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Pressure gradient in upward adiabatic flows of gas-liquid mixtures in vertical pipes

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Two-phase Flow Working Party on behalf of the Internal Flow Panel:

Prof. B.J.Azzopardi	— University of Nottingham, UK
Mr S.C. Curzons	— Rolls-Royce plc, Derby, UK
Dr L.Daniels	— Hyprotech, UK
Mr G. McNulty	— BHR Group Ltd, UK
Dr R.M. Rolston	— United Biscuits, UK
Mr G.H. Walter	— Independent, UK.

The Internal Flow Panel first met in 1979 and had the following membership:

Chairman	
Dr J.A. Eaton	— National University of Ireland, Galway, Ireland
Vice-Chairman	
Mr D.A. Campbell	— Independent, UK
Members	
Dr W.R. Geddes	— BNFL Consultancy Services, UK
Mr A.J. Green	— BHR Group Ltd, UK
Professor J.L. Livesey	— Independent, UK
Dr P.J.G. Long	— Cambridge University Engineering Department, UK
Dr S.J. Murray	— W.S. Atkins Science and Technology, UK
Dr. J.T. Turner	— University of Manchester, UK
Professor D.H. Freeston*	— Geothermal Institute, University of Auckland, New Zealand.

* Corresponding Member.

The technical work involved in the assessment of the available information and the construction and subsequent development of the Data Item and database was undertaken by

Prof. G.F. Hewitt	— Imperial College of Science, Technology and Medicine, London, UK
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with the assistance of Mr A. Laforgia and Mr S.J. Pugh.

The person with overall responsibility for the work in this area was Mr S.J. Pugh, Head of Thermofluids Group.

(Continued on inside back cover)

PRESSURE GRADIENT IN UPWARD ADIABATIC FLOWS OF GAS-LIQUID MIXTURES IN VERTICAL PIPES

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PRESSURE GRADIENT IN UPWARD ADIABATIC FLOWS OF GAS-LIQUID MIXTURES IN VERTICAL PIPES

1. NOTATION AND UNITS

The SI system of units should be used throughout all the calculations to ensure compatibility with the dimensional quantities in the text.

		SI
A	cross-sectional area of pipe	m^2
B	parameter used in Chisholm equations	—
$B_{1..2}$	parameters defined in Appendix D	—
$B_{3..5}$	parameters defined in Appendix E	—
C	confidence level, or parameter used in Lockhart and Martinelli correlation (Appendix H)	—
C_o	weighted mean distribution factor	—
$C_{1..9}$	parameters defined in Appendix E	—
D	pipe inside diameter	m
E	parameter in Friedel correlation (Appendix I)	—
E_1, E_2	parameters given by Equations (5.8) and (5.9)	—
$\bar{E}$	mean correction factor within selected batch of N data points given by $\bar{E} = 10^{\bar{e}}$	—
ΔE	uncertainty factor within selected batch of N data points given by $\Delta E = 10^{\Delta e}$	—
e	logarithmic error ratio defined by $\log_{10}$ (measured value of friction component of pressure gradient/calculated value of friction component) (Equation (8.1))	—
$\bar{e}$	mean value of e within batch of N points, given by $\bar{e} = \frac{1}{N} \sum_{1}^N e$	—
Δe	tolerance on $\bar{e}$ appropriate to specified confidence and probability given by $\Delta e = \sigma_{be} k_C$	—
F	parameter in Friedel correlation (Appendix I)	—
f	Fanning friction factor, given by $\tau_o / (\frac{1}{2} \rho U^2)$	—
F_G	wall function, see Equation (6.7)	—
f_H	value of Fanning friction factor used in homogeneous theory	—
Fr	Froude number, in Friedel correlation (Appendix I) and Olujic correlation (Appendix J)	—
g	acceleration due to gravity, = 9.81 m/s^2	m/s^2
H	parameter in Friedel correlation (Appendix I)	—
J	parameter defined in Appendix J	—

k_C	factor on standard deviation giving tolerance Δe	–
L	Chexal-Lellouche fluid parameter, defined in Appendix E	–
$L_{1..2}$	parameter defined in Appendix J	–
$\dot{M}_G$	mass flow rate of gas or vapour component of mixture	kg/s
$\dot{M}_L$	mass flow rate of liquid component of mixture	kg/s
$\dot{M}$	mass flow rate of whole mixture	kg/s
$\dot{m}$	mass flux of whole mixture, $= 4(\dot{M}_G + \dot{M}_L)/\pi D^2$	kg/s m ²
N	number of data points within selected batch	–
n	Blasius exponent defined in Appendix L	–
P	pipe perimeter	m
P	probability	–
p	pressure*	N/m ² (Pa)
dp/dz	pressure gradient	N/m ³ (Pa/m)
dp_f/dz	component of pressure gradient due to friction	N/m ³ (Pa/m)
dp_g/dz	component of pressure gradient due to gravity	N/m ³ (Pa/m)
dp_a/dz	component of pressure gradient due to accelerational change	N/m ³ (Pa/m)
Re_G	Reynolds number defined by $Re_G = x\dot{m}D/\eta_G$	–
Re_{GO}	Reynolds number defined by $Re_{GO} = \dot{m}D/\eta_G$	–
Re_H	Reynolds number defined by $Re_H = \dot{m}D/\eta_H$	–
Re_L	Reynolds number defined by $Re_L = (1-x)\dot{m}D/\eta_L$	–
Re_{LO}	Reynolds number defined by $Re_{LO} = \dot{m}D/\eta_L$	–
Re_{TP}	Reynolds number defined by $Re_{TP} = \dot{m}D/\eta_{TP}$	–
S	slip ratio ($= u_G/u_L$)	–
s	standard deviation of e within selected batch of N points, given by	–
$s = \sqrt{\frac{1}{N} \sum_0^N (\bar{e} - e)^2}$		
s_o	parameter in Beggs and Brill correlation (Appendix J)	–
U	superficial velocity (volume flow per unit cross-sectional area of pipe)	m/s
u	velocity	m/s
u_{mix}	flow velocity in Beggs and Brill correlation (Appendix G)	m/s
u_C	factor proportional to deviation of sample percentile from population percentile corresponding to confidence level C	–
u_p	number of standard deviations, σ_{be} , between percentile and estimated mean of population	–
$\dot{V}$	volumetric flowrate	m ³ /s
V_{GL}	weighted mean drift velocity (Equation (5.10))	m/s
We	parameter in Friedel correlation (Appendix F)	–

x	mass quality of gas component defined by $x = \dot{M}_G / (\dot{M}_G + \dot{M}_L)$	–
X	Martinelli parameter defined by $\sqrt{(dp_f/dz)_L / (dp_f/dz)_G}$	–
Y	ratio of gas volume flowrate to liquid volume flowrate	–
y	parameter in Beggs and Brill correlation (Appendix G)	–
Z_1	parameter given by Equation (5.12)	–
z	pipe axial length, positive in direction of flow	m
α	angle of inclination of pipe to horizontal	deg
β	gas volume flow fraction given by Equation (6.5)	–
Γ	coefficient used in Chisholm method, defined by Equation (6.8)	–
ε	equivalent sand roughness height	m
ε_G	void fraction (fraction of cross-section of pipe occupied by gas phase)	–
ε_L	hold up (fraction of cross-section of pipe occupied by liquid phase)	–
η_G	dynamic viscosity of gas or vapour phase	N s/m ²
η_L	dynamic viscosity of liquid phase	N s/m ²
η_H	mean dynamic viscosity in homogeneous model	N s/m ²
η_{TP}	two-phase mean dynamic viscosity	N s/m ²
ρ_G	density of gas or vapour phase	kg/m ³
ρ_H	density defined by Equation (5.2) for use with homogeneous correlations	kg/m ³
ρ_L	density of liquid phase	kg/m ³
ρ_{TP}	two-phase density defined by Equation (3.2)	kg/m ³
σ_{be}	unbiased estimate of standard deviation of e , defined by $\sigma_{be} = Ks$, see Section 10.2	–
σ	interfacial tension	N/m
τ_o	wall shear stress	N/m ²
ϕ^2	two-phase multiplier defined in Section 3	–

* In this Data Item, “pressure” is taken to be absolute or static pressure (that is, it does not include any dynamic component). This is the pressure that would be measured by a pressure tapping flush with the wall of a constant cross-section pipe in which fully developed flow was occurring.

