



Non-Newtonian fluids: tube viscometry worked example

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Dr P.G.Long	— Cambridge University
Dr S.J.Murray	— W.S. Atkins Consultants Ltd
Dr J.T.Turner	— University of Manchester

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Dr N.J. Alderman — Aspen Technology

with the assistance of Mr S.J. Pugh. The person with overall responsibility for the work in this subject area was Mr S.J. Pugh, Head of Thermofluids Group.

NON-NEWTONIAN FLUIDS: TUBE VISCOMETRY – WORKED EXAMPLE

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	2
2.1 Purpose and Scope	2
2.2 Use of Spreadsheets	3
3. WORKED EXAMPLE	4
3.1 Do Pressure Loss - Flow Rate Data Correspond To Laminar Flow Conditions In The Tube - Step (b)	6
3.2 Do Pressure Loss - Flow Rate Data Correspond To Isothermal Conditions In The Tube - Step (c)	9
3.3 Correcting Pressure Loss Data For End Effects - Step (d)	11
3.3.1 Use of multiple tubes (correction applied)	11
3.3.2 Use of longest available tube (correction not applied)	18
3.3.3 Comparison of corrected data with uncorrected data	19
3.4 Correcting the <i>Flow Rate</i> Data for Wall Slip - Step (e)	19
3.4.1 Use of multiple diameter tubes (correction applied)	19
3.4.2 Use of largest diameter tube (correction not applied)	28
3.4.3 Comparison of corrected data with uncorrected data	29
3.5 Obtaining The Corrected Flow Curve - Step (f) to (i)	30
4. DERIVATION AND REFERENCES	35
4.1 Derivation	35
4.2 References	35
APPENDIX A DATA FOR THE CORRECTED FLOW CURVE OF SKETCH 3.14	36

NON-NEWTONIAN FLUIDS: TUBE VISCOMETRY – WORKED EXAMPLE

1. NOTATION AND UNITS

		SI Units
a, b, c	constants in Equation (3.7)	–
D	tube diameter	m
K	consistency coefficient	$\text{N s}^n/\text{m}^2$ (Pa s^n)
K_0	consistency coefficient at reference temperature, θ_0	$\text{N s}^n/\text{m}^2$ (Pa s^n)
L	tube length	m
m	exponent in wall slip equation, see Table 3.14	–
n	exponent in power law or Herschel-Bulkley model	–
n'	local slope of $\ln \tau_w$ versus $\ln (8V/D)$	–
Q	volumetric flowrate	m^3/s
S	slip factor	$\text{m}^{2+m}\text{s/kg}$
V_{slip}	slip velocity	m/s
β	viscosity-temperature coefficient	–
$\dot{\gamma}_w$	shear rate at tube wall	s^{-1}
$\dot{\gamma}_{\text{nom}}$	nominal shear rate	s^{-1}
Δp	pressure loss along the tube	N/m^2 (Pa)
Δp_c	corrected pressure loss	N/m^2 (Pa)
Δp_e	end effect pressure loss	N/m^2 (Pa)
θ	temperature	K
θ_0	reference temperature	K
λ	thermal conductivity	W/m K
μ	dynamic viscosity	N s/m^2 (Pa s)
μ_0	dynamic viscosity at reference temperature, θ_0	N s/m^2 (Pa s)
ρ	fluid density	kg/m^3
τ_w	shear stress at tube wall	N/m^2 (Pa)
τ_y	yield stress	N/m^2 (Pa)

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2. INTRODUCTION

2.1 Purpose and Scope

This Data Item is the fourth in a group on the flow characteristics of non-Newtonian fluids. It contains a practical worked example that illustrates how to obtain, using a laboratory-scale tube viscometer, the relevant viscometric data necessary for frictional pressure loss estimation for full-scale pipeflow.

Tube viscometry is one of the five viscometric methods used for flow curve determination of non-Newtonian fluids. Further details can be found in Section 6 of ESDU 95012², which also provides background guidance on non-Newtonian flow characterisation and viscometer selection.

