



An introduction to lateral static aeroelasticity: controllability, loads and stability

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AN INTRODUCTION TO LATERAL STATIC AEROELASTICITY: CONTROLLABILITY, LOADS AND STABILITY

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AN INTRODUCTION TO LATERAL STATIC AEROELASTICITY: CONTROLLABILITY, LOADS AND STABILITY

1. INTRODUCTION

This Data Item introduces lateral static aeroelasticity in which the effects of structural deformation on lateral stability, controllability and loading in steady lateral manoeuvres are considered.

The analysis of this Item is in line with the approach of Item 96037^[8], which provided a qualitative introduction to longitudinal aeroelasticity, where the effects of symmetric deformations on longitudinal controllability, quasi-static loads, static stability, trim characteristics and quasi-steady pull-out manoeuvres of both swept and unswept configurations are described. In this Item, like the previous Item, consideration is given to what happens when deformation, relative to a reference state, of the various components, wing, tailplane, fin, fuselage and engine nacelles and pylons, but here the deformations are antisymmetric, giving rise to changes in lateral stability, controllability and in the antisymmetric loading distributions.

The effects of deformations are quantified using the concept of modified derivatives in which the stability derivatives associated with rigid aeroplane stability and controllability are changed to incorporate deformation effects, assuming all dynamic effects can be neglected, a reasonable assumption in lateral stability motions and lateral manoeuvres. The modified derivatives are derived in qualitative terms to illustrate their behaviour, and the important roles of divergence speeds and reversal speeds are identified.

The effects of structural flexibility on the lateral stability characteristics are assessed by noting the changes in the approximations for static stability, spiral mode, rolling subsidence and Dutch roll in terms of the lateral stability derivatives when modified derivatives are used.

The steady lateral manoeuvres considered are those identified in 'Loading on a rigid aeroplane in steady lateral manoeuvres', Data Item 01010^[11], namely, a 'steady horizontal turn', 'steady horizontal straight motion with an engine out' and 'steady horizontal straight sideslipping flight'. It is shown that although the overall loads on a configuration with a flexible wing, fin and fuselage are the same as for the rigid configuration, the loading distributions on wing and fin, as well as the effectiveness of the aileron and rudder, are affected by structural flexibility.

Related Items that deal with loading in quasi-steady longitudinal manoeuvres are Items 94009^[6], which concentrates on loading aspects, and 94045^[7], which deals with shear force, bending moment and torque. Longitudinal static aeroelasticity analysis with assumed modes and normal modes are dealt with in Items 97032^[9] and 99033^[10], respectively.

2. NOTATION

Any coherent system of units may be used.

$A_1, A_2, \dots, A_5$	coefficients of stability quartic (see Appendix A)
a_{Fin}, a_ζ	rate of change of sideforce coefficient with sideslip angle and rudder angle, respectively ($= dC_Y/d\beta, dC_Y/d\xi$)
b, b_{Fin}	wing span and fin height, respectively
C_Q	dynamic pressure coefficient ($= (W/S)/(\frac{1}{2}\rho V_{trim}^2)$)
C_L, C_Y	lift and sideforce coefficients, respectively ($= L/(\frac{1}{2}\rho V_{trim}^2 S), Y/(\frac{1}{2}\rho V_{trim}^2 S_{Fin})$)
c_{Fin}, c_W	chord of fin and wing, respectively
D	drag
$EI\{z_f\}$	bending moment stiffness distribution
e_{Fin}, e_W	denotes distance between aerodynamic centre and flexural axis as a fraction of chord, for fin and wing, respectively
e_x, e_z	constant-of-inertia ratios (see Item 67002 ^[2])
e_ζ, e_ξ	denotes distance from flexural axis of centre of lift due to rudder and aileron, respectively, as fraction of fin chord and wing chord
$GJ\{z_f\}$	torsional stiffness distribution
g	acceleration due to gravity
h_{Fin}	distance in z-direction of aeroplane mass centre to aerodynamic centre of fin
h_ζ, h_ξ	$= (e_{Fin} + e_\zeta)$ and $(e_W + e_\xi)$, respectively
i_x, i_z	inertia parameters (see Item 67002 ^[2])
k_y, k_θ	bending stiffness about built-in root and torsional stiffness about flexural axis, respectively
L, N	overall moments about axes Ox and Oz , respectively
L	aeroplane lift
L_p, L_r, L_v	aero-normalised rolling moment derivatives due to p, r and v , respectively ($= (\partial L/\partial p)/(\frac{1}{2}\rho V_{trim}^2 Sb^2), (\partial L/\partial r)/(\frac{1}{2}\rho V_{trim}^2 Sb^2), (\partial L/\partial v)/(\frac{1}{2}\rho V_{trim}^2 Sb)$)
$(L_W)_\alpha, (L_W)_\xi$	dimensional wing lift curve slopes due to incidence and aileron, respectively
L_ζ, N_ζ	aero-normalised rolling and yawing moment derivatives due to rudder deflection ($= (\partial L/\partial \zeta)/(\frac{1}{2}\rho V_{trim}^2 Sb), (\partial N/\partial \zeta)/(\frac{1}{2}\rho V_{trim}^2 Sb)$)
L_ξ, N_ξ	aero-normalised rolling and yawing moment derivatives due to aileron deflection ($= (\partial L/\partial \xi)/(\frac{1}{2}\rho V_{trim}^2 Sb), (\partial N/\partial \xi)/(\frac{1}{2}\rho V_{trim}^2 Sb)$)
l_{Fin}, l_{Nose}	distances in x-direction from aeroplane mass centre to aerodynamic centres of fin and nose of aeroplane, respectively

