

Flat panels in shear – post-buckling analysis

Associated software: ESDUpac A0205

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THE PREPARATION OF THIS DATA ITEM

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FLAT PANELS IN SHEAR – POST-BUCKLING ANALYSIS

PREFACE

This Data Item introduces a Fortran computer program, ESDUpac A0205, for the analysis of a flat, stiffened panel forming the web of a beam, the web having buckled under shear. The program can provide either (i) a detailed analysis of the strengths and buckling loads of the individual components of a single panel or (ii) the minimum plate reserve factor, the minimum stiffener reserve factor, the minimum flange reserve factor, the ratio of nominal shear stress to initial plate buckling stress and the weight of the panel (excluding the flanges) per unit run for a number of different user-specified panels, so allowing a comparison of their performance to be made. The program is a computerised form of the analysis performed in Data Item No. 77014 (Derivation 9). The initial buckling analysis of the plate applies only to the elastic range but the post-buckling analysis takes account of inelastic behaviour in the plate and stiffeners. The analysis is only applicable to panels fabricated from isotropic materials.

The Data Item consists of two sections. Section 1 describes the application of the program and its theoretical basis. Section 2 describes the use of the program, its input and output, and gives example calculations using the program.

The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable program for use within ESDUview. Both versions can be found in ESDUscope (the CD version of the Series), on the internet version of the Series, and on disk in the Series Software Volume.

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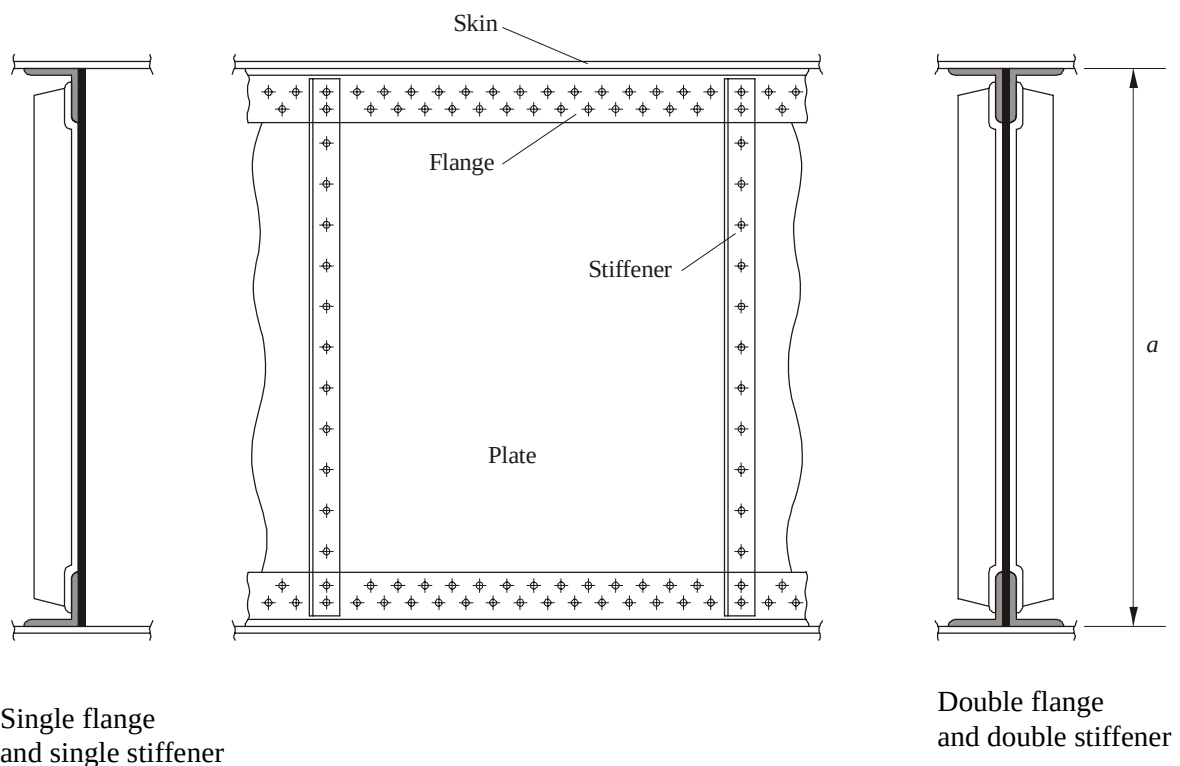
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FLAT PANELS IN SHEAR – POST-BUCKLING ANALYSIS

1. APPLICATION OF THE PROGRAM AND THEORETICAL CONSIDERATIONS

1.1 Introduction

This Data Item introduces a Fortran computer program, ESDUpac A0205, for the analysis of a flat, stiffened panel forming the web of a beam, the web having buckled under shear. A panel is defined as a thin plate, or sheet, reinforced by a system of longitudinal and transverse members referred to as flanges and stiffeners respectively, as shown in Sketch 1.1. The panels are assumed to be part of a beam, as in an aircraft spar. The program provides an analysis of either (i) the detailed component loads, shear and direct stresses, and buckling and failure stresses (as applicable) in the plate, flanges, stiffeners and fasteners of a single specified panel or (ii) the minimum plate reserve factor*, the minimum stiffener reserve factor, the minimum flange reserve factor, the ratio of nominal shear stress to initial plate buckling stress and the weight of the panel (excluding the flanges) per unit run for a chosen number of predefined panels in which the plate thickness, stiffener pitch, stiffener thickness and stiffener height may each take up to five different values. The Data Item is a computerised form of the analysis performed in Data Item No. 77014 (Derivation 9). In addition, use is made of associated ESDU Data Items, Derivations 1, 5 to 8, 10 to 12 and 14.



Sketch 1.1 Typical panel layout

* A "reserve factor" is defined as the ratio of allowable load to applied load.

The action of a web in shear may be explained with the aid of the simple structure shown in Sketch 1.2a, consisting of an initially rectangular frame of stiff bars, pinned at the corners and braced internally by two slender diagonals of equal size. As long as the applied shear load, Q_a , is small, the two diagonals will carry equal and opposite loads. At a certain value of Q_a , the compression diagonal will buckle (Sketch 1.2a) and thus lose its ability to take any additional load. Consequently, if Q_a is further increased, the additional diagonal bracing force must be furnished by the tension diagonal; at high applied loads, the load in the tension diagonal will be so large that the load in the compression diagonal will be negligible by comparison. An analogous change in the state of stress will occur if the internal bracing is replaced by a thin plate (see Sketch 1.2c).