The behaviour of a fluid in pipeflow can be characterised by its flowcurve over the appropriate shear rate/shear window for the pipeline application at the same temperature (and pressure). In this Item, the calculation procedure for the generation of the flow curve obtained with a tube viscometer is demonstrated, following the methods of Section 6.1 of ESDU 95012.

The purpose of this Data Item is to guide the user, by way of example, through the various procedures needed to obtain the flow curve. These procedures ensure that the experimental data are corrected for end effect and wall slip whilst ensuring that the viscometric data are obtained under laminar flow and isothermal conditions.

The methods of this Item are applicable to isothermal, fully-developed flow through straight pipes of circular cross-section, where flow properties are measured under steady-state shear conditions. These conditions are necessary in order to develop the flow curve, which can then be used to predict full-scale flow pressure drop of the tested fluid under the same conditions.

The calculation procedure assumes that the flow curve of the fluid can be described by a power-law relationship such as the following:

$$\tau = K\dot{\gamma}^n.$$

In this equation, which models shear-thinning and shear-thickening fluids, K is the constituency coefficient (in units of Pa s^n) and n is the power-law exponent.

Guidance on the flow characteristics of non-Newtonian fluids is given in ESDU 97034⁶, which provides a guide to their identification and classification, and ESDU 95012.

The layout of this Item is as follows:

Section 3 describes the step-by-step procedure for obtaining the corrected flow curve. Here, an example data set used throughout this Item is introduced.

Sections 3.1 and 3.2 address the question of whether the pressure drop (Δp) versus flow rate (Q) data correspond to laminar flow and isothermal conditions in the tube viscometer.

Section 3.3 describes the step-by-step procedure for correcting the Δp data for end effects.

Section 3.4 describes the step-by-step procedure for checking for the presence of wall slip and correcting the Q data for wall slip if it is found to be present.

Section 3.5 gives the methodology for calculating the corrected flow curve, which is then used for any

subsequent analysis such as the prediction of full-scale frictional pressure loss estimation in pipeflow using ESDU 91025.

The methods of this Item can be applied using a spreadsheet following the step-by-step procedures of Section 3.

2.2 Use of Spreadsheets

The calculation procedure can be carried out conveniently using a spreadsheet program such as Microsoft® Excel. For the purposes of this worked example, Excel 2000 was used.

The spreadsheet user should be familiar with the plotting capabilities of the program (using both linear and logarithmic scales), and the features within the spreadsheet that allow simple curve-fitting to the data (or adding 'Trendlines', which is the terminology used by Excel).

In Excel 2000, lines can be fitted to data sets and superimposed on the charts by selecting the data set (or a single point in the set) and selecting "Add Trendline..." from the Options menu that appears when the right-hand mouse button is pressed (assuming a right-handed mouse configuration).

The "Add Trendline" menu that subsequently appears allows a number of simple equation types to be fitted to the data, expressed as "Trend/Regression type" under the "Type" tab.

Within this calculation procedure, "Linear" (straight line), "Polynomial" and "Power" types are used, as described below. For the "Polynomial" type, it is necessary to select the order of the polynomial equation (which is 2 in all instances used in this worked example).

Having selected an equation type, it is necessary to view the equation and its coefficients. This is achieved by clicking the "Options" tab on the "Add Trendline" menu, and checking the box "Display equation on chart".

The user should be aware that the number of significant figures shown by default in the displayed equations may not be appropriate for the purposes of the calculations in this Item. It is recommended that the user program the resulting equation in the spreadsheet and, by plotting the resulting data on the same chart, compare the calculated line with the automatic trendline. For some types of equation, particularly higher-order polynomials, the number of significant figures shown by default in the displayed equations is not sufficient to reproduce the line to engineering accuracy: it is therefore necessary to over-ride the default for the displayed number of significant figures to show a larger number.

In Excel 2000, the number of significant figures shown in an equation can be altered as follows:

1. Selected the equation by clicking once within it with the right-hand mouse button.
2. Select "Format Data Labels..." from the resulting menu.
3. Select the category "Number" within the "Number" tab.
4. Select the number of decimal places within the "Decimal places" field. Note that this is not the same as the number of significant figures but is adequate for the purpose.

