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Sound radiation from orthotropic rectangular plates

Part I: Estimation by asymptotic method

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SOUND RADIATION FROM ORTHOTROPIC RECTANGULAR PLATES

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SOUND RADIATION FROM ORTHOTROPIC RECTANGULAR PLATES

1. NOTATION

		<i>Units</i>	
		<i>SI</i>	<i>British</i>
A_{mn}	amplitude of transverse deflection of plate	m	in
a	length of plate in x-direction	m	in
a_{min}	smaller value of a or b	m	in
b	length of plate in y-direction	m	in
c	see Sketch 8.11		
c_0	velocity of sound in air	m/s	in/s
c_L	longitudinal wave velocity	m/s	in/s
D	bending stiffness of isotropic plate	N m	lbf in
$D(\theta)$	bending stiffness of orthotropic plate	N m	lbf in
D_{ij}	elements of bending stiffness matrix	N m	lbf in
D_x	bending stiffness in x-direction	N m	lbf in
D_y	bending stiffness in y-direction	N m	lbf in
D_{xy}	torsional stiffness of plate about x- and y-axes, Equation (7.1)	N m	lbf in
d	distance along sine wave corrugations in one wavelength	m	in
E	Young's modulus for material of isotropic plate	Pa	lbf/in ²
E_L	Young's modulus for wood in longitudinal direction	Pa	lbf/in ²
E_R	Young's modulus for wood in radial direction	Pa	lbf/in ²
E_T	Young's modulus for wood in tangential direction	Pa	lbf/in ²
E_{cx}	Young's modulus of core material of sandwich	Pa	lbf/in ²
E_m	Young's modulus of matrix of lamina	Pa	lbf/in ²
E_x	Young's modulus in x-direction for orthotropic plate	Pa	lbf/in ²
E_y	Young's modulus in y-direction for orthotropic plate	Pa	lbf/in ²

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E_1	Young's modulus in 1-direction	Pa	lbf/in ²
E_2	Young's modulus in 2-direction	Pa	lbf/in ²
E_{1f}	Young's modulus of fibre in direction of fibres	Pa	lbf/in ²
E_{2f}	Young's modulus of fibre in direction perpendicular to fibres	Pa	lbf/in ²
e	ratio E_x/E_y		
$\bar{F}^2$	mean squared value of applied force, Equation (9.2)	N ²	lbf ²
f	frequency	Hz	Hz
f_a	amplitude of corrugations	m	in
f_c	critical frequency of wave coincidence	Hz	Hz
f_{c1}	critical frequency of wave coincidence (lower value), Equation (7.21)	Hz	Hz
f_{c2}	critical frequency of wave coincidence (upper value), Equation (7.22)	Hz	Hz
f_k	upper limit of frequency	Hz	Hz
f_{mn}	natural frequencies of plate	Hz	Hz
f_0	lower limit of frequency	Hz	Hz
G	shear modulus for material of isotropic plate	Pa	lbf/in ²
G_{LR}	shear modulus for wood (shear in the plane formed by longitudinal and radial directions, see Sketch 8.1)	Pa	lbf/in ²
G_m	shear modulus of matrix of lamina	Pa	lbf/in ²
G_{xy}	shear modulus in xy plane orthotropic plate	Pa	lbf/in ²
G_{12}	shear modulus in 1,2-plane	Pa	lbf/in ²
G_{12f}	shear modulus for fibre in 1,2-plane	Pa	lbf/in ²
H	height of stiffeners	m	in
h	thickness of plate	m	in
h_e	equivalent thickness of orthotropic plate	m	in
I	second moment of area	m ⁴	in ⁴
I_{cx}	second moment of area per unit width of core of sandwich about y-axis	m ³	in ³
k	acoustic wavenumber	m ⁻¹	in ⁻¹

k_b	bending wave vector in plate	m^{-1}	in^{-1}
k_{bx}	component of bending wave vector in x -direction	m^{-1}	in^{-1}
k_{by}	component of bending wave vector in y -direction	m^{-1}	in^{-1}
k_{mn}	wavenumber vector = $[m\pi/a, n\pi/b]$	m^{-1}	in^{-1}
L	perimeter of plate	m	in
M	number of laminae		
m	modal number in x -direction		
N	number of modes		
$N_{\Delta\omega}$	number of modes in frequency band $\Delta\omega$		
n	modal number in y -direction		
n_d	modal density	modes/ (rad/s)	modes/ (rad/s)
P_0	sound power output from plate	W	lbf in/s
Q	correction factor for radiation efficiencies for clamped plate		
Q_{ij}	elements of reduced stiffness matrix	Pa	lbf/in ²
$\bar{Q}_{ij}$	elements of transformed reduced stiffness matrix	Pa	lbf/in ²
S	surface area of plate	m ²	in ²
V_f	volume fraction of fibre in lamina		
V_m	volume fraction of matrix in lamina		
St_c	non-dimensional number = $2\pi f_c S / (c_0 L)$, defined in Equation (5.4)	—	—
s	spacing between ribs or corrugations	m	in
t	time	s	s
t	thickness of stiffeners	m	in
t_c	thickness of corrugated core	m	in
$\bar{v}^2$	mean squared velocity of surface of plate	m ² /s ²	in ² /s ²
x, y, z	coordinates of plate		
z_k	coordinate of edges of laminae	m	in
$\bar{z}$	position of neutral axis	m	in

w	transverse deflection of plate (in z -direction)	m	in
w_h	mass per unit area of honeycomb core	kg/m ²	*
α	dimensionless wavenumber in x -direction		
β	dimensionless wavenumber in y -direction		
γ	dimensionless frequency parameter, see Equation (6.16)		
γ_{12}	shear strain		
ε	boundary constraint factor		
ε_x	direct strain in x -direction		
ε_y	direct strain in y -direction		
ε_1	direct strain in 1-direction		
ε_2	direct strain in 2-direction		
η	dimensionless loss factor of plate		
ζ_1	Halpin-Tsai factor for E_2 estimation		
ζ_2	Halpin-Tsai factor for G_{12} estimation		
θ	angle of bending wave propagation in plate	rad	rad
λ_b	wavelength for panel motion	m	in
ν	Poisson's ratio for material of isotropic plate		
ν_{12}	Poisson's ratio of lamina for load in 1-direction, contraction in 2-direction		
ν_{21}	Poisson's ratio of lamina for load in 2-direction, contraction in 1-direction		
ν_{12f}	Poisson's ratio for fibre for load in 1-direction, contraction in 2-direction		
ν_{LR}	Poisson's ratio for wood, longitudinal load, radial contraction		
ν_{RL}	Poisson's ratio for wood, radial load, longitudinal contraction		
ν_m	Poisson's ratio of matrix		
ν_{xy}	Poisson's ratio for load in x -direction, contraction in y -direction		
ν_{yx}	Poisson's ratio for load in y -direction, contraction in x -direction		

ψ	transformation angle, angle between x -axis and 1-axis (angle between fibre and plate edge)	rad	rad
ρ	density of plate	kg/m ³	*
ρ_0	density of air	kg/m ³	*
σ	radiation efficiency		
$\tilde{\sigma}, \bar{\sigma}$	average radiation efficiencies of plate (radiation efficiency may be expressed in the decibel scale (db) which means $10 \log_{10}(\tilde{\sigma})$ or $10 \log_{10}(\bar{\sigma})$)		
$\tilde{\sigma}_{is}$	average radiation efficiency of isotropic plate		
σ_1	direct stress in 1-direction, see Section B.2.1, Appendix B	Pa	lbf/in ²
σ_2	direct stress in 2-direction	Pa	lbf/in ²
ω	angular frequency	rad/s	rad/s
ω_{mn}	natural frequencies of plate in bending	rad/s	rad/s
ω_0	centre frequency of bandwidth $\Delta\omega$	rad/s	rad/s
1, 2, 3	principal material coordinates		

* In British units, for the system to be coherent the mass in lb must be divided by the constant 386.4. In the case of a density known, for example, to be 0.097 lb/in³ the value to be used should be $0.097/386.4 = 2.510 \times 10^{-4}$ lbf s² in⁻⁴, where lbf is the unit of force.

2. INTRODUCTION

The presentation of information on average radiation efficiencies of orthotropic plates is the aim of this Item. Two estimation methods are provided. In the first method, design curves, which enable approximate efficiencies to be estimated for a range of plate dimensions and material properties, are described in Section 5 and shown as Figures 1 to 13. In the second method, more accurate values of the average radiation efficiencies may be obtained by using the computer programs described in Section 11 and included with this Item. The computer programs may be used for plate dimensions outside the range specified for the design curves. Both methods have to be interpreted in relation to the limitations listed in Section 10. The methods used in this Item for estimating the sound radiation efficiency of rectangular plates have been validated in Part II, issued separately as ESDU 02010.

Structures which vibrate produce sound and there is frequently a need to estimate the sound power radiated by different elements in a structure. Plates or panels are amongst the most commonly occurring structural elements and it is the radiation of sound from rectangular plates which is the subject of this Data Item. Sound radiation from baffled, rectangular, isotropic plates has already been considered in ESDU 00002 (Derivation 9). That Item is restricted to isotropic plates, where the material properties, such as Young's modulus, are independent of the direction in which they are measured. This Item enables the work on sound radiation to be extended to orthotropic plates and to sandwich or ribbed plates which have equivalent orthotropic properties.

The extent of the sound radiation is often characterised by the radiation efficiency or radiation ratio as it is sometimes called (References 24 and 40). The radiation efficiency, σ , is defined as (Derivations 4 and 9, Reference 37):

$$\sigma = \frac{P_0}{\rho_0 c_0 S \langle v^2 \rangle}, \quad (2.1)$$

where P_0 is the sound power radiated by the plate, ρ_0 is the density of the surrounding air, c_0 is the velocity of sound in air, and $\langle v^2 \rangle$ is the mean square velocity of the plate, averaged in time over the surface S of the plate. The radiation efficiency may be regarded as the ratio of the sound power radiated by the plate to the sound power radiated by a rigid piston in an infinite baffle with the same area as the plate and the same mean square velocity.

The normal modes of the plate are characterised by modal numbers m and n , and these integers are required in order to determine the natural frequencies of the plate (Reference 28). Similarly, the modal shapes depend on m and n and, for a plate simply-supported around the edges, take the form of a combination of sine terms. For each mode of vibration of the plate there is a radiation efficiency which may be referred to as a modal radiation efficiency for specified values of m and n (Derivation 4).

The motion of a plate which is vibrating because of a particular type of excitation, such as an impact at a point on the plate, comprises many modes of vibration. Consequently, it is more useful to consider an average radiation efficiency, $\bar{\sigma}$, which accounts for sound radiation from all the excited modes of vibration of the plate. Whilst the average radiation efficiency provides a good indication of how a plate radiates sound, it is the sound power radiated by the plate that many designers of structures require. The sound power depends upon the type of excitation of the plate and on other factors, including the average radiation efficiency and the mechanical damping of the plate. This Data Item considers how sound power can be estimated, particularly with regard to sandwich plates in Section 9.