Thus, a panel buckled in shear deforms into a set of inclined waves and carries, when these are fully developed, a tensile stress, approximately parallel to the waves, together with a smaller, perpendicular compressive stress (see Sketches 1.2c and 1.2d). This leads to a compressive load and a bending moment in the panel flanges and a compressive load in the stiffeners. Thus the stress distribution can be described as a combination of pure shear stress and a “diagonal tension field” acting in the general direction of the buckles (see Sketches 1.2b and 1.2d). The proportion of the total shear reacted by diagonal tension is termed the diagonal tension factor, k , and values for this factor, for flat panels, are given by the formula $\tanh[0.5 \log_{10}(q_a/q_b)]$.

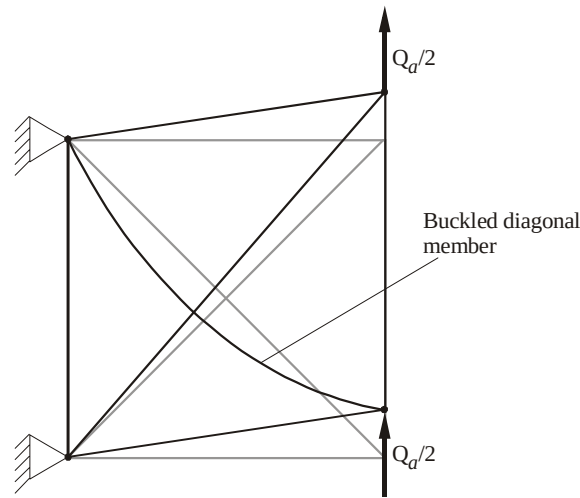
The initial buckling analysis of the plate performed by the program applies only to the elastic range but the post-buckling analysis takes account of inelastic behaviour in the plate and stiffeners. The analysis is only applicable to panels fabricated from isotropic materials. The program performs an analysis of the post-buckled stress system in the panel as a beam element subjected to an applied transverse shear load, Q_a . In addition to this, there will be an end load, P_{fm} , in the flanges due to the overall bending moment, M_b , on the beam in the plane of the plate. The end load, P_{fm} , is assumed to be constant along the flanges; however, in reality, P_{fm} , varies along the flanges. The applied transverse shear load, Q_a , and the overall bending moment on the beam in the plane of the plate, M_b , are assumed to be in a fixed ratio to one another. The program provides data on the following aspects.

- (i) Initial buckling of the plate.
- (ii) Strength of the plate.
- (iii) Strength of the stiffeners.
- (iv) Strength of the flanges.
- (v) Post-buckled stiffness of the panel.
- (vi) Weight of the panel.

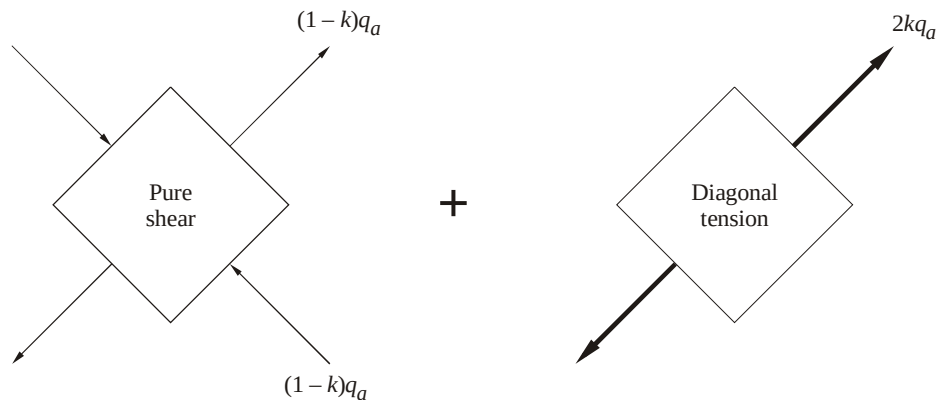
It should be noted that the program does not provide a complete analysis of the flanges and a more precise analysis, making use of form factors, should be carried out by the user as appropriate.

The relationship between the stresses in the panel and the applied load is not an explicit one. As a result of this, the program goes through an iterative process for each of the components of the panel in turn in order to determine the shear load, Q , that would result in a stress in the component under consideration equal to the allowable stress of the relevant material. The ratios of those loads to the actual applied load are the reserve factors for each of the components of the panel.

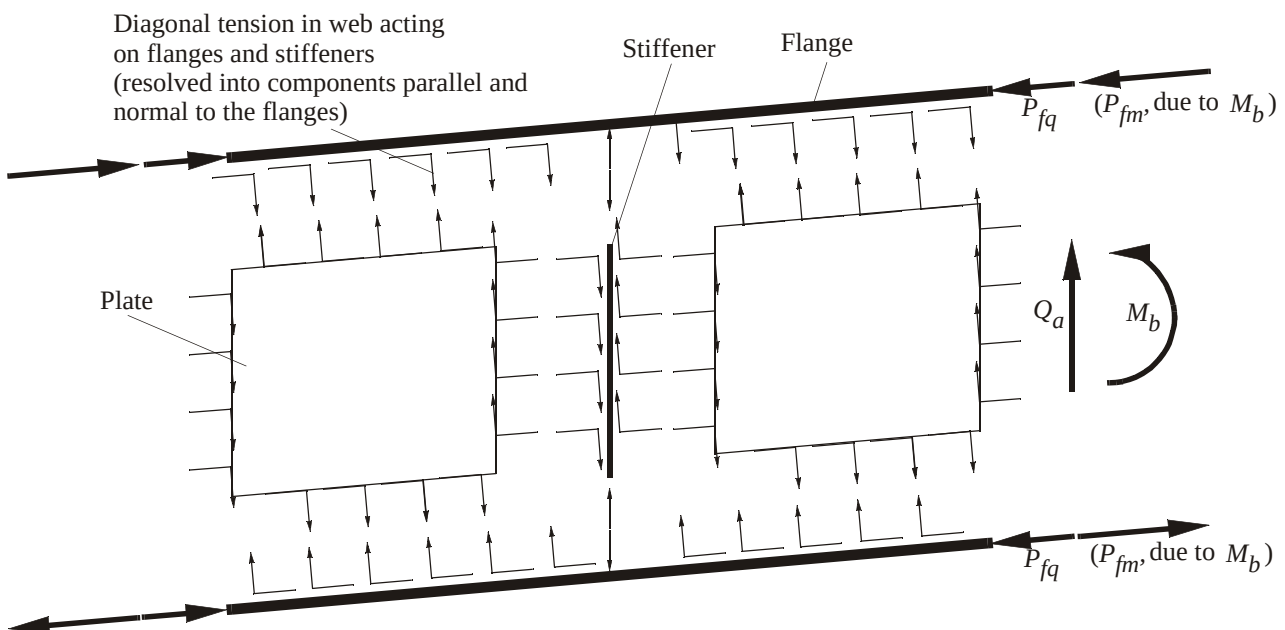
Section 1.3 provides a general guide to the application of the program and limitations on its use. Section 1.4 gives a description of the different aspects of the analysis and the detailed considerations associated with them.



