

Thickness selection for the flanges and web of a composite I-section beam subject to bending and shear

Associated software: ESDUpac A0313

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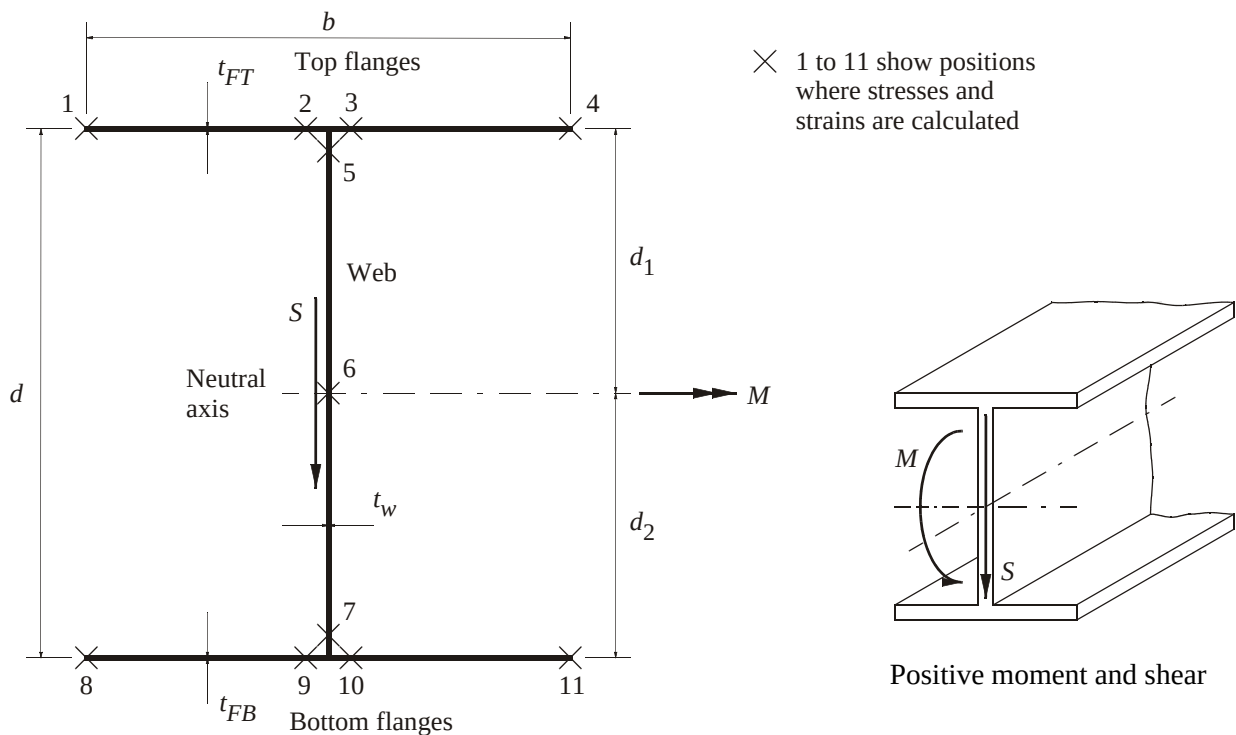
THICKNESS SELECTION FOR THE FLANGES AND WEB OF A COMPOSITE I-SECTION BEAM SUBJECTED TO BENDING AND SHEAR

This Data Item, together with ESDUpac A0313 provides a computer program to determine the flange and web thicknesses for laminated composite I-section beams that are required to withstand given bending moments and shear forces.

The Item consists of two parts. Part 1 describes the theoretical solution to the title problem. Part 2 describes the program, its input and output and example calculations that demonstrate the use of the program.

1. INTRODUCTION

This Item introduces a Fortran 77 computer program that calculates the thicknesses for the flanges and webs of laminated composite I-section beams that are required to withstand or transmit given bending moments and shear forces. The cross-sectional geometry of the beam is shown in Sketch 1.1.