ℓ	local lift
m	mass or mass distribution
N_p, N_r, N_v	aero-normalised yawing moment derivatives due to p , r and v , respectively ($= (\partial N/\partial p)/(\frac{1}{2}\rho V_{trim}^2 S b^2)$, $(\partial N/\partial r)/(\frac{1}{2}\rho V_{trim}^2 S b^2)$, $(\partial N/\partial v)/(\frac{1}{2}\rho V_{trim}^2 S b)$)
O	origin of axis system for overall aeroplane
O {...}	relates to order of magnitude of {...}
p, q, r	aeroplane angular velocities about x-, y-, z-directions, respectively
S, S_{Fin}	wing area and fin area, respectively
T	torque distribution
Th	engine thrust
V_{trim}	trimmed velocity of aeroplane in flight direction
u, v, w	aeroplane velocities in the x-, y-, z-directions, respectively
W	aeroplane weight
x, y, z	body axis directions for overall aeroplane with respect to origin O
x_f, y_f, z_f	axes associated with flexural axis, with origin at built-in root
Y	overall force in y-direction
Y_{SFin}	fin sideforce distribution
Y_p, Y_r, Y_v	aero-normalised sideforce derivatives due to p , r and v , respectively ($= (\partial Y/\partial p)/(\frac{1}{2}\rho V_{trim}^2 S b)$, $(\partial Y/\partial r)/(\frac{1}{2}\rho V_{trim}^2 S b)$, $(\partial Y/\partial v)/(\frac{1}{2}\rho V_{trim}^2 S)$)
Y_ξ, Y_ζ	aero-normalised sideforce derivatives due to ξ and ζ , respectively ($= (\partial Y/\partial \xi)/(\frac{1}{2}\rho V_{trim}^2 S)$, $(\partial Y/\partial \zeta)/(\frac{1}{2}\rho V_{trim}^2 S)$)
$y_e(z_f), \gamma_e(z_f)$	distributions of bending and twist deformations along and about the flexural axis
$y_\gamma(z_f), y_\theta(z_f)$	assumed bending and twisting mode shapes, respectively
α	aeroplane angle of attack
β	aeroplane sideslip angle
Γ	dihedral angle
γ_e	bending of flexural axis due to structural deformation
ζ	rudder angle or damping ratio
η_{Fin}	distance in z_f -direction between aerodynamic centre of fin and effective fin root as a fraction of fin height (see Equation (5.11))
θ	aeroplane pitch angle
θ_e	twist about flexural axis due to structural deformation

Λ	sweep angle
λ	eigenvalue of stability characteristic equation
μ	relative density parameter (= $(W/g)/(\frac{1}{2}\rho S b)$)
ξ	aileron angle
ρ	air density
τ	unit of time in aero-normalised system (= $m/(\frac{1}{2}\rho V_{trim} S)$)
ϕ	aeroplane bank angle
ψ	aeroplane yaw angle
ω_N	natural frequency of oscillation

