

Transonic data memorandum

Example of the application of constrained multivariate optimisation techniques to the design of aerofoil sections

Design point	: Dual
Geometry variation	: Upper and lower surfaces, leading edge shape, and leading and trailing edge flap deflections
Initial aerofoil	: RAE 2822
CFD	: Euler code and BVGK

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Transonic Aerodynamics Committee, which first met in 1960 and now has the following membership:

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EXAMPLE OF THE APPLICATION OF CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES TO THE DESIGN OF AEROFOIL SECTIONS

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1. INTRODUCTION

This Data Item presents an example which illustrates the use of mixed CFD options (BVGK for transonic flow conditions and an Euler code for supersonic) in a dual design point study using the constrained multivariate optimisation technique CODAS for the design of aerofoil sections.

To demonstrate the capability of the method fully, two disparate design points have been deliberately selected, one transonic ($M = 0.71$, $C_L = 0.75$, $R = 20 \times 10^6$) calculated by viscous BVGK (Reference 3) and the other supersonic ($M = 1.40$, $C_L = 0.05$) calculated by an inviscid Euler code (Reference 2). The basic aerofoil is RAE 2822. The optimisation objective is to minimise a linear combination of the C_D values* at the two design points. Reduction of the objective function is to be sought by changes in the upper and lower surfaces, the leading edge shape, and by deflection of simple leading and trailing edge flaps hinged at 15% and 75% chord, respectively.

The Data Item is one of a series intended to illustrate the process of using such optimisation techniques, to indicate their advantages and to reveal the kinds of problems that can arise. To facilitate this, the Data Item is in narrative form.

An introduction to the application of constrained multivariate optimisation techniques to the design of aerofoil section shapes is given in Data Item No. 99019 (Reference 5). Data Item No. 99020 (Reference 6) gives examples of single design point optimisation in which there are no problems from the CFD method. Data Item No. 00022 (Reference 7) shows the effect of problems within the CFD method upon the reliability of the multivariate optimisation technique. Data Item No. 00023 (Reference 8) compares the use of the (inviscid) Euler code option with BVGK (in its inviscid mode).

* That is, the objective function = $(W_1.C_{D1} + W_2.C_{D2})$, where $W_1 + W_2 = 1$.

2. NOTATION

2.1 Algebraic

C_D	drag coefficient	
$C_{D_1}, C_{D_2}, \dots$	drag coefficients at design points 1, 2, ...	
C_L	lift coefficient	
C_m	pitching moment coefficient (about quarter-chord point, positive nose-up)	
C_p	pressure coefficient	
c	aerofoil chord	
$\bar{H}_{0.99}$	transformed boundary layer shape parameter at 99% chord on upper surface	
$LRDSC$	parameter determining magnitude of Leading Edge Analytical Function	
M	free stream Mach number	
M_{local}	local Mach number (at edge of boundary layer where appropriate)	
R	Reynolds Number, based on chord and freestream conditions	
t	maximum thickness of aerofoil, <i>i.e.</i> , maximum value of $z_U(x) - z_L(x)$	
$W_1, W_2, \dots$	weightings applied in objective function to values of $C_{D_1}, C_{D_2}, \dots$ at design points 1, 2, ...	
x	chordwise coordinate	
z	normal coordinate	
$z_L(x)$	normal coordinate of lower surface of aerofoil at x	
$z_U(x)$	normal coordinate of upper surface of aerofoil at x	
α	angle of incidence of reference line (<i>i.e.</i> for aerofoil section with leading and trailing edge flaps undeflected)	degree
δz	normal coordinate of an incremental surface	
ρ_0	leading edge radius	

2.2 Computational

NFE	number of calls by RQPMIN to CFD routine
NFEMAX	maximum number of calls by RQPMIN to CFD routine (set in input data)
RES	residual in inviscid-flow element, maximum value in BVGK, average value in Euler code
VRES	maximum residual in viscous-flow element

2.3 Definitions

CFD	“Computational Fluid Dynamics”, programs used to calculate the flow around aerofoil sections. In the example in this Data Item the chosen CFD methods are an Euler code (Reference 2) and BVGK (Reference 3). The generic term “CFD” is used except where the behaviour under discussion is specific to BVGK or the Euler code.
CODAS	“Constrained Optimisation Design of Aerodynamic Surfaces”, a design method for aerofoils and wings that couples CFD and numerical optimisation methods (see Reference 5).
Optimiser	Software capable of solving constrained multivariate optimisation problems.
RQPMIN	The chosen optimiser in CODAS (see Reference 4).