For polynomial equations with large powers, it is necessary to ensure that the numbers are in "scientific" format prior to selecting the number of decimal places.

3. WORKED EXAMPLE

The step-by-step procedure for obtaining the *corrected* flow curve using tube viscometry data is outlined in Tables 6.1 and 6.2 of ESDU 95012² and is reproduced in the combined form shown in Table 3.1.

TABLE 3.1 Procedure for obtaining the corrected flow curve using tube viscometry

Step	Description	Section
(a)	Measure Δp as a function of Q or vice versa.	3
(b)	Reject Δp versus Q data affected by turbulent flow conditions.	3.1
(c)	Reject Δp versus Q data affected by viscous heating.	3.2
(d)	Correct Δp data for end effects.	3.3
(e)	Correct Q data for wall slip.	3.4
Using corrected Δp versus Q data, carry out steps (f) to (i)		
(f)	Calculate τ_w from Equation (3.2).	3.5
(g)	Plot τ_w versus $8V/D$ on logarithmic coordinates and estimate the slope n' at each $8V/D$ value.	3.5
(h)	Calculate $\dot{\gamma}_w$ from Equation (3.13).	3.5
(i)	Plot τ_w against $\dot{\gamma}_w$ to give the corrected flow curve.	3.5

To demonstrate the step-by-step procedure outlined in Table 3.1 for obtaining the *corrected* flow curve, the Δp versus Q data set (from Reference 1) for a 0.5% aqueous solution of partially hydrolysed polyacrylamide solution given in Table 3.2 was adopted.

This data set was obtained at 25°C using a tube viscometer fitted with an interchangeable horizontal length of precision-bored straight tube through which the test fluid was passed from a reservoir. Here, a 4 × 4 matrix of tube lengths and diameters was used, the dimensions of which are given in Table 3.2. In carrying out the sixteen experiments*, it is important to ensure that the same four different tube lengths are used for each of the four different tube diameters.

The data set given in Table 3.2 constitutes **Step (a)** of the step-by-step procedure outlined in Table 3.1.

* It should be noted the minimum number of experiments needed for the step-by-step procedure outlined in Table 3.1 is actually eight. Here, the same four different tube lengths must be used for each of the two different tube diameters.