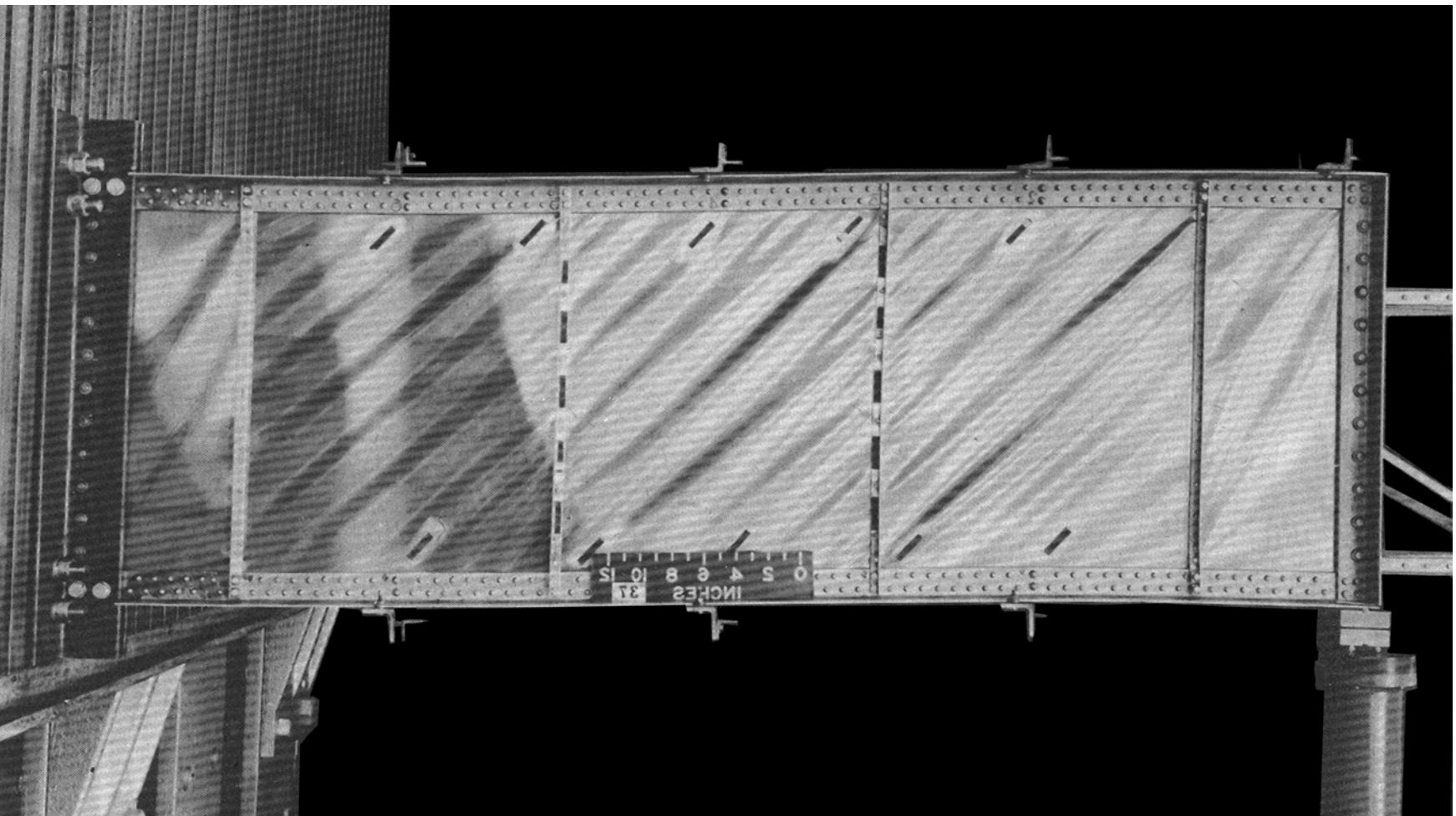
Sketch 1.2a Frame in shear braced by diagonal links



**Sketch 1.2b Stress systems in diagonal tension panel
(assuming the angle of the diagonal tension stress to be 45°)**



Sketch 1.2c Internal and external loads in diagonal tension panel (exploded view)



**Sketch 1.2d Typical diagonal tension beam
(from NACA Technical Note No. NACA TN-2661)**

1.2 Notation

Both SI and British units are quoted but any coherent system of units may be used.

A_f	cross-sectional area of flange and associated skin excluding associated plate, see Section 1.3 and Table 2.3	m ²	in ²
A_{fe}	cross-sectional area of flange, associated skin and associated plate, see Table 2.3	m ²	in ²
A_s	cross-sectional area of stiffener excluding associated plate (but including any pad that is present)	m ²	in ²
A_{se}	cross-sectional area of stiffener and associated plate (including any pad that is present)	m ²	in ²
a	full depth of panel excluding skin thickness, see Sketch 1.1	m	in
a_p	depth of web used for buckling analysis, see Table 2.3	m	in
b	stiffener pitch or spacing	m	in
b_p	width of plate used for buckling analysis, see Table 2.2	m	in
d	flange or stiffener free (unattached) height, see Tables 2.3 and 2.2 respectively	m	in
E	elastic modulus of component material	N/m ²	lbf/in ²
E_T	tangent modulus of component material (see Derivation 8)	N/m ²	lbf/in ²
e_m	manufacturing eccentricity of stiffener	m	in
F	flange or stiffener reserve factor		
F_{fc}	reserve factor relating to maximum compressive stress in flanges		
F_{fmin}	minimum flange reserve factor taken from F_{fc} , F_{fr} and F_{ft}		
F_{fr}	reserve factor relating to maximum stress at innermost rivet line of flanges		
F_{ft}	reserve factor relating to maximum tensile stress in flanges		
$F_{o,x}$, F_{pir} , F_{sd} , F_{sfc} , $F_{s, oT}$, F_{sz} , F_T	reserve factors relating to $f_{o,x}$, f_{pir} , f_{sd} , f_{sfc} , $f_{s, oT}$, f_{sz} and f_T respectively		
F_{smin}	minimum stiffener buckling reserve factor taken from $F_{o,x}$, F_{pir} , F_{sd} , F_{sfc} , $F_{s, oT}$, F_{sz} and F_T (see Sections 1.4.3.1 to 1.4.3.3)		
f	direct stress in component	N/m ²	lbf/in ²
f_{as}	allowable extreme fibre stress of stiffener material	N/m ²	lbf/in ²
f_{np} , f_{ns}	stress at which $E_T = E/2$ for plate and stiffener materials respectively (see Derivation 8)	N/m ²	lbf/in ²
$f_{o,x}$	overall flexural buckling stress of stiffener/plate combination	N/m ²	lbf/in ²

