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Prediction of sound attenuation in a refracting turbulent atmosphere with a Fast Field Program

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THE PREPARATION OF THIS DATA ITEM

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PREDICTION OF SOUND ATTENUATION IN A REFRACTING TURBULENT ATMOSPHERE WITH A FAST FIELD PROGRAM

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	3
3. THEORETICAL BACKGROUND	4
3.1 Wind and Temperature Gradients	7
3.2 Ground Impedance	9
4. ATMOSPHERIC TURBULENCE	10
5. INPUT PARAMETER SENSITIVITY	12
6. APPLICATION, ACCURACY AND LIMITATIONS	15
7. COMPUTER PROGRAMS	17
7.1 Data Input and Output	17
8. DERIVATION AND REFERENCES	21
9. EXAMPLE	22
APPENDIX A – ESDU DATA FILE	28
A.1 NOTATION	28
A.2 SOURC E AND RECEIVER GEOMETRY	30
A.3 DATA FILE STRUCTURE STANDARD	31

PREDICTION OF SOUND ATTENUATION IN A REFRACTING TURBULENT ATMOSPHERE WITH A FAST FIELD PROGRAM

1. NOTATION

		Units
		SI
A	matrix of boundary conditions	
B	source term vector	
$c(0)$	speed of sound at ground level	m/s
$c(z)$	speed of sound as a function of height, z	m/s
D	depth of air layer	m
d_1	rate of change of sound speed with height in Equation (3.12)	s ⁻¹
d_2	sound speed parameter in Equation (3.13)	m/s
d_3	refraction parameter in Equation (3.13)	m
f	frequency	Hz
h	maximum separation between direct and reflected paths	m
h_{lm}	height of lower boundary of air layer	m
h_{um}	height of upper boundary of air layer	m
K	horizontal component of wave number	m ⁻¹
k	wave number	m ⁻¹
L_0	scale of turbulence for a Gaussian spectrum, also known as turbulence length	m
m	layer number, also used as a subscript	
N	size of Fast Fourier Transform (FFT)	
n	index in FFT	
P	kernel of Hankel Transform	

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p	sound pressure	Pa
R	gas constant for air*	J/kg K
$R \uparrow, R \downarrow$	non-dimensional amplitudes of up-going and down-coming waves respectively	
r	horizontal distance from the source	m
$T(z)$	absolute temperature as a function of height, z	K
$V(z)$	wind speed as a function of height, z	m/s
$\mathbf{X}$	vector containing unknown amplitudes $R \uparrow, R \downarrow$	
x, y, z	Cartesian coordinates with origin at base of source	
Z	normalized acoustic impedance of ground	
z_r	height of receiver	m
z_s	height of source	m
α	inverse effective depth of porous surface layer	m^{-1}
β	vertical component of wave number ($= \sqrt{k^2 - K^2}$)	m^{-1}
Γ	mutual coherence function	
γ	ratio of specific heat capacities of air	
δ	delta function	
ζ	phase covariance between direct and reflected paths	
$\langle \mu^2 \rangle$	fluctuating index of refraction for a Gaussian spectrum, also known as turbulence parameter	
ρ	density of air	kg/m^3
σ	variance of phase fluctuation along a path	
σ_e	effective flow resistivity	$\text{Pa s/m}^{2\dagger}$
σ_V	rms value of wind velocity fluctuation	m/s
σ_T	rms value of temperature fluctuation	K
χ	angle between sound propagation direction and wind vector (see Sketch 3.2)	deg

* The gas constant value is 287.052 87 J/kg K.

† In Equation (3.14), σ_e is given in Pa s/m^2 . Elsewhere in the text and for input to the computer program σ_e is in kPa s/m^2 .