Suffixes

be	refers to best estimate value
G	refers to gas or vapour flowing alone at same mass flow rate as gas or vapour component of two-phase mixture
GO	refers to gas or vapour flowing alone at same mass flow rate as complete two-phase mixture
L	refers to liquid phase flowing alone at same mass flow rate as liquid component of two-phase mixture
LO	refers to liquid phase flowing alone at same mass flow rate as complete two-phase mixture

2. INTRODUCTION

This Data Item is one of a group on the prediction of pressure gradient, and hence pressure drop, in two-phase gas-liquid mixture flows. It provides techniques for the assessment of the frictional and gravitational pressure gradient for fluids flowing upwards in smooth vertical pipes. This assessment requires a knowledge of the phase flow rates, phase physical properties and pipe diameter. It also provides methods for estimation of the accuracy of the predictions.

Two-phase gas-liquid flow is of increasing importance in a wide range of applications including chemical and power plant, refrigeration systems, offshore petroleum pipelines, *etc.* Such flows are complex in nature and it is notoriously difficult to obtain accurate predictions of pressure drop. There are two main methods.

- (1) The use of more or less arbitrary empirical correlations, which usually attempt to relate the pressure drop in the two-phase situation to that which would occur in the single-phase flow of one of the phases flowing alone in the pipe. Development of empirical correlations was the main focus of work on two-phase flow in the 1950s to 1970s. Perhaps the best illustration of the problems of this approach to prediction is to compare the results obtained from the published correlations; these predictions are often widely different from each other, let alone from the available data. The correlations are generally based on limited data and systematic comparisons within the data were difficult before the widespread availability of computers. Nevertheless, these correlations are still widely used in the design of new systems, and in predicting the performance of existing systems.
- (2) In recent years, there has been increasing use of *phenomenological models* which recognise the differences between the respective *flow patterns*, where a flow pattern is defined as being a characteristic type of distribution of the interface between the gas and the liquid phases. Typical flow patterns for vertical flow are shown in Sketch 2.1. These range from bubble flow, through slug flow, and churn flow to annular flow. At very high liquid mass fluxes, there occur large structures in the entrained droplet field in the gas core in annular flow which are characteristic of the fifth flow pattern, *wispy annular flow*. As will be obvious, frictional pressure losses may follow very different laws depending on the type of flow pattern encountered. Thus, the phenomenological approach attempts to identify the flow pattern and then to develop specific relationships for each pattern in the hope of obtaining more general results.

Though, from a scientific point of view, the phenomenological approach is clearly preferable, it should be recognised that there is still a large measure of empiricism in this approach. First of all, the means for assessing the flow pattern are not entirely unequivocal and, secondly, the phenomenological models for each flow pattern depend in themselves on empirical relationships for such quantities as wall friction, interfacial friction, bubble size, slug frequency, slug velocity, *etc.* It does not follow, therefore, that these models always offer significant increases in accuracy compared to the empirical models. Recognising that the empirical models are widely used in industry, it has been felt that, for the purposes of this present Data Item, an assessment of their accuracy with recommendations on the ranges of parameters over which they are the best suited serves a useful purpose.

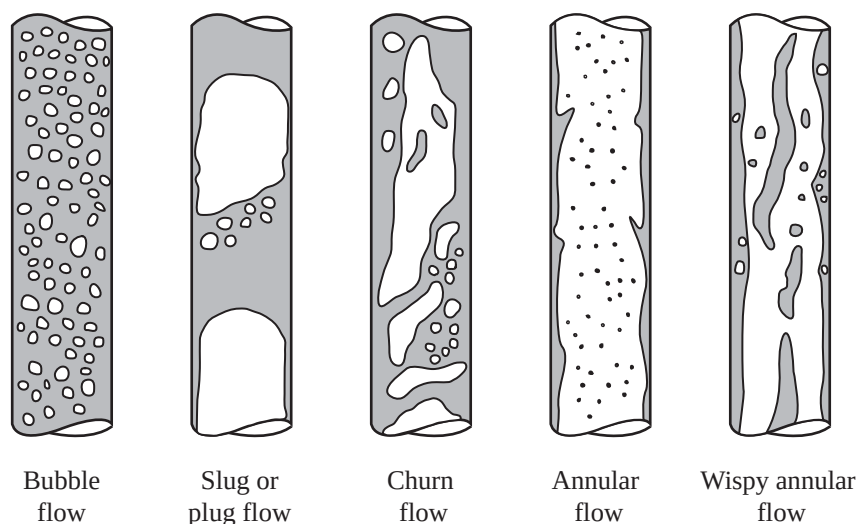
Future work is planned on assessing the accuracy of phenomenological models. It should be emphasized that the Data Item is strictly for vertical upward flows; vertical downward flows are very different due to the fact that gravity acts in the same direction as the flow. For instance in downward flow, there is a greater tendency for annular flow to exist. Application of downward flow do occur but are far less common than those for upward flows. This is reflected in the relative paucity of data for downward flows.

Computational Fluid Dynamics (CFD) methods are widely applied to predicting fluid flow systems. However, even for single-phase turbulent flows, the turbulence models used in CFD are empirical in nature and lack generality. For gas-liquid flow, where there is a combination of turbulence and deformable

interfaces, CFD methods are not usually applicable. Nevertheless, CFD modelling can sometimes be helpful in cases such as approximate prediction of bubble flows in three-dimensional geometries. However, such cases are rather rare; certainly, for the majority of cases covered by this Data Item, CFD methods are not yet applicable and are unlikely to be so in the immediate future.

In this Data Item, we define *pressure* (p) as the mean actual local pressure of the moving medium (i.e. the “static” pressure). This pressure (p) would be expected to be constant over the pipe cross section and would be that measured by a pressure tapping flush with the wall. We define pressure gradients as being the rate of change (dp/dz) of p with distance along the pipe. In practice, this is measured by determining the change in the pressure Δp measured by two sensitive wall pressure tapings spaced at Δz apart along the pipe. For fully developed flow, it is assumed that $(dp/dz) = \Delta p / \Delta z$ (through it should be noted that reaching fully developed flow is more difficult than for single phase flow, sometimes requiring lengths equivalent to many hundreds of pipe diameters to achieve; this is an important factor to consider in assessing experimental data).

The pressure gradient (dp/dz) is conventionally considered to consist of three components; namely frictional, gravitational and accelerational pressure gradients. Sometimes, it is convenient to refer to (dp/dz) as “total pressure gradient”. However, it should be stressed that this total pressure gradient is not the same as a gradient of total pressure, where total pressure is the pressure measured where the fluid is brought to rest (e.g. that measured by a pitot tube facing into the flow).