f_{p1}, f_{p2}	stresses at points on plate material stress-strain curve	N/m ²	lbf/in ²
f_{pir}	inter-rivet buckling stress of plate material acting with stiffener	N/m ²	lbf/in ²
f_{pr}	tensile stress in plate normal to flange cross-section at flange-to-plate rivet line	N/m ²	lbf/in ²
f_s	average applied compressive stress in stiffener at centre of its length	N/m ²	lbf/in ²
f_{s1}, f_{s2}	stresses at points on stiffener material stress-strain curve	N/m ²	lbf/in ²
f_{sd}	local buckling stress of stiffener outstand (stiffener element with dimension d_s in Table 2.2)	N/m ²	lbf/in ²
$f_s f_c$	value of f_s at which forced crippling of stiffener occurs	N/m ²	lbf/in ²
f_{sh}	maximum compressive stress at heel of stiffener	N/m ²	lbf/in ²
$f_{s, oT}$	torsional-flexural buckling stress of stiffener/plate combination	N/m ²	lbf/in ²
f_{sz}	local buckling stress of stiffener flange (stiffener element with dimension z_s in Table 2.2)	N/m ²	lbf/in ²
f_T	pure torsional buckling stress of stiffener/plate combination	N/m ²	lbf/in ²
f_{tf}, f_{tp}	tensile strengths of flange and plate materials respectively	N/m ²	lbf/in ²
G	shear modulus of component material	N/m ²	lbf/in ²
G_S, G_T	secant and tangent moduli respectively of plate	N/m ²	lbf/in ²
h_e	distance between centroids of flanges (area A_{fe})	m	in
I_{CF}	identifier for form of analysis		
I_{Cm}	identifier for plate and stiffener material stress-strain curve (as denoted by appropriate suffixes)		
ID_F	flange dimensions/section properties indicator		
ID_{type}	stiffener type (see Table 2.2)		
$I_{a1,2}$	indicator for attached or integral stiffener (see Table 2.1d)		
$I_{e1,2}$	indicator for the type of stiffener outstand with height d_s and thickness t_s (see Table 2.1d)		
I_f	second moment of area of flange (area A_f) about axis through its centroid normal to plate	m ⁴	in ⁴
$I_{f1,2}$	indicator for single or double flange (see Table 2.1a)		
I_{fe}	second moment of area of flange including associated skin and associated plate (area A_{fe})	m ⁴	in ⁴
$I_{o1,2}$	indicator for open- or closed-section stiffener (see Table 2.1d)		
I_{pse}	polar moment of inertia of stiffener and associated plate about axis through its shear centre	m ⁴	in ⁴
$I_{s1,2}$	indicator for single- or double-sided stiffener (see Table 2.1d)		

$I_{u1,2}$	indicator for whether stiffener is asymmetrical or symmetrical about y-axis (see Table 2.1d)		
$I_{x,s}$	second moment of area of stiffener (area A_s) about axis through its centroid parallel to plate	m ⁴	in ⁴
$I_{x,se}$	second moment of area of stiffener and associated plate (area A_{se}) about axis through its centroid parallel to plate	m ⁴	in ⁴
J_s	torsional constant of stiffener cross-section (area A_s) (see Table 2.2)	m ⁴	in ⁴
J_{se}	torsional constant of stiffener and associated plate cross-section (area A_{se}) (see Table 2.2)	m ⁴	in ⁴
K	buckling stress coefficient defined by $q_b = KE_p(t_p/b_p)^2$ (see Derivation 7)		
k	diagonal tension factor		
k_r	stiffener-to-plate attachment factor (see Table 2.1b)		
k_s	radius of gyration of stiffener (area A_s)	m	in
k_{se}	radius of gyration of stiffener-plate combination (area A_{se})	m	in
l'	equivalent pin-ended column length of stiffener	m	in
l_{sr}	rivet pitch along stiffener	m	in
M_b	overall bending moment on beam in plane of plate	N m	lbf in
M_f	maximum local bending moment in flange	N m	lbf in
m_p, m_s	material characteristics of plate and stiffener materials respectively (see Derivation 8)		
N	number of panels selected for output (for multiple panel analysis, $I_{CF} = 1$)		
n_1, n_2, n_3, n_4	number of plate thicknesses, stiffener pitches, stiffener thicknesses and stiffener heights, respectively, to be considered in multiple panel analysis		
P	allowable load in component (as denoted by appropriate suffixes)	N	lbf
P_{fm}	applied direct (end) load in flange due to overall bending moment on beam, $P_{fm} = M_b/h_e$	N	lbf
P_{fq}	flange compressive load component due to diagonal tension field	N	lbf
P_{fpr}	maximum resultant shear flow on attachments of flange (in stiffener region)	N/m	lbf/in
$P_{s ult}$	failing end load of stiffener and associated plate, $P_{s ult} = f_{o,x} A_{se}$	N	lbf
Q	allowable shear load on panel parallel to stiffeners for failure of each of components (as denoted by appropriate suffixes)	N	lbf
Q_a	applied shear load on panel parallel to stiffeners	N	lbf