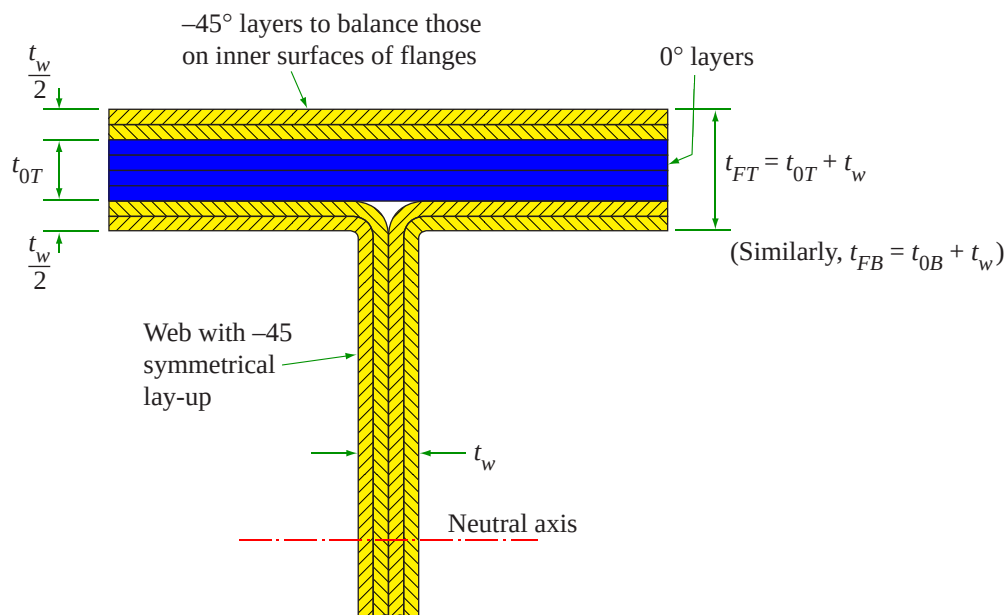


Sketch 1.1 Cross-section of beam

The section is treated as “thin-walled” so that the top and bottom flange thicknesses, t_{FT} and t_{FB} , and the web thickness, t_w , are small compared with the overall beam width, b , and depth, d . These overall dimensions are prescribed by the user, and the program provides efficient thicknesses for the flanges and web, for given bending moments and shear forces applied in the plane of the web, and for a specified flexural rigidity.

The flat plates that form the flanges and web are made from a number of thin orthotropic layers of continuous fibre-reinforced composite material with perfect bonding assumed between the layers. Because the section is taken to be thin-walled it is assumed that the applied bending moments give rise to longitudinal strains that are constant over the width of the flanges and vary linearly over the depth of the web. The shear forces

produce shear strains that are nearly constant over the depth of the web and vary linearly over the width of the flanges. It is well known that a 0° lay-up is best for reacting direct loads and that a $\pm 45^\circ$ fibre lay-up is best for shear loads. It is therefore advantageous to place the majority of 0° layers in the top and bottom flanges of the beam, and to use the $\pm 45^\circ$ lay-up in the web.



Sketch 1.2 Lay-up scheme for I-section

Sketch 1.2 shows the various layers in the cross-section. It is noted that the laminates that make up the beam are all of the same unidirectional composite plies layed-up in 0° and/or $+45^\circ$ and -45° layers. It can be seen that the connection between the web and flanges is made by continuing the web layers around to the inner faces of the flanges. Continuous $\pm 45^\circ$ layers are added to the outer flange faces as shown. These layers provide transverse strength to the flanges, and prevent the separated web layers from splitting at the top and bottom of the web. Because the web lay-up is constrained to be symmetric, this arrangement means that two of the flanges will have a symmetrical lay-up while the other two will be antisymmetric. It is, however, assumed in the analysis that all the flanges are symmetric and it is anticipated that any coupling terms that arise in the constitutive equations for the antisymmetric flanges will be small and can be neglected.

The loading on the beam is assumed to consist of given combinations of bending moment and shear force at the critical section of the beam along its span. The user provides values for the moments and shear forces for up to ten different load cases and the computer program calculates flange and web thicknesses for given overall beam section width and depth. Positive moments generate tensile stresses in the top flange. The sign of the shear forces is immaterial because the related local stresses in the flanges appear with equal magnitudes but opposite signs at locations 2, 3, 9 and 10, see Sketch 1.1. The web stresses may vary over the depth of the web, owing to the position of the neutral axis, but the strictly $\pm 45^\circ$ lay-up will ensure that the same reserve factors arise, albeit in different layers.

The analysis to obtain the layer thicknesses for the flanges and web of the I-section is controlled by a set of constraints that finally determines the detailed solution for the various stacking sequences. The constraints are as follows.