3. BACKGROUND

The starting point for the calculation of the radiation efficiency is the Rayleigh radiation integral (Reference 11) which gives the sound pressure in the far field produced by a moving element on a surface. Maidanik (Derivation 2) and, subsequently, Wallace (Derivation 4), Gomperts (Reference 30) and Leppington *et al.* (Derivation 5) applied Rayleigh's approach to the case of a simply-supported (or hinged) plate in an infinite baffle for a particular mode of vibration of the plate. Both Wallace and Gomperts provided graphically several examples of radiation efficiencies in terms of a frequency function. Both developed Rayleigh's integral to estimate the sound power, P_0 , radiated by the plate into a half space. The radiation efficiency σ relates P_0 to $\langle v^2 \rangle$, the mean square velocity of the plate, averaged over the surface area of the plate. The radiation efficiency of the plate at frequencies above the coincidence frequency is of the order of one for all modes of vibration, indicating that the entire plate is radiating sound. Below the coincidence frequency the plate is a less effective radiator and the radiated sound power depends upon the mode of vibration of the plate. At low frequencies the edges alone may be responsible for the radiation of sound and at much lower frequencies the cancellation effects in the centre of the plate are more complete and the corners are the only effective sound sources (see Section 7 of Derivation 9). Wallace and Gomperts obtained identical results for the radiation efficiencies of the simply-supported, baffled plate.

Leppington *et al.* (Derivation 5) used a mathematically different approach from Wallace or Gomperts and their method allowed asymptotic evaluation to be made in the ranges above, below and in the region of wave coincidence for a given mode of vibration of the plate. The approximate equations so derived are essentially high frequency approximations in the ranges considered. In a real situation a plate will be vibrating simultaneously in several modes and a radiation efficiency which averages the effects of all modes is practically of great significance. The advantage of the approximate method of Leppington *et al.* is that it could be used to obtain formulae for the modally-averaged radiation efficiencies without the lengthy, numerical integration which would otherwise be required. Their average radiation efficiency is strictly valid at high frequencies, or where there are sufficient modes of vibration within a specified frequency band (Derivation 9). Their method may be applied in models of Statistical Energy Analysis (Reference 48). Using Statistical Energy Analysis, Maidanik (Derivation 2) had earlier developed expressions for the average radiation efficiency of a simply-supported plate. His expression for the radiation efficiency for the condition below coincidence leads to values similar to those obtained by Leppington *et al.*

Radiation of sound from plates with boundary conditions other than simply-supported has been less often studied. Nikiforov (Reference 20) was able to estimate the sound power generated by a plate, whether it was clamped at the edges, simply-supported, or free. Although he derived no expressions for the radiation efficiency, he was able to estimate that the sound power for the clamped plate was twice that for the simply-supported plate in the region below wave coincidence. Gomperts (Reference 30) looked at a variety of combinations of supports, involving guided, free and clamped, but did not consider the case where all sides are clamped. Even with two sides clamped and the two other edges free or simply-supported the radiation efficiency was increased relative to the entirely simply-supported case. Leppington *et al.* (Derivation 7) attempted a solution of the problem by an indirect approach in which they devised a series of corrections to be applied to the radiation efficiencies of the single mode of vibration for a simply-supported, baffled plate. Their analysis allowed a range of end conditions for the plate from simply-supported to clamped. In their method a moment is in effect applied along the edges, which can vary from zero (for the simply-supported plate) to a finite value (for the clamped plate). To account for this variation, Leppington *et al.* (Derivation 7) introduced an edge constraint factor ϵ which varied from 0 (for the clamped plate) to ∞ for the simply-supported plate. An infinite series of resonant frequencies exists for a given value of ϵ . In ESDU 00002 on isotropic plates, correction factors, which can be applied to the radiation efficiencies of simply-supported plates for different values of ϵ , are provided in a graphical form. Hence the radiation efficiencies of plates with a range of boundary conditions (including the fully-clamped condition) can be estimated for a given mode of vibration. The same correction factors have been applied to the modally-averaged radiation efficiencies of the simply-supported plates (Derivations 9 and 10).

Thus ESDU 00002 provides the means of estimating the averaged radiation efficiencies of plates supported in a wide range of different ways. In practical cases it is not normally known how the plate is supported. In some situations a vibration test may have been carried out on the plate so that the natural frequencies, and the integer numbers which define the modal shape, will be known from the experiment. The computer program (ESDUpac A0002), issued with the Item, provides a method for estimating ε for situations in which the natural frequencies and modal numbers are known. The value of ε obtained may then be used with the design charts to estimate the modally-averaged radiation efficiency. Earlier, Lomas and Hayek (Reference 31) had developed a similar approach to Leppington *et al.* in which a variable edge constraint was allowed, but their method was not pursued as far as the estimation of radiation efficiencies for clamped plates.

To obtain the modally-averaged radiation efficiencies for the clamped plate (or for a specified boundary condition) there are, based on the method of Leppington *et al.*, two possible approaches. Firstly, an integration could be carried out which involves all the corrected modal radiation efficiencies in a specified frequency band. Secondly, it could be assumed that the correction factors are constant for all modes and hence only one correction factor is required to be applied to the average radiation efficiency for the simply-supported plate. Although not suggested by Leppington *et al.* such an assumption would appear to be reasonable at higher frequencies and for the case when ε is small, *i.e.*, for boundary conditions close to the fully-clamped case.

In this Item work on sound radiation from isotropic plates has been extended to orthotropic plates. The vibration of orthotropic plates has been considered in detail in the scientific literature, see, for example, Hearmon (Reference 18), Leissa (Reference 23), Dickinson and di Blasio (Reference 34), Yu and Cleghorn (Reference 42) or Dalaei and Kerr (Reference 46). See also ESDU 83036 and 90016 for methods of estimating the natural frequencies of orthotropic plates (References 32 and 41). Sound radiation from ribbed plates was first considered by Maidanik (Derivation 2) who used Statistical Energy Analysis (SEA) to estimate sound radiation from the ribs on the plate. His method is not easy to apply and his ribbed plate system is only approximately orthotropic. Hugin and Ohlrich (Derivation 10) have also used SEA by combining the approximate formulae of Leppington *et al.* (Derivation 5) with the modal density and bending stiffness of the orthotropic plate. However, their approach is different from the one adopted in this Item.

4. THE NATURE OF ORTHOTROPIC PLATES

The orthotropic plates which are considered in this Item are special types of anisotropic structures. In the case of orthotropic plates the material properties, in particular the Young's modulus, are different in two directions at right angles. Steel sheet produced by a rolling mill is an example of a homogeneous, orthotropic material (Reference 38). Because of the method of manufacture, the stiffness of the steel in the direction of the rollers is slightly different from that in the transverse direction. Wood is also an example of a homogeneous, orthotropic material (Reference 12). Such materials as wood, or crystals like quartz, have been referred to as 'naturally orthotropic' (Reference 29)

The present Item is concerned with specially orthotropic, rectangular plates, *i.e.*, plates which have edges aligned with the principal material axes, see Appendix B, Derivations 6 or 8. In terms of the xy -coordinate system created by the edges of the rectangular plate, the material constants are E_x and E_y , Young's moduli in the x - and y -directions, respectively, the shear modulus, G_{xy} , and the Poisson's ratio, ν_{xy} , for a load applied in the x -direction and a contraction in the y -direction. The Poisson's ratio, ν_{yx} , is obtained from the relationship:

$$E_x/E_y = \nu_{xy}/\nu_{yx} \quad (4.1)$$

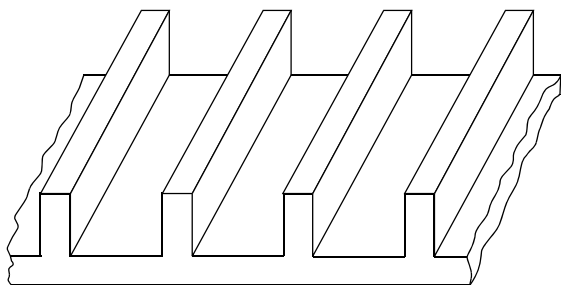
Composite plates, such as printed circuit boards and glass/aluminium reinforced laminates (GLARE) (Reference 33) may have their strengthening fibres in two directions at right angles and would fall within the category of specially orthotropic laminates considered in this Item. Specially orthotropic laminates have either a single layer of specially orthotropic material or multiple specially orthotropic layers symmetrically distributed about the laminate middle surface. In laminated plates each layer has fibres in a matrix, *e.g.* glass or carbon fibres in an epoxy matrix. The constituents of the composite plate, such as the fibres or the matrix, have their own material properties which can be used to calculate the overall properties of the composite. As the overall properties are not the same as those of the constituents, they are referred to as the effective (Derivation 8) or the apparent material properties (Derivation 6), such as the effective Young's moduli and the effective Poisson's ratios. The properties of the laminates discussed in this Item are derived on the basis of classical laminate theory (Derivation 8, Reference 43).

Apart from laminates, the other group of plates, which can be treated as orthotropic plates, includes stiffened or corrugated plates (Derivation 2, Reference 27). Ribbed, stiffened or corrugated plates exhibit different elastic properties in different directions, even though the material of the plate may be entirely isotropic. Such plates may be regarded as 'structurally or technically orthotropic' (Reference 29).

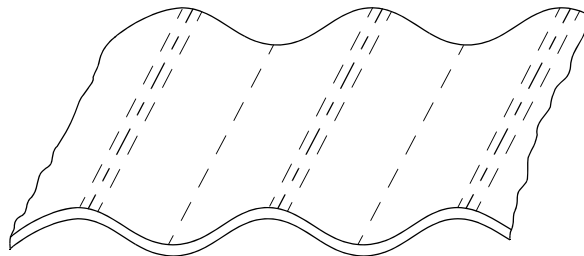
Structurally orthotropic plates, such as ribbed or stiffened plates, may be replaced by an equivalent orthotropic plate of constant thickness which has the same stiffness characteristics. The method of elastic equivalence can be applied, provided that several conditions are fulfilled. For a ribbed plate, for example, to be regarded as equivalently orthotropic the spacing of the ribs must be small relative to the bending wavelength to ensure homogeneity of stiffness. (See Section 10 for a further discussion.) In the case of separate stiffeners attached to a plate, it is assumed that a perfect bond exists between the plate and the stiffeners.

Configurations which may be regarded as equivalent orthotropic plates are shown in Sketch 4.1: (a) ribbed plate, (b) 'circular' corrugated, (c) stiffened corrugated, (d) trapezoidal corrugated and (e) stiffened honeycomb. For the integrally stiffened plate shown in Sketch 4.1a, in which the ribs and plate are made of the same material, the plate is more flexible for bending about an axis parallel with the ribs than about the axis at right angles to the ribs. In the case of stiffened plates the material properties may be measured by loading the plate, as described in References 15 to 17, 22 and 25.