Q_b	shear load on panel parallel to stiffeners at which plate first buckles, $Q_b = q_b h_e t_p$	N	lbf
Q_d	applied shear load on panel parallel to stiffeners at which no permanent deformation is permitted	N	lbf
q	allowable shear stress in plate (as denoted by appropriate suffixes)	N/m ²	lbf/in ²
q_a	nominal applied shear stress in plate, $q_a = Q_a / h_e t_p$	N/m ²	lbf/in ²
q_b	shear stress at which plate first buckles	N/m ²	lbf/in ²
R_{dg}	plate reserve factor based on general permanent deformation, $R_{dg} = q_{dg} h_e t_p / Q_d$		
R_{dl}	plate reserve factor based on local permanent deformation, $R_{dl} = q_{dl} h_e t_p / Q_d$		
R_{min}	minimum plate reserve factor taken from R_{dg} , R_{dl} , R_p and R_{pfr}		
R_p	plate reserve factor based on failure away from edges, $R_p = q_p h_e t_p / Q_a$		
R_{pfr}	plate reserve factor based on failure at the flange joint (innermost rivet line), $R_{pfr} = q_{pfr} h_e t_p / Q_a$		
t	component thickness	m	in
t_{2f}, t_{2p}, t_{2s}	0.2 per cent proof stresses in tension of flange, plate and stiffener materials respectively	N/m ²	lbf/in ²
W	weight per unit length of panel (excluding flanges)	kg/m	lb/in
w	dimension of flange, stiffener or pad	m	in
w_p	effective width of plate acting with stiffener, see Table 2.2	m	in
x_1	distance of shear centre from centroid of stiffener and associated plate in x-direction	m	in
$\bar{y}_f$	distance between centroid of flange (area A_f) and mid-plane of skin	m	in
$\bar{y}_{fe}$	distance between centroid of flange (area A_{fe}) and mid-plane of skin	m	in
$\bar{y}_{fie}$	distance between centroid of flange (area A_{fe}) and innermost fibre of flange	m	in
y_{fr}	distance of innermost rivet/bolt row from heel of flange (see Table 2.3)	m	in
$\bar{y}_{fre}$	distance between centroid of flange (area A_{fe}) and innermost rivet line	m	in
y_{pe}, y_{ne}	algebraic distances from neutral axis to extreme fibres above and below axis (see Derivation 14)	m	in
$\bar{y}_s$	distance between centroid of stiffener (area A_s) and mid-plane of plate in y-direction	m	in
$\bar{y}_{se}$	offset of neutral axis of stiffener and associated plate from mid-plane of plate	m	in

z_s	dimension of stiffener (see Table 2.2)	m	in
z_{sr}	distance between first and second rows of stiffener fasteners (see Table 2.2)	m	in
β	coefficient for effective width of plate acting with stiffener, $\beta = 0.5(1 - k)$		
Γ_s	total warping constant* of stiffener about its shear centre ($\Gamma_{s1} + \Gamma_{s2}$)	m ⁶	in ⁶
Γ_{s1}	primary warping constant* of stiffener about its shear centre	m ⁶	in ⁶
Γ_{s2}	secondary warping constant* of stiffener about its shear centre	m ⁶	in ⁶
$\epsilon_{p1}, \epsilon_{p2}$	strains at points on plate material stress-strain curve		
$\epsilon_{s1}, \epsilon_{s2}$	strains at points on stiffener material stress-strain curve		
ν_s	Poisson's ratio for stiffener material		
ρ	density of component material	kg/m ³	lb/in ³

1.2.1 Suffixes

<i>c</i>	compression
<i>d</i>	permanent deformation
<i>e</i>	effective value calculated including contribution from plate
<i>f</i>	flange
<i>foot</i>	stiffener foot, excluding plate (for integral stiffeners only, see Table 2.2)
<i>g</i>	general
<i>h</i>	heel of flange of stiffener
<i>l</i>	local
<i>m</i>	moment loading
<i>p</i>	plate
<i>pad1, pad2</i>	pad between stiffener and plate
<i>r</i>	rivet or fastener row
<i>s</i>	stiffener
<i>sk</i>	skin
<i>t</i>	tension
<i>ult</i>	ultimate stress
<i>x, y</i>	local axes parallel and perpendicular to plate respectively (see Table 2.2)

* The warping constant can be obtained using the Struct 00.07 series of Items, Item No. 77023, or Derivation 13, depending on the stiffener type.

1.3 General Application and Limitations of the Analysis

The program is based on the analysis given in Derivation 9 and associated Items; the majority of the data on which those Data Items are based were derived from experiment and therefore include the effects of any non-uniformity in the stress pattern in the panel. The analysis is based on the assumption that the panel is loaded in pure shear but will usually be applied to beams in which the flanges carry tensile or compressive loads. The data strictly apply to panels with $0.2 \leq b_p/a_p \leq 1.0$ with a well developed tension field, q_a/q_b greater than about 2, in which case the effect of initial irregularities of manufacture is not significant. For loads that give a less well developed tension field, initial irregularities of the plate may become significant. In this region it may be more efficient to design a panel in which the stiffeners are of reduced stiffness and the plate of increased thickness so that failure occurs by simultaneous buckling of the plate and stiffeners.

The program performs a detailed stress, strength and buckling analysis of the panel plate, flanges, stiffeners and fasteners. This can be performed either (i) for one specific panel, in which case full details of the analysis will be output, or (ii) for up to 625 panels of different plate thickness, stiffener pitch, stiffener thickness and stiffener height (each may take up to 5 values) in which case results, including panel weights, will be given for a specified number of panels.

The initial buckling analysis of the plate applies only to the elastic range but the post-buckling analysis takes account of inelastic behaviour in the plate and stiffeners. The analysis is based on the assumption that the stiffeners and flanges are manufactured from materials having the same values of modulus of elasticity as the plate. The program will accept data for stiffeners and flanges with different moduli from that of the plate but it is assumed that they are sufficiently similar for the loading in these members to be calculated using a simple direct ratio between their moduli and that of the plate. The stiffeners and flanges may be defined by either their dimensions or properties. The analysis is only applicable to panels fabricated from isotropic materials.

When applying the data to a beam in bending it is necessary to determine an effective width of skin acting as part of the flange area A_f ; here an effective width of skin equal to the flange width is used. However, the program can be used to analyse panels with no skin attached to the flanges. For the compressive flange, the effective width of skin acting as part of it is likely to be governed by considerations of compressive instability and reference should be made to Data Items dealing with compressive buckling of plates. As stated earlier, the program does not provide a complete analysis of the flanges and a more precise analysis, making use of form factors, should be carried out by the user as appropriate.

1.4 The Parts of the Analysis and Detailed Considerations

The program considers the analysis of a single panel in five parts as detailed in Sections 1.4.1 to 1.4.5. In addition an analysis can be performed to obtain the strength versus weight of a number of panels as detailed in Section 1.4.6.

Section 1.4.1 - Initial buckling of plate.

Section 1.4.2 - Strength of plate.