Sketch 2.1 Flow patterns in vertical upwards flow

In the previous Data Item on horizontal flow¹⁰⁷, pressure gradient in horizontal adiabatic flows was considered. This was a particularly simple case in that (with the horizontal flow orientation) the *gravitational* contribution to pressure gradient is zero. In vertical flows, this is clearly not the case and prediction of pressure gradient includes both the *frictional* term and a *gravitational* term which is related to the fluid density. For adiabatic flows in both horizontal and vertical channels, the third component of pressure gradient (the *accelerational pressure gradient*) is usually negligible unless the flow velocities are very high. Thus, in approaching the problem of predictions for vertical flows, attention has to be given not only to the frictional component but also to the density. The density is related to the fraction of the pipe which is occupied by the two phases (i.e., the *void fraction* representing the fraction of the gas phase present in the channel and the *hold-up*, representing the fraction of the liquid fraction in the channel). Clearly, the

sum of the void fraction and the hold-up must be unity. Thus, in considering empirical approaches to the prediction of pressure gradients in vertical flow, attention has to be given to correlations for void fraction or hold-up. A large number of correlations exist in the literature for hold-up and care has to be exercised in choosing the most rational ones for practical application. A major difficulty is that there is always an interaction between the frictional and gravitational terms. In principle, for any given correlation for frictional pressure gradient, one may choose any arbitrary correlation for void fraction (hold-up). This gives a large number of options. The approach we have taken in preparing this Data Item is as follows:

- (1) We have collected a large data base of pressure gradients and void fraction data and arranged this into a convenient form for comparisons with models.
- (2) We have compared measured and predicted values of fluid density (calculated from the void fraction/hold-up data) and presented these comparisons independently. However, the availability of data for which both pressure gradient and void fraction were measured is limited (390 points in the current work).
- (3) For the main data base (consisting of 5984 data points) we have compared total pressure gradient by combining frictional pressure gradient correlations with rationally-chosen hold-up correlations. Thus, for example, if a given researcher developed both hold-up and frictional pressure gradient correlations, then it was logical to take the two correlations together in predicting total pressure gradient. Similarly, for homogeneous-model type correlations, then the (simple) homogeneous model can be used to calculate the hold-up value.

In what follows, in Section 3, the basic generic methodologies for the empirical prediction of total pressure gradient are reviewed. Section 4 describes the development of the data base for data on vertical two-phase flows and Sections 5 and 6 describe the chosen correlations for void fraction (hold-up) and frictional pressure gradient respectively. Section 7 describes the combinations of gravitational and frictional pressure gradient used and Section 8 reviews the bases for the comparisons against the data base. In Section 9, the results of the performance comparisons with the data base are reviewed and, in Section 10, the methodology for applying the results is described. Finally, in Section 11, some worked examples are given for the calculation of total pressure gradient for air-water and steam-water flows. These worked examples are chosen to be identical to those used for the horizontal case¹⁰⁷ for comparison purposes.

3. CALCULATION METHODS

Pressure gradient in steady-state two-phase gas-liquid flow is often represented in terms of the combined momentum equation (see for instance: Hewitt and Hall-Taylor⁷⁸, 1970, Hewitt⁷⁹, 1982, Lahey and Moody⁸⁷, 1997, Whalley¹⁰², 1987 and Collier and Thome⁶⁷, 1994):

$$-\frac{dp}{dz} = \frac{\tau_o P}{A} + \frac{d}{dz} \left[\frac{(1-x)^2}{\rho_L(1-\epsilon_G)} + \frac{x^2}{\rho_G \epsilon_G} \right] \dot{m}^2 + g \rho_{TP} \sin \alpha \quad (3.1)$$

where dp/dz is the gradient of static pressure (Pa/m), τ_o the wall shear stress (N/m²), P the pipe perimeter (m), A the cross-sectional area of the pipe (m²), $\dot{m}$ the total mass flux of the fluid (kg/m² s), x the quality (fraction of the mass flow which is in the form of gas or vapour), ϵ_G the void fraction (fraction of the cross-section of the pipe occupied by the gas phase), ρ_L and ρ_G the liquid and gas densities (kg/m³), g the acceleration due to gravity (m/s²) and α the angle of inclination of the pipe from the horizontal (degrees). The two-phase density, ρ_{TP} , is defined as:

$$\rho_{TP} = \rho_L(1 - \epsilon_G) + \rho_G \epsilon_G. \quad (3.2)$$

The three terms of the right hand side of Equation (3.1) are designated as being the *frictional*, *accelerational* and *gravitational* pressure gradients respectively, written as:

$$-\frac{dp}{dz} = -\left(\frac{dp_f}{dz} + \frac{dp_a}{dz} + \frac{dp_g}{dz}\right). \quad (3.3)$$

The accelerational pressure gradient (dp_a/dz) is often very small; in the data considered here, the changes in quality, x , (due to flashing, say), gas density, ρ_G , (due to expansion under the pressure gradient) and void fraction, ϵ_G , are small. However the magnitude of this term should be checked, especially in cases where the change of pressure along the pipe is a significant portion of the absolute pressure (greater than 10%, say). Calculations on the database described in Section 4 indicated that the accelerational pressure gradient was typically less than 1% of the total pressure gradient for the adiabatic flow considered here. Of course, if significant changes in quality are induced as a result of evaporation or condensation, then significant accelerational pressure gradients can occur. However, in dealing with the database discussed here, this term has also been ignored, leaving only the frictional (dp_f/dz) and gravitational (dp_g/dz) to be considered. For a round horizontal pipe it follows that:

$$-\frac{dp_f}{dz} = \frac{\tau_o P}{A} = \frac{\tau_o(\pi D)}{(\pi D^2/4)} = \frac{4\tau_o}{D}. \quad (3.4)$$

Generally, there are two commonly used empirical approaches for calculating gas-liquid phase pressure drops.

- (1) To treat the flow as a *homogeneous mixture* to which is assigned a characteristic density and viscosity. The pressure gradient is then calculated using standard single-phase relationships or, alternatively, by the assignment of special friction factors for the two-phase flow.
- (2) The use of *two-phase multipliers* which relate the pressure gradient in the two-phase flow to that for a hypothetical single-phase flow of either of the phases flowing alone in the pipe or for the total flow in the pipe of a fluid with the properties of the gas or liquid. The following relationships are commonly used in this context:

$$-\frac{dp_f}{dz} = -\phi_G^2 \left(\frac{dp_f}{dz}\right)_G = -\frac{\phi_G^2 2x^2 \dot{m}^2}{\rho_G D} f_G, \quad (3.5)$$

$$-\frac{dp_f}{dz} = -\phi_{GO}^2 \left(\frac{dp_f}{dz}\right)_{GO} = -\frac{\phi_{GO}^2 2\dot{m}^2}{\rho_G D} f_{GO}, \quad (3.6)$$

$$-\frac{dp_f}{dz} = -\phi_L^2 \left(\frac{dp_f}{dz}\right)_L = -\frac{\phi_L^2 2\dot{m}^2 (1-x)^2}{\rho_L D} f_L, \quad (3.7)$$

$$-\frac{dp_f}{dz} = -\phi_{LO}^2 \left(\frac{dp_f}{dz}\right)_{LO} = -\frac{\phi_{LO}^2 2\dot{m}^2}{\rho_L D} f_{LO}. \quad (3.8)$$

In Equations (3.5) to (3.8), the multipliers ϕ_G^2 and ϕ_L^2 multiply the respective frictional pressure gradients for the gas and liquid phases flowing alone in the pipe. The multipliers ϕ_{GO}^2 and ϕ_{LO}^2 multiply the pressure gradients for a flow at the *total* mass flow rate having the gas and liquid properties respectively, and f_G , f_{GO} , f_L and f_{LO} are the respective friction factors. The single-phase equivalent friction factors can

be calculated if the geometries and physical properties are specified; normally, friction factor correlations include the effect of pipe roughness. Most of the data considered here are for relatively smooth pipes. The effect of roughness in two-phase flow appears to be much less than in single-phase flow. It is recommended, therefore, that the roughness be taken as zero in carrying out two-phase flow calculations though this would obviously not be appropriate as the quality closely approaches zero or unity. The friction factor relationships used in the present work are stated in Appendix M.