Subscripts

<i>DR</i>	relates to Dutch roll oscillation
<i>div</i>	denotes divergence
<i>E</i>	denotes external
<i>F</i>	fuselage
<i>Fin</i>	fin
<i>f</i>	relates to flexural axis
<i>Nac</i>	nacelle
<i>Nose</i>	nose of aeroplane
<i>rev</i>	denotes reversal
<i>rigid</i>	denotes rigid aeroplane
<i>T</i>	tailplane
<i>trim</i>	denotes trimmed flight
<i>unswept</i>	denotes unswept flexural axis
<i>W</i>	wing
α	relates to angle of attack
ζ	relates to rudder
ξ	relates to aileron
<i>2D</i>	refers to two-dimensional

3. GENERAL DESCRIPTION

3.1 Scope and Applicability

In line with Item 96037^[8] which provided an introduction to symmetric static aeroelasticity, the same simplifications in terms of aeroplane configuration and flight condition are introduced for asymmetric static aeroelasticity in the present Item.

- (i) The aeroplane is assumed to be a classically-configured transport type aeroplane which restricts the wing, tailplane and fin to medium and high aspect ratios, typically ≥ 4 , and up to moderate sweep angles, typically $\Lambda \leq 35^\circ$.
- (ii) The aeroplane flight condition is restricted to subsonic, asymmetric flight at either a steady level flight condition or a quasi-steady asymmetric manoeuvring condition. All Mach number effects are ignored. Values in flight of the angle of incidence, pitch angle, flight path angle and sideslip angle are assumed small. The contribution to lift from the thrust is neglected in line with the qualitative nature of the discussion.
- (iii) Wing, tailplane, fin and wing pylon flexibilities are assumed to take the form of bending and twist about an elastic, or flexural axis. Chordwise deformations are ignored.
- (iv) Fuselage flexibility is assumed to be limited to the aft fuselage; the front fuselage including the wing-fuselage junction is assumed to be rigid.
- (v) Weight distributions of all parts of the aeroplane are assumed unaffected by deformation effects.

3.2 Reference Shape, Axis System and Motion

The basic aeroplane *datum shape*, known as the '*jig shape*', is the shape of the aeroplane when all deformations are zero throughout the structure, *i.e.* when all loads and internal stresses are zero. The *reference shape* is the shape of the aeroplane at a specified flight condition and deformations at other flight conditions are then estimated relative to this flight condition. These definitions are fully described in Item 96037^[8] and will be used in this Item.

The axis system is defined relative to the aeroplane reference shape and the Attached or Structural axis system used here is the same as that used in Item 96037^[8], namely the axes are fixed in the rigid front fuselage. Sketch 3.1 illustrates this system which shows the aeroplane in steady level trimmed flight and in perturbed asymmetric flight. The system is of the right handed Cartesian type, Oxyz, with the x- and z-axes chosen in the plane of symmetry of the aeroplane, the x-axis pointing forward, the z-axis pointing downwards and the y-axis pointing to starboard.

