N , the number of measured results, is obtained from the Performance Charts.

- (5) Calculate the tolerance, Δe , from

$$\Delta e = \sigma_{be} k_C. \quad (8.4)$$

- (6) Evaluate the uncertainty factor, ΔE , from

$$\Delta E = 10^{\Delta e}. \quad (8.5)$$

Alternatively, for 90% confidence and a probability of 99% ($C = 90\%$ and $P = 99\%$), ΔE may be obtained for values of s and N from Figure 5.

9. EXAMPLES

Two examples are given. The first, for steam and water in saturated equilibrium, illustrates the application of the Chisholm and Friedel correlations. The second, for a gas oil mixture, illustrates the application of the Dukler *et al.* correlation. In both cases, Method Selection Chart I was used.

9.1 Steam/Water Example

Saturated steam of 75 per cent mass quality flows adiabatically at a mass flow rate, $\dot{M} = 0.2 \text{ kg/s}$ in a horizontally drawn steel tube of internal diameter $D = 0.01 \text{ m}$. The pressure is 100 bar. Find the pressure gradient in the tube given the following information.

$$\begin{aligned}\rho_L &= 688.7 \text{ kg/m}^3, \\ \rho_G &= 55.5 \text{ kg/m}^3, \\ \eta_L &= 86.9 \times 10^{-6} \text{ N s/m}^2, \\ \eta_G &= 20.36 \times 10^{-6} \text{ N s/m}^2 \text{ and} \\ \sigma &= 12 \times 10^{-3} \text{ N/m}.\end{aligned}$$

The properties are appropriate to steam and water in saturated equilibrium at 100 bar.

Calculation Instructions

1. From the specified data,

$$x = 0.75$$

$$\text{and} \quad \dot{m} = \frac{4\dot{M}}{\pi D^2} = \frac{4 \times 0.2}{\pi (0.01)^2} = 2546 \text{ kg/s m}^2.$$

2. Method Selection Chart I (based on mass flux and quality) is used. The flow pattern is not specified so the Method Selection Chart II is not applicable. Chart 1 (Table 11.1) suggests that, on the basis of the value of s , the Lockhart and Martinelli (1949) method of Appendix B is the best. However, some element of judgement is required. Examining Selection Charts I it is seen that the methods in Appendix D (Friedel) and Appendix G (Chisholm) are the next best. It is useful to examine the values of N , $\bar{E}$ and s (from the performance charts) for the various correlations for the range selected.

Method	Values from Performance Charts for $x = 0.75$ and $1000 < \dot{m} < 3200 \text{ kg s/m}^2$		
	N	$\bar{E}$	s
B (Lockhart and Martinelli)	14	0.586	0.106
D (Friedel)	14	0.827	0.151
G (Chisholm)	14	0.95	0.175

Examining these figures, it is seen that, though it gives the smallest values of s , the Lockhart and Martinelli correlation has a value of $\bar{E}$ which is far from unity. The judgement between the other two correlations is a fine one but, on average, the Chisholm correlation has values of $\bar{E}$ nearer to unity. However, both correlations were used for illustration.

3. Calculate pressure gradient using the Chisholm correlation (see Appendix G)

$$\begin{aligned}
 (1) \quad Re_L &= \frac{(1 - 0.75) \times 2546 \times 0.01}{86.9 \times 10^{-6}} = 7.32 \times 10^4, \\
 Re_G &= \frac{0.75 \times 2546 \times 0.01}{20.36 \times 10^{-6}} = 9.38 \times 10^5, \\
 Re_{LO} &= \frac{2546 \times 0.01}{86.9 \times 10^{-6}} = 2.93 \times 10^5 \text{ and} \\
 Re_{GO} &= \frac{2546 \times 0.01}{20.36 \times 10^{-6}} = 1.25 \times 10^6.
 \end{aligned}$$

Neither Re_L nor Re_G violates conditions for applicability (that is, the conditions $Re_L > 1000$ and $Re_G > 1000$ are met).

$$\begin{aligned}
 (2) \quad f_{LO} &= 0.046 \times (2.93 \times 10^5)^{-0.2} = 3.71 \times 10^{-3} \text{ and} \\
 f_{GO} &= 0.046 \times (1.25 \times 10^6)^{-0.2} = 2.78 \times 10^{-3}.
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad \left(\frac{dp_f}{dz} \right)_{LO} &= \frac{-2 \times 2546^2 \times 3.71 \times 10^{-3}}{688.7 \times 0.01} = -6984 \text{ Pa/m and} \\
 \left(\frac{dp_f}{dz} \right)_{GO} &= \frac{-2 \times 2546^2 \times 2.78 \times 10^{-3}}{55.5 \times 0.01} = -64938 \text{ Pa/m.}
 \end{aligned}$$

$$(4) \quad \Gamma = \sqrt{-64938 / -6984} = 3.05.$$

(5) According to Table G2.1 (in Appendix G),

$$B = 55/\dot{m}^{0.5} = 55/2546^{0.5} = 1.09.$$

$$(6) \quad n = \frac{\log_{10}\left(\frac{3.05^2 \times 55.5}{688.7}\right)}{\log_{10}\left(\frac{20.36 \times 10^{-6}}{86.9 \times 10^{-6}}\right)} = 0.1986.$$

$$(7) \quad \phi_{LO}^2 = 1 + (\Gamma^2 - 1) \{ Bx^{(2-n)/2} (1-x)^{(2-n)/2} + x^{(2-n)} \} \\ = 7.9484.$$

$$(8) \quad \frac{dp_f}{dz} = 7.9484 \times (-6984) = -5.55 \times 10^4 \text{ Pa/m.}$$

(9) Based on Performance Chart I:

$$\left(\frac{dp_f}{dz}\right)_{be} = 0.95 \times (-5.55 \times 10^4) = -5.27 \times 10^4 \text{ Pa/m.}$$

(10) Using Performance Chart I, and with $C = 90\%$ and $P = 99\%$, the uncertainty factor may be obtained either from Equations (8.2) - (8.5) above, or directly from Figure 5, which gives $\Delta E = 4.1$.

(11) The observed frictional component of pressure gradient will lie

$$\text{between} \quad -5.27 \times 10^4 \times 4.1 = -2.16 \times 10^5 \text{ Pa/m}$$

$$\text{and} \quad -5.27 \times 10^4 / 4.1 = -1.28 \times 10^4 \text{ Pa/m}$$

with a probability of 99 per cent for 90 per cent confidence.

4. Calculate pressure gradient using the Friedel correlation (see Appendix D)

$$(1) \quad Re_{LO} = \frac{2546 \times 0.01}{86.9 \times 10^{-6}} = 2.93 \times 10^5 \text{ and} \\ Re_{GO} = \frac{2546 \times 0.01}{20.36 \times 10^{-6}} = 1.25 \times 10^6.$$

$$(2) \quad f_{LO} = 0.046 \times (2.93 \times 10^5)^{-0.2} = 3.71 \times 10^{-3} \text{ and} \\ f_{GO} = 0.046 \times (1.25 \times 10^6)^{-0.2} = 2.78 \times 10^{-3}.$$