Section 1.4.3 - Strength of stiffeners.

Section 1.4.4 - Strength of flanges.

Section 1.4.5 - Post-buckled stiffness of panel.

Section 1.4.6 - Multiple panel analysis.

1.4.1 Initial Buckling of Plate

The program includes the option for the user to specify an independently calculated value for the initial plate buckling stress, q_b . If that option is not chosen, the program calculates the elastic stress at which the plate first buckles based on the data provided in Derivation 7. That derivation provides data computed for the initial buckling stress of flat panels that form a continuous beam of the type illustrated in Sketch 1.2d. The data relate to panels in which the flanges provide either clamped or simply-supported conditions at the top and bottom of the panel (see Table 2.3); the stiffeners are of identical section and equally spaced, and their thickness, t_s , is equal to, or greater than t_p . The program will, however, analyse panels for which t_s is less than t_p , in which case a warning message is given in the program output.

The program calculates the nominal applied shear stress in the plate, q_a , the initial plate buckling shear stress, q_b (unless a value has been entered by the user, in which case it is that value that is used) and their ratio. These values are given by program outputs P1, P2 and P3.

1.4.2 Strength of Plate

The analysis considers the plate strength at two points, (i) remote from its edges and (ii) at the plate-to-flange joint.

1.4.2.1 Plate strength remote from the edges

The buckled plate is subjected to a combination of shear, tensile and bending stresses, the effects of which are most severe at the crests of the waves. Therefore yielding first occurs at these crests and is evidenced by a slight waviness of the plate after unloading, without appreciable damage to the form of the whole panel. With increased loading the yielding progresses through the thickness of the plate with the waviness becoming sufficient to cause appreciable permanent deformation of the whole panel. For ductile materials fracture of the whole plate, by tearing in a direction normal to the principal tensile stress, will not occur until, over some region of the plate, the average stress through the thickness is sufficient to cause failure.

The program assesses the likelihood of failure and permanent deformation away from the edges by calculating the nominal shear stress at failure, q_p , the nominal shear stress for local permanent deformation, q_{dl} , and the nominal shear stress for general permanent deformation, q_{dg} . Failure and general permanent deformation have been assessed using the principal stress criterion in conjunction with the tensile strength, f_{tp} , and the 0.2 per cent proof stress of the plate material, t_{2p} , respectively. Local permanent deformation has been assessed by using the shear strain energy criterion of von Mises and Hencky with the 0.2 per cent proof stress, t_{2p} . The stress q_p should be compared with $Q_d/h_e t_p$ and q_{dl} and q_{dg} should be compared with $Q_d/h_e t_p$. These comparisons are given in program outputs P4, P5 and P6.

The data used for these calculations have been obtained from Figure 1 of Derivation 9. They have been derived on the assumption that the panel boundary members are rigid and therefore take no account of any non-uniformity of the stress pattern in the plate or of the effects of stress concentrations such as those that occur at rivet holes. Therefore in practice the plate may fail at stresses slightly below the values obtained.

1.4.2.2 Plate strength at plate-to-flange joint

In practice the stress distribution in the plate along the joint with the flange is different from that prevailing near the centre of the plate. The program calculates the nominal shear stress at failure of the plate at the innermost rivet line, q_{pfr} . This is given in program output section P7. It has been assessed by comparing the tensile strength of the plate material with both the shear strain energy criterion of von Mises and Hencky and the principal stress criterion and outputting the value that results in failure at the lowest stress.

The data used for these calculations have been obtained from Figure 2 of Derivation 9. They have been derived on the assumption that stress concentration effects due to flexibility of the flanges or the presence of rivet holes may be neglected. Small effects due to the variation of the diagonal tension factor, k , have been eliminated by taking the relevant minimum values. In the presence of stress concentrations a more detailed calculation of the maximum stress can be made by taking account of the three individual components of the stress field and applying the appropriate stress concentration factor. The three components are a diagonal stress, $2kq_a$ (assuming the angle of the diagonal tension stress to be 45°), a pure shear stress, $(1 - k)q_a$ (see Sketch 1.2b), and a tensile stress, f_{pr} . The value of the diagonal tension factor, k , and the angle of the diagonal tension stress may be obtained from Derivation 3.

1.4.3 Strength of Stiffeners

After the plate has buckled the stiffeners are required to support compressive loads due to the diagonal tension field and to resist out-of-plane forces associated with the plate buckles. The stiffeners are therefore prone to failure (i) by buckling, (ii) by forced crippling associated with the plate buckle, and (iii) by failure of their attachments. The stiffeners may be attached to one side of the plate only, referred to as single stiffeners, or else to both sides, back to back, referred to as double stiffeners, see Table 2.2.

When double stiffeners are used, the average compressive stress throughout the stiffener section, f_s , is obtained using the formulae in Derivation 3. When single stiffeners are used the reduced stiffness implied by the offset of the stiffener centroid from the plate is to some extent counter-acted by the tendency of the diagonal tension field to keep the stiffener straight. Thus the average stress is similar to that for a double stiffener of the same area, A_s . For a double stiffener the stress at the heel, f_{sh} , is the same as the average stress, f_s , but for a single stiffener it is different. A conservative estimate of the stress at the heel of a single stiffener is obtained by replacing the area A_s by $A_s/[1 + (\bar{y}_s^2 A_s/I_{x,s})]$. This is done by the program. Both f_s and f_{sh} are given in program output S1 for comparison with the buckling and crippling stresses of the stiffener and plate that are output by the program and discussed in the following sections.

1.4.3.1 Buckling of stiffeners

Four aspects of stiffener stability are considered in the analysis: (i) overall flexural buckling, (ii) pure torsional buckling, (iii) torsional-flexural buckling, and (iv) local buckling. A number of effects are taken into account when considering stiffener stability. Firstly, an effective width of the adjacent plate may be considered to act with the stiffener. The effective widths of plate acting with the various stiffeners are given in Table 2.2; however, for torsional buckling the effective plate width is taken as $[(w_s - z_{sr})/2] + z_{sr}$ and w_s for single- and double-sided attached stiffeners respectively and as $30t_p$ for integral stiffeners. Additionally, the effective length of the stiffener is affected by the facts that the tension field gives some restraint to the stiffener against overall flexural buckling and that the attachment to the flanges also provides some further constraint. Since the stiffener load arises from the plate, single stiffeners are eccentrically loaded and both single and double stiffeners are subject to the eccentric loading effects of manufacturing inaccuracies. For the stiffener/plate combination, the program calculates the effective width of the plate, w_p , the effective cross-sectional area, A_{se} , the effective second moment of area, $I_{x,se}$, the effective radius of gyration, k_{se} , the offset of the neutral axis, $\bar{y}_{se}$, and the reduced equivalent pin-ended strut length, l' . The program calculates w_p and l' using the formulae in Derivation 2. Values of these effective parameters are given in section S2 of the output for reference purposes.