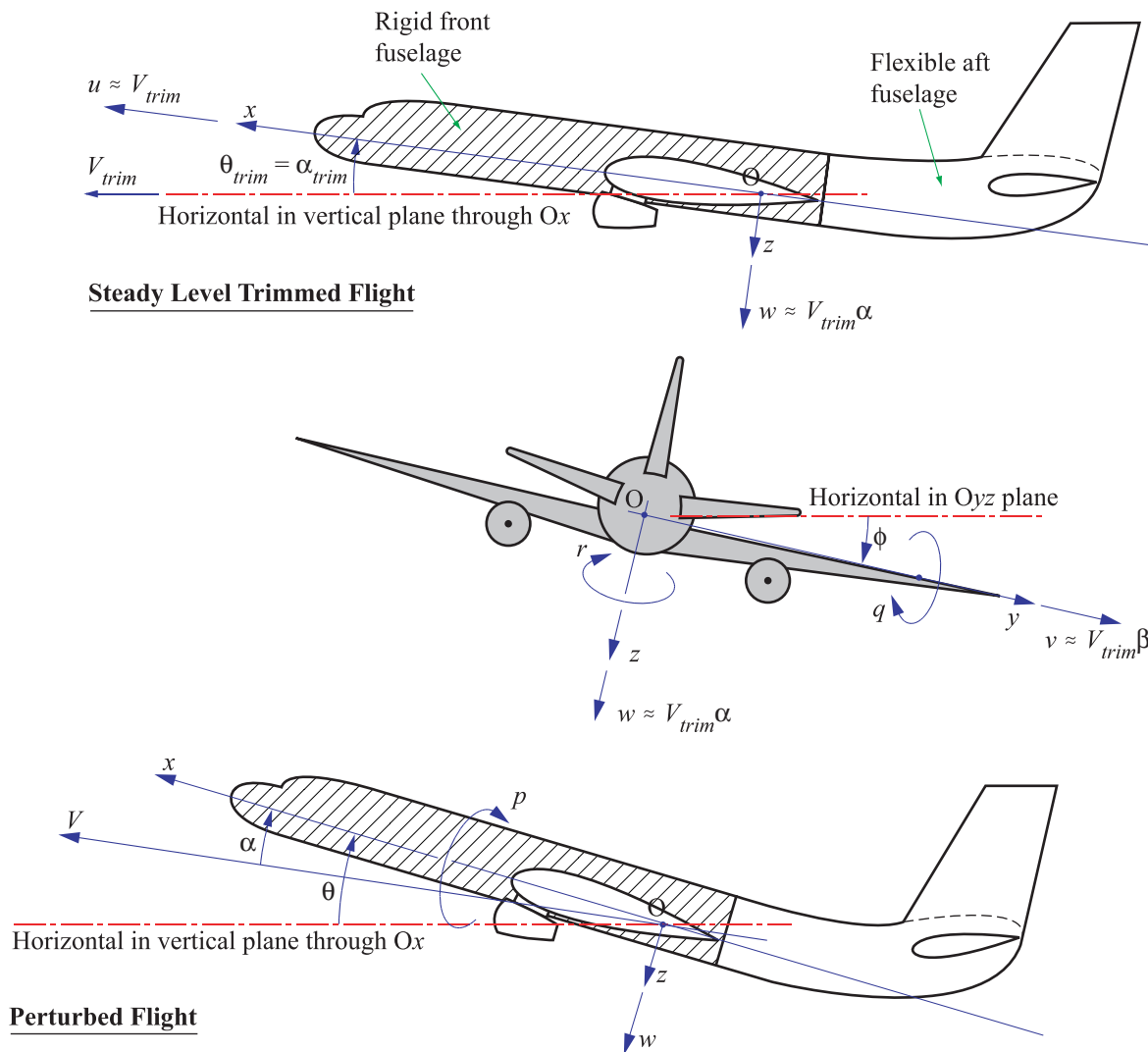
The aeroplane angular velocities about axes Ox, Oy and Oz, are respectively denoted by p , q and r .

The components of the linear velocity of the aeroplane in the x-, y- and z-axis directions are respectively denoted by u , v and w , and the overall resultant velocity of the aeroplane, relative to air, is denoted by V_{trim} .

The velocities in the y- and z-directions, v and w , are respectively related to the angle of sideslip, β , and angle of attack, α , by

$$w = u \tan \alpha \quad (3.1)$$

$$\text{and} \quad v = V_{trim} \sin \beta \quad (3.2)$$



Sketch 3.1 Attached axis system

The resultant aeroplane velocity, V_{trim} , is given by

$$V_{trim} = \left(u^2 + v^2 + w^2 \right)^{1/2} \approx \left(u^2 + \left(V_{trim} \sin \beta \right)^2 + \left(u \tan \alpha \right)^2 \right)^{1/2}. \quad (3.3)$$

For small α and β , Equations (3.1), (3.2) and (3.3) reduce to

$$w = V_{trim} \alpha, \quad v = V_{trim} \beta \quad \text{and} \quad u = V_{trim}. \quad (3.4)$$

3.3 Controls

A conventional aeroplane is manoeuvred laterally by, usually, all controls,

- (i) the two primary longitudinal controls of *throttle*, which alters the thrust setting of the engines, and *forward and aft stick, or yoke, movement*, which rotates the elevator, together with secondary trim control of the elevator,
- and (ii) and the two primary lateral controls of *lateral stick, or yoke, movement*, which changes the aileron setting angle, ξ , and *rudder pedal movement*, which alters the rudder setting angle, ζ , together with secondary trim control of the rudder.