- Allowable unidirectional layer strains in tension, compression and shear, as described in more detail in Section 1.3.2.1.
- Buckling constraints for flanges and web. The formulae utilized to estimate these critical stresses are presented in Sections 1.3.2.2 and 1.3.2.3.
- Satisfaction of a minimum stipulated value for the overall flexural rigidity of the beam, $\overline{EI}$, as outlined in Section 1.3.2.4.

The computer program is based on the theory given in Derivation 2. The user is required to provide the overall beam dimensions and loading, and the mechanical properties for a single unidirectional fibre-reinforced material. With this information, rough estimates of the flange and web thicknesses are obtained by assuming a uniform shear load in the web and uniform end-loads in the flanges. The procedure is known as “initial sizing”, and takes into account web and flange buckling and fibre failure. At this stage the top and bottom flanges have the same thickness.

Having obtained a rough first estimate for the design, the various thicknesses are then adjusted using a more precise direct and shear load distribution across the section and more precise equations for flange and web buckling. The ensuing reserve factors are utilized to adapt thicknesses to satisfy the constraints. This re-design procedure thickens or thins layers according to certain rules that are based on expectations of how the layers will best reinforce the beam. These are referred to as heuristic design rules and allow for failure due to excessive strains, flange and web buckling, and low flexural rigidity.

The design procedure is iterative, and is terminated when the minimum reserve factor is between 0.99 and 1.05 inclusive, indicating that any one layer just reaches one of the allowable strain values of the material, or that buckling is about to occur, or that the minimum required overall flexural rigidity of the beam is about to be reached. No other optimisation is performed and it is not possible to ensure that all of the above conditions are met at the same time because of the complicated interaction between them when the thicknesses are changed. However, the procedure guarantees that the associated reserve factors are all greater than or equal to 0.99 and the final design will generally be an efficient solution to the problem.

The analysis provides two sets of thicknesses, one in which the top and bottom flange thicknesses are constrained to be equal, denoted Configuration 1, and the other where these thicknesses are determined independently and may be unequal, denoted Configuration 2.

1.1 Scope and Limitations

For a composite I-beam of given overall dimensions, constructed from strictly 0°, +45° and –45° layers of a single unidirectional fibre-reinforced material, and required to withstand specified bending moments and shear forces, the program determines efficient flange and web thicknesses for the beam. The program also ensures that the section satisfies a given stiffness requirement, specified by the user in terms of a minimum overall flexural rigidity. The compressed flange and the web of the beam will satisfy conservatively-formulated buckling constraints.

Using the input data provided, the program will determine an efficient design for a beam within the following constraints.

- (i) I-beams constructed from a number of thin orthotropic layers of continuous, unidirectional, fibre-reinforced composite material.
- (ii) Beams with symmetrical balanced webs consisting of at least four layers at angles of $+45^\circ$ and -45° ; two on either side of the centre-line of the web. Note that this implies that the web thickness can only be increased in multiples of four times the standard ply thickness.

Beams with flanges that may be assumed to have symmetrical lay-ups about their mid-planes, consisting of layers at angles of 0° , $+45^\circ$ and -45° , and that may be treated as specially orthotropic laminates within the realm of classical laminate plate theory.

Beams in which the connection between the web and the flange is made by continuing the web layers around to the inner faces of the flanges. Note that this means that the outermost layers of the flanges must be at $+45^\circ$ and -45° to maintain the balanced lay-up of the flanges. Shear lag effects on the flange-web intersection stresses are ignored.

Beams for which first approximations for flange and web buckling stresses are considered acceptable. Other types of buckling, for example flexural-torsional buckling, should be checked subsequently.

1.2 Terms and Notation

1.2.1 Terms

Configuration 1	The case where the thicknesses of the top and bottom flanges are constrained to be equal.
Configuration 2	The case where the thicknesses of the top and bottom flanges may differ.
<i>DERIVED</i> design	The design in which the thicknesses are based purely on the buckling, strain limitation and flexural rigidity requirements; <i>cf.</i> <i>PRACTICAL</i> design.
<i>PRACTICAL</i> design	The design in which the thicknesses obtained in the <i>DERIVED</i> design have been increased to accommodate an integer number of standard ply thicknesses, while satisfying the buckling, strain limitation and flexural rigidity requirements; <i>cf.</i> <i>DERIVED</i> design.