As will be seen from Equations (3.1) to (3.3), the gravitational pressure gradient in vertical flows is given by:

$$-\frac{dp_g}{dz} = g\rho_{TP} = g[\rho_L(1 - \epsilon_G) + \rho_G\epsilon_G]. \quad (3.9)$$

Thus, to calculate gravitational pressure gradient, a correlation is required for the void fraction (ϵ_G) or hold-up ($1 - \epsilon_G$). Correlations for void fraction/hold-up are either of the *direct* type where hold-up is correlated in terms of system parameters or *indirect* forms where the *slip ratio* is itself correlated. The slip ratio is defined as:

$$S = \frac{u_G}{u_L} = \frac{x(1 - \epsilon_G)\rho_L}{(1 - x)\epsilon_G\rho_G} \quad (3.10)$$

where u_G and u_L are the velocities of the gas and liquid phases respectively, x is the quality (fraction of the mass flow which is in the form of the gas phase) and ρ_L and ρ_G the liquid and gas densities respectively. The relationship between ϵ_G and S follows from Equation (3.10) and is:

$$\epsilon_G = \frac{x}{x + S(1 - x)\rho_G/\rho_L} \quad (3.11)$$

4. DEVELOPMENT OF DATABASE

In a previous study by ESDU (ESDU Data Item 76018¹⁰⁵, 1976), comparisons of frictional pressure gradient were made with a database of 2210 measurements. In correlating frictional pressure gradient, no distinction was made in that Data Item between horizontal, vertically upwards and vertically downwards flows. The database consisted of 57% horizontal flow cases, 42% vertically upwards flow cases and 1% of vertically downwards flow cases. In ESDU Data Item 01014¹⁰⁷, comparisons are presented between a database of 6453 data points for pressure drop in horizontal two-phase flow, with a selection of correlations. In the present Data Item, this exercise has been extended to the case of vertical flows. As explained in Section 3, the gravitational pressure gradient ($g\rho_{TP}$) may be a significant component of the total pressure gradient in vertical flow. Thus in comparing total pressure drop measurements, it is important to take account of ($g\rho_{TP}$). For total pressure gradient, the following sources were used:

- (1) A database set up at the Harwell Laboratory of the United Kingdom Atomic Energy Authority (UKAEA) in the 1970s and used by the Heat Transfer and Fluid Flow Service (HTFS) at Harwell as a basis for their proprietary correlations for two-phase flow.
- (2) A database compiled at Stanford University* (Aziz and Petalas⁵⁹, 1994). This database is valuable in that it contains a wealth of industrial data (particularly for vertical flows), in addition to some of the older data incorporated in the earlier databases (for instance, that of Dukler⁶⁹, 1969).

* A copy of this data bank was obtained (with Stanford University's permission) through Prof. H. Müller-Steinhagen, formerly of the University of Surrey and now at the University of Stuttgart.

- (3) The experimental data of Owen⁹⁷ for air-water flow in a 32mm diameter tube. This data is probably unique in that it is for essentially *fully developed* flows which are only achieved in two phase flow after very long tube lengths (in the case of Owen, the tube length was over 20 m (i.e. over 600 tube diameters)).

Of the 5984 points in the selected data base, around 10% had their origin in the Stanford data base, 15% in the data of Owen and the remaining 75% from the Harwell data base.

Within the data, pipe internal diameter, D , varied from 5 mm to 200 mm, and gas density, ρ_G , from 0.44 to 164 kg/m³.

In the Stanford Database, some 390 of the total pressure gradient measurements were associated with measurements of void fraction (ϵ_G) and from these values, the two-phase density ρ_{TP} could be calculated from Equation (3.2). This meant that direct comparisons could be made between a limited set of experimental two-phase density measurements with values predicted from various correlations (see Section 5 for selected correlations).

The sources for the data base on vertical flows are listed in Table 19.1 which also gives details of the origin of the data points.

5. CHOICE OF CORRELATIONS FOR VOID FRACTION

As was explained above, a crucial factor in assessing pressure gradient in vertical flows is the in-situ fluid density in the channel, ρ_{TP} ; to calculate ρ_{TP} from Equation (3.2), one must have a value for ϵ_G , the void fraction, and estimation of ϵ_G is thus an important step in determining pressure gradient. It should be noted that calculation of ϵ_G is also important in its own right; the holdup ϵ_L of liquid in the channel (where $\epsilon_L = 1 - \epsilon_G$) can be an important factor, for instance, where the liquid phase is toxic and represents a hazard.

There is a wide range of correlations for void fraction ϵ_G in the literature. Here, we have chosen some of the more widely used ones - as important factor in the choice being where the void fraction correlation is directly linked to the correlations for functional pressure gradient.

The correlations tested were as follows:

- A. Homogeneous models.** For the homogeneous model, the two fluids are assumed to be travelling at the same velocity so that the void fraction is given simply by the ratio β of the gas volume flow rate to the total volume flow rate:

$$\epsilon_G = \frac{U_G}{U_G + U_L} = \frac{x\rho_L}{x\rho_L + (1-x)\rho_G} = \beta \quad (5.1)$$

where U_G and U_L are the superficial velocities of the gas and liquid phases. A mean density for the homogeneous mixture is given by:

$$\rho_H = \left[\frac{x}{\rho_G} + \left(\frac{1-x}{\rho_L} \right) \right]^{-1} \quad (5.2)$$

and the gravitational pressure gradient for the homogeneous model is given simply by $g\rho_H$.

- B. Lockhart and Martinelli (1949)³⁹.** The correlation by Lockhart and Martinelli is in the form of a graphical relationship between ϵ_G and the Martinelli parameter X which is defined as:

$$X = \sqrt{\left(\frac{dp_f}{dz} \right)_L / \left(\frac{dp_f}{dz} \right)_G} \quad (5.3)$$

where $(dp_f/dz)_L$ and $(dp_f/dz)_G$ are the frictional pressure gradients for the flow of the liquid and gas phases alone in the channel. The graphical correlation is fitted closely by the equation:

$$\epsilon_G = \frac{\phi_L - 1}{\phi_L} \quad (5.4)$$

where ϕ_L is the friction multiplier (see Equation (3.7)) for which relationships are given in Appendix B.

- C. Chisholm (1967)⁶⁴.** This correlation is in the simple form:

$$S = (\rho_L/\rho_H)^{1/2}. \quad (5.5)$$

It should be noted that this Equation has a limited range of applicability and should not be used without consulting Appendix C.

D. Premoli *et al.* (1971)⁹⁴. This correlation has the form:

$$S = 1 + E_1 \left(\frac{Y}{1 + YE_2} - YE_2 \right) \quad (5.6)$$

where

$$Y = \frac{\beta}{1 - \beta} = \left(\frac{x}{1 - x} \right) \frac{\rho_L}{\rho_G} \quad (5.7)$$

Parameters E_1 and E_2 are given by

$$E_1 = 1.578 Re^{-0.19} \left(\frac{\rho_L}{\rho_G} \right)^{0.22} \quad (5.8)$$

$$E_2 = 0.0273 We Re^{-0.51} \left(\frac{\rho_L}{\rho_G} \right)^{-0.08} \quad (5.9)$$

where

$$Re = \frac{\dot{m}D}{\eta_L}$$

$$We = \frac{\dot{m}^2 D}{\sigma \rho_L}$$

in which D is the tube diameter and σ the surface tension.

