

Transonic data memorandum

Examples of the application of constrained multivariate optimisation techniques to the design of aerofoil sections

Design point	: Single
Geometry variation	: Upper and lower surfaces and camber line variations
Initial aerofoils	: NACA 0012 and RAE 2822
CFD	: BVGK (viscous flow)

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THE PREPARATION OF THIS DATA ITEM

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EXAMPLES OF THE APPLICATION OF CONSTRAINED MULTIVARIATE OPTIMISATION TECHNIQUES TO THE DESIGN OF AEROFOIL SECTIONS

Design point : Single
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1. INTRODUCTION

This Data Item presents examples of the application of the constrained multivariate optimisation technique CODAS to the inverse design of aerofoil sections, *i.e.* determining the shape of aerofoil which has a specified pressure distribution at a given flow condition (M , C_L or α , Reynolds number).

The Data Item is one of a series intended to illustrate the process of using such optimisation techniques, to indicate their advantages and to reveal the kinds of problems that can arise. To facilitate this, the Data Item is in narrative form.

An introduction to the application of constrained multivariate optimisation techniques to the design of aerofoil section shapes is given in Data Item No. 99019 (Reference 4).

2. NOTATION

2.1 Algebraic

C_D	drag coefficient	
C_L	lift coefficient	
C_p	pressure coefficient	
c	aerofoil chord	
$\bar{H}$	transformed boundary layer shape parameter	
$\bar{H}_{0.99}$	transformed boundary layer shape parameter at 99% chord on upper surface	
M	free stream Mach number	
n	number of points defining target pressure distribution	
R	Reynolds number, based on chord and free stream conditions	
t	maximum thickness of aerofoil, <i>i.e.</i> maximum value of $z_U(x) - z_L(x)$	
x	chordwise coordinate	
x_{max}	value of x for which $z_U(x) - z_L(x)$ reaches its maximum value	
z	normal coordinate	
z_c	normal coordinate of incremental camber line	
α	angle of incidence of reference line	degree
δz	normal coordinate of an incremental surface	
ρ_0	leading edge radius	

Subscripts

<i>current</i>	denotes stage reached at any point in optimisation search
<i>L</i>	denotes lower surface of aerofoil
<i>target</i>	denotes required pressure distribution
<i>U</i>	denotes upper surface of aerofoil

2.2 Computational

NFE	number of calls by RQPMIN to CFD routine
NFEMAX	maximum number of calls by RQPMIN to CFD routine (set in input data)
NI	number of RQPMIN iterations
NIMAX	maximum number of RQPMIN iterations (set in input data)

2.3 Definitions

CFD	“Computational Fluid Dynamics”, programs used to calculate the flow around aerofoil sections. In the examples in this Data Item the chosen CFD method is BVGK (see Reference 1). The generic term “CFD” is used except where the behaviour under discussion is specific to BVGK.
CODAS	“Constrained Optimisation Design of Aerodynamic Surfaces”, a design method for aerofoils and wings that couples CFD and numerical optimisation methods (see References 2 and 4).
Optimiser	Software capable of solving constrained multivariate optimisation problems.
RQPMIN	The chosen optimiser in the examples in this Data Item (see Reference 3).

3. EXAMPLES

3.1 Objective Function

In principle, use of CODAS for the inverse design of aerofoil sections is no different from any other use of CODAS for aerofoil design. CODAS works by changing the values of a set of geometric variables which define the aerofoil in a search for a minimum value of an objective function. To minimise drag, the objective function is taken as C_D ; to find an aerofoil section which has a specified (*i.e.* “target”) pressure distribution, the objective function needs to be a measure of the closeness of fit of the current pressure distribution to the target. To this end it is taken as the mean of the squares of the deviations,

$$\frac{1}{n_U + n_L} \sum_1^{n_U + n_L} (C_{p \text{ current}} - C_{p \text{ target}})^2.$$

Note that this relates simply to the individual x/c positions at which the target pressure distribution is specified.