1.2.2 Notation

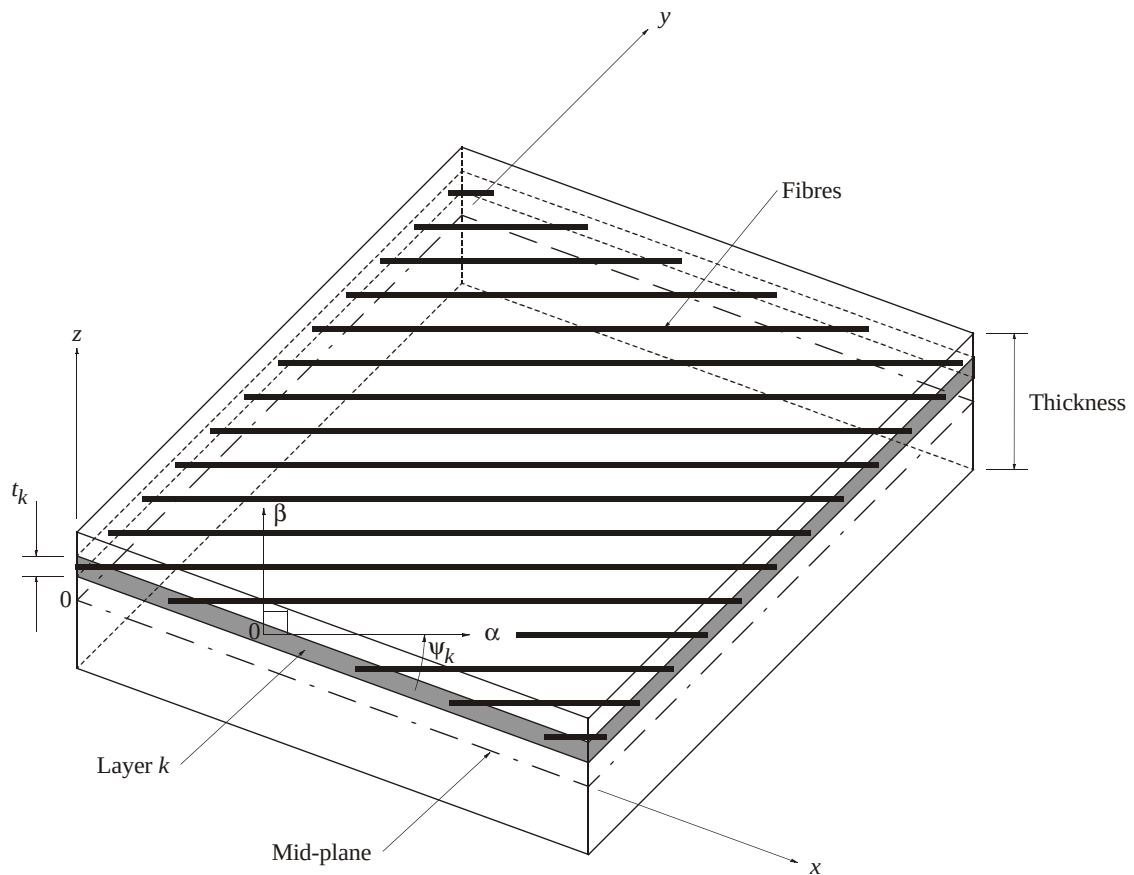
Both SI and British units are quoted but any coherent system of units may be used.

b	beam width	m	in
$D_{11}, D_{12}, D_{22}, D_{33}$	flexural rigidities in 3×3 matrix linking moments and curvatures	N m	lbf in
d	beam depth	m	in