Care should be exercised in using this correlation out of the range of its derivation. (See Appendix D.)

E. Chexal and Lellouche (1986)¹⁷. This correlation is of the drift flux form as follows:

$$\varepsilon_G = \frac{U_G}{C_o(U_G + U_L) + V_{GL}} \quad (5.10)$$

where C_o is a weighted mean distribution factor and V_{GL} is the weighted mean drift velocity. Essentially, the correlation takes the form of very complex correlations for C_o and V_{GL} . Further details are given in Appendix E. In the present work, the correlation was programmed into a

spreadsheet and the results checked against a program produced by the authors.

F. Zuber and Findlay (1965)¹⁰⁴. This correlation is again of the drift flux type with ε_G given by

$$\varepsilon_G = \frac{\beta}{C_o + Z_1} \quad (5.11)$$

where β is given by Equation (5.1), C_o has a value of 1.13 and Z_1 is given by the expression:

$$Z_1 = \frac{1.41 \dot{m} [\sigma g (\rho_L - \rho_G) / \rho_L^2]^{1/4}}{\rho_H} \quad (5.12)$$

Further details are given in Appendix F.

6. CHOICE OF CORRELATIONS FOR FRICTIONAL PRESSURE GRADIENT

The correlations tested are as follows.

- A. **Homogeneous Models.** In this class of models, the fluid is treated as a single-phase flow with suitably averaged properties. All the correlations use identical average density, ρ_H , given by Equation (5.2).

The four correlations considered here differ in their definition of mixture viscosity. Details of the homogeneous-type correlations are given in Appendix G. The correlations selected had the following characteristics.

- A1: **Dukler et al.** (1964)²². In this correlation, mean viscosity was calculated according to the expression:

$$\eta_H = \eta_G x \frac{\rho_H}{\rho_G} + \eta_L (1 - x) \frac{\rho_H}{\rho_L}. \quad (6.1)$$

- A2: **Isbin** (1958)³⁴ used a reciprocal mean viscosity defined as follows:

$$\eta_H = \left[\frac{x}{\eta_G} + \left(\frac{1-x}{\eta_L} \right) \right]^{-1}. \quad (6.2)$$

- A3: **Cicchitti et al.**²¹ (1960). Here, a linear averaged viscosity was taken as follows:

$$\eta_H = x\eta_G + (1-x)\eta_L. \quad (6.3)$$

- A4: **Beattie and Whalley**⁹ (1982). Here, a more complex form of relationship for viscosity was employed as follows:

$$\eta_H = \eta_L (1 - \beta)(1 + 2.5\beta) + \eta_G \beta, \quad (6.4)$$

where β is the gas volume flow fraction (the void fraction in homogeneous flow) given by:

$$\beta = \frac{\rho_L x}{\rho_L x + \rho_G (1 - x)}. \quad (6.5)$$

- B. **Lockhart and Martinelli** (1949)³⁹. Though obtained in the first half of the twentieth century, this correlation is still very widely used; it relates the pressure drop multipliers ϕ_L and ϕ_G to the Martinelli parameter X defined as follows:

$$X = \sqrt{\left(\frac{dp_f}{dz} \right)_L \left/ \left(\frac{dp_f}{dz} \right)_G \right.} \quad (6.6)$$

where $(dp_f/dz)_L$ and $(dp_f/dz)_G$ are the single-phase frictional pressure gradients for the liquid and gas phase flowing alone in the pipe respectively. In the original paper, the relationships between the pressure drop multipliers and X were presented in graphical form. However, Chisholm⁶⁴ (1967) provided convenient analytical expressions (see Appendix H).

- C. **Friedel** (1979)²⁴. The widely used Friedel correlation is written in terms of the multiplier ϕ_{LO} (see Equation (3.8)), where ϕ_{LO} is related to a number of dimensionless parameters in a complex correlation involving seven fitted constants. Details are given in Appendix I.
- D. **Beggs and Brill** (1973)¹¹. This correlation is used extensively in the oil industry and is, strictly speaking, a homogenous type of correlation. However, instead of relying on single-phase friction factor relationships involving Reynolds numbers with redefined densities and viscosities (as with the homogeneous models characterised under correlation A above), a specific correlation for friction factor is derived. This is in terms of phase holdup and so a correlation for phase holdup is also given as part of the package. Details are given in Appendix J.

- E. **Chexal and Harrison** (1999)¹⁶. This correlation takes the additive form:

$$\left(\frac{dp_f}{dz}\right) = F_G \left(\frac{dp_f}{dz}\right)_G + (1 - F_G) \left(\frac{dp_f}{dz}\right)_L \quad (6.7)$$

where F_G is a “wall function” which is related in a complex way to the system parameters. Details are given in Appendix K.

- F. **Chisholm** (1973)^{19, 20}. Chisholm related ϕ_{LO} to a parameter Γ defined as follows:

$$\Gamma = \sqrt{\left(\frac{dp_f}{dz}\right)_{GO} / \left(\frac{dp_f}{dz}\right)_{LO}} \quad (6.8)$$

where $(dp_f/dz)_{GO}$ and $(dp_f/dz)_{LO}$ are the frictional pressure gradients for a flow at the total mass flow rate having the gas and liquid properties respectively. This correlation has the form:

$$\phi_{LO}^2 = 1 + (\Gamma^2 - 1) \{ Bx^{(2-n)/2} (1-x)^{(2-n)/2} + x^{2-n} \} \quad (6.9)$$

where n is the exponent on the Blasius-type equation for friction factor and B is a parameter which is correlated in terms of Γ and mass flux, $\dot{m}$. Details are given in Appendix L.

7. COMBINATIONS OF GRAVITATIONAL AND FRICTIONAL PRESSURE GRADIENT

The pressure gradient data were compared in terms of *total pressure gradient* (gradient of static pressure). As was explained in Section 2, the choice of frictional and gravitational pressure gradient correlations to form the basis of the comparisons is crucial. The combinations tested were as follows:

<i>Combination</i>	<i>Frictional Pressure gradient</i>	<i>Gravitational Pressure gradient</i>
A1	Homogeneous (Dukler, G1)	Homogeneous (A)
A2	Homogeneous (Isbin, G2)	Homogeneous (A)
A3	Homogeneous (Cicchitti, G3)	Homogeneous (A)
A4	Homogeneous (Beattie & Whalley, G4)	Homogeneous (A)
B	Lockhart and Martinelli (H)	Lockhart and Martinelli (B)
C	Chisholm (L)	Chisholm (C)
D1	Friedel (I)	Premoli (D)
D2	Beggs and Brill (J)	Premoli (D)
E1	Chexal and Harrison (K)	Chexal and Lellouche (E)

(Note: The letters in parentheses after each method refer to the designation of their respective Appendix.)

The logic of the selection of combinations of correlations was as follows:

- (1) It is clear that homogeneous correlations for frictional pressure gradient should be combined with homogeneous correlations for gravitational pressure gradient.
- (2) Where given authors have developed correlations for both frictional pressure gradient and gravitational pressure gradient, then it is consistent to use the pairing (Lockhart and Martinelli, Chisholm and Chexal and co-workers).
- (3) Where frictional pressure gradient correlations were developed independently of gravitational pressure gradient correlations, then some choices have to be made. The Beggs and Brill correlation is used conventionally in combination with the Premoli correlation for vertical flows and it seems reasonable to also use the Friedel correlation in combination with the Premoli correlation since they are of similar degrees of complexity.

8. BASES OF PERFORMANCE COMPARISONS AGAINST DATABASE

There are considerable errors in estimating pressure drop from all empirical correlations. Here, the objective is to provide the designer with a realistic tool for estimating the errors involved in each case. The following method is used.