The overall flexural buckling stress and the failing end load of the stiffener/plate combination, $f_{o,x}$ and $P_{s,ult}$ respectively, are calculated using Derivation 14 and given in section S3 of the output. The overall flexural buckling stress, $f_{o,x}$, should be compared to stress f_s , given in section S1. In the single panel analysis the comparison is represented by the factor $F_{o,x} = f_{o,x} / f_s$.

The pure torsional buckling stress, f_T , and the torsional-flexural buckling stress, $f_{s,OT}$, of the stiffener attached to the plate are calculated using formulae from Derivations 4, 10 to 12 and 14 and are given in sections S4 and S4A of the output respectively. Stiffeners attached to a plate have a shear centre at a calculated point along the mid-plane of the plate (see Table 2.2). Therefore the pure torsional buckling stress, f_T , and the torsional-flexural buckling stress, $f_{s,OT}$, for the stiffener/plate combination are calculated using the warping constant and polar moment of inertia about the shear centre, and using the effective torsional constant of the stiffener, J_{se} . The f_T and $f_{s,OT}$ values should be compared to f_s , given in section S1. In the single panel analysis the comparison is represented by the factors $F_T = f_T/f_s$ and $F_{s,OT} = f_{s,OT}/f_s$. In practice the plate provides additional restraint against torsional deformation because of the diagonal tension stress so that the simple estimate given here is likely to give values below the actual torsional and torsional-flexural buckling strengths. There is no torsional-flexural buckling for stiffeners with symmetry about the y-axis (that is, stiffener types 7 to 10 and 13 to 14 of Table 2.2) because the fibres of the stiffener in contact with the plate cannot flex in the plane of the plate and flexure in the direction perpendicular to the plate is independent of torsion. The torsional and torsional-flexural buckling analysis for closed-section stiffeners (that is, stiffener types 11 and 12 of Table 2.2) are not performed by the program because torsional instability is unlikely ever to be critical for such stiffeners.

The local buckling stresses of the stiffener flange (as applicable) and outstand, f_{sz} and f_{sd} respectively, are calculated using Derivation 14 and given in section S5 of the output. These stresses should be compared to f_s , given in section S1. In the single panel analysis these comparisons are represented by the factors $F_{sd} = f_{sd}/f_s$ and $F_{sz} = f_{sz}/f_s$.

1.4.3.2 Forced crippling of stiffeners

Failure of the stiffener by forced crippling occurs when the buckled plate, assisted by compressive load in the stiffener, forces the stiffener to deform locally with a plate buckle. The forced crippling stress of the stiffener, f_{sfc} , is calculated using the formulae in Derivation 3. The value of f_{sfc} is given in section S5A of the output and it should be compared to f_s for double stiffeners and f_{sh} for single stiffeners, both given in section S1. In the single panel analysis the comparison is represented by the factors $F_{sfc} = f_{sfc}/f_{sh}$ for single stiffeners and $F_{sfc} = f_{sfc}/f_s$ for double stiffeners. For integral stiffeners (that is, stiffener types 15 to 20 of Table 2.2), forced crippling of the stiffener is not considered by the program.

1.4.3.3 Failure of the stiffener attachments

The analysis considers the attachment of the stiffeners at two points: (i) at the stiffener-to-plate interface, and (ii) at the flange-to-stiffener interface.

If the plate and stiffener are riveted together, there exists the possibility of buckling between the rivets. The tension caused by the buckled plate tending to force the stiffener and plate apart may cause the rivets to fail. Where double-sided stiffeners are used, inter-rivet buckling of the plate material acting with the stiffener is highly unlikely owing to the clamping effect of the stiffeners; therefore, for panels with double-sided stiffeners, the inter-rivet buckling stress of the plate is not calculated by the program. However, for panels with single-sided stiffeners, inter-rivet buckling of the plate material acting with the stiffener is more likely and for such panels the possibility of buckling between the rivets must therefore be considered. The inter-rivet buckling of the plate material acting with the stiffener is calculated using Derivation 14. The value of this stress is given in section S6 of the output and it should be compared to the stress at the heel of the stiffener, f_{sh} , given in section S1. In the single panel analysis the comparison is represented by the factor $F_{pir} = f_{pir}/f_{sh}$. The locations of the rivets on the stiffener flange are specified in the program input.

If a pad is present between the stiffener and the plate, and if the plate thickness is less than the pad thickness, then inter-rivet buckling of the plate will occur before that of the pad. Conversely, if the plate is thicker

than the pad, the plate will constrain the pad; therefore, a buckling analysis of the pad is not performed by the program as buckling of the pad is unlikely ever to be critical.

For integral stiffeners (that is, stiffener types 15 to 20 of Table 2.2) inter-rivet buckling cannot arise due to the absence of rivets and so it is not considered by the program.

The flange-to-stiffener rivets or bolts are required to transmit the compressive load in the stiffener. The average stress in the stiffener decreases towards the ends of the stiffener owing to the increase in the effective width of the plate acting with the stiffener. The load taken by the flange-to-stiffener rivets, P_{fsr} , which is the total load on the stiffener (or both stiffeners if they are double-sided) (excluding the associated plate) at the flanges, is calculated by the program taking account of these effects by using a formulation provided by Derivation 9. This load is given in section S7 of the output.

It should be noted that the strength of the flange-to-stiffener attachments is highly dependent on the local configuration of the joints. A detailed analysis of the joints is beyond the scope of the program and should therefore be performed separately as necessary.

1.4.4 Strength of Flanges

The components of the diagonal tension field parallel to and normal to the flanges introduce into the flanges a direct compressive load, P_{fq} , to add to the applied end load, P_{fm} , and a local bending moment, M_f .

Note that the loads in the upper and lower flanges, P_{fm} , due to the overall bending moment on the beam in the plane of the plate are equal in magnitude and opposite in direction, see Sketch 1.2c. Moreover, P_{fm} is assumed to be constant along the flanges, whereas in fact it varies along the flanges.