The two main types of control system, *manual control systems* for smaller aeroplanes and *powered control systems* for both larger aeroplanes and for high speed aeroplanes are described in Item 96037^[8] and apply equally to lateral controls as well as longitudinal controls. In both systems when it is assumed that the control system is rigid, a simple gearing ratio exists between forward and aft stick displacement and change in elevator angle, lateral stick displacement and change in aileron angle and rudder pedal displacement and rudder angle. Hinge moments in the case of manual control systems and artificial 'feel' systems in the case of powered controls provide stick and pedal forces that provide feedback to the pilot. Independent control of elevator and rudder trim tabs is also available to the pilot to alter the elevator and rudder hinge moments, and hence the stick and pedal forces, without altering the tailplane lift and fin force, respectively.

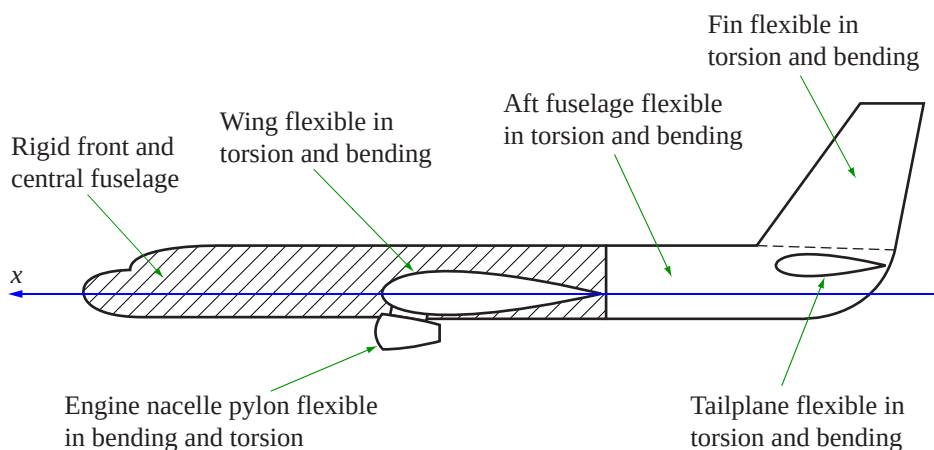
A lateral stick movement to starboard produces negative aileron deflection, *i.e.* an upward aileron deflection on the starboard half-wing and a corresponding downward aileron deflection on the port half-wing, and will generate a positive roll rate, p , about the x -axis of the aeroplane. A lateral stick movement to port will produce identically opposite results.

A starboard pedal forward displacement and corresponding port pedal rearward displacement will produce a negative rudder deflection angle, *i.e.* a starboard rudder deflection angle, generating a positive yaw rate, r , about the z -axis of the aeroplane.

4. FLEXIBLE AEROPLANES

In Item 96037^[8] the deformation of the aeroplane components, wing, tailplane, fuselage and engine nacelle pylons, under symmetric static loading, resulted in symmetric wing bending and twist, symmetric fuselage bending, symmetric tailplane bending and twist and symmetric bending and twist of a pair, or pairs, of engine nacelle pylons (one of each pair of nacelles on either side of the fuselage). These deformations depend upon the loading on the various surfaces, which in turn depend upon the stability and motion of the aeroplane in the quasi-steady symmetric manoeuvre.

In lateral motion, asymmetric loading comprises symmetric plus antisymmetric loadings in which the antisymmetric deformations are superimposed on the symmetric deformations. Antisymmetric loading on the aeroplane components results in antisymmetric wing bending and twist, rear fuselage bending and torsion, antisymmetric tailplane bending and twist, fin bending and twist and antisymmetric engine nacelle pylon bending and twist. The flexible aeroplane model for the purpose of this Item is illustrated in Sketch 4.1.