For each point in the database, a quantity e is defined as follows:

$$e = \log_{10} \left[\left(\frac{dp}{dz} \right)_{f, exp} / \left(\frac{dp}{dz} \right)_{f, calc} \right] \quad (8.1)$$

where $(dp / dz)_{f, exp}$ is the experimental value of the pressure gradient, and $(dp / dz)_{f, calc}$ is the value calculated from the particular correlation. A mean value $\bar{e}$ for the logarithmic error ratio is defined as

follows:

$$\bar{e} = \frac{1}{N} \sum_{1}^N e \quad (8.2)$$

where N is the number of data points in the batch being considered. A mean correction factor, $\bar{E}$, for the batch of N data points to which Equation (8.2) has been applied is given by:

$$\bar{E} = 10^{\bar{e}}. \quad (8.3)$$

A standard deviation of e within the selected batch of N data points is defined as follows:

$$s = \sqrt{\frac{1}{N} \sum_{1}^N (\bar{e} - e)^2}. \quad (8.4)$$

For a perfect fit, $\bar{E} = 1$ and $s = 0$. In what follows, the data are categorised in terms of N , $\bar{E}$ and s . The use of these parameters in estimating the uncertainty of a prediction of pressure drop from any of the correlations is described in detail in Section 10.

9. RESULTS OF PERFORMANCE COMPARISONS AGAINST DATABASE

The data for void fraction and pressure gradient are presented in several ways as follows:

- (1) **Performance charts.** Here, values of the statistical parameters N , $\bar{E}$ and s (see Section 8) are presented for ranges of quality (x , the fraction by mass of gas or vapour in the flow) and mass flux ($\dot{m}$, in $\text{kg/m}^2\text{s}$).
- (2) **Graphical representation of results.** Here, the data are parted in two forms:
 - (a) Plots against specific quantities of the ratio of measured values to those calculated, examples of the specific quantities being mass flux and quality.
 - (b) Plots of the distribution of the number points lying within certain ranges of error.
- (3) **Method selection charts.** In these charts, methods are arranged in order of decreasing performance in terms of standard deviation, s for the identical ranges of quality and mass flux to those used in the Performance Charts. Thus, for a particular range of parameters, the best available method can be selected.

Some general points can be made about the presented data, which are common to both the comparisons for void fraction and for total pressure gradient:

- (1) Where there is a stated limitation in the range of the correlation, data points may exist within the data bank within particular ranges of parameter that are outside those limitations. Within Performance Charts, such points are indicated by the letter “R” and, of course, the correlation should not be used within those ranges. Limitations in the range can also lead to a gross reduction in the number of data points captured for particular ranges of conditions and, again, great care should be taken in application of the correlation in these cases. Such cases are easily identified by examination of Performance Charts and comparing the numbers of points for the various correlations.
- (2) In some cases, under some headings, there are no points within the range. For example, there may be no points in the particular flow regime for particular range of quality (see Performance Chart II). Such points are designated as “W” (*i.e.* Without) in the respective charts.
- (3) In some cases, there is only one data point in the range for the given correlation. In these cases, the standard deviation cannot, of course, be determined and the values of s in these cases are designated as “I” (*i.e.* Impossible to determine).
- (4) In the graphical presentations of the second type, the distribution of error between predicted and measured values for the “best” correlation has a range change between 90-100% to 100-200%, hence the apparent peaks in the outer zones. This has been done to produce a compact presentation.

9.1 Comparisons of Void Fraction Data

To be consistent with the overall objective of this Data Item, namely to address the prediction of pressure gradient, it was decided to compare the void fraction data in terms of the mixture density ρ_{TP} rather than void fraction itself. In comparison to the large number (5984) data points available for total pressure gradient, the same data sources yielded only 390 data points for void fraction so the comparisons were less extensive. All the data used were for air-water flows and were from the Stanford database.

The method selection chart for two-phase density is given in Table 13.1. The corresponding Performance Charts are given as Tables 14.1 to 14.6.

The data for void fraction are compared with the predictions of the homogeneous model in Figure 1, which shows a plot of the ratio of predicted to actual density as a function of quality (x). Not surprisingly, the homogeneous model predicts the density well at very low qualities (where this density approaches that of the liquid phase). However, the density becomes much larger (more than a factor 10) than the predicted value for high qualities. This is due to the fact that there is slip in the phases, with the liquid phase travelling more slowly and its content in the tube being higher than had equal velocities of the two phases been assumed as in the homogeneous model; this results in a higher in-site density. Fortunately, in many applications, the total pressure gradient is dominated by the frictional component at high quality and these large errors in density (and hence gravitational pressure gradient) often have a small influence on the total pressure gradient prediction. It is commonly assumed that the homogeneous model fits better at high mass fluxes and this is apparently confirmed by the data shown in Figure 2. However, there is a link, within the data, between mass flux and quality. Only rarely is data at high qualities (x) obtained at high mass flux.

The best overall predictions were obtained using the correlation of Premoli *et al.* (1971), the comparisons being shown in Figure 3. Though there is some considerable scatter, the predictions are reasonable over the full range of quality.

A surprising result from the comparisons is shown in Figure 4; the multi-parameter correlation of Chexall and Lellouch (1986) also shows large discrepancies at high qualities. This is probably because the data used in the development of the correlation was focused in the low quality region.

On the whole, therefore, one can say that reasonable predictions of the data, with standard deviations of the order of 10-20% can be obtained using the Premoli *et al.* (1971) correlation and this is the one recommended for generic use.

9.2 Total Pressure Gradient

Method selection charts for total pressure gradient (*i.e.* gradient of static pressure) are given in Tables 13.2 to 13.5 and performance charts are given in Tables 15.1 to 15.9, 16.1 to 16.9, 17.1 to 17.9 and 18.1 to 18.9.

Both the method selection charts and the performance charts are given for four cases as follows:

- (1) For steam-water data (3906 data points). (Table 13.2, Tables 15.1 to 15.9).
- (2) For air-water data points (1863 data points). (Table 13.3, Tables 16.1 to 16.9).
- (3) For other data (mainly hydrocarbon (215 data points)). (Table 13.4, Tables 17.1 to 17.9).
- (4) For all of the data (5984 data points). (Table 13.5, Tables 18.1 to 18.9).

From the method selection charts, certain features of the available data become clear:

- (1) There is a dearth of data at low mass flux for steam-water flows.
- (2) There is a dearth of data at high mass flux for air-water flows.
- (3) There is a dearth of data at high quality and high mass flux for hydrocarbon flows.