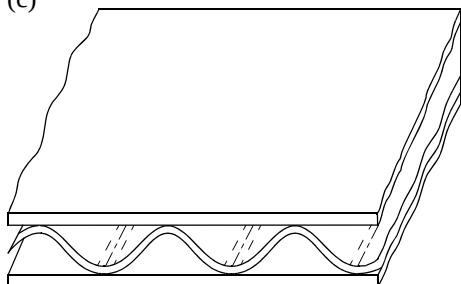
(a)



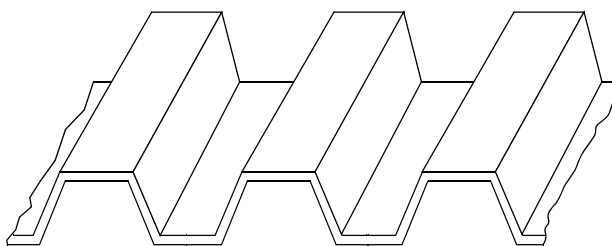
(b)



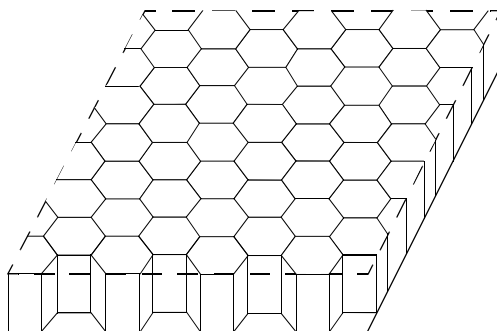
(c)



(d)



(e)



(Honeycomb panel between plates)

Sketch 4.1 Equivalent orthotropic plates: (a) ribbed plate, (b) circular corrugated, (c) stiffened corrugated, (d) trapezoidal corrugated and (e) stiffened honeycomb

5. DESIGN CURVES FOR AVERAGE RADIATION EFFICIENCIES

The aim of the design curves, described in this Section, is to provide a relatively simple method of obtaining average radiation efficiencies in one-third octave bands without recourse to the computer programs. The design curves and the computer programs are based on the same theory, see Section 7. The design curves provide a useful indication of how the average radiation efficiencies are influenced by the dimensions and material properties of the plate.

Idealised design curves for the average radiation efficiencies of isotropic, rectangular plates have been presented by Ver and Holmer (Reference 24) and by Norton (Reference 40), mostly based on the work of Maidanik (Derivation 2), and intended to cover all sizes of plates.

The design curves presented in this Item refer to isotropic and orthotropic plates of different sizes. The plates are simply-supported at all edges. The design curves for isotropic plates, and for orthotropic plates with a small degree of orthotropy, are shown in Figure 1, where the average radiation efficiency, $\bar{\sigma}$, in the form $10\log\bar{\sigma}$ (in dB) is plotted against the non-dimensional frequency parameter, f/f_c . The frequency, f_c , is the critical frequency of wave coincidence and for an isotropic panel is given by:

$$f_c = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho h}{D}}, \quad (5.1)$$

where c_0 is the velocity of sound in air, ρ is the density of the panel material, h is the thickness of the panel and the bending stiffness, D , is

$$D = \frac{Eh^3}{12(1 - \nu^2)} \quad (5.2)$$

in terms of the Young's modulus or modulus of elasticity, E , of the material and the Poisson's ratio, ν . The coincidence frequency is the frequency at which the phase velocity of the bending waves in the plate equals the phase velocity of sound in air, c_0 . For more explanation, see Derivation 9.

The design curves for orthotropic plates are shown in Figures 2-13 for the simply-supported plates. Four values of D_x/D_y are considered (2.0, 4.0, 6.0 and 8.0) and for each value of D_x/D_y there are three values of D_{xy}/D_y of 0.5, 1.0 and 2.0.

In general the design curves presented in this Item are based on the relationship between six non-dimensional variables:

$$\bar{\sigma} = F\left(St_c, \frac{D_x}{D_y}, \frac{D_{xy}}{D_y}, \frac{a}{b}, \frac{f}{f_{c2}}\right), \quad (5.3)$$

where D_x , D_y and D_{xy} are the bending stiffnesses of the plate and St_c is a non-dimensional parameter, at the coincidence frequency, and is defined as:

$$St_c = \frac{2\pi f_{c2}}{c_0} \cdot \frac{S}{L}, \quad (5.4)$$

where S is the surface area of the plate and L is its perimeter. The frequency, f_{c2} , is a coincidence frequency

of the orthotropic plate:

$$f_{c2} = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho h}{D_y}} \quad (5.5)$$

and is equal to f_c for isotropic plates, see Equation (5.1).

The average radiation efficiency in the one-third octave band, $\bar{\sigma}$, is presented in the design graphs as a function of the dimensionless frequency ratio f/f_{c2} and the four parameters St_c , D_x/D_y , D_{xy}/D_y and the aspect ratio of the plate, a/b . The ratio D_x/D_y is a measure of the orthotropy of the plate. The bending stiffnesses, D_x and D_y , are described in more detail in Section 7. It is shown in Section 8 that D_x and D_y may be estimated from formulae for particular orthotropic plates. The stiffness, D_{xy} , is also often estimated from formulae.

For an isotropic plate $D_x = D_y = D_{xy} = D$. The design curves of Figure 1 apply to an isotropic plate, but can also be used for plates with a small degree of orthotropy in which $D_x = D_y$ and $0.8D < D_{xy} < 1.2D$. For orthotropic plates for a given ratio of D_x/D_y , three sets of curves are presented for values of the ratio D_{xy}/D_y of 0.5, 1 and 2.

The radiation efficiencies depend quite strongly on St_c and in the design curves for a given ratio of D_x/D_y there are several curves for constant values of St_c . Of the plate dimensions the thickness, h , influences the St_c parameter via f_{c2} . The aspect ratio, a/b , has a less significant effect and each design curve can be used for a range of aspect ratios from 0.2 to 5.

In the use of the design curves, interpolation is required for intermediate values of D_x/D_y , D_{xy}/D_y and St_c . Accuracy should be within ± 2 dB for $10 \log \bar{\sigma}$.

The design curves apply to simply-supported plates. For plates which are clamped at the edges, 2 dB should be added to the values of $10 \log \bar{\sigma}$ in the range $f < f_{c2}$.

5.1 Example of the Use of the Design Curves

For an equivalent orthotropic plate, $D_x = 180$, $D_y = 95$ and $D_{xy} = 92$ N m, the density of the material is 2710 kg/m^3 , $a = b = 0.8$ m and the thickness h is 3 mm. Obtain the average radiation efficiencies in the one-third octave bands from 2000 Hz to 10 000 Hz.

The upper value of the coincidence frequency, f_{c2} , is obtained from Equation (5.5).

$$f_{c2} = \{344^2/(2\pi)\} \sqrt{2.71 \times 3/95} = 5509 \text{ Hz.}$$

$$\frac{D_x}{D_y} = 1.89 \text{ and } \frac{D_{xy}}{D_y} = 0.97.$$

The non-dimensional parameter St_c is given by:

$$St_c = 2\pi \times 5509 \times 0.8^2 / (344 \times 3.2) = 20.1.$$

The nominal centre frequencies of the one-third octave bands are 2000, 2500, 3150, 4000, 5000, 6300, 8000 and 10000 Hz. The values of the ratio f/f_{c2} (i.e., $f/5509$) corresponding to the centre frequencies are 0.363, 0.454, 0.572, 0.726, 0.908, 1.144, 1.452 and 1.815, respectively.

Figure 3, for which $D_x/D_y = 2$ and $D_{xy}/D_y = 1$, may be used with the curve marked $St_c = 20$ to obtain estimates of $\bar{\sigma}$ with sufficient accuracy. Greater accuracy could be obtained by linear interpolation between Figure 3 and Figure 1 ($D_x/D_y = 1$).

The estimates of $10 \log \bar{\sigma}$ are -13.1, -11.5, -7.9, -2.0, 2.8, 3.8, 2.0 and 1.5 dB.

The final results are:

$\frac{1}{3}$ -octave centre frequency (Hz)	2000	2500	3150	4000	5000	6300	8000	10000
$\bar{\sigma}$	0.049	0.071	0.16	0.63	1.91	2.40	1.58	1.41

6. CALCULATION OF RADIATION EFFICIENCY

The sound radiated from the plate is a consequence of the vibration of the plate. Some background information is presented in Section 6.1. The theory for the vibration of the plates is based on linear elasticity and small deflections.

6.1 Vibration of Rectangular Plates

The rectangular plate which will be considered is simply-supported at all four edges, lies in the xy plane with the origin of the axes at its centre, and has sides of lengths a and b in the x - and y -directions. The thickness of the plate is h . The material of the plate has a density, ρ , Young's modulus, E , and Poisson's ratio, ν , and these are the normal constants of the material. For an isotropic plate $E = E_x = E_y$ and $\nu = \nu_{xy} = \nu_{yx}$. The bending stiffness, D , for an isotropic plate is a function of E and is given by Equation (5.2).

Since the plate has finite dimensions, the standing waves in the plate are only considered with the associated natural frequencies and spatial distributions of the vibration amplitudes, known as the natural modes of the plate. Among the different types of structural waves that can form standing waves in the plate, the bending (or flexural) waves are responsible for sound radiation.