TABLE 3.2 Pressure loss versus volumetric flowrate data*

D = 1.91E-04m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)
43075	3.66E-11	60248	3.66E-11	121675	3.66E-11	132687	3.66E-11
60780	7.60E-11	85266	7.60E-11	172853	7.60E-11	188555	7.60E-11
70593	1.03E-10	99146	1.03E-10	201280	1.03E-10	219590	1.03E-10
92259	1.96E-10	129784	1.96E-10	264011	1.96E-10	288075	1.96E-10
101141	2.48E-10	142347	2.48E-10	289738	2.48E-10	316163	2.48E-10
115897	3.33E-10	163236	3.33E-10	332566	3.33E-10	362924	3.33E-10
125741	3.97E-10	177174	3.97E-10	361150	3.97E-10	394133	3.97E-10
134583	4.53E-10	189700	4.53E-10	386852	4.53E-10	422198	4.53E-10
146329	5.18E-10	206350	5.18E-10	421043	5.18E-10	459533	5.18E-10
153309	6.01E-10	216224	6.01E-10	441269	6.01E-10	481616	6.01E-10
D = 2.66E-04m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)
34640	9.98E-11	48384	9.98E-11	97548	9.98E-11	106362	9.98E-11
45265	1.85E-10	63372	1.85E-10	128142	1.85E-10	139754	1.85E-10
56581	3.09E-10	79349	3.09E-10	160788	3.09E-10	175388	3.09E-10
62989	4.22E-10	88386	4.22E-10	179232	4.22E-10	195519	4.22E-10
71474	5.63E-10	100375	5.63E-10	203757	5.63E-10	222291	5.63E-10
82061	7.55E-10	115346	7.55E-10	234408	7.55E-10	255754	7.55E-10
89786	8.80E-10	126283	8.80E-10	256832	8.80E-10	280237	8.80E-10
96154	1.03E-09	135288	1.03E-09	275271	1.03E-09	300368	1.03E-09
101760	1.12E-09	143230	1.12E-09	291567	1.12E-09	318162	1.12E-09
115813	1.40E-09	163133	1.40E-09	332396	1.40E-09	362742	1.40E-09
D = 6.26E-04m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)
10874	3.41E-10	14865	3.41E-10	29141	3.41E-10	31701	3.41E-10
18924	1.59E-09	26127	1.59E-09	51892	1.59E-09	56512	1.59E-09
25648	3.43E-09	35574	3.43E-09	71078	3.43E-09	77444	3.43E-09
29293	4.68E-09	40706	4.68E-09	81529	4.68E-09	88848	4.68E-09
31110	5.34E-09	43267	5.34E-09	86750	5.34E-09	94545	5.34E-09
34134	6.52E-09	47530	6.52E-09	95448	6.52E-09	104039	6.52E-09
36864	7.84E-09	51378	7.84E-09	103293	7.84E-09	112600	7.84E-09
40201	9.67E-09	56081	9.67E-09	112887	9.67E-09	123071	9.67E-09
42016	1.07E-08	58643	1.07E-08	118117	1.07E-08	128780	1.07E-08
44438	1.22E-08	62060	1.22E-08	125093	1.22E-08	136394	1.22E-08
49575	1.55E-08	69313	1.55E-08	139916	1.55E-08	152573	1.55E-08
51100	1.69E-08	71463	1.69E-08	144302	1.69E-08	157360	1.69E-08
D = 1.10E-03m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)	Δp (Pa)	Q (m ³ /s)
6537	6.49E-10	8813	6.49E-10	16956	6.49E-10	18415	6.49E-10
13505	1.21E-08	18460	1.21E-08	36188	1.21E-08	39366	1.21E-08
15944	1.73E-08	21888	1.73E-08	43149	1.73E-08	46960	1.73E-08
17705	2.32E-08	24356	2.32E-08	48146	2.32E-08	52411	2.32E-08
20145	3.15E-08	27786	3.15E-08	55117	3.15E-08	60017	3.15E-08
22059	3.90E-08	30478	3.90E-08	60594	3.90E-08	65993	3.90E-08
23969	4.73E-08	33168	4.73E-08	66070	4.73E-08	71969	4.73E-08
26752	6.20E-08	37085	6.20E-08	74047	6.20E-08	80673	6.20E-08
29693	7.78E-08	41232	7.78E-08	82508	7.78E-08	89908	7.78E-08
31602	9.04E-08	43924	9.04E-08	87996	9.04E-08	95898	9.04E-08
33845	1.04E-07	47089	1.04E-07	94463	1.04E-07	102957	1.04E-07

* Obtained for 0.5% aqueous solution of partially hydrolysed polyacrylamide at 25°C using a tube viscometer fitted with tubes of differing lengths and diameters (Data adapted from Cohen and Metzner¹)

3.1 Do Pressure Loss - Flow Rate Data Correspond To Laminar Flow Conditions In The Tube - Step (b)

Step (b) of the step-by-step procedure outlined in Table 3.1 makes use of the criterion for the laminar flow limit of power law non-Newtonian fluids given by Equation (6.1) of ESDU 95012, which is reproduced here as:

$$\tau_w > \frac{\rho D^2}{8} \left[\frac{n}{404} \frac{\left(\frac{1+3n}{4n} \right)^2}{(n+2)^{n+1}} \right] \left(\frac{8V}{D} \right)^2. \quad (3.1)$$