Examining the method selection charts and the performance charts, and examining representations of the

comparisons in graphical form, one may make a number of observations as follows:

- (1) The homogeneous models based on an arbitrary definition of viscosity perform notably badly, generally over-predicting the pressure gradient except at very low qualities where it is governed by the gravitational component. This is illustrated, by way of example, by Figure 5 where all the data are plotted. However, for specific fluid pairs, the performance may be better (for instance, the Isbin model is better for steam-water than for air-water).
- (2) Comparisons with the whole of the data sets and the traditional Lockhart-Martinelli (1949) correlation reveal the trends expected from earlier reports, namely a larger deviation at high mass fluxes (see Figure 6).
- (3) Overall, the best results are obtained using the combination of the Friedel (1979) correlation for frictional pressure gradient coupled with gravitational pressure gradients predicted from the correlation of Premoli *et al.* (1971). The results obtained using this combination are illustrated in Figure 7. Note that even in this “best” case, the scatter is still very large. Typical standard deviations are of the order of 10-20%.
- (4) The most recent correlation combination considered is the use of the Chexall and Lellouch (1986) correlation for void fraction (and hence two phase density and gravitational pressure gradient) together with the Chexall and Harrison (1999) correlation for the frictional pressure gradient. The results obtained by comparing this correlation pair with all the data are shown in Figure 8. Very considerable scatter occurs, particularly at low mass flux. To be fair, however, it should be pointed out that the Chexall and Harrison correlation was primarily developed for steam-water flows and the performance is much better for this case as is illustrated in Figure 9. However, as is illustrated in Figure 10, even for the selected cases of steam-water flows, the Chexall-Lellouch/Chexall-Harrison combination does not perform better than the earlier correlation pair of Friedel/Premoli *et al.*
- (5) The correlation of Beggs and Brill (1973) (used in combination with the correlation of Premoli *et al.* to determine the two-phase density) shows large scatter when compared with all the data (Figure 11) but shows less scatter when compared with the mainly-hydrocarbon data set (Figure 12). This is consistent with the fact that this correlation is often used in the oil industry for prediction of pressure gradient in pipelines and risers. The superiority of the Beggs and Brill correlation for this data set is confirmed by the performance charts (see 13.4).
- (6) Even when compared with the overall best correlation pair (Friedel/Premoli *et al.*), the air-water data shows considerable scatter as is illustrated in Figure 13. Most of the data falls fairly close to the predictions but there are both positive and negative deviations which may occur due to the closeness of flow pattern transitions etc. It is only by considering the actual flow patterns and basing models on such flow patterns that these discrepancies could be investigated more objectively.

Figure 14 shows the distribution of errors, taking all the data and best overall correlation pair (Friedel/Premoli *et al.*). As will be seen, though the vast bulk of the data points lie within a reasonable error band, there are data where the predictions are in error by a large factor. As was mentioned above, this may be accounted for by the relatively unsophisticated nature of general correlations for multiphase flow (not taking account of detailed flow regimes) or due to errors in measuring or recording data (bearing in mind that two phase pressure gradient measurement needs to be carried out with care).

10. METHODOLOGY FOR APPLICATION OF RESULTS

This section gives procedures for aligning the predicted values of dp_f/dz with the experimental data and for calculating a margin for error (assuming a normal distribution of error) about this best estimate value in accordance with any value of confidence level and probability that the user may nominate. Details of the statistical methods used here are given in ESDU FMI2¹⁰⁷ (1997).

10.1 Best Estimate Pressure Gradient

In general, the best estimate value of pressure gradient $(dp_f/dz)_{be}$, is obtained from the product of the value of dp_f/dz calculated from the method in use and the mean correction factor, $\bar{E}$, obtained from the appropriate location in the Performance Chart corresponding to that method. However, a lower limit of $|(dp_f/dz)_{LO}|$ is set on $|(dp_f/dz)_{be}|$.

Thus,

$$\left(\frac{dp_f}{dz}\right)_{be} = \bar{E} \left(\frac{dp_f}{dz}\right) \quad (10.1)$$

or, if $(dp_f/dz)_{LO} > (dp_f/dz)$,

$$\left(\frac{dp_f}{dz}\right)_{be} = \bar{E} \left(\frac{dp_f}{dz}\right)_{LO}.$$

The reciprocal argument that $|(dp_f/dz)_{GO}|$ should be the upper limit of $|(dp_f/dz)_{be}|$ is not valid because, at high qualities, the magnitude of pressure gradient can exceed $|(dp_f/dz)_{GO}|$.

10.2 Uncertainty Factor

The uncertainty factor, ΔE , determines the upper and lower limits of pressure gradient between which the actual pressure gradient can be expected to lie if the confidence level C and probability P are specified. Thus the actual pressure gradient can be expected to lie in the range

$$\left(\frac{dp_f}{dz}\right)_{be} \Delta E \quad \text{and} \quad \left(\frac{dp_f}{dz}\right)_{be} / \Delta E.$$

The procedure for estimating ΔE is as follows.

- (1) Obtain K from Figure 15 from the specified value of N .
- (2) Calculate the best estimate standard deviation σ_{be} from

$$\sigma_{be} = Ks \quad (10.2)$$

where s , the standard deviation for the batch of data, is obtained from the Performance Charts.

- (3) Nominate a confidence C and a probability P and obtain u_C and u_P respectively from Figure 16. Values of $C = 90\%$ (giving $u_C = 1.25$) and $P = 99\%$ (giving $u_P = 2.3$) are suggested.
- (4) Evaluate k_C from

$$k_C = \frac{u_P + u_C \left\{ \frac{1}{N} \left[1 - \frac{u_C^2}{2(N-1)} \right] + \frac{u_P^2}{2(N-1)} \right\}^{0.5}}{1 - \frac{u_C^2}{2(N-1)}} \quad (10.3)$$

where N , the number of measured results, is obtained from the Performance Charts.

- (5) Calculate the tolerance, Δe , from

$$\Delta e = \sigma_{be} k_C. \quad (10.4)$$

- (6) Evaluate the uncertainty factor, ΔE , from

$$\Delta E = 10^{\Delta e}. \quad (10.5)$$

Alternatively, for 90% confidence and a probability of 99% ($C = 90\%$ and $P = 99\%$), ΔE may be obtained for values of s and N from Figure 17.

11. EXAMPLES

Two examples are given. The first, for steam and water in saturated equilibrium, illustrates the application of the Beattie and Whalley correlations. The second, for an air-water mixture, illustrates the application of the Chexall and Harrison correlation.

11.1 Steam/Water Example

Saturated steam of 75 per cent mass quality flows adiabatically at a mass flow rate, $\dot{M} = 0.3 \text{ kg/s}$ in a vertical drawn steel tube of internal diameter $D = 0.01 \text{ m}$. The pressure at this given locality is 100 bar abs. Find the frictional pressure gradient in the tube given the following information.

$$\begin{aligned}\rho_L &= 688.7 \text{ kg/m}^3, \\ \rho_G &= 55.5 \text{ kg/m}^3, \\ \eta_L &= 86.9 \times 10^{-6} \text{ N s/m}^2, \\ \eta_G &= 20.36 \times 10^{-6} \text{ N s/m}^2 \text{ and} \\ \sigma &= 12 \times 10^{-3} \text{ N/m.}^*\end{aligned}$$

The properties are appropriate to steam and water in saturated equilibrium at 100 bar.

Calculation Instructions

1. From the specified data,

$$\begin{aligned}x &= 0.75 \\ \text{and} \quad \dot{m} &= \frac{4\dot{M}}{\pi D^2} = \frac{4 \times 0.3}{\pi (0.01)^2} = 3819 \text{ kg/s m}^2.\end{aligned}$$

2. The Method Selection Chart for the steam-water data set (Table 13.2) suggests that, on the basis of the value of s , the Beattie and Whalley (1982) method of Appendix G is the best. However, some element of judgement is required. Examining Selection Charts I it is seen that the Isbin method (1959) is the next best. It is useful to examine the values of N , $\bar{E}$ and s (from the performance charts) for the various correlations for the range selected.

* Note that σ is not actually used in this selected correlation, though several of the correlations available do require it.

Method	Values from Steam-Water Performance Charts for $x = 0.75$ and $3200 < \dot{m} < 10000 \text{ kg s/m}^2$		
	N	$\bar{E}$	s
A4 (Beattie and Whalley)	8	1.017	0.056
A2 (Isbin)	8	1.019	0.057
C (Chisholm)	8	1.034	0.059

Both the Beattie and Whalley and the Isbin methods have small values of s and values of $\bar{E}$ close to unity. Therefore both correlations were used for illustration. (Note that the Chisholm correlation performed only marginally less well.)