The expression for the displacement of the vibrating plate, as a function of x , y and time, t , should be in the form:

$$w_{mn} = A_{mn} \sin(m\pi x/a) \sin(n\pi y/b) \sin(\omega t), \quad (6.1)$$

where A_{mn} is the amplitude of the transverse deflection of the plate. The vector $\mathbf{k}_{mn} = [m\pi/a, n\pi/b]$ denotes the mode wavenumber or plate wavenumber vector. Each of the functions w_{mn} satisfies the boundary conditions and is the standing wave solution of the bending wave equation for the isotropic plate:

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \quad (6.2)$$

if

$$|\mathbf{k}_{mn}| = |\mathbf{k}_b|, \quad (6.3)$$

where $\mathbf{k}_b$ is the bending wavenumber vector at frequency ω with the magnitude

$$k_b = |\mathbf{k}_b| = \omega^{0.5} (\rho h / D)^{0.25}. \quad (6.4)$$

The shape of the vibrating surface depends upon the modal integers m and n , corresponding to the x - and y -directions, respectively. Both the integers m and n increase from 1, as is the case in Derivations 5 and 9, and References 32 and 41. It should be noted, however, that some writers, such as Hearmon (Reference 18) and Warburton (Reference 14), use integer series that start at 2 and represent the number of nodal lines in the plate that exist during free vibration, including the edges as nodal lines. The Equation (6.3) defines for each mode (m, n) the eigenfrequency, ω_{mn} , or natural frequency of the plate in bending:

$$\omega_{mn} = \pi^2 \{D/(\rho h)\}^{0.5} (m^2/a^2 + n^2/b^2), \quad (6.5)$$

where $\omega_{mn} = 2\pi f_{mn}$ rad/s and f_{mn} are the natural frequencies in Hz. The bending stiffness, D , is given by Equation (5.2)

6.2 Radiation Efficiencies for Plate Modes

This section refers to the modal sound radiation efficiencies which are subsequently applied in the calculation of the averaged sound radiation efficiencies of isotropic as well as orthotropic plates. In ESDU 00002 the radiation efficiencies for individual modes have been presented based on the method of Wallace (Derivation 4). In this Item the approximate formulae of Leppington *et al.* (Derivation 5) have been used, as they can be applied to models based on Statistical Energy Analysis. The formulae of Leppington *et al.* are for asymptotic conditions in the sense that the exact expressions are investigated as ka and kb tend to infinity, where k is the acoustic wavenumber. The modal radiation efficiencies for plates are considered as functions of the dimensionless components of $\mathbf{k}_{mn}$, the plate wavenumber vector. The dimensionless components α and β are:

$$\alpha = m\pi/(ka) \quad (6.6)$$

and

$$\beta = n\pi/(kb). \quad (6.7)$$

The radiation efficiency is defined for different regions in the wavenumber vector space. Below coincidence there are formulae which cover the ranges of α and β . Wave coincidence occurs when $\alpha^2 + \beta^2 = 1$. Thus:

$$\{m^2\pi^2/(k^2a^2) + n^2\pi^2/(k^2b^2)\} = 1. \quad (6.8)$$

Below coincidence ($\alpha^2 + \beta^2 > 1$) the formulae from Derivation 5 are stated for the different ranges of α and β .

For $\alpha > 1$ and $\beta > 1$,

$$\sigma = (\pi k^2 \alpha \beta ab)^{-1} \left\{ \frac{2\alpha\beta(2p^4 + p^2 - 3\alpha^2\beta^2)}{p^4(\alpha^2 - 1)(\beta^2 - 1)} + \frac{2p^4 + p^2 + 3\alpha^2\beta^2}{p^5} \ln \left| \frac{\alpha\beta + p}{\alpha\beta - p} \right| \right\}, \quad (6.9)$$

where $p^2 = \alpha^2 + \beta^2 - 1$. The panel modes which radiate in this region are referred to as corner modes (Derivations 2 and 9, Reference 24).

For $\alpha > 1$ and $\beta < 1$,

$$\sigma = \frac{2\alpha^2 + \beta^2 - 1}{ka\alpha(\alpha^2 + \beta^2 - 1)^{3/2}}. \quad (6.10)$$

In the region $\alpha > 1$, $\beta < 1$ the radiation is from edge modes with the edges parallel to the y-axis applicable.

For $\alpha < 1$ and $\beta > 1$,

$$\sigma = \frac{2\beta^2 + \alpha^2 - 1}{kb\beta(\alpha^2 + \beta^2 - 1)^{3/2}}. \quad (6.11)$$

In this case the radiation is from edge modes which are parallel with the x-axis.

For $\alpha^2 + \beta^2 > 1$, $\alpha < 1$ and $\beta < 1$,

$$\sigma = \frac{(2\alpha^2 + \beta^2 - 1)/(ka\alpha) + (2\beta^2 + \alpha^2 - 1)/(kb\beta)}{(\alpha^2 + \beta^2 - 1)^{3/2}}. \quad (6.12)$$

Above coincidence ($\alpha^2 + \beta^2 < 1$),

$$\sigma = (1 - \alpha^2 - \beta^2)^{-1/2}. \quad (6.13)$$

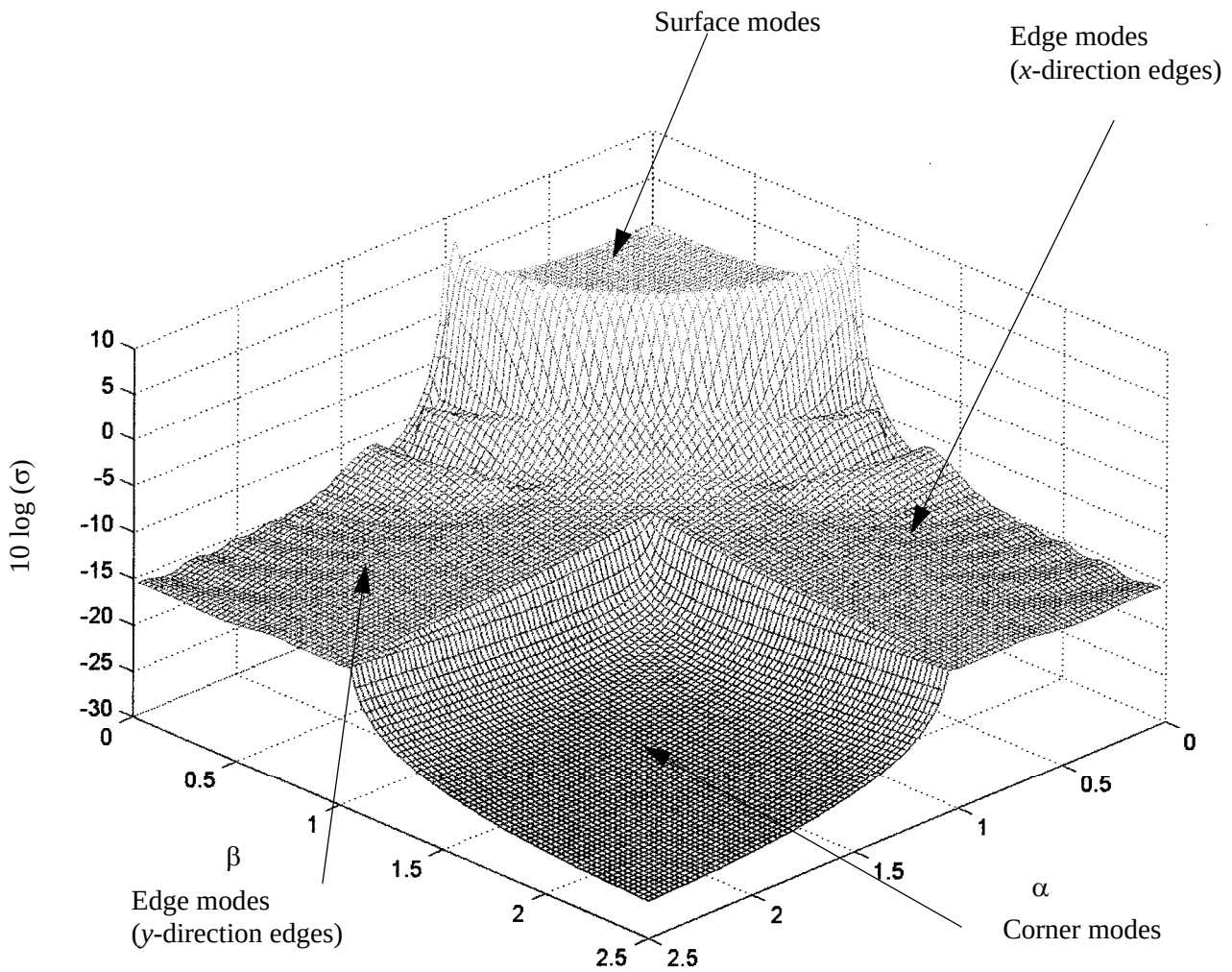
Above coincidence the plate is radiating like an infinite plate, hence the modes are called surface modes.

At coincidence ($\alpha^2 + \beta^2 = 1$),

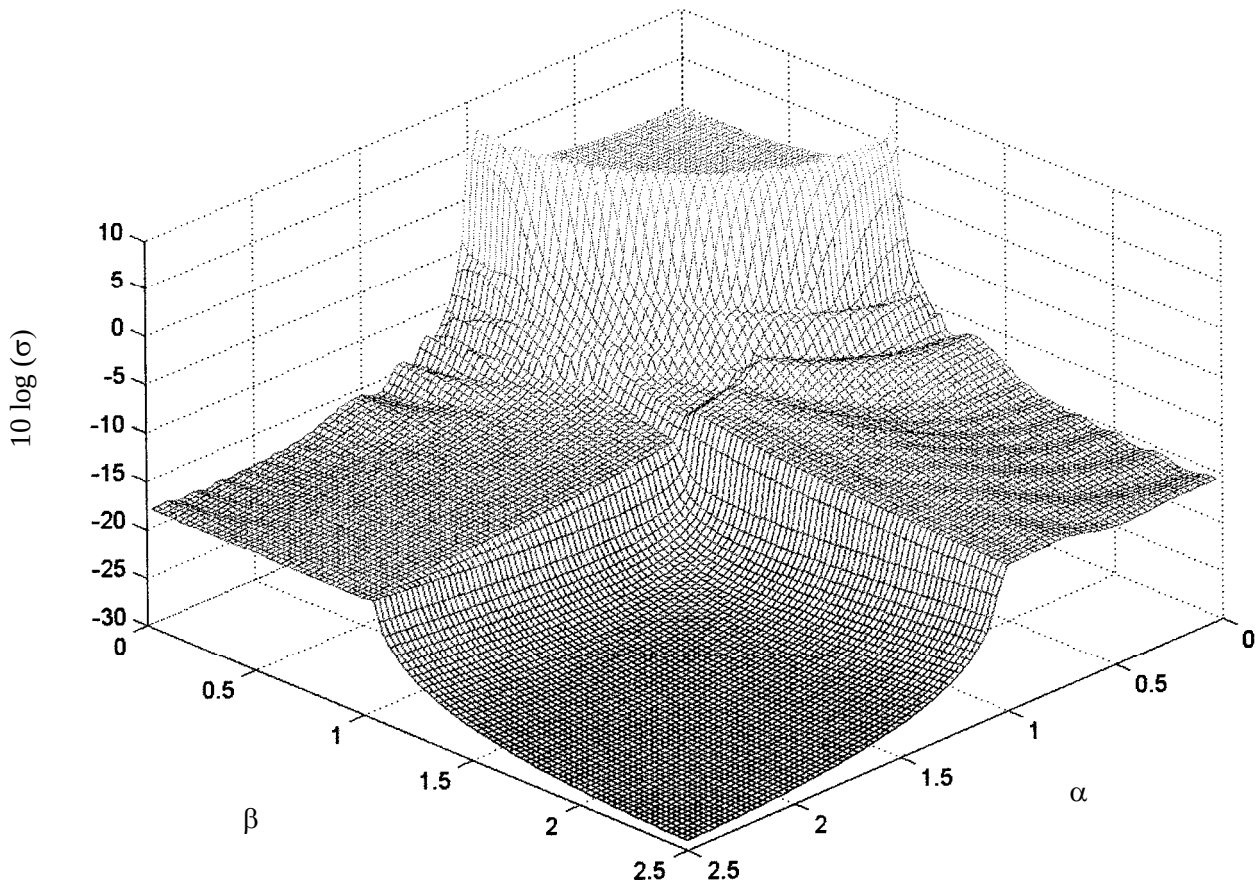
$$\sigma = \left(\frac{2}{15}\right) \left(\frac{ka}{\pi\alpha}\right)^{1/2} \left(5 - \frac{a\beta}{b\alpha}\right). \quad (6.14)$$

The radiation efficiency σ is not entirely a smooth function of α and β . There are discontinuities at the frequencies where $\alpha = 1$, $\beta = 1$ and particularly at $\alpha^2 + \beta^2 = 1$. Leppington *et al.* (Derivation 5) provide various corrections and transition formulae which go some way to reducing the abrupt changes in σ , but they cannot be completely removed. Consequently, smoothing functions have to be introduced near the discontinuities in order to ensure that σ is a continuous function of α and β .