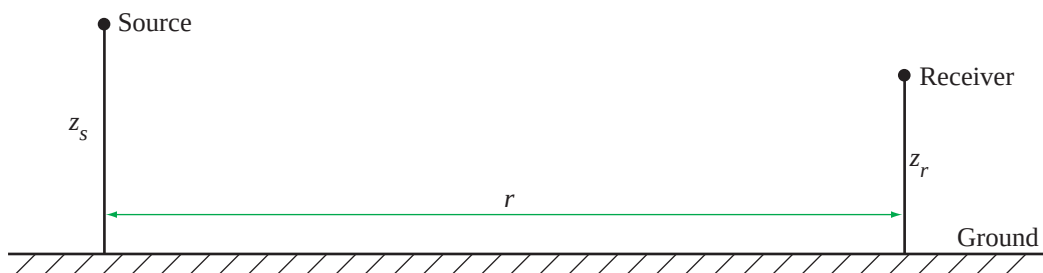
2. INTRODUCTION

This Item provides a means of estimating the effect of wind and temperature gradients in the presence of air turbulence on sound propagation between points located above a flat surface using a version of the Fast Field Program (FFP) developed at the Open University, UK (Derivation 1). The Item can be used to predict the sound attenuation from a point source. The attenuation is expressed as that relative to the free-field noise level at a point 1 m from the source in the direction of sound radiation. The Item can also be used to predict the ground reflection correction relative to free-field levels at that location allowing the user to convert a frequency spectrum measured at the location to a free-field spectrum, or to infer a measured spectrum from a free-field spectrum.

If it is assumed that the characteristics of the atmosphere do not vary significantly with horizontal distance between points located above flat ground with uniform acoustic impedance, the FFP (References 4 and 5) is an accurate method for estimating sound attenuation between these points. It uses as its starting point a Hankel Transform of the wave equation to decouple the horizontal and vertical dependence of the sound pressure and expresses the horizontal variation as a Fast Fourier Transform. Thus, the FFP can calculate the sound pressure level as a function of range in a single transform.

The user is required to input the source-receiver geometry, the impedance characteristics of the ground and data relevant to prevailing meteorological conditions. Wind and temperature gradients need to be expressed in terms of the variation of the speed of sound with height while the atmospheric turbulence and relative humidity are assumed constant. Attenuation is output in dB at frequencies defined by the user. Up to ten receiver locations may be specified. Level differences in dB between the first receiver location specified and all subsequent locations are also output.

A geometric representation of the source and receiver is shown in Sketch 2.1



Sketch 2.1 Source – receiver geometry

3. THEORETICAL BACKGROUND

The classical wave equation describes the propagation of sound pressure from a point source at the origin as

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -4\pi\delta(x,y,z), \quad (3.1)$$

where δ represents a delta-function source of unit strength.

For simple harmonic motion, this equation becomes the Helmholtz equation

$$\nabla^2 p^2 + k^2 p = -4\pi\delta(x,y,z) \quad (3.2)$$

which, for an axisymmetric case in cylindrical coordinates is

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} + k^2 p = -\frac{2}{r} \delta(r) \delta(z - z_s), \quad (3.3)$$

with the source at $r = 0$ and $z = z_s$.