Sketch 4.1 Flexible aeroplane model

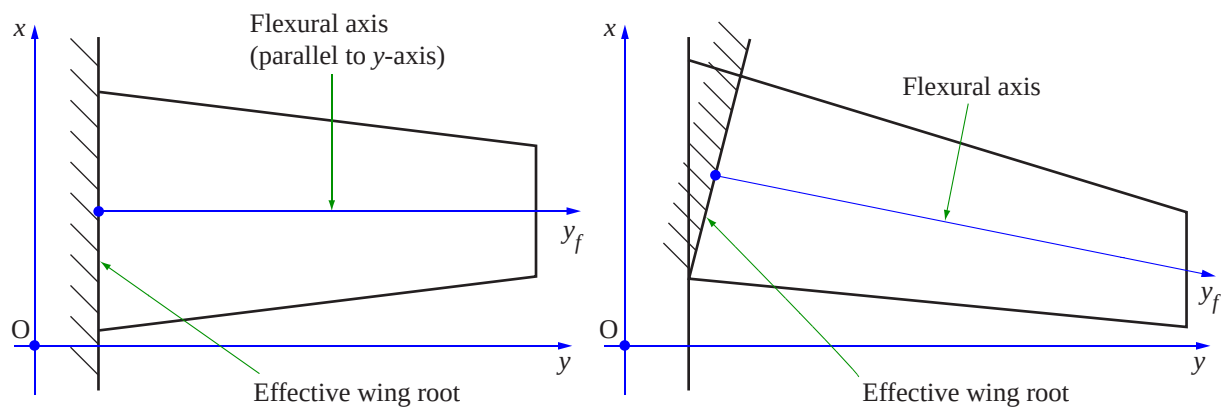
The antisymmetric bending and torsion of the various components of the aeroplane can be represented by the superposition of a set of semi-rigid modes or, alternatively, by a set of normal modes.

4.1 Component Flexibility

4.1.1 Wing

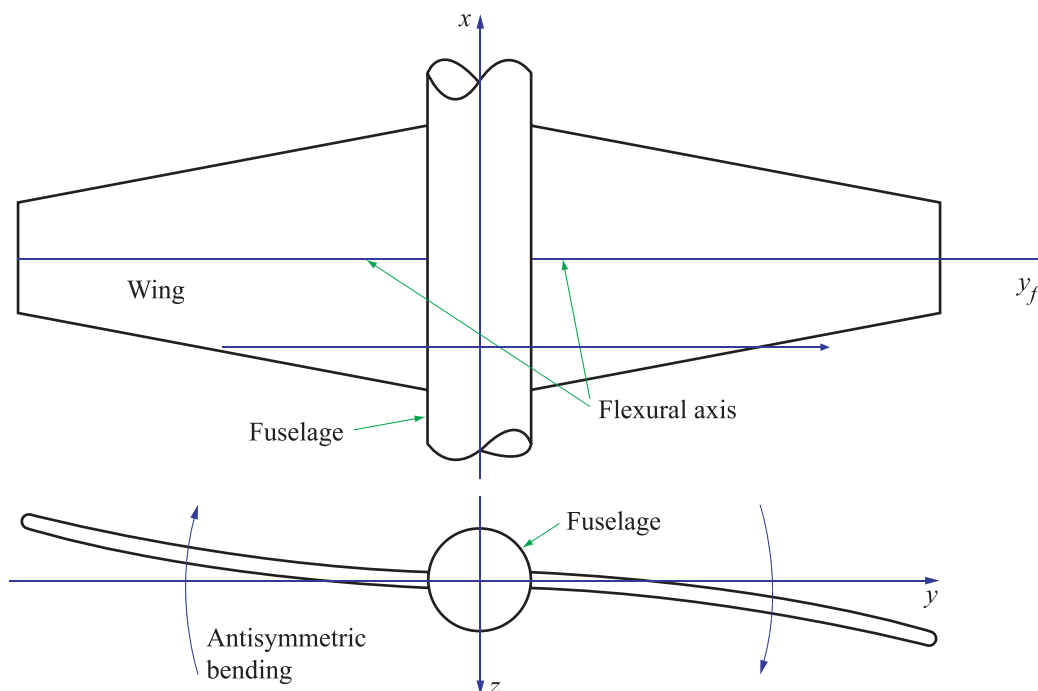
In line with the analysis of Item 96037^[8], each flexible half-wing acts as a cantilever built-in to a central rigid fuselage. Each half-wing bends about an effective wing root and twists about a flexural axis which is normal to the wing root.

For an unswept half-wing the effective wing root is parallel to the aeroplane centre-line while for a swept half-wing the effective wing root lies at an angle to the aeroplane centre-line. Sketch 4.2 illustrates both an unswept and a swept half-wing in which the flexural axis is denoted by y_f . The unswept half-wing therefore bends in a plane parallel to the y - z plane of the aeroplane and twists about an axis in the y -direction, whereas the swept half-wing bends in the y_f - z plane, which is swept relative to the y -axis, and twists about the y_f -axis.