3.2 Target Pressure Distribution

There is considerable flexibility in the way that the target pressure distribution may be defined. The target is simply quoted in the input file as the number of upper and lower surface points (n_U and n_L), then a list of n_U points, in the form $(x/c, C_p)$, followed by n_L similar points. Typical possibilities are as follows:

- the entire pressure distribution, as in the first example (Section 3.3),
- the entire upper surface pressure distribution ($n_L = 0$),
- the pressure distribution over a part of the upper surface from (say) 7.5% to 95% chord, as in the second example (Section 3.4),
- the pressure distribution over a number of separate regions around the aerofoil upper and lower surfaces.

In principle, these may be coupled with constraints on various aspects of the pressure distribution, *e.g.* set an upper surface maximum suction level near the leading edge in addition to defining the pressure distribution further downstream. This facility was not available at the time of the present work.

3.3 First Example: Design to a Complete Pressure Distribution

The first example demonstrates the ability of CODAS to obtain the shape of an aerofoil section which will provide a specified complete pressure distribution.

The target pressure distribution is taken as that obtained from a BVGK calculation of the flow around the RAE 2822 section at $M = 0.73$, $\alpha = 2$ degree, Reynolds number = 6.5×10^6 with transition at 5% chord on both surfaces. The inverse design process is started from a NACA 0012 section at a value for α of 1.5 degree (corresponding roughly to the expected C_L) and the section is modified by increments to the upper and lower surface ordinates defined by second derivatives of δz with respect to x at eight chordwise positions ($x/c = 0, 0.06, 0.22, 0.4, 0.6, 0.8, 0.94$ and 1.0). The values at $x/c = 0$ and 1.0 are held at a value of 0 throughout. Hence, there are 12 variable geometric parameters plus α .

Figures 1 to 4 show the pressure distribution at various stages through the development of the aerofoil section. The target distribution is depicted by square symbols. The search process may be considered in five phases, as follows.

- (i) The shock wave is moved rearwards to its required position, driven by the need to reduce the effect on the objective function of the region of large pressure deviations between the positions of the shock wave on the initial aerofoil and the target. This is achieved in the first iteration* (Figure 1).
- (ii) The pressure deviations on the upper surface, forward of the shock wave, are reduced to a level fairly close to the target. This is achieved within three more iterations, to NI = 3 (Figure 1).
- (iii) In the next five iterations (to NI = 8, Figure 2), the lower surface distribution is altered to approximate to the form of the target whilst slight improvements are made to the fit on the upper surface.
- (iv) After four more iterations (to NI = 12, Figure 3) all of the features of the target distribution are represented, the worst fit being in a region on the lower surface near the trailing edge.
- (v) Subsequent iterations provide a “fine tuning” process. An example at NI = 66 is shown in Figure 4. The fit is remarkably close considering how few points are used to define the incremental surfaces.

The aerofoil section shapes at the end of each of these phases (*i.e.* for NI = 0, 3, 8, 12 and 66), together with the starting aerofoil (NACA 0012), are shown in Figure 5. A comparison of the final aerofoil section (*i.e.* after 66 iterations) with the aerofoil (RAE 2822) corresponding to the target pressure distribution is shown in Figure 6.

Figure 7 shows the CODAS convergence histories of the objective function, α , C_L and C_D . The objective function decreases fairly rapidly over the first 8 iterations, then continues to decrease at a lower rate to about 42 iterations, after which little further improvement is achieved. In fact, the objective function then appears to behave erratically, with occasional wild excursions from the attained level – to which it always returns. In this example, the run stopped at NI = 67 because NFE reached the set limit (NFEMAX = 2500). At this point the objective function is of the order of 10 times the lowest value achieved so far (0.0000809, reached at NI = 66). It is necessary to be aware of the possibility of this behaviour and to select an appropriate solution when a run stops because NFE or NI reaches its limit.

The CODAS convergence histories of α , C_L and C_D show that both C_L and C_D are more-or-less constant after about NI = 12 but that α continues to change throughout, albeit slowly. It must be remembered that α is one of the independent variables and is working together with the other independent variables to achieve the required pressure distribution.

Figure 8 shows the history of various geometric characteristics of the aerofoil as the search proceeded, confirming that most of the changes are made in the first 12 iterations.