d_1, d_2	distances, see Sketch 1.1	m	in
E_α, E_β	moduli of elasticity of layers in α - and β -directions respectively, see Sketch 1.3	N/m ²	lbf/in ²
$\overline{EI}$	overall flexural rigidity of beam	N m ²	lbf in ²
$G_{\alpha\beta}$	shear modulus of layers in α - and β -directions, see Sketch 1.3	N/m ²	lbf/in ²
LC	number of load cases		
M	applied bending moment	N m	lbf in
N_c	critical buckling load per unit width of flange	N/m	lbf/in
OP	output specifier		
RF	reserve factor		
S	applied shear force	N	lbf
t_{0B}	combined thickness of 0° layers in bottom flanges, see Sketch 1.2	m	in
t_{0T}	combined thickness of 0° layers in top flanges, see Sketch 1.2	m	in
t_{FB}	total thickness of bottom flanges, see Sketch 1.2	m	in
t_{FT}	total thickness of top flanges, see Sketch 1.2	m	in
t_k	thickness of k th layer, see Sketch 1.3	m	in
t_s	standard thickness of ply	m	in
t_w	thickness of web	m	in
$0(x, y, z)$	plate main reference axes, see Sketch 1.3		
$0(\alpha, \beta, z)$	principal axes of orthotropy of a layer, see Sketch 1.3		
γ	shear strain		
ϵ	direct strain		
λ	half-wavelength of buckle	m	in
$\nu_{\alpha\beta}$	Poisson's ratio for layer resulting from load applied in α -direction with induced strain in β -direction		
ψ_k	angle of principal axis of orthotropy, α_k , of k th layer measured from plate reference axis, $0x$, see Sketch 1.3	deg	deg

1.2.3 Suffixes

a, all	allowable values
B	bottom flanges
c	compressive strain values
F, FL	flanges
i	index
k	refers to k th layer
min	minimum
T	top flanges
t	tensile strain values
w	web
x, y, z	relate to x -, y - and z - directions respectively, see Sketch 1.3
α, β	relate to α - and β - directions respectively, see Sketch 1.3



Sketch 1.3 Laminated plate geometry

1.3 Theory

1.3.1 The Netting Analysis

In this first estimate for the flange and web thicknesses, the applied bending moment is assumed to be reacted entirely by the 0° fibres in the equal top and bottom flanges, whilst the applied shear force is taken entirely by the web. In each case the resulting loads are assumed to be uniformly distributed. At this stage no account is taken of the flexural rigidity requirements.

Thus for the flanges, the end-load is simply given by the value of M/d , and this is used to calculate three thicknesses for the 0° layers; one for tensile failure, one for compressive failure, and one for buckling. The lowest value of these three is selected for the initial design so that the heuristic rules that are applied next may have the opportunity of improving each mode of failure. For the web, the shear load can be represented by direct tension and compression loads at $\pm 45^\circ$ to the direction of shear, which of course correspond with the fibre directions. Then, web thicknesses are calculated for tensile failure and for compressive failure. A third thickness is calculated for shear web buckling and the lowest of the three thicknesses is used for the initial design.

The beam thicknesses that result for the initial design are calculated with the maximum values for M and S . See also the note following Table 2.1 in Section 2.2.1. If any ply thickness has a value smaller than the standard unidirectional ply thickness specified in the input, the specified input value, t_s , is used in further computations.

1.3.2 More Precise Analysis

The netting analysis described above provides a starting point for the design procedure in which various heuristic rules are employed. However, in calculating the various failure modes, more precise analyses are required for the calculation of layer stresses and strains and for flange and web buckling. These rules and formulae are utilized to thicken or thin the 0° layers in the flanges and the $\pm 45^\circ$ layers in flanges and web. This gives what is referred to in this Item as the '*DERIVED* design'. Once the minimum reserve factor has satisfied the convergence criterion,

$$0.99 \leq RF \leq 1.05,$$

the thicknesses are increased to accommodate an integer number of unidirectional plies, based on the input value for standard ply thickness, t_s , and this gives the final solution, or the '*PRACTICAL* design'. Note that the convergence criterion holds strictly only for the *DERIVED* design and the failure mode and reserve factors for the *PRACTICAL* design may differ considerably.

1.3.2.1 Stresses and strains

Engineer's theory of bending, adapted for use with composite material, is applied to calculate the direct load distribution due to the bending moment, giving uniform loads in the flanges with a linear variation in the web. The shear force gives a linear variation of shear loading in the flanges (being zero at the free edges) and a parabolic variation in the web, with a maximum value at the neutral axis of the section. The layer stresses and strains that arise from the combined action of these loadings are obtained by applying elastic, small displacement, classical laminate theory to the flanges and web which are assumed to behave as flat plates. From a cursory inspection it is not possible to establish *a priori* where the maximum layer stresses and strains will occur; they are therefore sought by calculating stresses at eleven different positions on the beam cross-section: namely, at each of the four free edges of the top and bottom flanges; at the inner end of each flange on either side of the web; at the top and bottom of the web; and at the neutral axis of the section. These positions are shown in Sketch 1.1. Strain reserve factors are then calculated for each possible