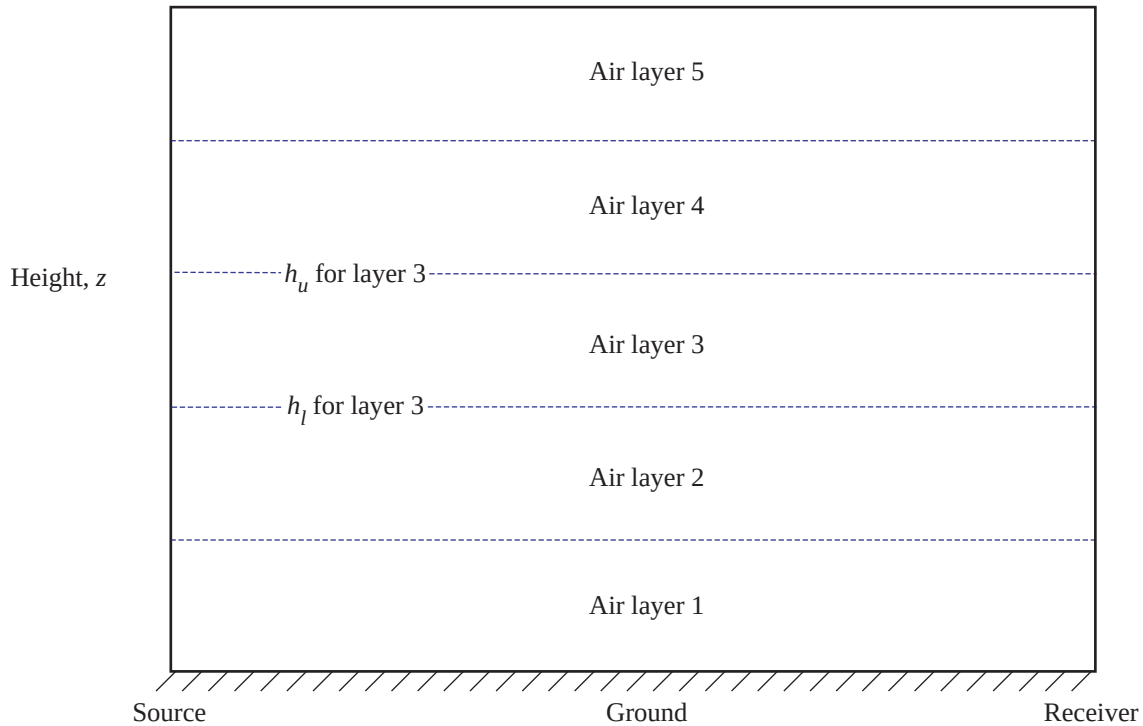
Equation (3.3) can be solved with a zero-order Hankel Transform

$$p(r,z) = \int_{-\infty}^{\infty} J_0(Kr) P(K,z) K dK, \quad (3.4)$$

where $P(K,z)$ satisfies the transformed wave equation

$$\frac{d^2 P(K,z)}{dz^2} + [k^2(z) - K^2] P(K,z) = -2\delta(z - z_s). \quad (3.5)$$

Equation (3.5) is evaluated numerically by dividing the atmosphere into horizontally uniform layers (see Sketch 3.1) where the sound speed is constant within each layer but can vary from one layer to another.



Sketch 3.1 Schematic of a layered atmosphere

The form assumed for the transformed pressure potential within the layer m takes the form

$$P_m = R \uparrow e^{i(h_l - z)\beta_m} + R \downarrow e^{i(z - h_u)\beta_m}, \quad (3.6)$$

where $\beta_m = \sqrt{k_m^2 - K^2}$

and $k_m = k(z_m)$.

If the source is positioned within the layer, the potential is given by

$$P_m = \frac{1}{\beta_m} e^{i(z_s - z)\beta_m} + R \uparrow e^{i(h_l - z)\beta_m} + R \downarrow e^{i(z_r - h_u)\beta_m}. \quad (3.7)$$

In this case, at $z_r = h_u$

$$P_m = \frac{1}{\beta_m} e^{i(z_s - z_r)\beta_m} + R \uparrow e^{iD\beta_m} + R \downarrow, \quad (3.8)$$

where $D = h_u - h_l$ is the depth of the layer.

The pressure field contains up-going and down-going components with amplitudes $R_{\uparrow}$ and $R_{\downarrow}$ which can be determined from the boundary conditions at the horizontal interfaces.

These boundary conditions are

- (i) continuity of vertical particle displacement and
- (ii) continuity of pressure.

At the interface at ground level ($m = 1, z = 0$) there is a further impedance boundary condition

$$\frac{dP}{dz} + \frac{1}{Z} k(0)P = 0, \quad z = 0. \quad (3.9)$$

where Z is the normalized acoustic surface impedance of the ground (see Section 3.2).