To apply Equation (3.1), it is necessary to convert all the Δp versus Q data in Table 3.2 to wall shear stress and nominal wall shear rate data using Equations (3.2) and (3.3) respectively:

$$\tau_w = \frac{D \Delta p}{4L} \quad (3.2)$$

$$\dot{\gamma}_{nom} = \frac{8V}{D} = \frac{32Q}{\pi D^3}. \quad (3.3)$$

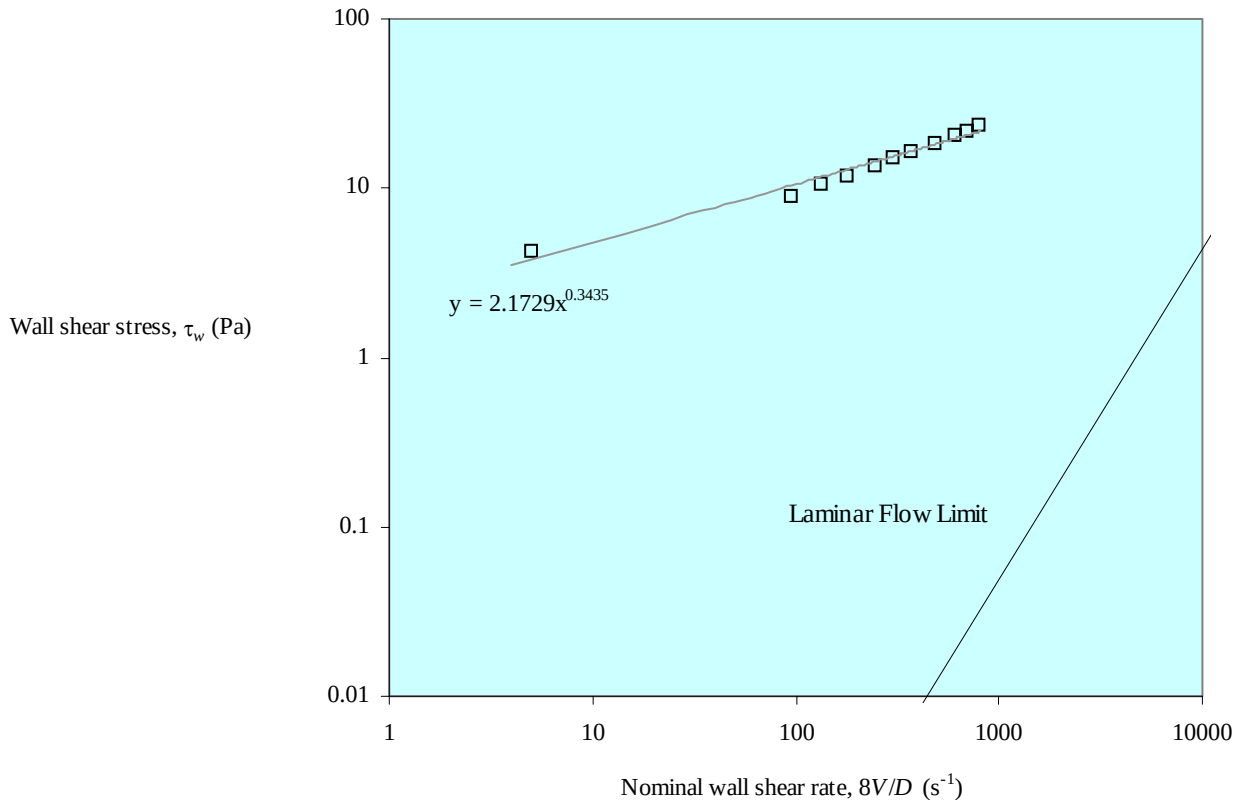
The wall shear stress and nominal wall shear rate data corresponding to Table 3.2 are given in Table 3.3.

TABLE 3.3 Wall shear stress versus nominal shear rate data corresponding to Table 3.2