The compressive load component from the applied shear load, P_{fq} , is calculated by the program and given in output section F2. The value of P_{fq} is derived from the formulae given in Derivation 2. The maximum local moment loading on the flange, M_f , is calculated by the program from data provided in Figure 13 of Derivation 9 and given in section F3 of the output. The data from which that figure was derived are based largely on experiment and, owing to the limited number of data available, it is not possible to determine where the maximum value of the bending moment occurs. In the absence of other information, it is suggested that the maximum bending moment should be assumed to occur both at the stiffeners and, with opposite sign, midway between them.

The stresses in the flanges due to the loads P_{fm} and P_{fq} and moment M_f are calculated by the program and given in section F4 of the output as (i) the maximum compressive stress in the flanges, f_{fc} , (ii) the maximum tensile stress in the flanges, f_{ft} , and (iii) the maximum stress at the innermost rivet or bolt line of the flanges, f_{fr} . The maximum stress in the compression flange, f_{fc} , occurs in the innermost fibre of the flange at the stiffeners, and the maximum stress in the tension flange, f_{ft} , occurs in the innermost fibre of the flange midway between the stiffeners. The value of f_{fr} is derived on the assumption that the stress concentration effects due to flexibility of the flanges or the presence of rivet holes may be neglected. It should be noted that the program does not consider the area lost owing to rivet holes when calculating the cross-sectional area of the flange, A_f .

1.4.4.1 Strength of the flange attachments

The rivets joining the plate to the flanges must transmit the shear complementary to that portion of Q_a carried by the plate in pure shear, together with the components of the diagonal tension field parallel to and normal to the flange. The resultant loading has a maximum value, P_{fpr} , in the region of the stiffeners. The program calculates the value of this load based on data from Figure 14 of Derivation 9 and gives it in section F5 of the output.

1.4.5 Post-buckled Stiffness of Panel

With the development of the diagonal tension field the overall in-plane stiffness of the panel is reduced. The reduction is represented by the secant modulus, G_S , and the tangent modulus, G_T , which are respectively the ratio of the nominal shear stress in the plate to the total shear strain of the panel, and the rate of increase of nominal shear stress in the plate with the total shear strain of the panel. The values of G_S and G_T are calculated by the program on the basis of data given in Figures 15 to 17 and Figures 18 to 20, respectively, of Derivation 9. These values are given in section P8 of the output. The program also calculates the panel shear deflection over the pitch of the stiffeners and the value for this is given in section P9 of the output.

1.4.6 Multiple Panel Analysis

In addition to the calculation of the stresses, loads and stiffnesses given in Sections 1.4.1 to 1.4.5 for a single panel, the program can also be used to assess the comparative weight of a number of panels.

For each panel the analysis provides

- (i) the distance between the flange centroids, h_e ,
- (ii) the panel weight per unit length, W , excluding the weight of the flanges,
- (iii) the minimum plate reserve factor, R_{min} , chosen from R_{dg} , R_{dl} , R_p and R_{pfr} ,
- (iv) the minimum stiffener buckling reserve factor, $F_{s min}$, chosen from $F_{o,x}$, F_{pir} , F_{sd} , F_{sfc} , $F_{s,oT}$, F_{sz} and F_T ,
- (v) the minimum flange reserve factor, $F_{f min}$, chosen from F_{fc} , F_{fr} and F_{ft} ,
- (vi) the ratio of nominal applied shear stress to initial plate buckling stress, q_a/q_b and
- (vii) the buckling stress coefficient, K .

These quantities are listed in a table for the specified number of panels in order of increasing weight/unit length, W . The analyses performed to obtain the stresses are those given in Sections 1.4.1 to 1.4.5. The calculation of the weight of the panel is approximate and intended to be used for comparative purposes only. It is based on Equations (1.1) to (1.3) which ignore the weight of the flanges, fasteners, cut-outs and other details. If a pad is present then it is taken into account when calculating the stiffener sectional properties and the weight per unit length of the panel, with the pad material properties assumed to be the same as those of the stiffener.

Panel weight per unit length, W ,

$$W = (W_s + W_p)/b, \quad (1.1)$$

where stiffener weight, $W_s = A_s h_e \rho_s$ and (1.2)

plate weight, $W_p = b a t_p \rho_p$. (1.3)

1.5 Derivation

1. ESDU The strength of struts. Data Item No. Struct 01.01.01, ESDU International plc, London, November 1943.
2. KUHN, P.
PETERSON, J.P.
LEVIN, L.R. A summary of diagonal tension. Part 1 – Method of analysis. Technical Note No. NACA TN-2661, National Advisory Committee on Aeronautics, Langley Field, Va, USA, October 1951.
3. KUHN, P. *Stresses in aircraft and shell structures*. Chapter 3, pp. 47-94. Published by McGraw-Hill Book Co. Inc., New York, 1956.
4. TIMOSHENKO, S.P.
GERE, J.M. *Theory of elastic stability*. Chapter 5, pp. 212-250. Second edition. McGraw-Hill Book Co. Inc., USA, 1961.
5. ESDU Local instability of struts with flat sides. Data Item No. Struct 01.01.08, ESDU International plc, London, October 1962.
6. ESDU Buckling in compression of sheet between rivets. Data Item No. Struct 02.01.08, ESDU International plc, London, October 1962.
7. ESDU Flat panels in shear. Buckling of long panels with transverse stiffeners. Data Item No. Struct 02.03.02, ESDU International plc, London, December 1964.
8. ESDU Generalisation of smooth continuous stress-strain curves for metallic materials. Data Item No. 76016, ESDU International plc, London, August 1976.
9. ESDU Flat panels in shear – Post-buckling analysis. Data Item No. 77014, ESDU International plc, London, July 1977.
10. ESDU Shear centre and primary warping constant for lipped and unlipped channel and Z-sections. Data Item No. 77023, ESDU International plc, London, September 1977.
11. ESDU Information on the use of Data Sheets of the 00.07 series. Data Item No. Struct 00.07.00, ESDU International plc, London, January 1983.
12. ESDU Flexural and torsional-flexural buckling of thin-walled open section struts. Data Item No. 89007, ESDU International plc, London, March 1989.
13. YOUNG, W.C. *Roark's formulas for stress and strain*. Chapter 9, pp. 366-369. Sixth edition. Published by McGraw-Hill Book Co. Inc., USA, 1989.
14. ESDU Struts: strength under flexural, local and inter-rivet buckling. Data Item No. 90002, ESDU International plc, London, August 1990.
15. DEV-ANAND, N. Method employed by computer program ESDUpac A0205 to interpolate between and extrapolate the curves in Data Item No. 77014 and the associated Data Items. ESDU Memorandum No. 118, March 2002.