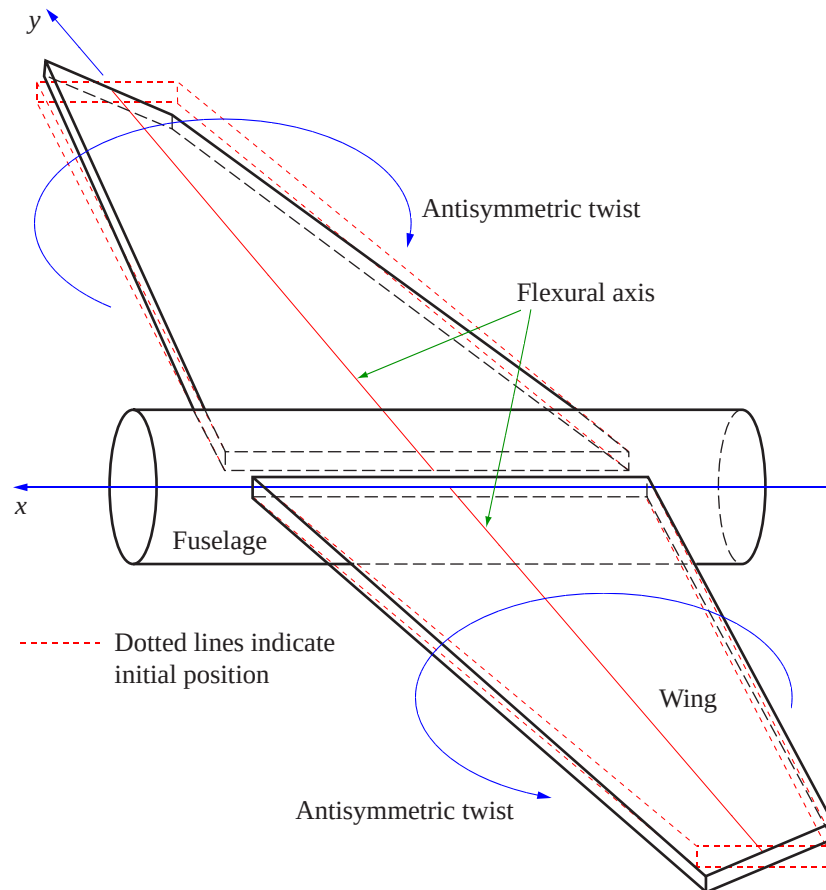
Sketch 4.2 Unswept and swept wing models

Antisymmetric bending, with zero twist, involves one half-wing bending upwards whilst the other bends downwards, as illustrated in the simplified model of a unswept configuration shown in Sketch 4.3.



Sketch 4.3 Antisymmetric bending of a simplified unswept configuration

Antisymmetric twist of an unswept wing with zero bending, for the same simplified configuration, is illustrated in Sketch 4.4 and shows twist relative to the effective wing root, one side of the half-wing twisting nose-up whilst the other twists nose-down.



Sketch 4.4 Twisting of a simplified unswept configuration

4.1.2 Fuselage

The flexible aeroplane model considered in this Item is illustrated in Sketch 4.1, in which the aft fuselage is considered flexible in bending, laterally (in the x - y plane) and longitudinally (in the x - z plane).

The aft fuselage can also twist about the x -axis.

4.1.3 Tailplane

The tailplane flexibility characteristics are identical with those of the wing in which the two halves of the flexible tailplane twist about the flexural axis of the tailplane and bend about an effective tailplane root. The tailplane root is built-in to the flexible aft fuselage.

4.1.4 Fin

The fin flexibility characteristics are akin to those of a half-wing but transposed to a vertical plane. The fin acts as a cantilever built-in to a fin root taken as perpendicular to the fin flexural axis. The fin will bend about an effective fin root and twist about the fin flexural axis. The fin root is built-in to the flexible aft fuselage.

4.1.5 Engine Nacelles and Pylons

Due to antisymmetric loading on the nacelles and pylons, the pylons supporting the engine nacelles will twist and bend in a highly coupled way, leading to nacelle displacement which includes mainly a twist about an axis parallel to the z-axis of the aeroplane and bending in a plane parallel to the aeroplane y-z plane. Changes in loading due to nacelle twist dominate whereas those due to bending are very small compared with wing bending effects and are generally ignored.