D = 1.91E-04 m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)
5.49	53.8	5.40	53.8	5.30	53.8	5.29	53.8
7.74	111.9	7.65	111.9	7.53	111.9	7.52	111.9
8.99	151.6	8.89	151.6	8.77	151.6	8.76	151.6
11.75	288.8	11.64	288.8	11.50	288.8	11.49	288.8
12.88	365.1	12.76	365.1	12.62	365.1	12.61	365.1
14.76	490.1	14.64	490.1	14.48	490.1	14.47	490.1
16.01	584.6	15.89	584.6	15.73	584.6	15.72	584.6
17.14	667.0	17.01	667.0	16.85	667.0	16.83	667.0
18.64	761.5	18.50	761.5	18.34	761.5	18.32	761.5
19.53	883.4	19.39	883.4	19.22	883.4	19.20	883.4
D = 2.66E-04 m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)
6.16	53.9	6.06	53.9	5.93	53.9	5.92	53.9
8.05	99.7	7.94	99.7	7.79	99.7	7.78	99.7
10.06	166.9	9.94	166.9	9.78	166.9	9.77	166.9
11.20	227.9	11.07	227.9	10.90	227.9	10.89	227.9
12.71	304.1	12.57	304.1	12.39	304.1	12.38	304.1
14.60	407.8	14.44	407.8	14.26	407.8	14.24	407.8
15.97	475.0	15.81	475.0	15.62	475.0	15.61	475.0
17.10	557.3	16.94	557.3	16.74	557.3	16.73	557.3
18.10	606.1	17.94	606.1	17.73	606.1	17.72	606.1
20.60	758.6	20.43	758.6	20.22	758.6	20.20	758.6
D = 6.26E-04 m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)
4.55	14.1	4.38	14.1	4.17	14.1	4.15	14.1
7.91	66.2	7.69	66.2	7.42	66.2	7.40	66.2
10.73	142.5	10.47	142.5	10.16	142.5	10.14	142.5
12.25	194.4	11.99	194.4	11.66	194.4	11.63	194.4
13.01	221.9	12.74	221.9	12.41	221.9	12.38	221.9
14.28	270.7	13.99	270.7	13.65	270.7	13.62	270.7
15.42	325.6	15.13	325.6	14.77	325.6	14.74	325.6
16.81	401.9	16.51	401.9	16.14	401.9	16.11	401.9
17.57	444.6	17.27	444.6	16.89	444.6	16.86	444.6
18.59	505.6	18.27	505.6	17.89	505.6	17.86	505.6
20.73	642.8	20.41	642.8	20.01	642.8	19.98	642.8
21.37	700.7	21.04	700.7	20.64	700.7	20.60	700.7
D = 1.10E-03 m							
L = 0.374m		L = 0.531m		L = 1.094m		L = 1.195m	
τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)	τ_w (Pa)	8V/D (s ⁻¹)
4.79	5.0	4.55	5.0	4.25	5.0	4.23	5.0
9.90	93.7	9.53	93.7	9.07	93.7	9.03	93.7
11.69	133.4	11.30	133.4	10.82	133.4	10.78	133.4
12.98	179.2	12.57	179.2	12.07	179.2	12.03	179.2
14.77	243.3	14.34	243.3	13.82	243.3	13.77	243.3
16.17	301.2	15.73	301.2	15.19	301.2	15.14	301.2
17.57	365.3	17.12	365.3	16.56	365.3	16.52	365.3
19.61	478.2	19.14	478.2	18.56	478.2	18.51	478.2
21.77	600.2	21.28	600.2	20.68	600.2	20.63	600.2
23.17	697.8	22.67	697.8	22.06	697.8	22.01	697.8
24.81	801.4	24.30	801.4	23.68	801.4	23.63	801.4

For each of the 16 tubes, a check has to be made on whether the τ_w versus $8V/D$ data are lying to the left of the laminar flow limit line. Any data found lying on or to the right of the line must be rejected as this is indicative of transitional or turbulent flow. The procedure for carrying out the check is demonstrated for the tube with dimensions of $D = 1.10 \times 10^{-3}$ m and $L = 1.195$ m as an example.

The τ_w versus $8V/D$ data for this tube are plotted as a double logarithmic plot in Sketch 3.1, where the line through the data points illustrates a power-law fit to the data points.



Sketch 3.1 Plot of τ_w versus $8V/D$ data and laminar flow limit for tube of $D = 1.10 \times 10^{-3}$ m and $L = 1.2$ m

Curve fitting the τ_w versus $8V/D$ data in Sketch 3.1 using

$$\tau_w = K \left(\frac{8V}{D} \right)^n, \quad (3.4)$$

yields a power law exponent, n , of 0.3435.