The radiation efficiency, σ , for isotropic plates has been calculated, as a function of α and β , for this Item and may be seen in Sketches 6.1 and 6.2. In these three-dimensional diagrams the different regions for the surface, edge and corner modes are clearly seen. The boundaries between the regions are also evident, particularly at $\alpha = 1$ and $\beta = 1$ and at $\alpha^2 + \beta^2 = 1$, the wave coincidence frequency. Sketch 6.1 refers to a square plate and the function is symmetrical. The aspect ratio is different in Sketch 6.2 ($a = 1.0$ m and $b = 2.0$ m) and the asymmetry in the edge modes may be observed.



Sketch 6.1 Radiation efficiency σ as a function of α and β for a simply-supported, baffled rectangular plate with $a = 1.0$ m and $b = 1.0$ m



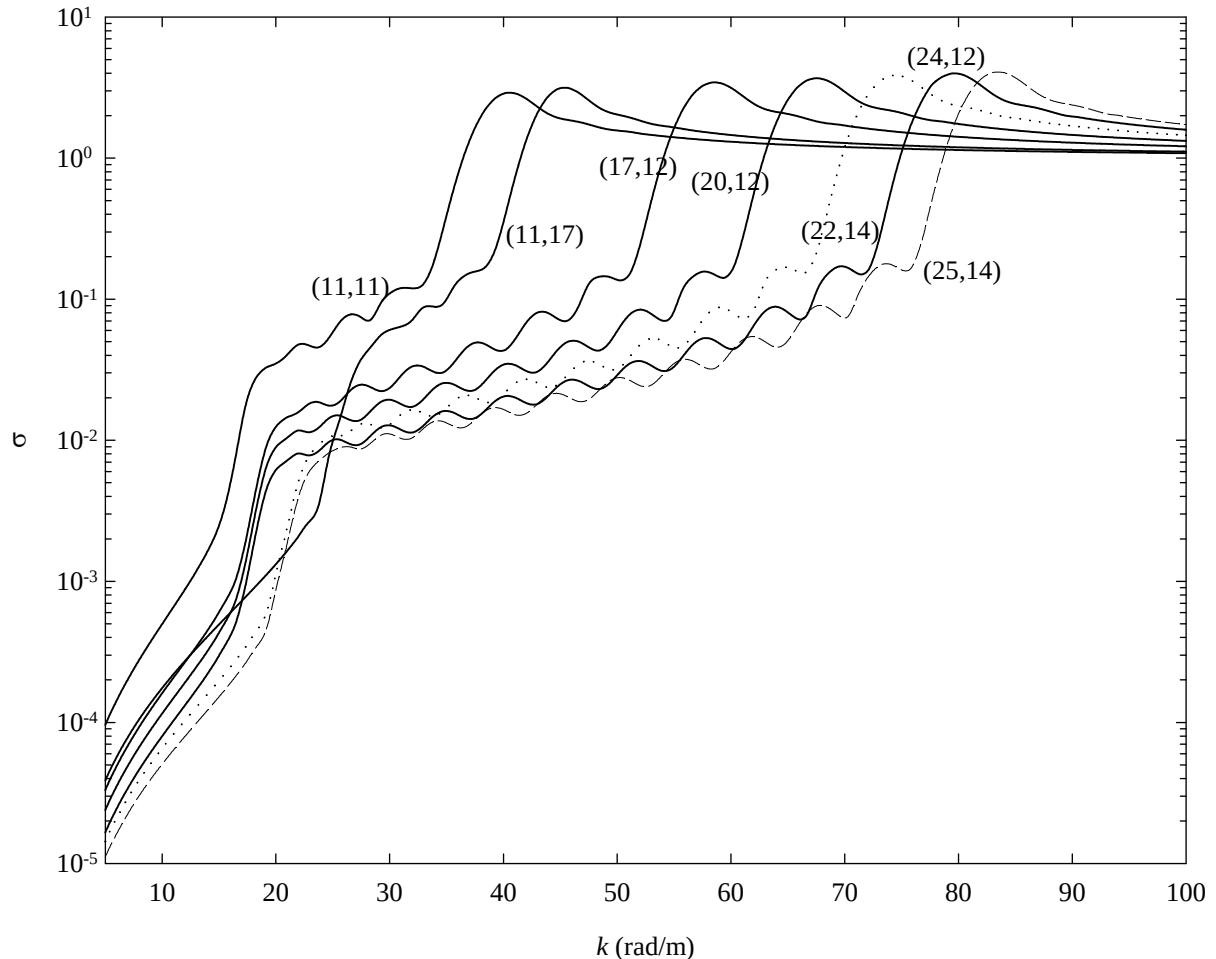
Sketch 6.2 Radiation efficiency σ as a function of α and β for a simply-supported, baffled rectangular plate with $a = 1.0$ m and $b = 2.0$ m

6.2.1 Examples of radiation efficiencies for simply-supported plates

This Item allows the user to estimate, using the programs issued with the Item, the radiation efficiencies for individual modes of vibration of the plate. For example, modal radiation efficiencies are presented in Sketch 6.3 for a plate with dimensions $a = 1.0$ m and $b = 2.0$ m for several combinations of the modal numbers m and n as a function of k , the acoustic wavenumber. The mode $m = 22$ and $n = 14$ may be directly compared with an example obtained by Leppington *et al.* (Derivation 5). For this case the values of k for which $\alpha = 1$ and $\beta = 1$ are 69.1 and 22.0 rad/m, respectively. The condition for $\beta = 1$ ($k = 22.0$ rad/m) coincides with the transition from corner modes to edge modes. In the region $\beta < 1$ and $\alpha > 1$ ($22.0 < k < 69.1$ [rad/m]), where the ripples are seen, the edge modes are dominant and Equation (6.10) is applicable. Wave coincidence for the panel occurs at $\alpha^2 + \beta^2 = 1$, for which $k = 72.5$ rad/m (3971 Hz for $c_0 = 344$ m/s), where the maximum value of σ is observed.

In the case of mode (11,17) the k values for $\alpha = 1$ and $\beta = 1$ are 34.6 and 26.7 rad/m, respectively. Thus the wavenumber range for the edge modes is smaller. Wave coincidence occurs at $k = 43.7$ rad/m.

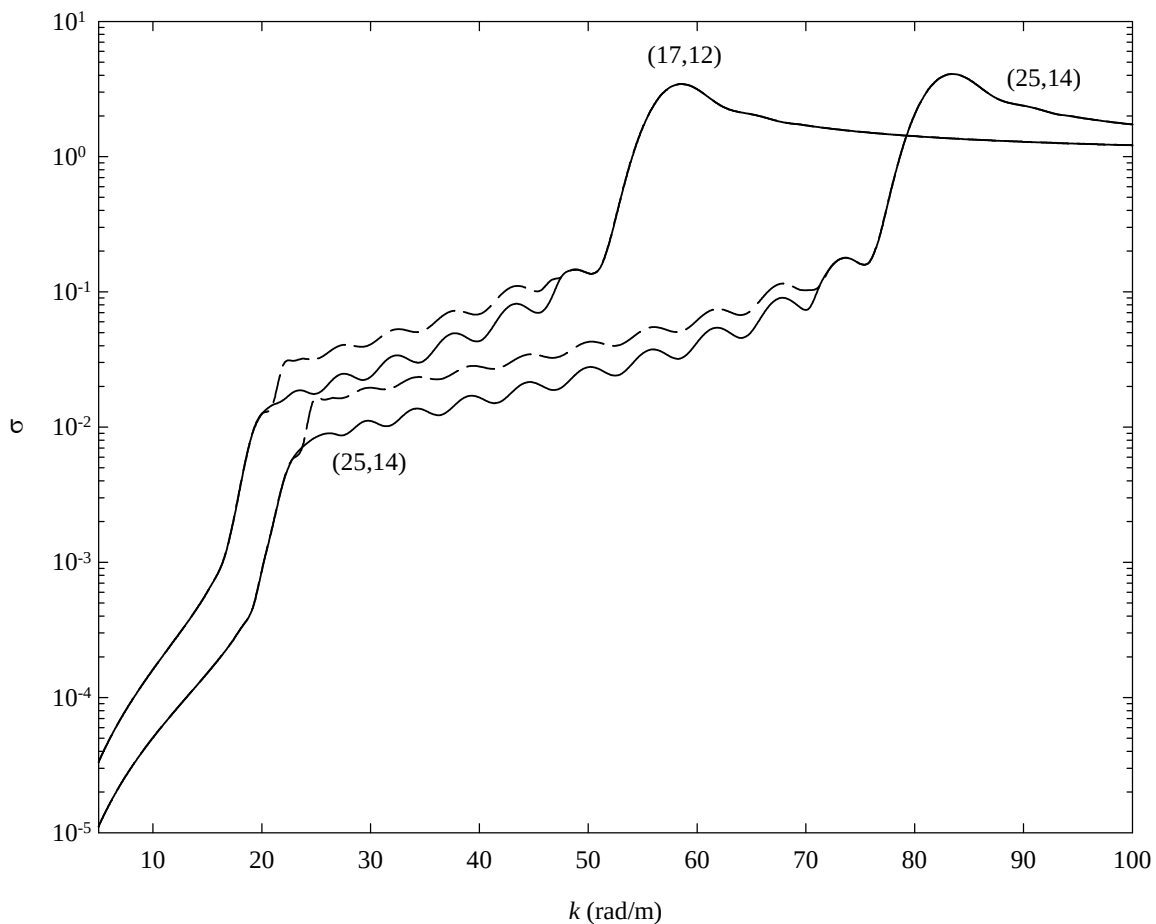
The radiation efficiencies for the modes, shown in Sketch 6.3, may be obtained for plates of any material and size, *i.e.*, they are independent of E , ρ , ν and thickness, h . However, the natural frequency (eigenfrequency) associated with each mode (see Equation (6.5)) depends on the material properties of the plate. If a material is assumed, *e.g.*, aluminium alloy with $E = 68.9$ GPa, $\nu = 0.33$, $\rho = 2710$ kg/m³, and the thickness is taken to be 2 mm, the natural frequency of the (22, 14) mode is 2582 Hz.