3. EXAMPLE

3.1 Definition of the Problem

3.1.1 Geometric parameters

In this example, the geometric parameters to be varied in the search for the configuration that will give the minimum value for the objective function are

- the upper and lower surfaces,
- the leading edge shape, and
- deflection of simple leading and trailing edge flaps.

Increments to the upper and lower surfaces, δz , are defined by second derivatives of δz with respect to x at eight chordwise positions (in this case they are selected as $x/c = 0, 0.06, 0.2, 0.4, 0.6, 0.8, 0.94$, and 1.0). The values of these second derivatives at the ends (*i.e.* $x/c = 0$ and 1.0) are set at zero (thereby ensuring no change of curvature of the surfaces at the ends, in particular the leading edge radius, results from these parameters). The increments to the surfaces are then defined by just six variables for the upper surface changes and six for the lower surface. It should be noted that the same effects could have been achieved by increments to the thickness distribution and the camber surface. However, it was thought that direct changes to the separate surfaces might be the more efficient process.

In CODAS, an incremental leading edge shape variable (referred to as the “Leading Edge Analytical Function”) is available in the form shown in Figure 1, the magnitude being controlled by a single parameter, $LRDSC$. In the present case this is applied only to the thickness distribution. It could also be applied to a single surface, or both surfaces, and also to the camber line. This parameter does affect the leading edge radius.

The flap deflections may, of course, be different at the two design points, resulting in a further four variables.

Finally, there are two values for the incidence, one for each of the design points (the option to let BVGK calculate at a value of C_L is not used here). Initial values for incidence (giving the required values for C_L) were determined from preliminary CODAS runs in which the only variable was incidence.

The problem therefore has a total of 19 variables. In the process of determining the direction of the next

step in the optimisation search, the optimiser has to calculate gradients of the objective function and constraint functions with respect to each of these variables. Each gradient requires one evaluation (two if the optimiser has switched to the use of central difference calculation in place of forward differences) of these functions. Inevitably, the greater the number of variables the greater the run time involved.

3.1.2 Geometric constraints

It is essential to restrict incremental changes to the surfaces so that the surfaces do not cross over, *i.e.* the upper surface does not become lower than the lower surface at some points. A constraint is available to ensure this, setting a minimum value for $z_U/c - z_L/c$ for values* of x/c from 0.001 to 0.999. However, this constraint is not always sufficient. Figure 2 shows the result of an optimisation search at $M = 1.4$, $C_L = 0.05$ with the minimum value of $z_U/c - z_L/c$ constrained to be greater than 0.0001. The convergence history shows an excellent reduction in C_D but, despite the constraint on $z_U/c - z_L/c$, the surfaces did cross over, presumably because the value of the constraint function was within the tolerance. It is necessary to be aware of this possibility and to be prepared to re-run the case with additional geometric constraints to avoid cross over, if necessary. In the example shown, specific $z_U/c - z_L/c$ constraints near $x/c = 0.15$ might be effective.

There have been problems in applying BVGK to thin sections with small leading edge radii† (see Reference 7) and preliminary runs for the present investigation suggested that these problems were likely to be more severe here. Consequently, minimum constraints were set for t/c and ρ_0/c of 0.05 and 0.0012, respectively.

3.1.3 Aerodynamic constraints

Four aerodynamic constraints were set in this problem, as follows.

- The two values of C_L , 0.75 at the first design point and 0.05 at the second.
- A maximum value of 1.25 for the Mach number upstream of the upper surface shock wave at the first (transonic) design point.
- A maximum value of 2.3 for $\overline{H}_{0.99}$ at the first (transonic) design point.

3.1.4 Objective function

Initial optimisation runs were made for an objective function consisting of a simple average of the values of C_D calculated at the two design points, *i.e.* $W_1 = W_2 = 0.5$. It must, however, be recalled that the CFD method used at the first (transonic) design point was BVGK (viscous), whereas that used at the second (supersonic) design point was an inviscid Euler code. Combination of these two CFD methods yields meaningful results provided that viscous effects on C_D in the supersonic case vary only slightly over the range of geometric variables involved – in which case the absence of the viscous contribution to the drag simply reduces the objective function by a nearly constant amount. This is true whatever weighting is being used.

Further runs were made to investigate the effect of varying the weightings applied to the C_D values, from ($W_1 = 0$, $W_2 = 1$) to ($W_1 = 1$, $W_2 = 0$).

* The actual values are 0.001 (0.001) 0.01 (0.01) 0.1 (0.1) 0.9 (0.01) 0.99 (0.001) 0.999; figures in brackets show the increments in the intervals.

† These problems only arise because CODAS has to use BVGK with a common case definition throughout the optimisation search. When making direct use of BVGK, techniques exist which allow difficult flow conditions to be calculated. These are not available within CODAS since they rely upon individual corrections specific to each separate case.