3.4 Second Example: Design to an Upper Surface Pressure Distribution

For the second example the target is a shock-free upper surface pressure distribution defined as shown in Figure 9; $M = 0.72$, $C_L = 0.77$, $R = 20 \times 10^6$. (This example is close to that of Section 4.1 in Reference 5.) The starting aerofoil is taken as RAE 2822 and a revised camber line is sought which will give the required pressure distribution. **Note** that the target pressure distribution is not defined for the first 7.5% and the last 5% of the chord.

* Note that, within CODAS, the starting point and the first iteration are both labelled NI = 0.

3.4.1 Incremental camber defined at 8 points (as in Section 3.3)

This series of runs used camber definition points at chordwise positions* $x/c = 0, 0.06, 0.22, 0.4, 0.6, 0.8, 0.94$ and 1.0 (as in the example in Section 3.3).

In the first run the parameters at $x/c = 0$ and 1 were kept constant and equal to zero. The run reached “Convergence B[†]” after 46 iterations with an objective function of 0.000550 . The resulting pressure distribution is shown in Figure 10a in comparison with the target. The pressure distribution is fairly close to the target except in the immediate vicinity of the rounded kink at $x/c = 0.6$.

A second run, including the parameter at $x/c = 1$ as a variable, produced less satisfactory results despite reaching “Convergence B[†]” after 40 iterations. The objective function was slightly larger, at 0.000654 . The resulting pressure distribution is shown in Figure 10b.

A third run, including the parameters at $x/c = 0$ and 1 as variables, produced the best results by a slight margin (see Figure 10c), stopping when the number of calls to BVGK reached NFEMAX (after 45 iterations). At this stage the objective function was 0.000421 . It is possible that setting a higher value for NFEMAX would result in some further improvement.

None of these runs resulted in as close a fit to the target pressure distribution as was achieved in Reference 5. However, the results in Reference 5 were achieved using roughly half as many points to define the changes in the aerofoil camber line as there are coordinates defining the basic aerofoil. It is remarkable how closely the target was matched here with the incremental camber surface defined at only 8 points across the chord.

3.4.2 Incremental camber defined at 4 to 16 cosine distributed points

The runs described in Section 3.4.1 were followed by a systematic series with the incremental camber defined at a number of points varying from 4 to 16, with a cosine distribution across the chord (resulting in different positions for each case, apart from the points at $x/c = 0$ and 1), and with all parameters variable. The intention was to determine how few points are necessary to achieve an acceptable match with the target pressure distribution. For convenience, the individual cases are referred to in the form “7-point aerofoil”.

The results are shown in Figures 11a to 11m and a comparison of the achieved levels of the objective function is shown in Figure 12. All results shown correspond to runs that reached “Convergence B[†]” except for those for the 12-point aerofoil in Figure 11h, which corresponds to a run stopped with a “No further progress possible” notice[‡]. In every case it was necessary to experiment with different starting points and control parameters in order to get solutions with objective functions as low as those shown. Even so, the uneven levels of objective function in Figure 12 suggest that the converged solutions for some cases still correspond to local optima and that further improvement may be possible. The low values for the 9-, 11-, 13- and 16-point aerofoils may represent the best possible but this cannot be certain without exhaustive research with many different starting points. The high value for the 8-point aerofoil and, to a lesser degree, the 10-point aerofoil appear to be out of line with the surrounding aerofoils but both cases were the subject of extensive attempts to find better solutions, without avail. It should be noted that the level of objective function for the 8-point aerofoil in Figure 10c, Section 3.4.1, is of the same order as that for the present 8-point aerofoil.

The degree to which the target pressure distribution is matched with the use of incremental camber surfaces defined at only 4 or 5 points (Figures 11a and 11b) is again remarkable. However, the closeness of fit

* Indicated by circle symbols on the $C_p = 0$ line across the subsequent charts.

† “Convergence B” indicates that RQPMIN has located a point which satisfies the constraints to the tolerance specified and a value of the objective function sufficiently smaller than the current value cannot be found (References 2 and 3).

‡ “No further progress possible” indicates that RQPMIN can make no further progress in finding a point that satisfies the currently active constraints. The result is still a valid one and, since it is better than other results for the 12-point aerofoil, may be taken as the solution.