Sketch 6.3 Radiation efficiencies for different modes (m, n) for simply-supported plate with $a = 1.0$ m and $b = 2.0$ m

6.2.2 Radiation efficiency for clamped plates

It is more realistic to consider plates to be clamped, rather than simply-supported. Leppington *et al.* (Derivation 7) have provided a method for estimating the edge mode radiation from clamped plates. Their method allows the calculation of a correction factor to be applied to the radiation efficiency of the simply-supported plate for plate boundary conditions that vary from the fully-clamped to the simply-supported.



Sketch 6.4 Comparison of radiation efficiencies of simply-supported and clamped rectangular plates with $a = 1.0$ m and $b = 2.0$ m (— simply-supported, --- clamped) for modes (17, 12) and (25, 14)

Leppington *et al.* introduced a parameter ϵ which was a measure of the compliance of a varying restoring couple acting on a hinged edge of the plate. The parameter ϵ could vary from 0 for the clamped condition to ∞ for the simply-supported case. The parameter ϵ influenced the value of the natural frequency of the plate and a correction factor Q applied to the modal radiation efficiency of the plate. Normally, ϵ is not known, but in ESDU 00002, a method is described for the estimation of ϵ based on an experimental modal analysis of the plate. Computer program ESDUpac A0002 enables ϵ to be determined from experimental values of the natural frequencies and corresponding values of m and n .

The effect of clamping the edges can be seen in Sketch 6.4 where the radiation efficiency is increased by, at the most, 2 dB. The increase in radiation is only for the edge modes; above coincidence, where the plate vibrates as an infinite plate irrespective of the boundary conditions, there is no change. It has been suggested before that the effect of clamping the edges of the plate is to increase the radiation efficiency by a factor of 2 or 3 dB (Derivation 2), compared to the simply-supported case.

6.3 Average Radiation Efficiency of Isotropic Panels

The radiation efficiency of a particular mode is of fundamental importance, but the most practical information is obtained when a large number of modes are averaged for a given plate. The modally-averaged radiation efficiency is obtained by considering the expressions for σ in Equations (6.9) to (6.14) as continuous functions of α and β . This will be a reasonable approach at high frequencies when the characteristic frequencies for the modes are closely spaced.

The formula for the average radiation efficiency, for the below coincidence condition, given by Leppington *et al.* may be written as:

$$\tilde{\sigma} = \frac{(a+b)\gamma^2}{\pi k a b (1-\gamma^2)^{0.5}} \left(\ln \left(\frac{1+\gamma}{1-\gamma} \right) + \frac{2\gamma}{1-\gamma^2} \right), \quad (6.15)$$

where

$$\gamma^2 = 1/(\alpha^2 + \beta^2). \quad (6.16)$$

For the region above coincidence, *i.e.*, $\alpha^2 + \beta^2 < 1$ or $\gamma > 1$, the expression for the average radiation efficiency is the same as that for the modal, see Equation (6.13). Thus:

$$\tilde{\sigma} = 1/(1 - \alpha^2 - \beta^2)^{0.5} = \gamma/(\gamma^2 - 1)^{0.5}. \quad (6.17)$$

At coincidence, the value of $\tilde{\sigma}$ is given by:

$$\tilde{\sigma} = k^{0.5} a^{0.5} (0.5 - 0.15 a/b) \text{ for } a < b \quad (6.18)$$

or

$$\tilde{\sigma} = k^{0.5} a^{0.5} (0.5 - 0.15 b/a) \text{ for } b < a. \quad (6.19)$$

7. AVERAGE RADIATION EFFICIENCY OF ORTHOTROPIC PLATES

The formulations made in Section 6.1 in relation to the vibration of rectangular plates apply in general also to orthotropic plates. However, instead of the bending wave equation valid for the isotropic plate, the equation describing the transverse bending of the plate lying in the xy -plane must be specified in the form:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \quad (7.1)$$

where D_x is the flexural rigidity of the plate in the x -direction (for bending in the xz -plane), D_y is the flexural rigidity in the y -direction (for bending in the yz -plane) and D_{xy} is the torsional rigidity of the plate. The expressions, which apply to a single orthotropic layer, for the flexural and torsional rigidities in terms of the Young's moduli in the x - and y -directions are:

$$D_x = E_x h^3 / \{ 12(1 - \nu_{xy} \nu_{yx}) \}, \quad (7.2)$$

$$D_y = E_y h^3 / \{ 12(1 - \nu_{yx} \nu_{xy}) \}, \quad (7.3)$$

$$D_{xy} = \frac{h^3}{12} [\nu_{xy} E_y / (1 - \nu_{yx} \nu_{xy}) + 2G_{xy}]. \quad (7.4)$$

In the case of an isotropic plate D_x , D_y and D_{xy} are equal to D , see Equation (6.2). For an isotropic material a standard relationship between the material constants is

$$G = E / \{ 2(1 + \nu) \}, \quad (7.5)$$

where G is the shear modulus of the material. With $\nu = \nu_{xy} = \nu_{yx}$, $E = E_y$ and $G = G_{xy}$ for an isotropic material, from Equations (7.4) and (7.5)

$$\begin{aligned} D_{xy} &= \frac{\nu E h^3}{12(1 - \nu^2)} + \frac{2E}{2(1 + \nu)} \cdot \frac{h^3}{12} \\ &= \frac{E h^3}{12(1 - \nu^2)}, \end{aligned}$$

which, from Equation (5.2), is D .

The magnitude of the bending wavenumber vector for the orthotropic plate depends on the direction of the bending wave propagation, θ :

$$k_b(\theta) = \omega^{0.5} (\rho h / D(\theta))^{0.25} \quad (7.6)$$

and the bending stiffness is given by:

$$D(\theta) = D_x \cos^4 \theta + 2D_{xy} \cos^2 \theta \sin^2 \theta + D_y \sin^4 \theta. \quad (7.7)$$

It should be noted that similar natural modes of vibration for isotropic and orthotropic plates of identical geometry are associated with different natural frequencies of the plates.

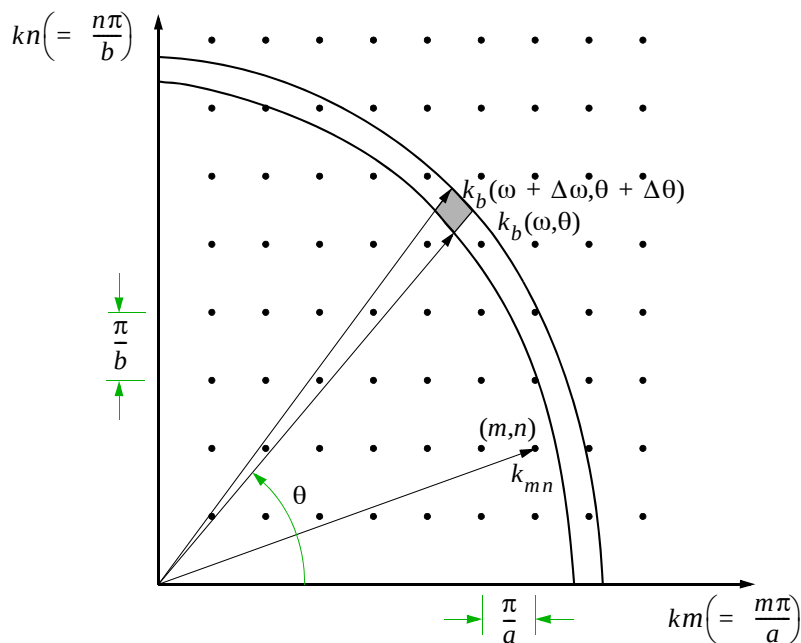
With the modal radiation efficiency, σ , and the bending wave number, k_b , known, it is possible to define the averaged radiation efficiency. The radiation efficiency, averaged over a frequency band $\Delta\omega$, may be defined as:

$$\bar{\sigma}(\omega_0, \Delta\omega) = \frac{\int_{\omega_0}^{\omega_0 + \Delta\omega} \int_0^{\pi/2} \sigma(\omega, \theta) k_b \frac{\partial k_b}{\partial \omega} d\theta d\omega}{\int_{\omega_0}^{\omega_0 + \Delta\omega} \int_0^{\pi/2} k_b \frac{\partial k_b}{\partial \omega} d\theta d\omega}, \quad (7.8)$$

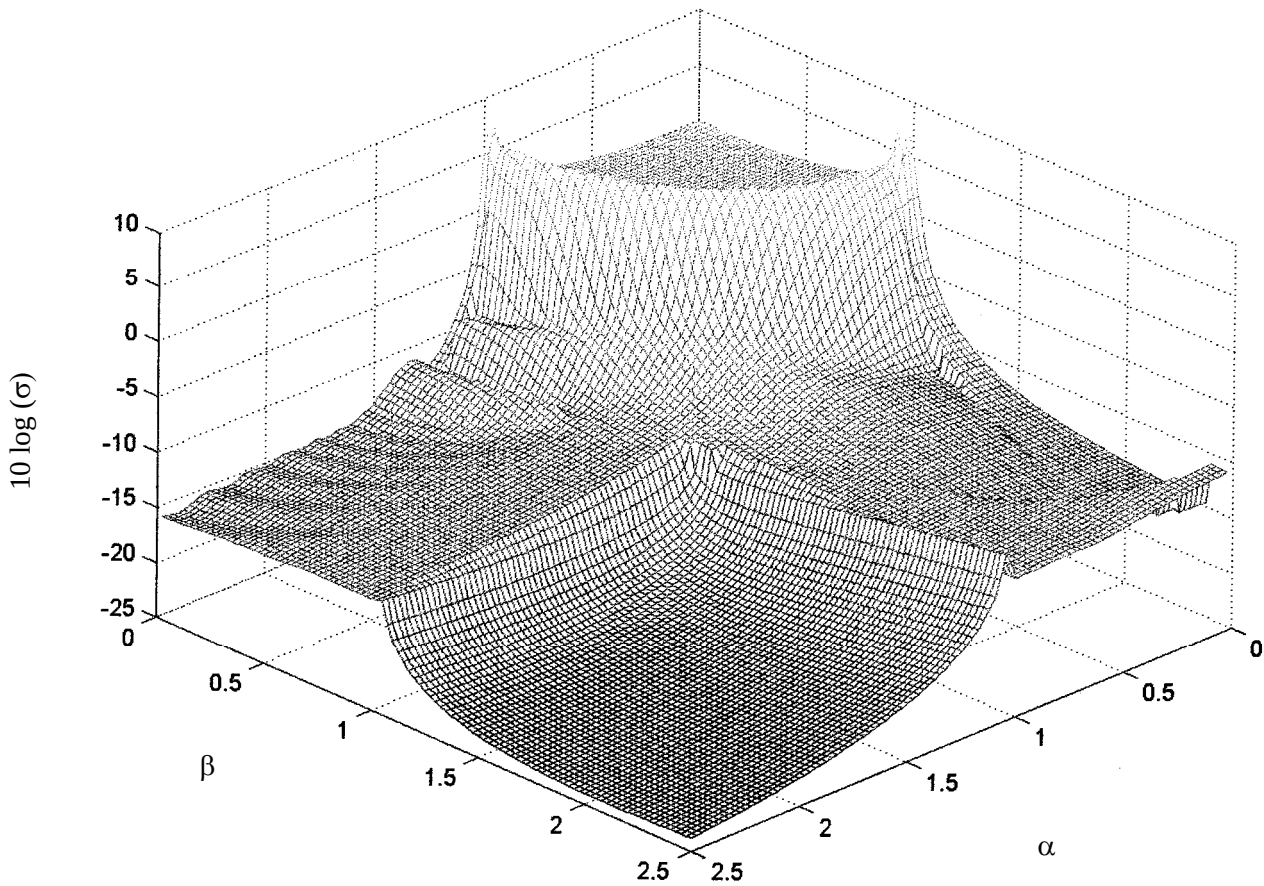
where $k_b = |\mathbf{k}_b|$ and $\mathbf{k}_b$ is the wavenumber vector for bending waves in the plate. The magnitude of $\mathbf{k}_b$ is given by Equation (7.6).