The boundary condition equations are set up as a set of linear equations with $R_{\uparrow}$ and $R_{\downarrow}$ as the unknowns.

These equations are put in the form of a global matrix equation

$$\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$$

where $\mathbf{X}$ is a vector containing the unknown amplitudes $R_{\uparrow}$ and $R_{\downarrow}$.

$\mathbf{A}$ contains the known terms from the boundary condition equations and $\mathbf{B}$ is the source term vector.

The global matrix equation is solved to obtain the amplitudes and then the wave potential for any receiver position can be evaluated in the appropriate layer using the relevant amplitudes from either Equation (3.6) or from Equation (3.7) if the source is positioned in the same layer as the receiver.

The pressure, $p(r_m, z)$ can then be calculated from the wave potential using the relationship given in Equation (3.4). The Bessel function can be replaced by its large argument approximation at distances larger than about two wavelengths and the integral then reduces to a Fourier integral. At lesser distances atmospheric refraction is not a significant factor.

The medium can be approximated by a discrete sum to a finite value of K and the pressure can be calculated by using a Fast Fourier transform as given by Equation (3.10).

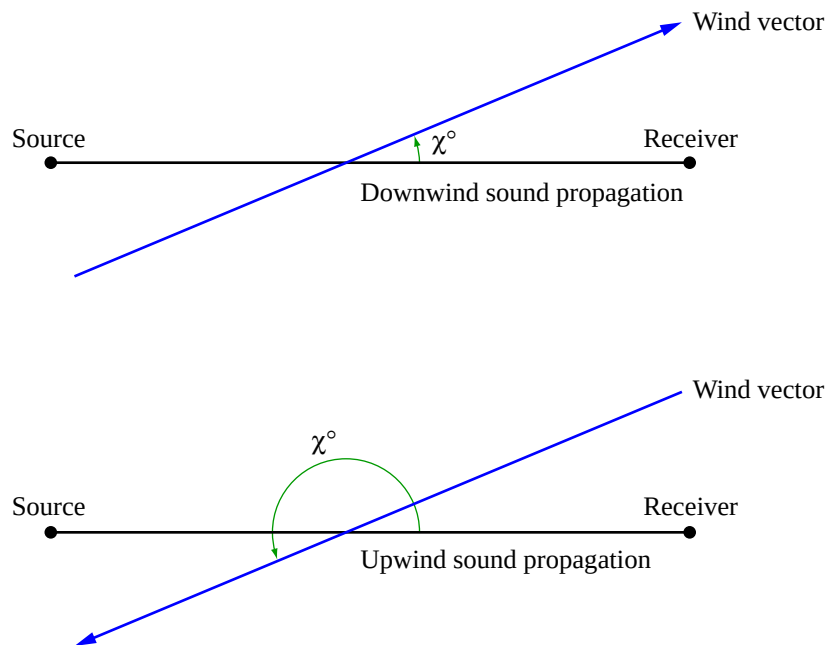
$$p(r_m, z) = \frac{\delta K N^{1/2}}{\sqrt{2\pi r_m}} \left[e^{-i\pi/4} \sum_{n=0}^{N-1} P(K_m, z) n^{-1/2} e^{\frac{2i\pi n m}{N}} + e^{-i\pi/4} \sum_{n=0}^{N-1} P(K_m, z) n^{-1/2} e^{\frac{-2i\pi n m}{N}} \right]. \quad (3.10)$$

3.1 Wind and Temperature Gradients

The effects of wind and temperature gradients are only considered through the resulting variation of sound speed with height. In SI units, within the range of normal atmospheric conditions, the effective sound speed, $c(z)$, at a height z can be described by

$$c(z) = \sqrt{\gamma RT(z)} + V(z) \cos \chi, \quad (3.11)$$

where $T(z)$ and $V(z)$ are the mean absolute temperature and wind speed at height z , and χ is the angle between the direction of interest and the wind vector (see Sketch 3.2).