3. Calculate pressure gradient using the Beattie and Whalley correlation (see Appendix G)

$$(1) \quad \rho_H = \left[\frac{0.75}{55.5} + \frac{0.25}{688.7} \right]^{-1} = 72 \text{ kg/m}^3.$$

$$(2) \quad \beta = \frac{688.7 \times 0.75}{688.7 \times 0.75 + 55.5 \times (1 - 0.75)} = 0.97.$$

$$(3) \quad \eta_H = 0.97 \times 20.36 \times 10^{-6} + (1 - 0.97)(1 + 2.5 \times 0.97) \times \{86.9 \times 10^{-6}\} = 28 \times 10^{-6}.$$

$$(4) \quad R_E = \frac{3819 \times 0.01}{28 \times 10^{-6}} = 13 \times 10^5.$$

$$(5) \quad f_H = 0.046 \times (13 \times 10^5)^{-2} = 2.7 \times 10^{-3}$$

$$(6) \quad \frac{dp_f}{dz} = \frac{-2 \times (3819)^2}{0.01 \times 72} \times 2.7 \times 10^{-3} = 111038 \text{ Pa/m}.$$

- (7) Using Performance Chart I, and with $C = 90\%$ and $P = 99\%$, the uncertainty factor may be obtained either from Equations (10.2) - (10.5) above, or directly from Figure 17, which gives $\Delta E = 1.62$.

- (8) The observed frictional component of pressure gradient will lie

$$\begin{aligned} \text{between} \quad & 111038 \times 1.62 = 180\,746 \text{ Pa/m} \\ \text{and} \quad & 111\,038/1.62 = 68\,215 \text{ Pa/m} \end{aligned}$$

with a probability of 99 per cent for 90 per cent confidence.

4. Calculate pressure gradient using the Isbin correlation (see Appendix G)

$$(1) \quad \eta_H = \left[\frac{0.75}{20.36 \times 10^{-6}} + \frac{(1 - 75)}{86.9 \times 10^{-6}} \right]^{-1} = 25.2 \times 10^{-6}.$$

$$(2) \quad R_E = \frac{3819 \times 0.01}{25.2 \times 10^{-6}} = 15 \times 10^5.$$

$$(3) \quad f_H = 0.046 \times [15 \times 10^5]^{-2} = 0.0026.$$

$$(4) \quad \frac{dp_f}{dz} = \frac{-2 \times (3819)^2}{0.01 \times 72} \times 2.6 \times 10^{-3} = 108187 \text{ Pa/m}.$$

- (5) Using Performance Chart I, and with $C = 90\%$ and $P = 99\%$, the uncertainty factor may be obtained from Equations (10.2) - (10.5) above, or directly from Figure 17, which gives $\Delta E = 1.63$.

- (6) The observed frictional component of pressure gradient will lie

$$\text{between} \quad 108187 \times 1.63 = 176104 \text{ Pa/m}$$

$$\text{and} \quad 108187/1.63 = 66463 \text{ Pa/m}$$

with a probability of 99 percent for 90 percent confidence.

11.2 Air-Water Example

Calculate the frictional component of pressure gradient for an air-water mixture flowing in a straight vertical pipe given the following information:

$$D = 0.04 \text{ m}, \quad \sigma = 7.2 \cdot 10^{-3} \text{ N/m},$$

$$\dot{m} = 30 \text{ kg/s m}^2, \quad \eta_L = 10.5 \cdot 10^{-4} \text{ N s/m}^2, \quad \rho_L = 1000 \text{ kg/m}^3,$$

$$x = 0.75, \quad \eta_G = 18 \times 10^{-6} \text{ N s/m}^2, \quad \rho_G = 2.4 \text{ kg/m}^3.$$

Calculation Instructions

- Using Method Selection Chart for the air-water data set, the best methods are those described in Appendices B and H (Lockhart and Martinelli) - Method B, Appendices E and K (Chexall-Lellouche and Chexall-Harrison) - Method E1, and Appendices D and A (Premoli *et al.* and Dukler *et al.*) - Method D1. As in Example 11.1, some judgement has to be exercised and the values of N , $\bar{E}$ and s are listed for the respective correlations as follows:

Method	Values from Air-Water Performance Chart I for $x = 0.75$ and $0 < \dot{m} < 32 \text{ kg s/m}^2$		
	N	$\bar{E}$	s
B (Lockhart and Martinelli)	24	1.885	0.108
E1 (Chexall and Harrison)	24	0.775	0.128
D1 (Friedel and Premoli)	24	1.78	0.132

Correlation D1 has the highest value of standard derivation (s) and a high value of $\bar{E}$, and is thus not chosen. Correlation B has the lowest value of standard deviation but gives a value of $\bar{E}$ far from unity. Correlation E1 has a somewhat higher standard deviation but has a value of $\bar{E}$ much closer to unity. Thus, correlation E1 is chosen as being the best overall.

- Calculate pressure gradient using the correlation model of Chexall and Harrison and Chexall and Lellouche. According to Appendix E, if the Chexall-Lellouche equations are solved iteratively then

$$\varepsilon_G = 0.958.$$

From Appendix K

$$Re_{GO} = \frac{30 \times 004}{18 \times 10^{-6}} = 66\,079.$$

$$Re_{LO} = \frac{30 \times 004}{10.5 \times 10^{-4}} = 1142.$$

No information is available on ε/D but the typical value of $\varepsilon/D = 5 \times 10^{-5}$ is taken for illustration.

$$(1) \quad \frac{1}{\sqrt{f_{GO}}} = -4 \log_{10} \left\{ 5 \times 10^{-5} + \frac{2.51}{66079} \left[1.14 - 2 \log_{10} \left[5 \times 10^{-5} + \frac{21.75}{(66079)^{0.9}} \right] \right] \right\}$$

$$\text{therefore } f_{GO} = 0.00475.$$

$$(2) \quad \frac{1}{\sqrt{f_{LO}}} = -4 \log_{10} \left\{ 5 \times 10^{-5} + \frac{2.51}{1142} \left[1.14 - 2 \log_{10} \left\{ 5 \times 10^{-5} + \frac{21.25}{1142^{0.9}} \right\} \right] \right\}$$

$$\text{therefore } f_{LO} = 0.01475.$$

$$(3) \quad F_G = 1 - \exp[-10.177(1 - 0.95)] = 337.$$

$$(4) \quad F_L = \exp[-5.993(1 - 0.95)] = 0.78.$$

$$(5) \quad \left(\frac{dp_f}{dz} \right)_L = \frac{-2 \times 0.01475}{0.04 \times 1000} [(1 - 0.75) \times 30]^2 \times 0.78 = 0.0325.$$

$$(6) \quad \left(\frac{dp_f}{dz} \right)_G = \frac{-2 \times 0.00475}{0.04 \times 2.4} ([0.75 \times 30]^2 \times 0.337) = 17.5.$$

$$(7) \quad \frac{dp_f}{dz} = 0.0325 + 17.25 = 17.5325 \text{ Pa/m}$$

(8) Using Performance Chart I, and with C = 90% and P = 99%, the uncertainty factor may be obtained from Equations (8.2) to (8.4), or directly from Figure 17, which gives $\Delta E = 1.42$.

(9) The observed friction component of pressure gradient will lie

$$\text{between } -17.5325 \times 1.42 = -24.9 \text{ Pa/m}$$

$$\text{and } -17.5325/1.42 = -12.3 \text{ Pa/m}$$

with a probability of 99 per cent for 90 per cent confidence.

12. DERIVATION AND REFERENCES

Derivation

The derivation lists selected sources that have assisted in the preparation of this Item.

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