The radiation efficiency, denoted by σ , is shown as a surface stretched over the wavenumber plane, for example, in Sketch 6.1

The interpretation of Equation (7.8) may be considered with the aid of the mode lattice diagram in the wave number plane, shown in Sketch 7.1, for a rectangular plate of length a and width b . The intersections of lines, $km = m\pi/a$ and $kn = n\pi/b$, represent the eigenmodes for the vibration of the thin, simply-supported, rectangular plate. The vector $\mathbf{k}_b(\omega, \theta)$ denotes the bending wavenumber vector and $\mathbf{k}_{mn} = [m\pi/a, n\pi/b]$ is defined as the wavenumber vector for the plate. The resonance points at certain frequencies, see the wave number diagram, are those points for which the bending wavenumber vector for the infinite plate equals the mode wavenumber vector at that frequency.



Sketch 7.1 Wavenumber diagram for an orthotropic plate



Sketch 7.2 Radiation efficiency σ of a simply-supported, baffled, orthotropic plate as a function of α and β for $E_x/E_y = 4$, $E_y = 191$ GPa, $a = 1.0$ m and $b = 1.0$ m

A three-dimensional representation of the radiation efficiency, σ , is shown in Sketch 7.2 as a function of α and β , where α and β are functions of θ and ω . The sketch is for an orthotropic plate in which $E_x/E_y = 4$. The asymmetry of the function, particularly of the edge modes, may be seen, even though the sketch refers to a square plate.

The number of modes, N , with natural frequencies less than ω is given by the number of points or intersections of the lines: $km = m\pi/a$ and $kn = n\pi/b$ in the first quadrant of the wavenumber plane, enclosed by the constant frequency contour (References 13 and 45). Since the 'area' associated with each mode is π^2/ab , the number of modes, N , can be determined from:

$$N(\omega) = (ab/\pi^2) \int_0^\omega \int_0^{\pi/2} k_b \frac{\partial k_b}{\partial \omega} d\theta d\omega. \quad (7.9)$$

The modal density, $n_d(\omega)$, defined as the rate of change of the number of modes with frequency, i.e.,

$$n_d(\omega) = \partial N(\omega)/\partial \omega, \quad (7.10)$$

determines the number of modes within a unit band of frequency ($\Delta\omega = 1$ rad/s).

For the case of the orthotropic plate, from Equation (7.9) and from Equation (7.6) for k_b :

$$N(\omega) = (ab/2\pi^2) \int_0^\omega \int_0^{\pi/2} \left[\frac{\rho h}{D(\theta)} \right]^{1/2} d\theta d\omega, \quad (7.11)$$

where $D(\theta)$ is given by Equation (7.7). Similarly, the number of resonant modes within the $\Delta\omega$ frequency band is:

$$N_{\Delta\omega} = (ab/2\pi^2) \int_{\omega_0}^{\omega_0 + \Delta\omega} \int_0^{\pi/2} \left[\frac{\rho h}{D(\theta)} \right]^{1/2} d\theta d\omega, \quad (7.12)$$

$$N_{\Delta\omega} = (ab/2\pi^2)(\rho h)^{0.5} \Delta\omega \int_0^{\pi/2} \frac{d\theta}{\sqrt{D(\theta)}} \quad (7.13)$$

and the modal density, n_d , can be expressed as:

$$n_d = (ab/2\pi^2)(\rho h)^{0.5} \int_0^{\pi/2} \frac{d\theta}{\sqrt{D(\theta)}}. \quad (7.14)$$

It should be noted that for the orthotropic plate the modal density does not depend on frequency. The formulae for the radiation efficiency, averaged in the $\Delta\omega$ frequency band, can be presented in the form:

$$\bar{\sigma}(\omega_0, \Delta\omega) = \frac{\frac{ab}{\pi^2} \int_{\omega_0}^{\omega_0 + \Delta\omega} \int_0^{\pi/2} \sigma(\omega, \theta) k_b \frac{\partial k_b}{\partial \omega} d\theta d\omega}{N_{\Delta\omega}} \quad (7.15)$$

or,

$$\bar{\sigma}(\omega_0, \Delta\omega) = \frac{1}{\Delta\omega} \int_{\omega_0}^{\omega_0 + \Delta\omega} \tilde{\sigma}(\omega) d\omega, \quad (7.16)$$

where

$$\tilde{\sigma}(\omega) = \frac{(ab/\pi^2)}{n_d} \int_0^{\pi/2} \sigma(\omega, \theta) k_b \frac{\partial k_b}{\partial \omega} d\theta \quad (7.17)$$

and n_d is the modal density. The radiation efficiency, $\sigma(\omega)$, can be interpreted as the radiation efficiency averaged within the unit band of frequency.

For the isotropic plate $\tilde{\sigma}(\omega)$ is reduced to:

$$\tilde{\sigma}_{is}(\omega) = (2/\pi) \int_0^{\pi/2} \sigma(\omega, \theta) d\theta, \quad (7.18)$$

as in the model of Leppington *et al.* (Derivation 5).

Examples of radiation efficiencies averaged over 1 rad/s are shown in Sketch 7.3. The sketch applies to plates of homogeneous, orthotropic material with properties similar to those of steel for which $E = E_y = 191$ GPa, $\nu = \nu_{xy} = 0.29$, $\rho = 7.97 \times 10^3$ kg/m³ and G is given by Equation (7.5). The sketch refers to a plate 3 mm thick for values of the ratio $E_x/E_y (= e)$ from 1 to 8. The curve $e = 1$ is the isotropic

case. Such an orthotropic sheet may not be practical, but the results provide a useful insight into the nature of orthotropic materials. For infinite orthotropic sheets there are two coincidence frequencies, f_{c1} and f_{c2} , given by (Reference 19):

$$f_{c1} = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho h}{D_x}} \quad (7.19)$$

and

$$f_{c2} = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho h}{D_y}}. \quad (7.20)$$

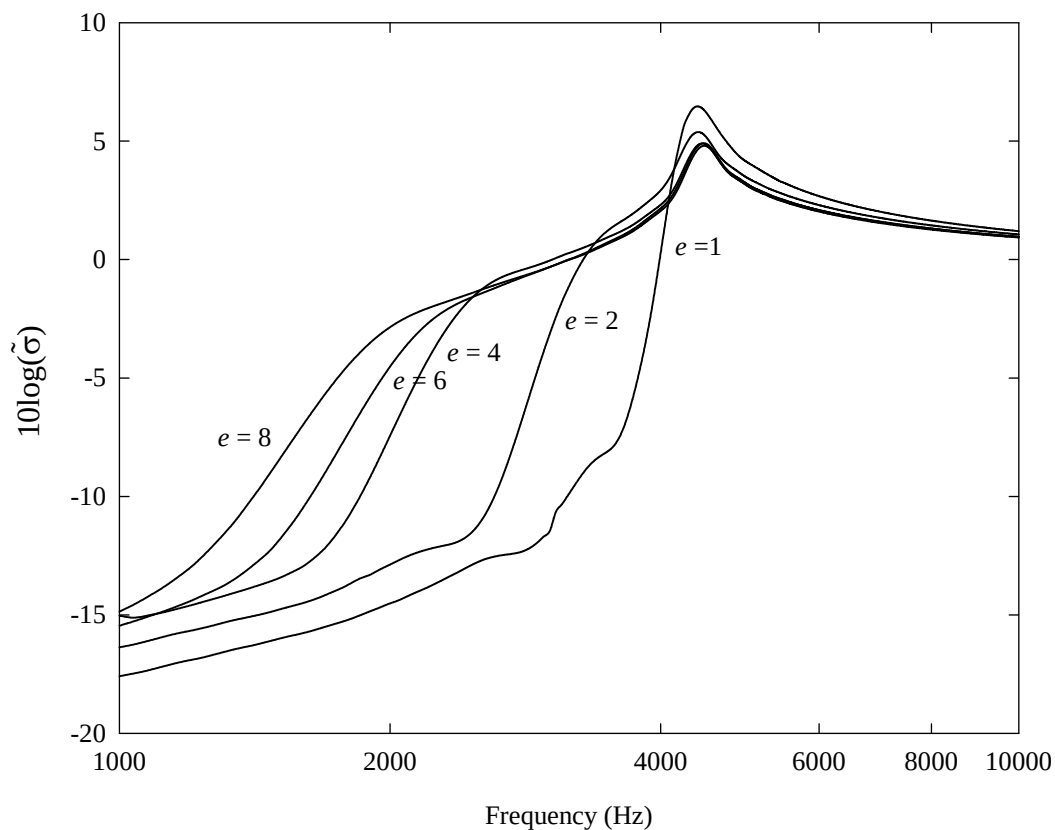
With Equation (7.2) for D_x , Equation (7.3) for D_y and Equation (4.1), f_{c1} , the lower coincidence frequency and f_{c2} , the coincidence frequency with a higher value, are given by:

$$f_{c1} = \frac{c_0^2}{2\pi} \sqrt{\frac{12\rho(1 - \nu^2/e)}{Eeh^2}} \quad (7.21)$$