Sketch 3.2 Downwind and upwind sound propagation

The user can select a linear sound speed profile, a logarithmic sound speed profile or the option of entering specific sound speed values at different heights.

For a linear profile, the effective sound speed is given by

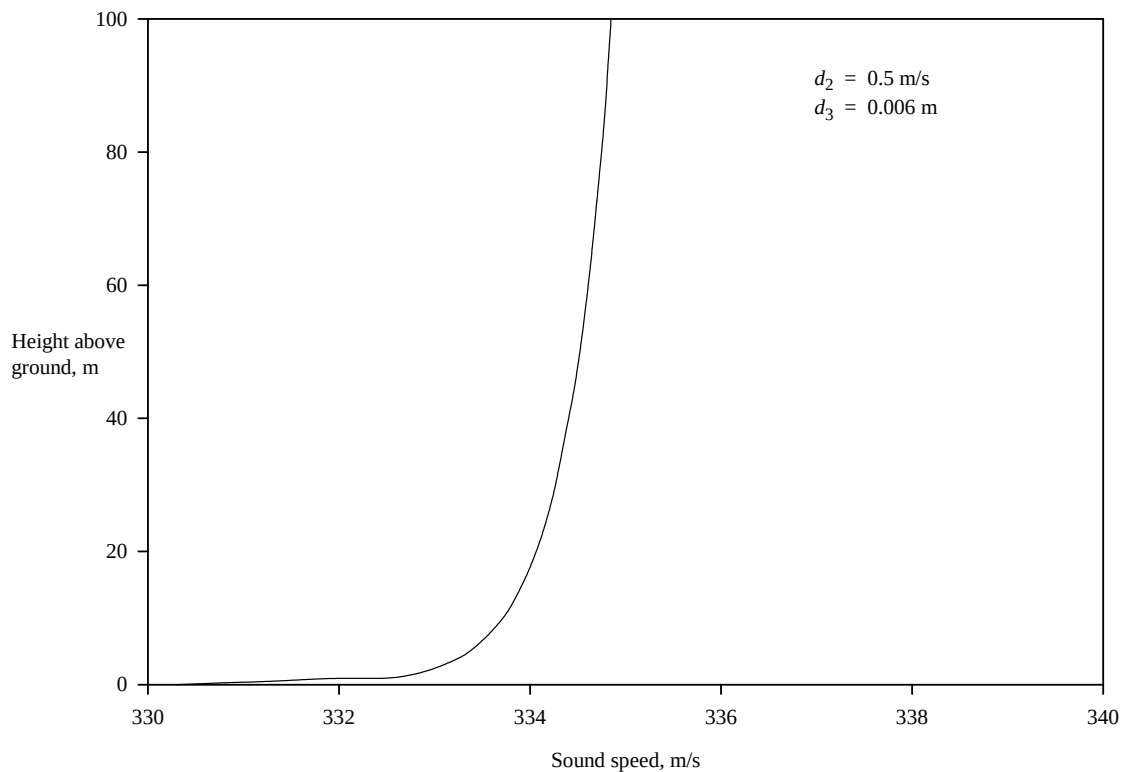
$$c(z) = c(0) + d_1 z. \quad (3.12)$$

For a logarithmic profile the effective sound speed is given by

$$c(z) = c(0) + d_2 \ln(z/d_3). \quad (3.13)$$

In addition to the sound speed at ground level the user is required to input the appropriate constants which represent the desired profile. A value of 0.006 m for d_3 is suggested in Reference 6 and may be used as a starting point for modelling the sound speed profile.

An illustration of a logarithmic sound speed profile is given in Sketch 3.3.



Sketch 3.3 Logarithmic sound speed profile

If the option to input sound speeds at different heights is to be exercised, the minimum height for the highest station should be the greater of

- one-tenth of the separation distance between source and receiver
- or
- twice the sum of source and receiver heights.

It should be noted that a very steep gradient or one which changes sign rapidly with height can sometimes lead to numerical instability.