Using Microsoft Excel, the curve fitting was carried out using the “Add Trendline” facility. Selecting a “Power” law line yields the equation $y = 2.1729x^{0.3435}$; this is equivalent to Equation (3.4). In this case, the number of significant figures returned by default by Excel is unnecessarily large: two significant figures on the values of K and n (or 2.2 and 0.34 respectively) are sufficient for this calculation. The user should, however, test the numerical sensitivity of the equation to the number of significant figures to prevent unexpected errors, as discussed in Section 2.2

Assuming a density of 1000 kg/m^3 for the aqueous solution, Equation (3.1) can be used to calculate the

laminar flow limit for a power-law fluid with $n = 0.3435$:

$$\tau_w = \frac{1000 \times (1.10 \times 10^{-3})^2}{8} \left[\frac{0.3435}{404} \frac{\left(\frac{1 + 3 \times 0.3435}{4 \times 0.3435} \right)}{\frac{0.3435 + 2}{(0.3435 + 2)^{0.3435 + 1}}} \right] \left(\frac{8V}{D} \right)^2$$

$$= 6.32 \times 10^{-8} \left(\frac{8V}{D} \right)^2.$$

With this equation, the laminar limit line can be then drawn as shown in Sketch 3.1. For this tube, all of the τ_w versus $8V/D$ data were found to lie to the left of the laminar limit line, i.e. all the τ_w versus $8V/D$ data correspond to laminar flow conditions.

This checking procedure can be repeated for each of the remaining 15 tubes. It will be found that all of the τ_w versus $8V/D$ data for each of the 16 tubes lie to the left of their respective laminar limit lines. Having checked the τ_w versus $8V/D$ data for each tube correspond to laminar flow in the tube, the next step is to check that this data set corresponds to isothermal conditions in the tube as outlined in Section 3.2.

3.2 Do Pressure Loss - Flow Rate Data Correspond To Isothermal Conditions In The Tube - Step (c)

Step (c) of the step-by-step procedure outlined in Table 3.1 makes use of the criterion for the viscous heating limit of power law non-Newtonian fluids given by Equation (6.2) of ESDU 95012 which is reproduced here as:

$$\tau_w < 2 \left[\exp \left\{ \frac{0.1}{2n} \right\} - 1 \right] n \lambda \frac{\theta_0}{\beta} \left(\frac{1 + 3n}{4n} \right)^n \left[\frac{4^{n+2}}{D^2} \right] \frac{1}{8V/D}, \quad (3.5)$$

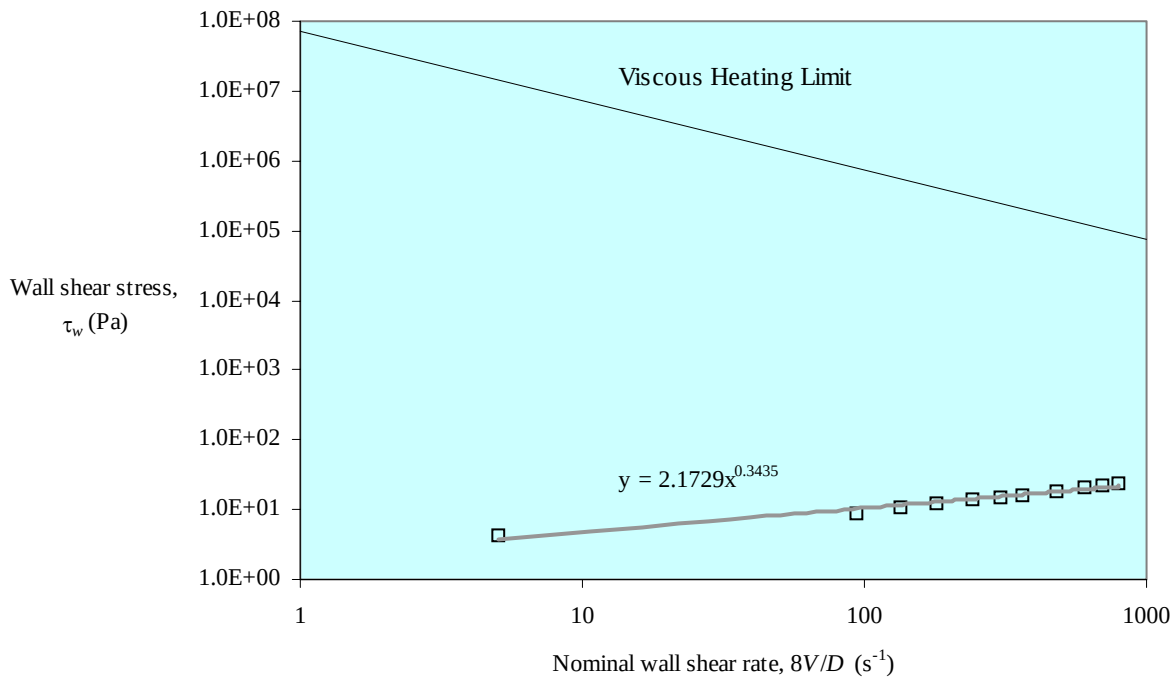
where λ is the thermal conductivity and β is the viscosity-temperature coefficient obtained from:

$$\beta = \frac{\ln(K/K_0)}{1 - \theta/\theta_0}, \quad (3.6)$$

in which K_0 is the consistency coefficient at a certain reference temperature, θ_0 . This criterion is based on the maximum temperature not exceeding the reference temperature by more than 10% (that is, $\theta/\theta_0 \leq 1.1$).

For each of the 16 tubes, a check has to be made on whether the τ_w versus $8V/D$ data given in Table 3.2 are below the viscous heating limit line. Any data found lying on or above the line must be rejected as this is indicative of the data being affected by significant temperature increases due to viscous heat dissipation. The procedure for carrying out the check is demonstrated using the same tube example ($D = 1.10 \times 10^{-3}$ m and $L = 1.195$ m) in Section 3.1.

The τ_w versus $8V/D$ data for this tube are plotted as a double logarithmic plot in Sketch 3.2.



Sketch 3.2 Plot of τ_w versus $8V/D$ data and viscous heating limit for the tube of $D = 1.10 \times 10^{-3}$ m and $L = 1.2$ m

From Section 3.1, the curve fit of the τ_w versus $8V/D$ data is the line through the data points in Sketch 3.2.

Taking the thermal conductivity of the aqueous solution to be that of water, the thermal conductivity at temperature $\theta = 25^\circ\text{C}$ can be obtained from ESDU 04002 (Derivation 3) as 0.607 W/m K.

To enable the use of Equation (3.6), the consistency coefficient ratio, K/K_0 is assumed to be similar to the viscosity ratio, μ/μ_0 for water. The criterion given by Equation (3.5) is based on the maximum temperature in $^\circ\text{C}$ not exceeding the reference temperature in $^\circ\text{C}$ by more than 10%, ($\theta = 1.1\theta_0$).

From ESDU 04002, at $\theta_0 = 25^\circ\text{C}$, $\mu_0 = 0.889 \times 10^{-3}$ Pa s and at $\theta = 1.1\theta_0$ (27.5°C), $\mu = 0.842 \times 10^{-3}$ Pa s.

It follows that $K/K_0 = \frac{0.842}{0.889} = 0.947$, and from Equation (3.6)

$$\beta = \frac{\ln(0.956)}{1 - (300.65/298.15)} = 6.49.$$

With n , λ and β known, Equation (3.5) can be used to calculate the viscous heating limit.

$$\begin{aligned} \tau_w &= 2 \left[\exp \left\{ \frac{0.1}{2 \times 0.3435} \right\} - 1 \right] \times 0.3435 \times 0.607 \times \frac{298.15}{6.49} \left(\frac{1 + 3 \times 0.3435}{4 \times 0.3435} \right)^{0.3435} \times \left[\frac{4^{2.3435}}{(1.097 \times 10^{-3})^2} \right] \frac{1}{8V/D} \\ &= 7.4 \times 10^7 \times \frac{1}{8V/D}. \end{aligned}$$

With this equation, the viscous heating line can be drawn as shown in Sketch 3.2. For this tube, all of the τ_w versus $8V/D$ data were found to lie well below the viscous heating line, that is, all the τ_w versus $8V/D$