and

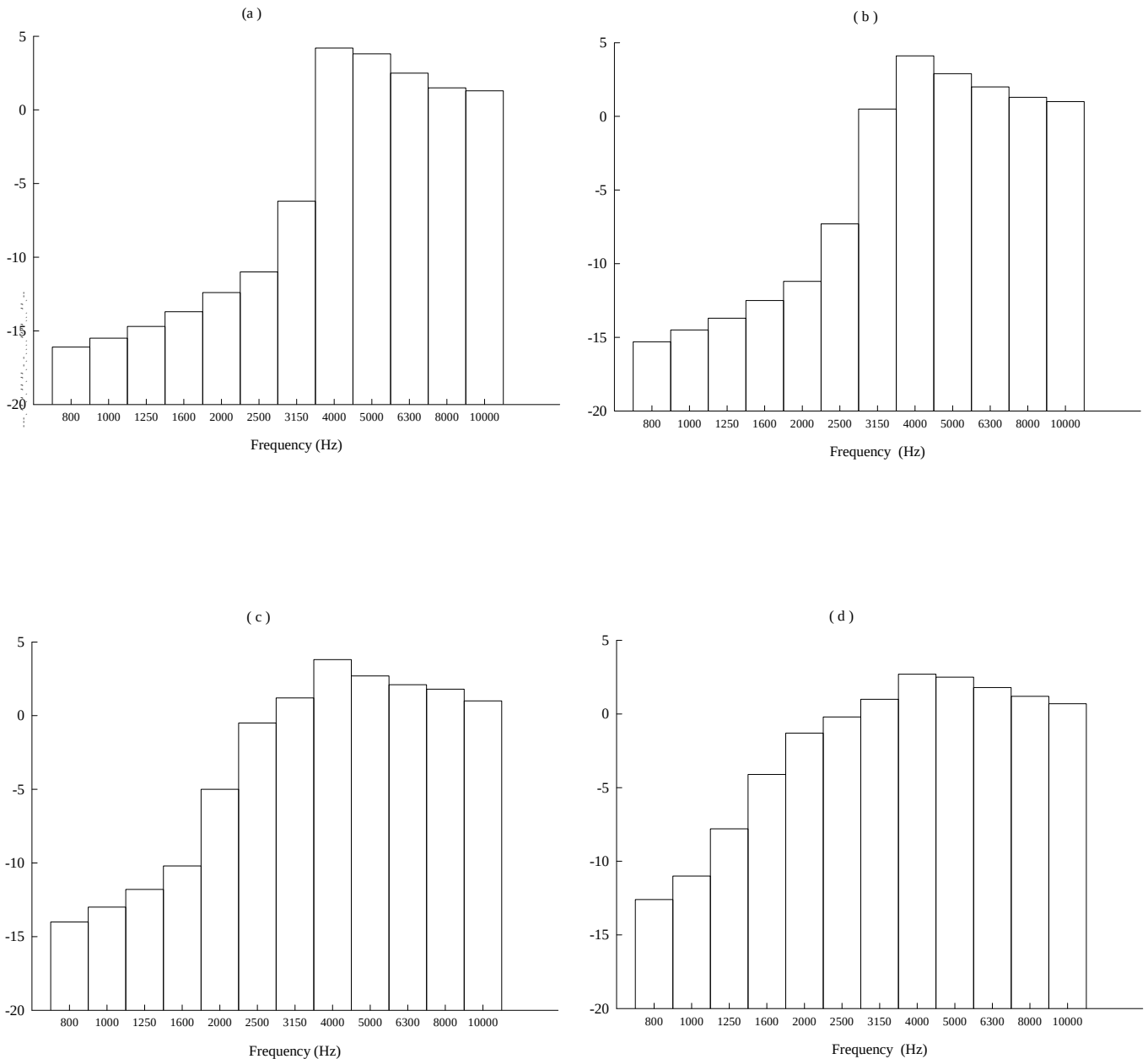
$$f_{c2} = \frac{c_0^2}{2\pi} \sqrt{\frac{12\rho(1 - \nu^2/e)}{Eh^2}}. \quad (7.22)$$

The lower coincidence frequencies, f_{c1} , equal 4252, 3075, 2198, 1801 and 1562 Hz for $e = 1, 2, 4, 6$ and 8, respectively. These coincidence frequencies appear to occur at the knee of the curves where the curves almost flatten out. The upper coincidence frequency, f_{c2} , does not vary greatly for all the cases. For the isotropic sheet $e = 1$ and $f_{c1} = f_{c2}$.



Sketch 7.3 Averaged radiation efficiencies of rectangular orthotropic plates for $E_x/E_y (= e) = 1, 2, 4, 6$ and 8, $E_y = 191$ GPa, thickness $h = 3$ mm, $a = 1.0$ m, $b = 2.0$ m

In general it will be more useful to have the radiation efficiencies averaged over a frequency band. In Sketch 7.4 results have been presented averaged over one-third octave bands (23% constant percentage bandwidth) for $e = 1, 2, 4$ and 8 and for a plate size $a = 1.0$ m, $b = 1.0$ m and $h = 3$ mm.



Sketch 7.4 Radiation efficiencies averaged in one-third octave bands for rectangular orthotropic plates of size $a = 1.0$, $b = 1.0$ and $h = 3$ mm with (a) $E_x/E_y (= e) = 1$, (b) $e = 2$, (c) $e = 4$ and (d) $e = 8$, with $E_y = 191$ GPa

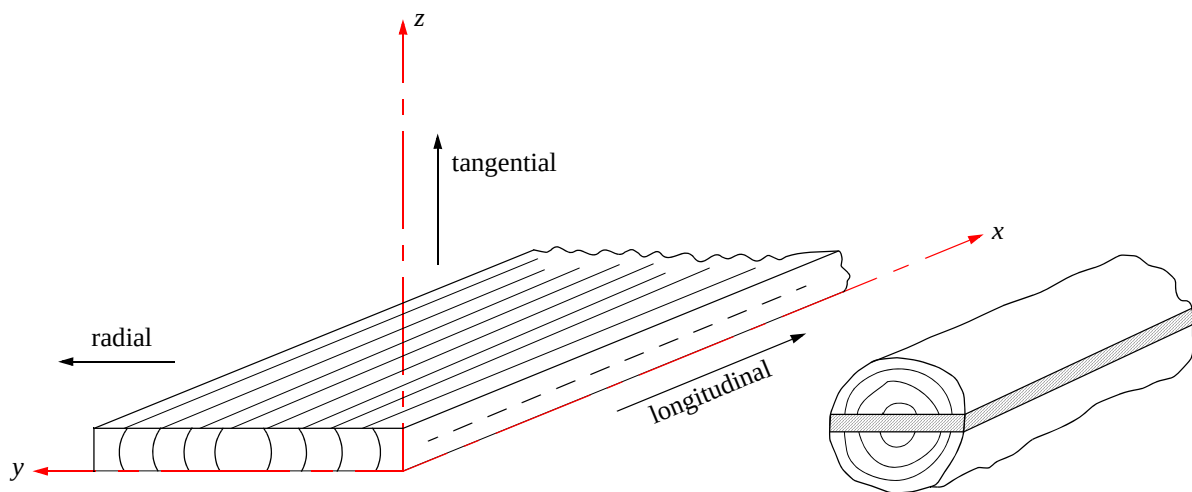
8. APPLICATIONS

In this section several examples are presented of radiation efficiencies for isotropic, orthotropic and equivalent orthotropic plates. In the case of the equivalent orthotropic plates, wherever possible, use has been made of previous ESDU Items. For equivalent orthotropic plates the bending stiffnesses D_x , D_y and D_{xy} are required. In the Table in Appendix A expressions for the bending stiffnesses are given for various systems which can be treated as equivalent orthotropic plates within the limitations specified. There may be other methods in the literature for converting ribbed and corrugated plates, *etc.*, into equivalent orthotropic plates.

All applications refer to plates in infinite baffles in which the baffle is flush with the plate. The plates are all simply-supported.

8.1 Average Radiation Efficiency of a Thin Plate of Wood

A useful introduction to the subject of sound radiation from orthotropic panels is provided by a wooden plate. Wood is one of the few examples of a naturally, orthotropic material, which is also approximately homogeneous. Consider a thin slice of wood of the type that may be used in a musical instrument. The wood is of spruce and the slice is a 'quarter cut', *i.e.*, a longitudinal section cut from the middle of the tree. For this cut the 'rings' of the tree, when seen from the end appear almost perpendicular to the longitudinal direction, as shown in Sketch 8.1.



Sketch 8.1 Spruce plate cut from a tree

Usually, wood has three applicable Young's moduli E_L , E_R and E_T , corresponding to the longitudinal, radial and tangential directions, respectively (Reference 12). As the slice of wood considered here is thin and is a quarter cut, it can be considered as specially orthotropic with the longitudinal and radial directions as the axes of orthotropy.

For spruce, the material properties are (Reference 12):

$$\begin{aligned}E_L &= 10.7 \text{ GPa}, \\E_R &= 0.71 \text{ GPa}, \\v_{LR} &= 0.38, \\G_{LR} &= 0.5 \text{ GPa}, \\\rho &= 450 \text{ kg/m}^3.\end{aligned}$$

The longitudinal and radial directions coincide with the x - and y -axes, respectively. The plate is considered to be simply-supported at its edges and $h = 4 \text{ mm}$.

The components of the bending stiffness D_x , D_y and D_{xy} are:

$$\begin{aligned}D_x &= E_x h^3 / \{12(1 - v_{xy} v_{yx})\} = E_L h^3 / \{12(1 - v_{LR} v_{RL})\} \\D_y &= E_y h^3 / \{12(1 - v_{yx} v_{xy})\} = E_R h^3 / \{12(1 - v_{RL} v_{LR})\}\end{aligned}$$

and

$$\begin{aligned}D_{xy} &= v_{xy} E_y h^3 / \{12(1 - v_{xy} v_{yx})\} + 2G_{xy} h^3 / 12 \\&= v_{LR} E_R h^3 / \{12(1 - v_{RL} v_{LR})\} + 2G_{LR} h^3 / 12.\end{aligned}$$

Hence, D_x , D_y and D_{xy} are calculated to be 57.62, 3.82 and 6.79 N m, respectively.

With the use of Equation (7.17) the radiation efficiency averaged over unity bandwidth has been computed and the results shown in Sketch 8.2 for various plate sizes. The radiation efficiencies averaged in one-third octave bands have been estimated for $a = 0.3 \text{ m}$, $b = 0.2 \text{ m}$ and $h = 4 \text{ mm}$ and are shown in Sketch 8.3. The critical frequency of wave coincidence associated with D_y and E_R is 12 928 Hz ($f_{c2} = (344^2/2\pi)(0.45 \times 4/3.82)^{0.5} = 12\,928 \text{ Hz}$) from Equation (7.20) and is shown quite clearly in Sketch 8.2, particularly for the slice of wood where $a = 0.3 \text{ m}$ and $b = 1.0 \text{ m}$. The second and lower coincidence frequency, associated with D_x or E_L is 3 329 Hz ($f_{c1} = (344^2/2\pi)(0.45 \times 4/57.62)^{0.5} = 3\,329 \text{ Hz}$) is less clearly shown but approximately occurs at the point of inflexion of the curves as σ increases.